$$\Psi^\dagger_\sigma(\bar r) = \sum_\alpha \langle \alpha|\bar r\rangle a^\dagger_{\alpha\sigma} = \sum_\alpha \varphi^*_\alpha(\bar r)\, a^\dagger_{\alpha\sigma}$$

$$\Psi_\sigma(\bar r) = \sum_\alpha \langle \bar r|\alpha\rangle a_{\alpha\sigma} = \sum_\alpha \varphi_\alpha(\bar r)\, a_{\alpha\sigma}$$

$$\Psi^\dagger_\sigma(\bar r) = \sum_{\bar k} \frac{1}{\sqrt V} e^{-i\bar k\bar r}\, a^\dagger_{\bar k\sigma}$$

$$\Psi_\sigma(\bar r) = \sum_{\bar k} \frac{1}{\sqrt V} e^{i\bar k\bar r}\, a_{\bar k\sigma}$$

- $\sigma \neq \sigma'$

$$\langle FS|\Psi^\dagger_\sigma(\bar r)\Psi^\dagger_{\sigma'}(\bar r')\Psi_{\sigma'}(\bar r')\Psi_\sigma(\bar r)|FS\rangle =$$

$$= \frac{1}{V^2}\sum_{\bar k_1\bar k_2\bar k_3\bar k_4} e^{-i\bar k_4\bar r} e^{-i\bar k_3\bar r'} e^{i\bar k_2\bar r'} e^{i\bar k_1\bar r} \langle FS|a^\dagger_{\bar k_4\sigma} a^\dagger_{\bar k_3\sigma'} a_{\bar k_2\sigma'} a_{\bar k_1\sigma}|FS\rangle$$

[NONVANISHING CONTRIBUTIONS ONLY FROM $\bar k_4=\bar k_1, \bar k_3=\bar k_2$]

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$$= \frac{1}{V^2}\sum_{\bar k_1\bar k_2} \cancel{e^{-i\bar k_1\bar r}}\, \cancel{e^{-i\bar k_2\bar r'}}\, \cancel{e^{i\bar k_2\bar r'}}\, \cancel{e^{i\bar k_1\bar r}} \langle FS|a^\dagger_{\bar k_1\sigma} a^\dagger_{\bar k_2\sigma'} a_{\bar k_2\sigma'} a_{\bar k_1\sigma}|FS\rangle$$

$$= \frac{1}{V^2}\sum_{\bar k_1\bar k_2} \Theta(k_F-|\bar k_1|)\,\Theta(k_F-|\bar k_2|) = \frac{1}{V^2}\Big(\sum_{\bar k}\Theta(k_F-|\bar k|)\Big)^2 =$$

$$= \frac{N^2}{V^2}\cdot\left(\frac{1}{2}\right)^2 = \frac{1}{4}n^2$$

- $\sigma = \sigma'$

$$\langle FS|\Psi^\dagger_\sigma(\bar r)\Psi^\dagger_\sigma(\bar r')\Psi_\sigma(\bar r')\Psi_\sigma(\bar r)|FS\rangle =$$

$$= \frac{1}{V^2}\sum_{\bar k_1\cdots\bar k_4} e^{-i\bar k_4\bar r - i\bar k_3\bar r' + i\bar k_2\bar r' + i\bar k_1\bar r}\langle FS|a^\dagger_{\bar k_4\sigma} a^\dagger_{\bar k_3\sigma} a_{\bar k_2\sigma} a_{\bar k_1\sigma}|FS\rangle =$$

$$= \frac{1}{V^2} \sum_{\bar{k}_1 \bar{k}_2} \Big[\langle FS | a^{\dagger}_{\bar{k}_1\sigma} a^{\dagger}_{\bar{k}_2\sigma} a_{\bar{k}_2\sigma} a_{\bar{k}_1\sigma} | FS \rangle + e^{i(\bar{k}_1 - \bar{k}_2)(\bar{r}-\bar{r}')} \langle FS | a^{\dagger}_{\bar{k}_2\sigma} a^{\dagger}_{\bar{k}_1\sigma} a_{\bar{k}_2\sigma} a_{\bar{k}_1\sigma} | FS \rangle \Big] =$$

[NONVANISHING CONTRIBUTIONS ONLY FROM $(\bar{k}_4 = \bar{k}_1, \bar{k}_3 = \bar{k}_2)$ AND $(\bar{k}_4 = \bar{k}_2, \bar{k}_3 = \bar{k}_1)$]

$$= \frac{1}{V^2} \sum_{\bar{k}_1 \bar{k}_2} \Big[\langle FS | a^{\dagger}_{\bar{k}_1\sigma} \big(a_{\bar{k}_1\sigma} a^{\dagger}_{\bar{k}_2\sigma} a_{\bar{k}_2\sigma} - \cancel{\delta_{\bar{k}_1 \bar{k}_2} a_{\bar{k}_2\sigma}} \big) | FS \rangle$$

$$- e^{i(\bar{k}_1 - \bar{k}_2)(\bar{r}-\bar{r}')} \langle FS | a^{\dagger}_{\bar{k}_2\sigma} \big(a_{\bar{k}_2\sigma} a^{\dagger}_{k_1\sigma} a_{\bar{k}_1\sigma} - \cancel{\delta_{\bar{k}_1 \bar{k}_2} a_{k_1\sigma}} \big) | FS \rangle$$

$$\left\{ \vec{x} := (\bar{r} - \bar{r}') \right\} =$$

$$= \frac{1}{V^2} \left(\frac{1}{2} N \right)^2 - \frac{1}{V^2} \sum_{\bar{k}_1 \bar{k}_2} e^{i(\bar{k}_1 - \bar{k}_2)\bar{x}} \, \theta(k_F - |\bar{k}_1|) \, \theta(k_F - |\bar{k}_2|) =$$

$$= \frac{1}{4} n^2 - \int \frac{d^3 k_1}{(2\pi)^3} e^{i \bar{k}_1 \bar{x}} \theta(k_F - |\bar{k}_1|) \int \frac{d^3 k_2}{(2\pi)^3} e^{-i \bar{k}_2 \bar{x}} \theta(k_F - |\bar{k}_2|)$$

$$\int \frac{d^3 k}{(2\pi)^3} e^{i\bar{k}\bar{x}} \theta(k_F - |\bar{k}_1|) = \frac{2\pi}{(2\pi)^3} \int_0^{k_F} dk\, k^2 \int_0^{\pi} d\theta \sin\theta \, e^{ikx\cos\theta} =$$

$$= \{ t = \cos\theta \} = \frac{1}{(2\pi)^2} \int_0^{k_F} dk\, k^2 \int_{-1}^{1} dt\, e^{ikxt} = \frac{1}{(2\pi)^2} \int_0^{k_F} dk\, k^2 \frac{e^{ikx} - e^{-ikx}}{ikx}$$

$$= \frac{2}{(2\pi)^2} \frac{1}{x} \int_0^{k_F} dk\, k \sin(kx) = \left\{ \begin{matrix} u = k & u' = 1 \\ v' = \sin kx & v = -\frac{1}{x}\cos kx \end{matrix} \right\} =$$

$$= \frac{2}{(2\pi)^2} \frac{1}{x} \left[-\frac{k}{x} \cos kx \Big|_0^{k_F} + \frac{1}{x} \int_0^{k_F} dk \cos kx \right] =$$

$$= \frac{2}{(2\pi)^2} \frac{1}{x} \left[-\frac{k_F}{x} \cos k_F x + \frac{1}{x^2} \sin k_F x \right] = \frac{1}{2\pi^2} \frac{\sin k_F x - k_F x \cos k_F x}{x^3}$$

THE CORR. FUNCTION IS:

$$\frac{1}{4}n^2 - \frac{1}{4\pi^4}\left(\frac{\sin k_F x - k_F x \cos k_F x}{x^3}\right)^2 = \circledast$$

(DIFFERENCE) $\longleftrightarrow$ (PAULI PRINCIPLE)

SMALL X EXPANSION:

$$\sin k_F x - k_F x \cos k_F x = k_F x - \frac{1}{6}k_F^3 x^3 + \frac{1}{5!}k_F^5 x^5 + - k_F x\left(1 - \frac{1}{2}k_F^2 x^2 + \frac{1}{4!}k_F^4 x^4\right) + \dots =$$

$$= \frac{1}{3}(k_F x)^3 + \left(\frac{1}{5!} - \frac{1}{4!}\right)(k_F x)^5 + \dots$$

$$\circledast = \frac{1}{4}n^2 - \frac{1}{4\pi^4}\left[\frac{1}{3}k_F^3 + \frac{1}{24}\left(\frac{1}{5} - 1\right)k_F^5 x^2 + \dots\right]^2 =$$

$$\left\{ n = \frac{2}{(2\pi)^3}\int_0^{k_F} dk\, k^2 \cdot 4\pi = \frac{2}{(2\pi)^3}4\pi\,\frac{1}{3}k_F^3 = \frac{1}{3\pi^2}k_F^3 \quad k_F^3 = 3\pi^2 n \right\}$$

$$= \frac{1}{4}n^2 - \frac{1}{4\pi^4}\frac{1}{9}k_F^6 + \frac{1}{4\pi^4}\cdot 2\,\frac{1}{3}k_F^3\,\frac{1}{24}\,\frac{4}{5}k_F^5 x^2 + \dots =$$

$$= \cancel{\frac{1}{4}n^2} - \cancel{\frac{1}{4\pi^4}\frac{1}{9}9\pi^4 n^2} + \frac{1}{180\pi^4}k_F^8 x^2 + O(x^4) =$$

$$\simeq \frac{1}{180\pi^4}9\pi^4 n^2 (k_F x)^2 = \frac{1}{20}n^2(k_F x)^2 =$$

$$= \frac{1}{20}n^2 x^2 (3\pi^2 n)^{\frac{2}{3}} = \underline{\frac{(3\pi^2)^{\frac{2}{3}}}{20}x^2 n^{\frac{8}{3}}}$$