

CLASSICAL FIELD THEORY IN THE TIME OF COVID-19
PROBLEM SHEET IV

Basing on the discussion of Example 2.4 in volume 2. of the book "Foundations of Differential Geometry" by Kobayashi and Nomizu and of Urbantke's paper "The Hopf fibration – seven times in physics" (J. Geom. Phys. **46** (2003) 125–150, Appendix A.1), reconstruct in detail the nontrivial principal bundle $U(1) \cong \mathbb{S}^1 \longrightarrow \mathbb{S}^3 \longrightarrow \mathbb{S}^2$ aka the **Hopf fibration**, and subsequently relate, based on Urbantke's paper "Two-level quantum systems: States, phases, and holonomy" (Am. J. Phys. **59** (1991) 503–509, Sections I, II, III), the application of that bundle in the description of the space of quantum states of the qubit, regarded (upon a suitable normalisation) as a total space of a certain bundle over the space of pure states.