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Correlation between ΔM_s and $B_{s,d}^0 \to \mu^+\mu^-$ in Supersymmetry at Large $\tan\beta$

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Abstract

Considering the MSSM with the CKM matrix as the only source of flavour violation and heavy supersymmetric particles at large $\tan\beta$, we analyze the correlation between *the increase* of the rates of the decays $B_{s,d}^0 \to \mu^+\mu^-$ and *the suppression* of ΔM_s, that are caused by the enhanced flavour changing neutral Higgs couplings to down-type quarks. We give analytic formulae for the neutral and charged Higgs couplings to quarks including large $\tan\beta$ resummed corrections in the $SU(2) \times U(1)$ limit and comment briefly on the accuracy of this approximation. For $0.8 \le (\Delta M_s)^{\mathrm{exp}}/(\Delta M_s)^{\mathrm{SM}} \le 0.95$ we find $6 \cdot 10^{-7} \ge BR(B_s^0 \to \mu^+\mu^-)^{\mathrm{max}} \ge 4 \cdot 10^{-8}$ and $1.4 \cdot 10^{-8} \ge BR(B_d^0 \to \mu^+\mu^-)^{\mathrm{max}} \ge 1 \cdot 10^{-9}$. For $(\Delta M_s)^{\mathrm{exp}} \ge (\Delta M_s)^{\mathrm{SM}}$ substantial enhancements of $B_{s,d}^0 \to \mu^+\mu^-$ relative to the expectations based on the Standard Model are excluded. With $(\Delta M_s)^{\mathrm{exp}} > 15.0/\mathrm{ps}$ a conservative analysis of $(\Delta M_s)^{\mathrm{SM}}$ gives $BR(B_s^0 \to \mu^+\mu^-) \lesssim 1.2 \cdot 10^{-6}$ and $BR(B_d^0 \to \mu^+\mu^-) \lesssim 3 \cdot 10^{-8}$. However, we point out that in the less likely scenario in which the squark mixing is so large that the neutral Higgs contributions dominate ΔM_s, the rates for $B_{s,d}^0 \to \mu^+\mu^-$ increase with increasing ΔM_s and the bounds in question are weaker. Violation of all these correlations and bounds would indicate new sources of flavour violation.

1 Introduction

The Minimal Supersymmetric Standard Model (MSSM), with large value of $\tan\beta$, the ratio of the two vacuum expectation values v_u/v_d, is a very interesting scenario. On the one hand, it is consistent with the unification of the top and bottom Yukawa couplings predicted by some $SO(10)$ GUT models. On the other hand, its predictions for rates of certain low energy processes can differ significantly from the ones of the Standard Model (SM) even for heavy sparticles and with the Cabibbo-Kobayashi-Maskawa (CKM) matrix being the only source of flavour and CP violation in the quark sector.

In the down-quark sector large supersymmetric effects originate from $\tan\beta$ enhanced flavour changing neutral currents (FCNC) mediated by Higgs scalars and generated at one loop by Higgs penguin-like diagrams with charginos and top-squarks. They have been first considered in [5] and subsequently found to *increase* by orders of magnitude the branching ratios of the rare decays $B_{s,d}^0 \to \mu^+\mu^-$ [1, 2, 3, 4] and to *decrease* significantly the B_s^0-$\bar{B}_s^0$ mass difference ΔM_s [6] relative to the expectations based on the SM. Since both these effects are caused by the same neutral Higgs boson mediated FCNC (see figs. 2 and 3), a correlation between them must exist [6]. This is particularly interesting as ΔM_s and $BR(B_{s,d}^0 \to \mu^+\mu^-)$ can in principle be measured at the Tevatron and B–factories in the coming years. It is the purpose of this letter to point out the consequences of this correlation.

Analyzing low energy processes in the MSSM with $\tan\beta \gg 1$ it is essential to take into account all potentially large effects in a consistent framework. Four such effects have been identified in the literature:

1) Modification of the tree-level relation between the MSSM Lagrangian mass parameters m_d, m_s, m_b determining the corresponding Yukawa couplings and the running ("measured") quark masses $\overline{m}_d$, $\overline{m}_s$, $\overline{m}_b$ [7].

2) Corrections to the CKM matrix, as a result of which elements of the physical CKM matrix, to be called V_{JI}^{eff}, differ from V_{JI} present in the original Lagrangian [8].

3) Enhanced flavour changing neutral Higgs boson penguins mentioned above.

4) Enhanced corrections to charged Higgs boson vertices [9].

Several steps towards including consistently all these effects in phenomenological analyses have been already made during the last years. In refs. [9, 10, 11] the effects 1) and 4) have been discussed in the context of the $\bar{B} \to X_s\gamma$ decay. In [12] the effects 1)-3) have been calculated in the $SU(2) \times U(1)$ symmetry limit in the context of $B_{s,d}^0 \to \mu^+\mu^-$ decays and $B_{s,d}^0$-$\bar{B}_{s,d}^0$ mixings confirming the increase of $BR(B_{s,d}^0 \to \mu^+\mu^-)$ and the suppression of ΔM_s pointed out in [1, 2, 3, 4] and [6], respectively.

In the following detailed analysis [13] we extend these analyses based on $SU(2) \times U(1)$ symmetry limit [1, 12] by calculating all the four effects in a more general effective La-

grangian approach, comparing the results, analytically and numerically, with the $SU(2) \times U(1)$ symmetry limit and thereby confirming and in certain cases correcting and generalizing analytical rules for inclusion of the large $\tan\beta$ effects presented in [9, 10, 11, 12]. As the analysis of [13] is long and technical, in the present letter we summarize compactly the results for all the four listed effects. We present numerical results based on the formalism of [13] and explain them qualitatively using the formulae obtained in the $SU(2)\times U(1)$ symmetry limit. This allows us to analyze in detail the correlation between $BR(B_{s,d}^0 \to \mu^+\mu^-)$ and ΔM_s pointed out in [6] taking into account the $\bar{B} \to X_s\gamma$ constraint.

During the completion of this letter a model independent analysis of rare processes in theories with the CKM matrix as the unique source of flavour and CP violation has been presented in [14]. While those authors also investigated large $\tan\beta$ effects in $BR(B_{s,d}^0 \to \mu^+\mu^-)$, ΔM_s and $\bar{B} \to X_s\gamma$, they have not analyzed the correlation between $BR(B_{s,d}^0 \to \mu^+\mu^-)$ and ΔM_s addressed here.

As the recent discussions in the literature [3, 14] show that the statements like "models in which flavour mixing is ruled only by the CKM matrix" or "models with minimal flavour violation" have different meaning in different papers, we would like to specify the structure of flavour violation in the MSSM version considered by us. While the flavour violation in the scenario considered is ruled by the CKM matrix, it should be emphasized that for split soft SUSY breaking masses of left-handed squarks belonging to different generations some flavour violation unavoidably appears in the up- or down-type (or in both) squark mass squared matrices. In our calculations we choose the soft SUSY breaking mass parameter m_Q^2 such that flavour violation appears in the up-type squark mass matrix. The scenario with flavour violation in the down-type squark mass matrix would require the inclusion of box and Higgs penguin diagrams with gluinos and is beyond the scope of this paper.

2   The effective Lagrangian

Let us consider the decoupling of sparticles in the limit of unbroken $SU(2) \times U(1)$ symmetry [1, 12]. The electroweak symmetry breaking is then taken into account after sparticles are integrated out. This approximation should be valid if the sparticle mass scale is larger than that of the Higgs boson sector (set by M_{H^+}). The absence of vacuum expectation values before decoupling implies neglecting the left-right mixing in the squark mass matrices even for non-vanishing $A_{u,d}$ and/or μ parameters.

In this approach below the sparticle mass scale the effective Lagrangians describing the neutral and charged Higgs boson couplings to the down- and up-type quarks have the form [15, 1]

$$\mathcal{L}_{\text{eff}}^{(d)} = -\epsilon_{ij}H_i^{(d)}\overline{d_R} \cdot (\mathbf{Y}_d + \Delta_d\mathbf{Y}_d) \cdot q_{jL} - H_i^{(u)*}\overline{d_R} \cdot \Delta_u\mathbf{Y}_d \cdot q_{iL} + \text{H.c.} \qquad (2.1)$$

$$\mathcal{L}_{\text{eff}}^{(u)} = -\epsilon_{ij} H_i^{(u)} \overline{u_R} \cdot (\mathbf{Y}_u + \Delta_u \mathbf{Y}_u) \cdot q_{jL} - H_i^{(d)*} \overline{u_R} \cdot \Delta_d \mathbf{Y}_u \cdot q_{iL} + \text{H.c.} \tag{2.2}$$

where $\epsilon_{21} = -\epsilon_{12} = 1$ and $\mathbf{Y}_{d,u}$ are Yukawa coupling matrices. The neutral and charged components of the two Higgs doublets are given in the standard way

$$H_1^{(d)} = \frac{v_d}{\sqrt{2}} + \frac{1}{\sqrt{2}} \left(H^0 \cos\alpha - h^0 \sin\alpha + iA^0 \sin\beta - iG^0 \cos\beta \right)$$

$$H_2^{(u)*} = \frac{v_u}{\sqrt{2}} + \frac{1}{\sqrt{2}} \left(H^0 \sin\alpha + h^0 \cos\alpha - iA^0 \cos\beta - iG^0 \sin\beta \right) \tag{2.3}$$

$$H_2^{(d)*} = H^+ \sin\beta - G^+ \cos\beta, \qquad H_1^{(u)} = H^+ \cos\beta + G^+ \sin\beta \ . \tag{2.4}$$

In these conventions

$$m_{d_J} = -\frac{v_d}{\sqrt{2}} y_{d_J}, \qquad m_{u_J} = \frac{v_u}{\sqrt{2}} y_{u_J} \tag{2.5}$$

where y_{d_J} and y_{u_J} are the Yukawa couplings. Here J is the flavour index with $d_1 \equiv d$, $d_2 \equiv s$, $d_3 \equiv b$ and similarly for the up-type quarks. Finally $v_d^2/\cos^2\beta = v_u^2/\sin^2\beta = 1/\sqrt{2}G_F \approx (246 \text{ GeV})^2$.

The loop induced terms $\Delta_d \mathbf{Y}_d$ and $\Delta_u \mathbf{Y}_u$ are always subleading in the large $\tan\beta$ limit and can be neglected. Diagrams giving rise to the correction $\Delta_u \mathbf{Y}_d$ are shown in figs. 1a and 1b. In the basis in which $\mathbf{Y}_d = \text{diag}(y_d)$, $\mathbf{Y}_u = \text{diag}(y_u) \cdot V$ where V is the CKM matrix, and neglecting y_u^2 and y_c^2, the correction $\Delta_u \mathbf{Y}_d$ has the structure [12]

$$(\Delta_u \mathbf{Y}_d)^{JI} = -y_{d_J} \left(\epsilon_0 \delta^{JI} + \epsilon_Y y_t^2 V^{3J*} V^{3I} \right). \tag{2.6}$$

The correction $\Delta_d \mathbf{Y}_u$ is generated by the diagrams shown in figs. 1c and 1d and has the form

$$(\Delta_d \mathbf{Y}_u)^{JI} = y_{u_J} V_{JI} \left(\epsilon_0' + \epsilon_Y' y_{d_I}^2 \right) \ . \tag{2.7}$$

The four quantities ϵ_0, ϵ_Y, ϵ_0', ϵ_Y' can be obtained by calculating the diagrams in fig. 1:

$$\epsilon_0 = -\frac{2\alpha_s}{3\pi} \frac{\mu}{m_{\tilde{g}}} H_2 \left(x^{Q/g}, x^{D/g} \right), \qquad \epsilon_Y = \frac{1}{16\pi^2} \frac{A_t}{\mu} H_2 \left(x^{Q/\mu}, x^{U/\mu} \right) \tag{2.8}$$

$$\epsilon_0' = -\frac{2\alpha_s}{3\pi} \frac{\mu}{m_{\tilde{g}}} H_2 \left(x^{Q/g}, x^{U/g} \right), \qquad \epsilon_Y' = \frac{1}{16\pi^2} \frac{A_b}{\mu} H_2 \left(x^{Q/\mu}, x^{D/\mu} \right) \tag{2.9}$$

where $x^{Q/g} \equiv m_Q^2/m_{\tilde{g}}^2$, $x^{D/g} \equiv m_D^2/m_{\tilde{g}}^2$, $x^{Q/\mu} \equiv m_Q^2/\mu^2$ etc., and m_Q^2, m_D^2, m_U^2, A_t, and A_b are the parameters of the soft supersymmetry breaking in the MSSM Lagrangian[1]. The function $H_2(x,y)$ is defined as

$$H_2(x,y) = \frac{x \ln x}{(1-x)(x-y)} + \frac{y \ln y}{(1-y)(y-x)} \ . \tag{2.10}$$

The eqs. (10),(15),(16) of [9] reduce to (2.8) and (2.9) in the $SU(2) \times U(1)$ symmetry limit.

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[1]Our convention [16] for A_u and A_d parameters is fixed by the form of the left-right mixing terms in the squark mass matrices which read $-m_u(A_u + \mu \cot\beta)$ and $-m_d(A_d + \mu \tan\beta)$ for the up and down squarks, respectively.
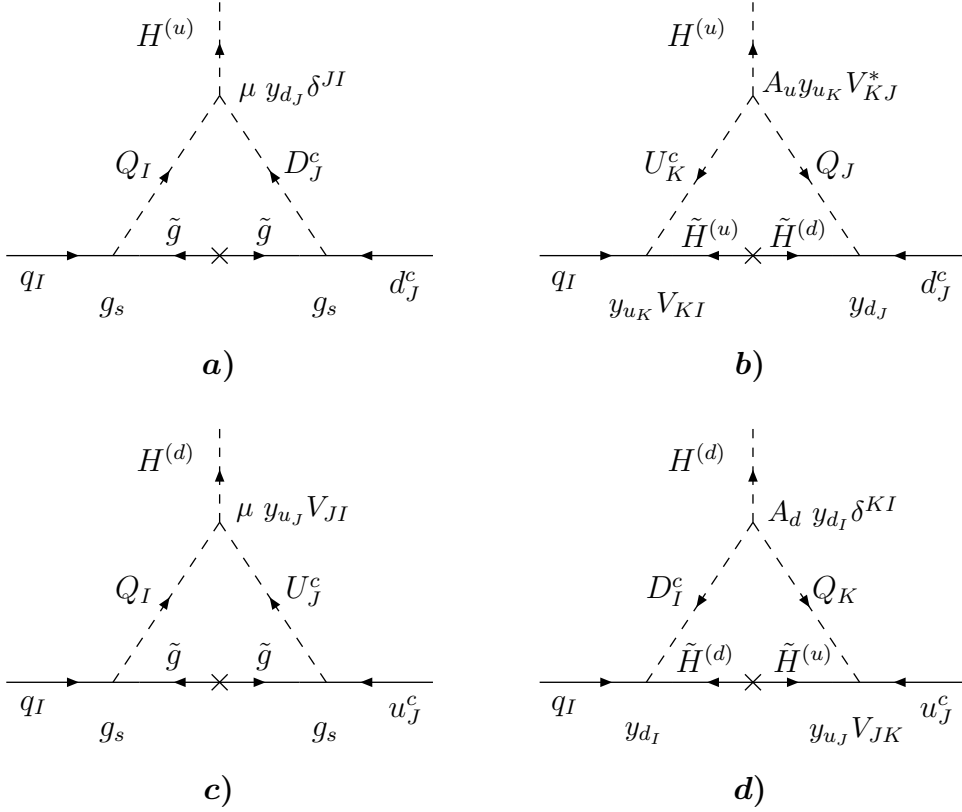
Figure 1: Vertex corrections in the $SU(2) \times U(1)$ symmetry limit. Diagrams a) and b) give rise to corrections $(\Delta_u \mathbf{Y}_d)^{JI}$, diagrams c) and d) to corrections $(\Delta_d \mathbf{Y}_u)^{JI}$.

3   Effective Parameters and Couplings

The mass matrices of the down- and up-type quarks can be obtained by replacing the neutral scalar fields in (2.1) and (2.2) by their vacuum expectation values. One finds that the down-type-quark mass matrix $\hat{M}_d$ receives $\tan\beta$ enhanced corrections both to the diagonal and non-diagonal entries, whereas the corresponding corrections to $\hat{M}_u$ are negligible. $\hat{M}_d$ is then diagonalized by the appropriate rotations of the d_L and d_R fields. Except for the charged Higgs boson H^+ couplings in which loop correction $\Delta_d \mathbf{Y}_u$ matters, the four effects listed in the Introduction result from performing these rotations on the d_L and d_R fields in the interaction vertices in (2.1) and (2.2).

In the full approach that goes beyond the $SU(2) \times U(1)$ symmetry limit [13], the corrections to $\hat{M}_d$ are found by calculating directly the self-energy diagrams of the down-type-quarks. The resulting formulae are rather complicated and are presented in [13] where also the derivation of the formulae in the $SU(2) \times U(1)$ limit is described in detail.

Below we give the formulae that summarize the effects 1)–4) in the $SU(2) \times U(1)$ symmetry limit. The quark fields in these formulae are mass eigenstates of the one-loop

corrected matrices $\hat{M}_d$ and $\hat{M}_u$ as opposed to the original fields in (2.1) and (2.2).

1. The original mass parameters m_{d_J} and m_{u_J} in (2.5), that enter the Feynman rules, are related to the *effective* running mass parameters $\overline{m}_{d_J}$ and $\overline{m}_{u_J}$ of the low energy theory through [7]

$$m_{d_J} = \frac{\overline{m}_{d_J}}{1 + \tilde{\epsilon}_J \tan\beta}, \qquad m_{u_J} \approx \overline{m}_{u_J} \tag{3.1}$$

with $\tilde{\epsilon}_J$ given by

$$\tilde{\epsilon}_J \equiv \epsilon_0 + \epsilon_Y y_t^2 V_{3J}^* V_{3J} \approx \epsilon_0 + \epsilon_Y y_t^2 \delta^{J3} . \tag{3.2}$$

It has been shown [10] that expressing m_{d_J} through $\overline{m}_{d_J}$ by means of (3.1) in the neutral and charged Higgs couplings resums for large values of $\tan\beta$ dominant supersymmetric corrections to all orders of perturbation theory. Such a resummation is necessary for obtaining reliable results. Note that in contrast to the corrections to m_b in (3.1), the ones to m_d and m_s do not depend on the top Yukawa coupling.

2. The original elements of the CKM matrix, V_{JI}, present in the Feynman rules of the MSSM are related to the *effective* CKM matrix V_{JI}^{eff} through [8, 1, 12, 13]

$$V_{JI} = V_{JI}^{\text{eff}} \left[\frac{1 + \tilde{\epsilon}_3 \tan\beta}{1 + \epsilon_0 \tan\beta}\right] \quad \text{for} \quad (JI) = (13), \ (23), \ (31) \text{ and } (32),$$
$$V_{JI} = V_{JI}^{\text{eff}} \quad \text{otherwise.} \tag{3.3}$$

It is V_{JI}^{eff} that has to be identified with the CKM matrix whose elements are determined from the low energy processes. Note that the elements $|V_{ub}|$ and $|V_{cb}|$, that are affected by these corrections are usually determined from tree level decays under the assumption that new physics contributions to the relevant branching ratios can be neglected. This assumption is violated in the case of supersymmetry at large $\tan\beta$. In other words, what experimentalists extract from tree level decays are $|V_{ub}^{\text{eff}}|$ and $|V_{cb}^{\text{eff}}|$ and not $|V_{ub}|$ and $|V_{cb}|$.

3. The effective Lagrangian describing flavour violating neutral Higgs interactions with down-type quarks is given by

$$\mathcal{L}_{\text{eff}}^{\text{off}-\text{diag}} = -\overline{(d_J)_R} \left[X_{RL}^S\right]^{JI} (d_I)_L S^0 - \overline{(d_J)_L} \left[X_{LR}^S\right]^{JI} (d_I)_R S^0 \tag{3.4}$$

with $S^0 = (H^0, h^0, A^0, G^0)$. In the case of B-physics the pairs $(J, I) = (3, 2), (3, 1), (2, 3)$ and $(1, 3)$ matter. We find [13]

$$\left[X_{RL}^S\right]^{JI} = \left[X_{LR}^S\right]^{IJ*} = \frac{g}{2M_W \cos\beta} \frac{\overline{m}_{d_J} V_{\text{eff}}^{3J*} V_{\text{eff}}^{3I}}{(1 + \tilde{\epsilon}_3 \tan\beta)(1 + \epsilon_0 \tan\beta)} \epsilon_Y y_t^2 \left(x_u^S - x_d^S \tan\beta\right) \tag{3.5}$$

where $x_d^S = (\cos\alpha, -\sin\alpha, i\sin\beta, -i\cos\beta)$, and $x_u^S = (\sin\alpha, \cos\alpha, -i\cos\beta, -i\sin\beta)$.

In the case of K-physics the pairs $(J, I) = (2, 1)$ and $(1, 2)$ matter and we find [13]

$$\left[X_{RL}^S\right]^{JI} = \left[X_{LR}^S\right]^{IJ*} = \frac{g}{2M_W \cos\beta} \overline{m}_{d_J} V_{\text{eff}}^{3J*} V_{\text{eff}}^{3I} \frac{(1 + \tilde{\epsilon}_3 \tan\beta)^2}{(1 + \epsilon_0 \tan\beta)^4} \epsilon_Y y_t^2 \left(x_u^S - x_d^S \tan\beta\right) . \tag{3.6}$$

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Note that the flavour violating couplings of G^0 vanish in this limit. Formulae (3.5)–(3.6) agree with the recent corrected version of [12] except that $V_{J3}^{\text{eff}*}$ in equation (10) of that paper should be replaced by $V_{3J}^{\text{eff}*}$.

4. The effective couplings of the charged Higgs ($H^\pm$) and Goldstone ($G^\pm$) bosons to quarks are given respectively by

$$\mathcal{L}_{\text{eff}}^{H^+} = \overline{(u_J)_R} \left[P_{RL}^H\right]^{JI} (d_I)_L H^+ + \overline{(u_J)_L} \left[P_{LR}^H\right]^{JI} (d_I)_R H^+ + h.c \tag{3.7}$$

$$\mathcal{L}_{\text{eff}}^{G^+} = \overline{(u_J)_R} \left[P_{RL}^G\right]^{JI} (d_I)_L G^+ + \overline{(u_J)_L} \left[P_{LR}^G\right]^{JI} (d_I)_R G^+ + h.c . \tag{3.8}$$

It is useful to define the parameters ϵ_{JI}^{HL}, ϵ_{JI}^{HR}, ϵ_{JI}^{GL} and ϵ_{JI}^{GR} through

$$\left[P_{RL}^H\right]^{JI} = \frac{g}{\sqrt{2}M_W} \cot\beta \, \overline{m}_{u_J} V_{JI}^{\text{eff}} (1 - \epsilon_{JI}^{HL}), \qquad \left[P_{LR}^H\right]^{JI} = \frac{g}{\sqrt{2}M_W} \tan\beta \, V_{JI}^{\text{eff}} \overline{m}_{d_I} (1 - \epsilon_{JI}^{HR}),$$
$$\tag{3.9}$$

$$\left[P_{RL}^G\right]^{JI} = \frac{g}{\sqrt{2}M_W} \overline{m}_{u_J} V_{JI}^{\text{eff}} (1 + \epsilon_{JI}^{GL}), \qquad \left[P_{LR}^G\right]^{JI} = -\frac{g}{\sqrt{2}M_W} V_{JI}^{\text{eff}} \overline{m}_{d_I} (1 + \epsilon_{JI}^{GR}). \tag{3.10}$$

Using $\tilde{\epsilon}_J$ defined in (3.2), we find in the $SU(2) \times U(1)$ symmetry limit [13]

$$\epsilon_{JI}^{HL} = \tan\beta \left(\epsilon_0' + \epsilon_Y' y_b^2 \delta^{I3}\right) + \Delta_{JI} \qquad \epsilon_{JI}^{HR} = \frac{\tilde{\epsilon}_J \tan\beta}{1 + \tilde{\epsilon}_J \tan\beta}, \qquad \epsilon_{JI}^{GL} = \epsilon_{JI}^{GR} = 0. \tag{3.11}$$

where

$$\Delta_{JI} = y_b^2 y_t^2 \frac{\epsilon_Y \epsilon_Y' \tan^2\beta}{1 + \epsilon_0 \tan\beta} \times \begin{cases} +1 & (J,I) = (1,3),(2,3) \\ -1 & (J,I) = (3,1),(3,2) \\ 0 & \text{otherwise} \end{cases} \tag{3.12}$$

In the $SU(2) \times U(1)$ symmetry limit vanishing of the corrections ϵ_{JI}^{GL} and ϵ_{JI}^{GR} to the charged Goldstone boson vertices expressed in terms of V^{eff} and physical masses $\overline{m}_{d_I}$ is required by gauge invariance [13]. The results for ϵ_{JI}^{HL} and ϵ_{JI}^{HR} agree with ref. [14], where the presence of Δ_{JI} has been pointed out.

We observe that the ϵ_{JI}^{HR} corrections to the vertices involving V_{ts}^{eff} and V_{td}^{eff} depend on the top Yukawa coupling y_t^2 while those to the vertices involving V_{cb}^{eff} and V_{ub}^{eff} do not. Note also that whereas the rule (3.11) for ϵ_{JI}^{HR} for ($J \neq 3, I$) and accidentally for $J = I = 3$ is equivalent to expressing in the tree level formulae m_{d_I} and V_{JI} through $\overline{m}_{d_I}$ and V_{JI}^{eff} by means of (3.1) and (3.3) respectively, for $J = 3$ and $I = 1,2$ it is more involved. Expressing in these cases only V_{JI} and m_{d_I} through V_{JI}^{eff} and $\overline{m}_{d_I}$, would give wrong results. In [9] explicit expressions for $[P_{RL}^{H(G)}]^{JI}$ with $J = 3$, $I = 1,2,3$ and for $[P_{LR}^{H(G)}]^{JI}$ with $J = 1,2,3$ and $I = 3$ have been given omitting the modifications of the CKM factors summarized in (3.3) - see the formula (17) of that paper. As discussed in [13], the particular couplings given in [9] agree with the formulae given above provided Δ_{JI} is set to zero and the CKM matrix V of [9] is identified with V^{eff} in $[P_{RL}^{H(G)}]^{JI}$ of that paper and with the original MSSM CKM matrix in $[P_{LR}^{H(G)}]^{JI}$. In spite of this inconsistency, in the special case of the

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dominant operator in the $\bar{B} \to X_s \gamma$ decay, the recipes for the inclusion of large $\tan\beta$ effects into Wilson coefficients formulated in eqs. (18) and (19) of that paper are accidentally correct provided all the CKM factors involved in this decay are identified with V_{eff} and Δ_{JI} is set to zero. However, as emphasized in [14] Δ_{JI} cannot be generally neglected for $|\epsilon_Y \tan\beta|$ and $|\epsilon_Y' \tan\beta|$ larger than 0.5 and it could be important for $\epsilon_0' \approx -\epsilon_Y'$ when the $\mathcal{O}(\tan\beta)$ term in ϵ_{JI}^{HL} is small.

As discussed in detail in [13], the approximations described here work rather well for the relation (3.1) between the original mass parameters m_{d_I} (i.e. the Yukawa couplings) and the running masses $\overline{m}_{d_I}$ and also for the relation between V and V_{eff}. The differences between the full and approximate calculation are usually smaller than 15% and are mainly due to neglecting in the $SU(2) \times U(1)$ symmetry limit some gauge coupling-dependent terms. The same remains true also for the flavour changing couplings X_{RL} and X_{LR} of the neutral scalars since their dominant parts originate from the rotations of d_L and d_R fields which are directly related to the corrections to the down-type quark mass matrix.

Let us record that typically $|\epsilon_0|$ and $|\tilde{\epsilon}_3|$ are $\sim 5 \times 10^{-3}$ and can reach $\sim 10^{-2}$ for very large values of $|\mu|$ and/or $|A_t|$. We have also checked that taking the $\bar{B} \to X_s \gamma$ constraint into account, values of the factor $(1 + \tilde{\epsilon}_3 \tan\beta)(1 + \epsilon_0 \tan\beta)$ entering the denominator of eq. (3.5), vary between 0.2 and 2 for $\tan\beta \approx 50$.

In the case of charged Higgs boson couplings the full calculation confirms the smallness of the corrections $\epsilon^{GL(R)}$ (typically $|\epsilon^{GL(R)}| \lesssim 0.05$). The approximate formulae (3.11) for ϵ^{HR} and especially for ϵ^{HL} are not as accurate as the ones for the couplings X_{RL} and X_{LR}. This is because triangle vertex diagrams with the chargino-neutralino pairs coupling to H^+ also play a role. However, in the case of the B_s^0-$\bar{B}_s^0$ mixing and of the decays $B_{s,d}^0 \to \mu^+\mu^-$ these corrections constitute only subdominant contribution to the relevant amplitudes and the inaccuracy of the approximation is not essential. Therefore, the approximate formulae we present in the following section give qualitatively correct picture of the dependence of the dominant corrections to the B_s^0-$\bar{B}_s^0$ and $B_{s,d}^0 \to \mu^+\mu^-$ amplitudes on the MSSM parameters. We stress however, that the results presented in fig. 4 are based on the complete calculation along the lines of [13].

4 ΔM_s and $B_{s,d}^0 \to \mu^+\mu^-$

1. In the supersymmetric scenario considered here, ΔM_s is given by

$$\Delta M_s = |(\Delta M_s)^{\text{SM}} + (\Delta M_s)^{H^\pm} + (\Delta M_s)^{\chi^\pm} + (\Delta M_s)^{\text{DP}}| \equiv (\Delta M_s)^{\text{SM}}|(1 + f_s)| \qquad (4.1)$$

(ΔM_s is by definition a positive definite quantity). Here, $(\Delta M_s)^{\text{SM}}$ represents the SM contribution, $(\Delta M_s)^{H^\pm}$ results from box-diagrams with top and $(H^\pm, H^\pm)$, $(H^\pm, W^\pm)$ and

$(H^\pm, G^\pm)$ exchanges and $(\Delta M_s)^{\chi^\pm}$ is the contribution of box diagrams with chargino and squarks. Finally, $(\Delta M_s)^{\mathrm{DP}}$ results from double Higgs penguin diagrams of fig. 2.

Explicit expressions for different contributions in terms of the Wilson coefficients of contributing operators and hadronic matrix elements can be found in [6, 13, 17]. With respect to our previous analysis in [6] we have now included all resummed large $\tan\beta$ corrections to the relevant couplings as discussed in the previous section.
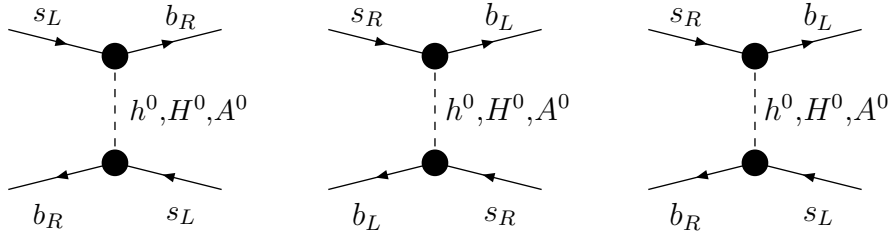
Figure 2: Double penguin diagrams contributing to ΔM_s.

In the scenario considered in [6] and here supersymmetric particles are heavier than the Higgs particles and the chargino box contribution $(\Delta M_s)^{\chi^\pm}$ is small. At large $\tan\beta$ the double penguin contribution $(\Delta M_s)^{\mathrm{DP}}$ is the dominant correction to $(\Delta M_s)^{\mathrm{SM}}$ but the charged Higgs box contribution can also be significant [6]. Both contributions have signs opposite to $(\Delta M_s)^{\mathrm{SM}}$. Consequently for large $\tan\beta$ one finds $(1 + f_s) < 1$ independently of the other supersymmetric parameters. For not too large values of $\tan\beta \lesssim 50$ and of the stop mixing parameter $A_t \lesssim M_{\mathrm{SUSY}}$ the contributions $(\Delta M_s)^{\mathrm{DP}}$ and $(\Delta M_s)^{H^+}$ are smaller than $(\Delta M_s)^{\mathrm{SM}}$ and one gets $0 < (1 + f_s) < 1$. Of interest is also the case $(1 + f_s) < 0$ corresponding to a very large negative $(\Delta M_s)^{\mathrm{DP}}$ that can be realized for some special values of supersymmetric parameters - large $\tan\beta \gtrsim 50$ and/or $A_t \gg M_{\mathrm{SUSY}}$. We will include this possibility in our analysis as it has quite different implications than the case $0 < (1 + f_s) < 1$.

The double penguin diagrams of fig. 2 give $\mathcal{O}(\tan^4\beta)$ correction to ΔM_s. The leading contribution comes from the last diagram that contributes to the Wilson coefficient C_2^{LR} of the operator $Q_2^{\mathrm{LR}} = (\overline{b_R}s_L)(\overline{b_L}s_R)$. Using the vertices of eq. (3.5) we find [13]

$$(\Delta M_s)^{DP} = \frac{G_F^2 M_W^2}{24\pi^2} M_{B_s} F_{B_s}^2 |V_{ts}^{\mathrm{eff}}|^2 P_2^{LR} C_2^{LR} \tag{4.2}$$

where

$$C_2^{\mathrm{LR}} \approx -\frac{G_F \overline{m}_b \overline{m}_{d(s)} \overline{m}_t^4}{\sqrt{2}\pi^2 M_W^2} \frac{\tan^4\beta \ \epsilon_Y^2 (16\pi^2)^2}{(1 + \tilde{\epsilon}_3 \tan\beta)^2 (1 + \epsilon_0 \tan\beta)^2} \left[\frac{\sin^2(\alpha - \beta)}{M_{H^0}^2} + \frac{\cos^2(\alpha - \beta)}{M_{h^0}^2} + \frac{1}{M_{A^0}^2} \right] \tag{4.3}$$

and $P_2^{LR} \approx 2.5$ includes the short distance NLO QCD corrections [17, 18, 19] and the

relevant hadronic matrix elements [20]. Details are given in [6, 13, 17]. C_2^{LR} in (4.3) agrees with the corrected version of [12].

For large $\tan\beta$ one has $M_{H^0} \approx M_{A^0}$, $\cos^2(\alpha - \beta) \approx 0$ and $\sin^2(\alpha - \beta) \approx 1$ and we find

$$(\Delta M_s)^{DP} = -12.0/ps \times \left[\frac{\tan\beta}{50}\right]^4 \left[\frac{P_2^{LR}}{2.50}\right] \left[\frac{F_{B_s}}{230 \text{ MeV}}\right]^2 \left[\frac{|V_{ts}|}{0.040}\right]^2$$

$$\times \left[\frac{\overline{m}_b(\mu_t)}{3.0\text{GeV}}\right] \left[\frac{\overline{m}_s(\mu_t)}{0.06\text{GeV}}\right] \left[\frac{\overline{m}_t^4(\mu_t)}{M_W^2 M_A^2}\right] \frac{\epsilon_Y^2(16\pi^2)^2}{(1 + \tilde{\epsilon}_3 \tan\beta)^2(1 + \epsilon_0 \tan\beta)^2} \quad . \quad (4.4)$$

We recall that for large $\tan\beta$ the H^0 and A^0 contributions to the first two diagrams in fig. 2 cancel each other [1, 6] and as the contribution of h^0 can be neglected in this limit, the total contributions of these two diagrams are very small.

2. At large $\tan\beta$ the branching ratios $BR(B_{s,d}^0 \to \mu^+\mu^-)$ are fully dominated by the diagrams in fig. 3 [1, 2, 3, 4]. Following [21] we find

$$BR(B_s^0 \to \mu^+\mu^-) = 2.32 \times 10^{-6} \left[\frac{\tau_{B_s}}{1.5 \ ps}\right] \left[\frac{F_{B_s}}{230 \text{ MeV}}\right]^2 \left[\frac{|V_{ts}^{\text{eff}}|}{0.040}\right]^2 \left[|\tilde{c}_S|^2 + |\tilde{c}_P|^2\right] \quad . \quad (4.5)$$

Here $\tilde{c}_S$ and $\tilde{c}_P$ are the dimensionless Wilson coefficients $\tilde{c}_S = M_{B_s} c_S$ and $\tilde{c}_P = M_{B_s} c_P$ with c_S and c_P being properly normalized (see [21]) Wilson coefficients of the operators

$$\mathcal{O}_S = m_b(\overline{b_R} s_L)(\bar{l}l), \quad \mathcal{O}_P = m_b(\overline{b_R} s_L)(\bar{l}\gamma_5 l). \quad (4.6)$$
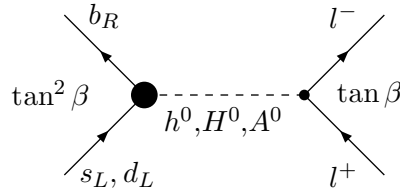

Figure 3: Dominant diagrams contributing to $B_{s,d}^0 \to l^+ l^-$ decays at large $\tan\beta$.

Using the vertices in (3.5) one finds from the diagrams of fig. 3 [12, 13]

$$c_S \approx -\frac{m_\mu \overline{m}_t^2}{4M_W^2} \frac{16\pi^2 \epsilon_Y \ \tan^3\beta}{(1 + \tilde{\epsilon}_3 \tan\beta)(1 + \epsilon_0 \tan\beta)} \left[-\frac{\sin(\alpha - \beta)\cos\alpha}{M_{H^0}^2} + \frac{\cos(\alpha - \beta)\sin\alpha}{M_{h^0}^2}\right] \quad . \quad (4.7)$$

$$c_P \approx -\frac{m_\mu \overline{m}_t^2}{4M_W^2} \frac{16\pi^2 \epsilon_Y \ \tan^3\beta}{(1 + \tilde{\epsilon}_3 \tan\beta)(1 + \epsilon_0 \tan\beta)} \left[\frac{1}{M_{A^0}^2}\right] \quad . \quad (4.8)$$

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In the large $\tan\beta$ limit the contribution of h^0 to c_S can be neglected and setting $M_{H^0}^2 \approx M_{A^0}^2$ we find from (4.7) and (4.8) that $|c_S| = |c_P|$ with c_P given in (4.8). Consequently

$$
\begin{aligned}
BR(B_s^0 \to \mu^+\mu^-) &= 3.5 \times 10^{-5} \left[\frac{\tan\beta}{50}\right]^6 \left[\frac{\tau_{B_s}}{1.5\ ps}\right] \left[\frac{F_{B_s}}{230\ \text{MeV}}\right]^2 \left[\frac{|V_{ts}^{\text{eff}}|}{0.040}\right]^2 \\
&\times \frac{\overline{m}_t^4}{M_A^4} \frac{(16\pi^2)^2 \epsilon_Y^2}{(1 + \tilde\epsilon_3 \tan\beta)^2 (1 + \epsilon_0 \tan\beta)^2}.
\end{aligned} \tag{4.9}
$$

This result agrees with [12]. Moreover one has

$$
\frac{BR(B_d^0 \to \mu^+\mu^-)}{BR(B_s^0 \to \mu^+\mu^-)} = \left[\frac{\tau_{B_d}}{\tau_{B_s}}\right] \left[\frac{F_{B_d}}{F_{B_s}}\right]^2 \left[\frac{|V_{td}^{\text{eff}}|}{|V_{ts}^{\text{eff}}|}\right]^2 \left[\frac{M_{B_d}}{M_{B_s}}\right]^5 \tag{4.10}
$$

that is, the ratio of the branching fractions can depend on the SUSY parameters only weakly through $|V_{td}^{\text{eff}}/V_{ts}^{\text{eff}}|$ which should be consistently determined from the unitarity triangle analysis [22, 13].

The presence of additional $\tan\beta$ dependence in the denominators of eqs. (4.4) and (4.9), not included in [6] and [1, 2, 3, 4], has been pointed out in [12]. While we confirm these additional factors, we would like to emphasize that depending on the sign of the supersymmetric parameter μ they can suppress ΔM_s^{DP} and $BR(B_s^0 \to \mu^+\mu^-)$ relative to the estimates in the papers in question, as stressed in [12], but can also provide additional enhancements.

3. Using (4.4) and (4.9) we find the correlation between the neutral Higgs contributions to $BR(B_s^0 \to \mu^+\mu^-)$ and ΔM_s^{DP} that we have pointed out in [6]:

$$
BR(B_s^0 \to \mu^+\mu^-) = \kappa\ 10^{-6} \left[\frac{\tan\beta}{50}\right]^2 \left[\frac{200\text{GeV}}{M_{A^0}}\right]^2 \left[\frac{|\Delta M_s^{DP}|}{2.12/ps}\right] \tag{4.11}
$$

where

$$
\kappa = \left[\frac{2.50}{P_2^{LR}}\right] \left[\frac{3.0\text{GeV}}{\overline{m}_b(\mu_t)}\right] \left[\frac{0.06\text{GeV}}{\overline{m}_s(\mu_t)}\right] \left[\frac{\tau_{B_s}}{1.5\ ps}\right] \approx 1 . \tag{4.12}
$$

This relation depends sensitively on M_{A^0} and $\tan\beta$ but it does not depend on ϵ_0 and $\tilde\epsilon_3$. From (4.10) a similar correlation between $BR(B_d^0 \to \mu^+\mu^-)$ and ΔM_s^{DP} follows.

In order to understand these results better, let us now assume that ΔM_s has been measured and that appropriate supersymmetric parameters can be found for which the MSSM considered here agrees with $(\Delta M_s)^{\text{exp}}$. If $0 < (1 + f_s) < 1$ this implies $(\Delta M_s)^{\text{exp}} < (\Delta M_s)^{\text{SM}}$. Then combining (4.1) and (4.11) we find

$$
\begin{aligned}
BR(B_s^0 \to \mu^+\mu^-) &= 8.5 \cdot 10^{-6} \kappa \left[\frac{\tan\beta}{50}\right]^2 \left[\frac{200\text{GeV}}{M_{A^0}}\right]^2 \left[\frac{(\Delta M_s)^{\text{SM}}}{18.0/ps}\right] \\
&\times \left[1 \mp \frac{(\Delta M_s)^{\text{exp}}}{(\Delta M_s)^{\text{SM}}} - \frac{|(\Delta M_s)^{H^\pm}|}{(\Delta M_s)^{\text{SM}}} + \frac{(\Delta M_s)^{\chi^\pm}}{(\Delta M_s)^{\text{SM}}}\right].
\end{aligned} \tag{4.13}
$$

with "∓" corresponding to $0 < (1 + f_s) < 1$ and $(1 + f_s) < 0$, respectively. Using (4.10) analogous expression for $BR(B_d^0 \to \mu^+\mu^-)$ can be found. In writing (4.13) we have taken into account that $(\Delta M_s)^{\mathrm{DP}}$ is always negative and that for large $\tan\beta$ $(\Delta M_s)^{H^\pm}$ is negative and $(\Delta M_s)^{\chi^\pm}$ is positive. Formula (4.13) is valid provided the expression in square brackets is positive and larger than 10^{-3}. Otherwise, other contributions, in particular those coming from Z^0-penguins have to be taken into account. In our numerical analysis we take them into account anyway.

Formula (4.13) demonstrates very clearly that if $(\Delta M_s)^{\mathrm{exp}}$ will turn out to be close or larger than the SM value, the order of magnitude enhancements of $BR(B_{s,d}^0 \to \mu^+\mu^-)$ in the scenario of the MSSM considered here with $0 < (1 + f_s) < 1$ will be excluded. On the other hand large enhancements of $BR(B_{s,d}^0 \to \mu^+\mu^-)$ are in principle still possible if the double-penguin contribution is so large that $(1 + f_s) < 0$ and the "+" sign in (4.13) applies. For $\tan\beta < 50$ obtaining $(1 + f_s) < 0$ and the right magnitude of ΔM_s requires $\mu < 0$ so that the couplings (3.5) are enhanced by the ϵ-factors in the denominator. $\mu < 0$ is excluded in particular scenarios like minimal SUGRA, in which the sign of A_t is fixed and $\mu < 0$ does not allow for satisfying the $\bar{B} \to X_s\gamma$ constraint [11], but cannot be excluded in general.

In order to find $(\Delta M_s)^{\mathrm{exp}}/(\Delta M_s)^{\mathrm{SM}}$ one has to deal with the non-perturbative uncertainties contained in the evaluation of $(\Delta M_s)^{\mathrm{SM}}$. The allowed range for $(\Delta M_s)^{\mathrm{exp}}/(\Delta M_s)^{\mathrm{SM}}$ can be obtained by varying all relevant SM parameters like m_t, V_{ts} and $F_{B_s}\sqrt{B_{B_s}}$. A conservative scanning of these parameters performed in [6] resulted in

$$ a\left[\frac{(\Delta M_s)^{\mathrm{exp}}}{15/ps}\right] \leq \frac{(\Delta M_s)^{\mathrm{exp}}}{(\Delta M_s)^{\mathrm{SM}}} \leq b\left[\frac{(\Delta M_s)^{\mathrm{exp}}}{15/ps}\right] \tag{4.14} $$

with $a = 0.52$ and $b = 1.29$. It is however clear that the numerical values of the parameters a and b depend on the error analysis and the difference $b - a$ should also become smaller as the uncertainties in the parameters m_t, V_{ts} and in particular in $F_{B_s}\sqrt{B_{B_s}}$ are reduced with time. For example, the very recent analysis using the Bayesian approach gives $a = 0.71$ and $b = 1.0$ [23] that correspond to the 95% probability range $15.1/ps \leq (\Delta M_s)^{\mathrm{SM}} \leq 21.0/ps$.

We illustrate the correlations in question in fig. 4 where we plot $BR(B_{s,d}^0 \to \mu^+\mu^-)$ as functions of $(\Delta M_s)^{\mathrm{exp}}/(\Delta M_s)^{\mathrm{SM}}$ for $\tan\beta = 50$ and $M_{A^0} = 200$ GeV by scanning the other MSSM parameters with the restriction that sparticles are heavier than 500 GeV and the $\bar{B} \to X_s\gamma$ constraint is satisfied. For each point in the MSSM parameter space V_{td}^{eff} is determined by the standard unitarity triangle analysis [6, 22, 13, 23]. $(\Delta M_d)^{\mathrm{exp}}$ and the parameter ε_K do not constrain the scan as the Higgs and supersymmetric corrections to these quantities are small in our scenario [6]. In the numerical analysis we have used the formulae from the full approach [6, 13] including $SU(2) \times U(1)$ breaking corrections. Still, the approximate formula (4.13) describes qualitatively the main features of the correla-
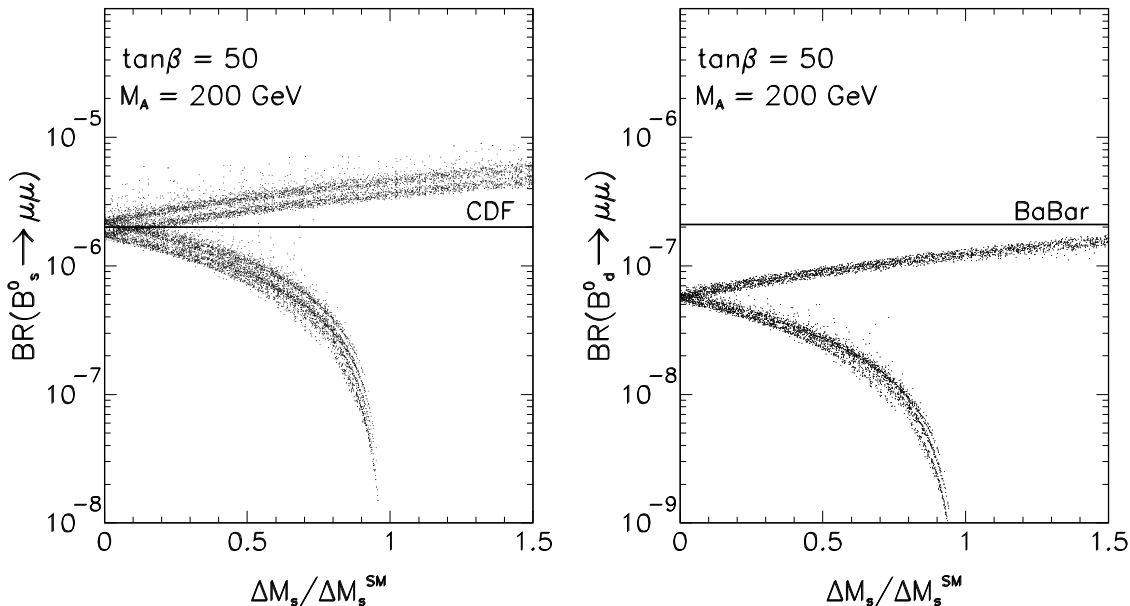
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Figure 4: Correlation between ΔM_s and $B^0_{s,d} \to \mu^+ \mu^-$ in the MSSM with flavour violation ruled by the CKM matrix. Lower (upper) branches of points correspond to $0 < 1 + f_s < 1$ $(1 + f_s < 0)$. Current experimental bounds: $BR(B^0_s \to \mu^+ \mu^-) < 2 \cdot 10^{-6}$ (CDF) [24] and $BR(B^0_d \to \mu^+ \mu^-) < 2.1 \cdot 10^{-7}$ (BaBar) [25] are shown by the horizontal solid lines.

tion. For sparticles heavier than 500 GeV the contribution of chargino-stop boxes to the formula (4.13) is negligible, $(\Delta M_s)^{\chi^\pm}/(\Delta M_s)^{\mathrm{SM}} \lesssim 0.03$. On the other hand, the contribution of the $H^\pm$ boxes can be substantial, $|(\Delta M_s)^{H^\pm}|/(\Delta M_s)^{\mathrm{SM}}$ can reach 0.65 due to the corrections $\epsilon^{HL(R)}$ described in section 3. This is contrary to the claim made in ref. [12] that the $\epsilon^{HL(R)}$ corrections are not important. We have checked that for charginos and stops as light as 150 GeV, $(\Delta M_s)^{\chi^\pm}/(\Delta M_s)^{\mathrm{SM}} \lesssim 0.2$ whereas $|(\Delta M_s)^{H^\pm}|/(\Delta M_s)^{\mathrm{SM}}$ can reach 0.3. Also, as follows from the scan based on the complete calculation, the typical values of $|(\Delta M_s)^{\mathrm{DP}}|$ are smaller for lighter sparticles.

For values of M_A and $\tan\beta$ shown in fig. 4 all points corresponding to the rather unlikely scenario with $1 + f_s < 0$ are eliminated by the combination of the lower limit (4.14) and the CDF upper bound $BR(B^0_s \to \mu^+ \mu^-) < 2 \times 10^{-6}$ [24] but this is not the case for heavier A^0 and/or smaller $\tan\beta$ values. Therefore for such points we can only use (4.10) to find

$$ BR(B^0_d \to \mu^+ \mu^-) < 3.6 \ (3.1) \cdot 10^{-8} \left[\frac{1.15}{F_{B_s}/F_{B_d}} \right]^2 \left[\frac{BR(B^0_s \to \mu^+ \mu^-)^{\exp}}{10^{-6}} \right] \qquad (4.15) $$

with the numerical factor corresponding to the analyses in [6] and [23], respectively. With

the current CDF bound one has the upper bound $BR(B_d^0 \to \mu^+ \mu^-) < 8\,(7) \cdot 10^{-8}$ which is still lower than the current BaBar bound [25].

For a more likely situation of $0 < 1 + f_s < 1$ and $(\Delta M_s)^{\text{exp}}$ satisfying (4.14) we get upper bounds on both branching ratios:

$$BR(B_s^0 \to \mu^+ \mu^-) \lesssim 1.2 \cdot 10^{-6}\,(8 \cdot 10^{-7}) \quad \text{for} \quad a = 0.52\,(0.71),$$
$$BR(B_d^0 \to \mu^+ \mu^-) \lesssim 3 \cdot 10^{-8}\,(2 \cdot 10^{-8}) \quad \text{for} \quad a = 0.52\,(0.71). \tag{4.16}$$

where the two values for the parameter a correspond to the analyses in [6] and [23], respectively. This should be compared with the SM values that are in the ballpark of $3 \cdot 10^{-9}$ and $1 \cdot 10^{-10}$, respectively. On the basis of our discussion of the contribution $(\Delta M_s)^{\chi^\pm}$, we would like to emphasize that the upper limits on $BR(B_{s,d}^0 \to \mu^+ \mu^-)$ obtained here for heavy sparticle spectrum cannot be significantly altered by lowering the sparticle masses.

5 Summary

In this letter we have analyzed ΔM_s and $BR(B_{s,d}^0 \to \mu^+ \mu^-)$ in the MSSM with the CKM matrix as the only source of flavour and CP violation. By considering heavy sparticle spectrum we have quantified the tight correlation between these quantities that exists for large values of $\tan \beta$. Our analysis shows that the neglect of this correlation in the analyses of $BR(B_{s,d}^0 \to \mu^+ \mu^-)$ at large $\tan \beta$ as done in the previous literature [1, 2, 3, 4, 12, 26] is not justified. The correlation in question leads to interesting upper bounds on $BR(B_s^0 \to \mu^+ \mu^-)$ and $BR(B_d^0 \to \mu^+ \mu^-)$ not considered sofar in the literature. In the most likely scenario with $0 < (1 + f_s) < 1$ the upper bounds are becoming very strong when the ratio $(\Delta M_s)^{\text{exp}}/(\Delta M_s)^{\text{SM}}$ approaches unity. For $(\Delta M_s)^{\text{exp}} \geq (\Delta M_s)^{\text{SM}}$ substantial enhancements of $BR(B_{s,d}^0 \to \mu^+ \mu^-)$ with respect to the values obtained in the SM are not possible within the MSSM scenario considered here. Therefore finding experimentally $BR(B_d^0 \to \mu^+ \mu^-)$ above $3 \cdot 10^{-8}$, that is one order of magnitude below the current limit, would be a strong signal of new sources of flavour violation [22].

As the upper bounds on $BR(B_{s,d}^0 \to \mu^+ \mu^-)$ discussed here are sensitive functions of the ratio $(\Delta M_s)^{\text{exp}}/(\Delta M_s)^{\text{SM}}$, their quantitative usefulness will depend on the value of $(\Delta M_s)^{\text{exp}}$ and on the accuracy with which $(\Delta M_s)^{\text{SM}}$ can be calculated. In this respect the present efforts of experimentalists to measure $BR(B_{s,d}^0 \to \mu^+ \mu^-)$ and ΔM_s and of theorists to calculate $F_{B_{d,s}}$ and the parameters $B_{d,s}$ appear even more important than until now.

Acknowledgments

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