Duality, Descent & Defects
The Higher Geometry and Category Theory of Charged Dynamics

The goal of this two-semester lecture course is an in-depth introduction into the conceptual framework and methodology of higher geometry and algebra employed in the modern study and modelling of phenomena involving dynamical distributions of (topological) charge, such as charged material points, loops, membranes *etc.* – from the definition of Dirac-Feynman amplitudes for charged dynamics in nontrivial topologies in terms of differential characters and a hierarchical geometrisation of the integral cohomology classes of the corresponding gauge fields (in the form of *n*-gerbes), through categorification of quantum-mechanically consistent field-theoretic (super)symmetries and the universal gauge principle for symmetries (also in field theories with non-tensorial couplings) modelled on group and groupoid actions and on more general correspondences (with a homological description of anomalies and classification of inequivalent reductions), all the way to a realisation of dualities by topological defects, construction of simplicial field theories over defect-stratified spacetimes and the corresponding higher categorial structures on the field bundle.

Below, we give a more detailed outline of the course

1. A lightning review of
   - the theory of fibre bundles with connective structure: vector bundles with Koszul connections, principal bundles with principal connection 1-forms and associated bundles with Crittenden connections, reduction and prolongation (generalised Stieffel-Whitney classes);
   - the theory of Lie groups and their Cartan differential calculus, group actions, the quotient-manifold theorem and the Cartan-Borel construction;
   - rudiments of category theory (universality and representability) and homological algebra (Čech and de Rham cohomology, sheaf cohomology, Dupont's simplicial cohomology, hypercohomology, Lie-algebra and group cohomology).
2. Classical field theory with tensorial resp. simplicial couplings (using the Beilinson-Deligne hypercohomology and its Cheeger-Simons model). Dirac-Feynman amplitudes, the Tulczyjew-Gawędzki-Kijowski-Szczyrba(s) first-order formalism and prequantisation through cohomological transgression.
3. Murray's gerbes – from the lifting gerbe and the Beilinson-Deligne hypercohomology to (weak) higher categories, *via* (simplicial) higher geometry. The Aharonov-Bohm effect (for pointlike charges, loops *etc.*) and beyond.
4. Symmetry analysis: configurational and (semi-)gauge symmetries, Noether-Poisson realisations and (classical) central extensions, comomenta and rigid-symmetry algebroids, categorification of symmetries.
5. Orbital field dynamics and non-linear realisations of internal and spacetime symmetries. The Ivanov-Ogievietskii (aka inverse Higgs) mechanism. Constructions in the invariant de Rham cohomology, gerbe objects in the category of groups, Nieuwenhuizen's FDA techniques and the rise of Stasheff's L_∞ structures *via* the Baez-Crans correspondence).
6. The universal gauge principle and gauge anomalies for symmetries modelled on group actions:
   - the standard formulation (association) and the underlying universal mixing construction of Cartan and Borel;
   - the Lie groupoid behind the standard formulation and its Higgs module – a MacKenzie-Moerdijk-Mrčun principal groupoid bundle;
   - one Lie groupoid to rule them all – the Atiyah gauge groupoid and the (extended) Atiyah short exact sequence;
   - the Kobayashi-Nomizu induction scheme for associated connections and covariant

derivatives, the minimal-coupling recipe for tensorial couplings, descent;

- equivariant cohomology models and their geometrisations for simplicial couplings, Dirac anomalies in Courant algebroids for symmetries under gauging;
- the gauge defect, twisted sectors and the ensuing simplicial field theory with defects.

7. Groupoidal symmetries and their gauging

- the special rôle of bisections and their relation to the tangent Lie algebroid;
- circumnavigating Fréchet: principaloid bundles and their foliated connections;
- the inextricable entwinement of groupoidal matter and radiation in the augmented Atiyah short exact sequence for the groupoidal gauge symmetry;
- the universal gauge principle for tensorial and simplicial field theories;
- the curved Yang-Mills-Higgs theory;
- Q-bundles and Poisson-σ-models.

8. Some advanced gauge field theory:

- the Anderson-Brout-Englert-Higgs-Guralnik-Hagen-Kibble-... mechanism;
- instantons;
- standard gauge fields *vs* gerbe-module connections;
- bi-chiral Kač-Moody symmetries in the Wess-Zumino-Novikov-Witten-Gawędzki model of 2*d* CFT;
- topological gauge field theory – a case study of the Chern-Simons theory in 3*d* in the presence of Wilson lines, of its reduction à la Alekseev and Malkin and of... its intricate relation to the WZNWG model (an example of \`holography').

9. Field theories with a statistical gradation:

- rudiments of supergeometry with supersymmetry (Z_2-graded manifolds, super-Harish-Chandra pairs, super-Cartan calculus and Lie supergroup actions);
- a recapitulation of the theory of spinor bundles with spin connections;
- Freed's inner-Hom superfield theories and their (quasi-)supersymmetry;
- super-σ-models and geometrisations in the Cartan-Eilenberg cohomology of Lie supergroups – supergerbes.

10. Dualities

- the (pre)symplectic description;
- the defect-duality correspondence;
- useful instantiations: the Hughes-Polchinski duality, T-duality *via* gauging of toroidal actions and the ensuing field-bundle topology change, S-duality in a weakly abelian gauge field theory, a sector-restricted duality between the 2d WZNWG CFT and the 3d CS TGFT – a bold step towards functorial quantisation, and more...

The language of the lecture course: English (most likely)

Requirements: The course is dedicated to 3rd-year undergrads from the individual student groups at the faculty of maths and physics, as well as to PhD students. It bases on the 1st-year course in algebra, the 2nd-year course in differential geometry and that in group theory, and on the two-semester monographic course "Methods of higher algebra in physics: from quadratic forms to spinor bundles" by the present lecturer, and so a working knowledge of the theories and methods discussed at length in these courses will be assumed freely (although not dogmatically).

Literature: to be provided in due course.