Advanced Quantum Mechanics of Many-Body Systems
Homework 4
(08 Dec 2024)

Problem 1
Derive the evolution equation for the density matrix in interaction representation $\partial_t \hat{\rho}_t = \frac{i}{\hbar}\left[\hat{\rho}_t, \hat{V}_t\right]$ used in the lecture.

Problem 2
Show that

$$2\pi\theta(t-t') = i\int_{-\infty}^{+\infty} dx \frac{e^{-i(t-t')x}}{x+i0^+} \ .$$

Problem 3
Derive the equation of motion for the causal Green's function $G^c_{AB}(t,t')$.

Problem 4
Consider a system of fermions interacting via a two-body potential. Derive the relation

$$\left[a_{\vec{k},\sigma}, \mathcal{H}\right]_- = (\epsilon_{\vec{k}} - \mu)a_{\vec{k},\sigma} + \sum_{\vec{q},\vec{k}'}\sum_{\sigma'} V_{\vec{q}}\, a^\dagger_{\vec{k}'+\vec{q},\sigma'} a_{\vec{k}',\sigma'} a_{\vec{k}+\vec{q},\sigma}$$

used in the lecture.