Problem 1

$$\hat{H} = \sum_{\bar{k}} \epsilon_{\bar{k}}\, a^{+}_{\bar{k}\sigma} a_{\bar{k}\sigma} \qquad \epsilon_{\bar{k}} = \frac{\hbar^2 k^2}{2m}$$

$$N = \langle FS | \sum_{\bar{k}\sigma} a^{+}_{\bar{k}\sigma} a_{\bar{k}\sigma} | FS \rangle \longrightarrow \frac{2S}{(2\pi)^2} \int d^2k\, \theta(k_F - |\bar{k}|) =$$

$$= \frac{2S}{(2\pi)^2} 2\pi \int_0^{k_F} dk\, k = \frac{2S}{2\pi} \frac{1}{2} k_F^2$$

$$n = \frac{N}{S} = \frac{k_F^2}{2\pi} \qquad \epsilon_F = \frac{\hbar^2}{2m} k_F^2 = \frac{\hbar^2}{2m} 2\pi n$$

$$U = \langle FS | \hat{H} | FS \rangle \longrightarrow \frac{2S}{(2\pi)^2} \int d^2k \frac{\hbar^2 k^2}{2m} \theta(k_F - |\bar{k}|) =$$

$$= \frac{2S}{(2\pi)^2} 2\pi \frac{\hbar^2}{2m} \int_0^{k_F} dk\, k^3 = \frac{\cancel{2}S}{2\pi} \frac{\hbar^2}{\cancel{2}m} \frac{1}{4} k_F^4 = \frac{S}{8\pi} \frac{\hbar^2}{m} k_F^4$$

$$U = \frac{2S}{\cancel{2\pi}} \frac{\hbar^2}{2m} \frac{1}{4} (2\pi)^{\cancel{2}} \left(\frac{N}{S}\right)^2 = \underbrace{2 \frac{\hbar^2}{2m} \frac{1}{4} (2\pi)}_{C_1} N^2 S^{-1}$$

$$p = -\left(\frac{\partial U}{\partial S}\right)_{\substack{T=0 \\ N}} = C_1 N^2 S^{-2} \qquad C_1 = \frac{\hbar^2}{2m} \pi$$

$$\kappa = -\frac{1}{V} \left(\frac{\partial V}{\partial p}\right)_{T,N} \xrightarrow[T=0]{d=2} -\frac{1}{S} \frac{1}{\left(\frac{\partial p}{\partial S}\right)_{T,N=0}} = -\frac{1}{S} \frac{1}{C_1 N^2 (-2S^{-3})} =$$

$$= \frac{1}{2C_1 N^2} S^2$$

$$\kappa = \frac{1}{2C_1} \frac{1}{n^2} = \frac{1}{2C_1} \left(\frac{2\pi}{\epsilon_F}\right)^2 \frac{(\hbar^2)^2}{(2m)^2}$$

$$\kappa = \frac{\cancel{2m}}{\cancel{\hbar^2} \cancel{2\pi}} \frac{(2\pi)^{\cancel{2}}}{\epsilon_F^2} \frac{\hbar^{\cancel{4}\,2}}{(2m)^{\cancel{2}}} = 2\pi \frac{\hbar^2}{2m} \frac{1}{\epsilon_F^2} = \pi \frac{\hbar^2}{m} \frac{1}{\epsilon_F^2}$$