A TWO-SEMESTER MONOGRAPHIC LECTURE ON

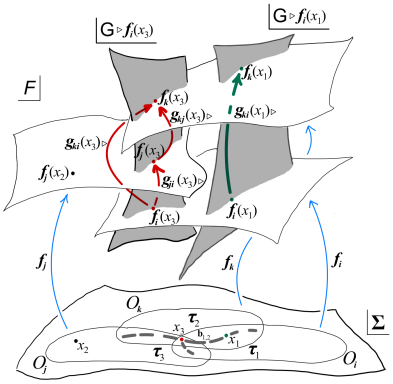
DUALITY, DESCENT & DEFECTS

THE HIGHER GEOMETRY AND CATEGORY THEORY OF CHARGED DYNAMICS

(FACULTY OF PHYSICS, UW, ACAD. YEAR 2024/25; 1100-DDD)

LECTURER: **RAFAŁ R. SUSZEK** [KMMF]

TIME & PLACE (WINTER SEMESTER): **TUESDAYS @ 1800–2000 IN LECTURE ROOM 2.23**



Goal: An in-depth introduction into the conceptual framework and methodology of higher geometry and algebra employed in the modern study of phenomena involving dynamical (extended) distributions of (topological) charge – from the definition of Dirac-Feynman amplitudes for charged dynamics in nontrivial spacetime and configurational topologies in terms of differential characters and a hierarchical geometrisation of the integral cohomology classes of the corresponding gauge fields (in the form of n-gerbes), through categorification of quantum-mechanically consistent symmetries and the universal (higher) gauge principle for symmetries modelled on group and groupoid actions and on more general correspondences (with a homological description of anomalies and classification of inequivalent reductions), all the way up to a realisation of dualities by topological defects, construction of simplicial field theories over defect-stratified spacetimes and the corresponding higher categorial structures on the configuration bundle.

Skeleton:

(1) A lightning introduction to/review of
- the theory of Lie groupoids and algebroids (axiomatics, examples, the group of bisections and its actions), the Cartan-type calculus, groupoid modules and orbispaces;
- the theory of fibre bundles with connective structure: vector bundles with Koszul connections, principal bundles with principal connection 1-forms and associated bundles with Crittenden connections, reduction and prolongation (generalised Stieffel-Whitney classes);
- rudiments of category theory (universality, representability and internalisation) and homological algebra (Čech and de Rham cohomology, sheaf cohomology, Dupont's simplicial cohomology, hypercohomology, Lie-algebra and group cohomology).

(2) Murray's (n-)gerbes – from the lifting gerbe and the Beilinson-Deligne hypercohomology to (weak) higher categories, *via* (simplicial) higher geometry. The (higher) Aharonov-Bohm effect and beyond.

(3) Classical field theory with tensorial resp. simplicial couplings (using the Beilinson-Deligne hypercohomology and its Cheeger-Simons model). Dirac-Feynman amplitudes, the Tulczyjew-Gawędzki-Kijowski-Szczyrba(s) first-order formalism and prequantisation through cohomological transgression.

(4) Symmetry analysis: configurational and (semi-)gauge symmetries, Noether-Poisson realisations and (classical) central extensions, comomenta and rigid-symmetry algebroids, categorification of symmetries.

(5) The universal gauge principle and gauge anomalies for symmetries modelled on group actions:
- the standard formulation (association) and the underlying Cartan-Borel universal mixing construction;
- the Lie groupoid behind the standard formulation and its Higgs module – a MacKenzie-Moerdijk-Mrčun principal groupoid bundle of the Higgs model;
- one Lie groupoid to rule them all – the Atiyah gauge groupoid and the associated short exact sequence;
- the Kobayashi-Nomizu induction scheme for associated connections and covariant derivatives, the minimal-coupling recipe for tensorial couplings, descent to configurational orbispaces;
- equivariant cohomology models and their geometrisations for simplicial couplings, Dirac anomalies in Courant algebroids for symmetries under gauging;
- the gauge defect, twisted sectors and the ensuing simplicial field theory with defects.

(6) Groupoidal symmetries and their gauging:
- the special rôle of bisections and their relation to the tangent Lie algebroid;
- circumnavigating Fréchet: principaloid bundles and their foliated connections;
- the inextricable entwinement of groupoidal matter and radiation in the augmented Atiyah short exact sequence for the groupoidal gauge symmetry;
- the universal gauge principle for tensorial and simplicial field theories;
- the curved Yang-Mills-Higgs theory;
- Q-bundles and the Poisson-σ-model.*

(7) Some advanced gauge field theory:
- the Anderson-Brout-Englert-Higgs-Guralnik-Hagen-Kibble-... mechanism;
- orbital field dynamics and non-linear realisations of internal and spacetime symmetries, the Ivanov-Ogievietskii (aka inverse Higgs) mechanism, constructions in the invariant de Rham cohomology, gerbe objects in the category of groups, Nieuwenhuizen's FDA techniques and the rise of Stasheff's L_∞-structures *via* the Baez-Crans correspondence);
- standard gauge fields *vs* gerbe-module connections;
- bi-chiral Kač-Moody symmetries in the Wess-Zumino-Novikov-Witten-Gawędzki model of $2d$ CFT;*
- topological gauge field theory – a case study of the Chern-Simons theory in $3d$ in the presence of Wilson lines, of its reduction à la Alekseev and Malkin and of... its intricate relation to the WZNWG model (an example of 'holography').*

(8) Field theories with the statistical gradation:
- rudiments of supergeometry with supersymmetry ($\mathbb{Z}/2\mathbb{Z}$-graded manifolds, super-Harish-Chandra pairs, super-Cartan calculus and Lie supergroup actions);
- a recapitulation of the theory of spinor bundles with spin connections;
- Freed's inner-Hom superfield theories and their (quasi-)supersymmetry;
- super-σ-models and geometrisations in the Cartan-Eilenberg cohomology of Lie supergroups – supergerbes.*

(9) Dualities:
- the (pre)symplectic description;
- the defect-duality correspondence;
- useful instantiations: T-duality *via* gauging of toroidal actions and the ensuing higher configuration-bundle topology change, the Hughes-Polchinski duality, S-duality in a weakly abelian gauge field theory*, a sector-restricted duality between the $2d$ WZNWG CFT and the $3d$ CS TGFT – a first step towards functorial quantisation*, and more*...

*Time permitting.

Literature: Listed on the lecture's homepage.

Requirements: The course is dedicated to 3rd-year undergrads from the individual student groups at the maths and physics faculties, as well as to PhD students. It bases on the 1st-year course in (linear) algebra, the 2nd-year course in differential geometry and that in group theory. Familiarity with the content of the two-semester monographic course "Methods of higher algebra in physics: from quadratic forms to spinor bundles" by the present lecturer may also be helpful but is not strictly necessary. Accordingly, a working knowledge about concepts and methods discussed at length in these courses shall be assumed freely (albeit non-dogmatically).

Homepage: `https://www.fuw.edu.pl/~suszek/2025DDD1.html`

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