Advanced Quantum Mechanics of Many-Body Systems
Homework 2

(25 Oct 2024)

Problem 1

Particles interact via a delta-function potential $V(r) = Ua^3\delta^{(3)}(r)$. Write down the second-quantized interaction operator in momentum space representation.

Problem 2

For a fermionic system evaluate:
(a) $a_4^\dagger a_5^\dagger a_2 |1110000\ldots\rangle$
(b) $a_1^\dagger a_5^\dagger a_2 |1110000\ldots\rangle$.

Problem 3

One-particle basis states of a Fermi system are labeled by $k \in \mathbb{N}$, and ϵ_k is known. Evaluate:
(a) $a_3^\dagger a_6^\dagger a_4 a_6^\dagger a_3 |11111000\ldots\rangle$,
(b) $\sum_k \epsilon_k a_k^\dagger a_k |11111000\ldots\rangle$.

Problem 4

Spin operator identity - Show that for the spin 1/2 operator the following identity holds

$$\hat{\mathbf{S}}_i^2 = \frac{3}{4}\hbar^2(\hat{n}_{i\uparrow} + \hat{n}_{i\downarrow} - 2\hat{n}_{i\uparrow}\hat{n}_{i\downarrow}),$$

where $\hat{n}_{i\sigma} = \hat{a}_{i\sigma}^\dagger \hat{a}_{i\sigma}$, and the subscript i marks the corresponding quantum state number.

Problem 5

Average of spin operator - Find $\langle \sum_i \hat{\mathbf{S}}_i^2 \rangle$ and $\langle \sum_i \hat{S}_i^z \rangle$ on a quantum state $|\Psi\rangle = \hat{a}_{1\uparrow}^\dagger \hat{a}_{1\downarrow}^\dagger \hat{a}_{2\uparrow}^\dagger \hat{a}_{3\uparrow}^\dagger \hat{a}_{3\downarrow}^\dagger \hat{a}_{4\downarrow}^\dagger |vac\rangle$.

Problem 6

A one-dimensional harmonic chain of lattice constant a consists of atoms of mass M and m ($M > m$) arranged alternately.
(a) Find and plot the phonon dispersion $\omega(k)$.
(b) What is the range of allowed excitation frequencies?
(c) Consider the special case $M = m$.