Primordial black holes and induced gravitational waves

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Faculty of Physics, University of Warsaw, Poland,12/12/2024

SSM
Scuola Superiore Meridionale

Introduction

PBH formation mechanisms

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4. Collapse through bubble collisions during a phase transition [M. Crawford and D. Schramm - 1982, S. Hawking et al. - 1982, H Kodoma et al. - 1982, I. Moss - 1994]

PBHs from collapse of primordial inhomogeneities

- Primordial Black Holes (PBHs) form in the early universe, out of the **collapse of enhanced energy density perturbations** upon horizon reentry of the typical size of the collapsing overdensity region. This happens when $\delta > \delta_{\rm c}(w \equiv p/\rho)$ [Carr - 1975].

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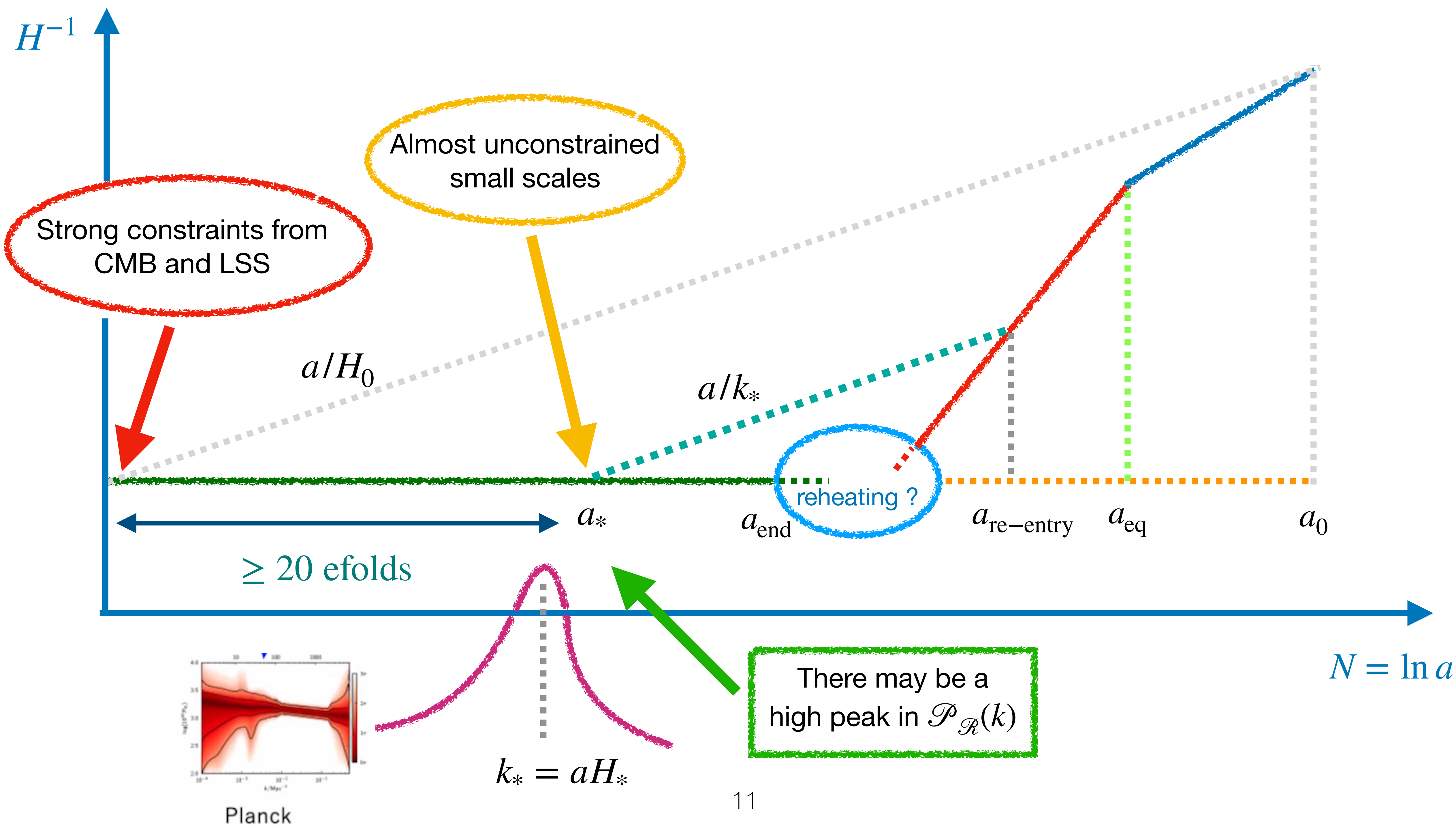

Why we study PBHs?

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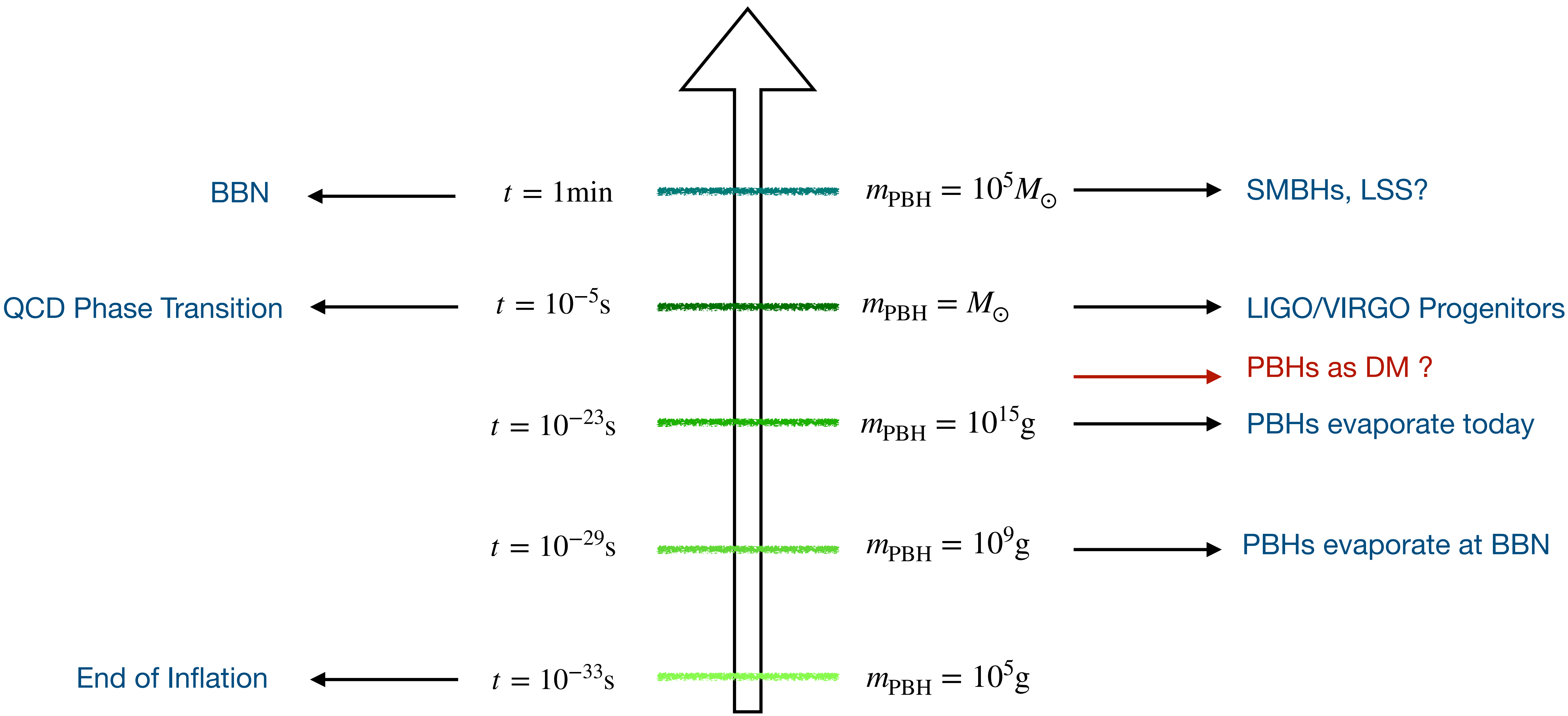

See for reviews in [Carr et al.- 2020, Sasaki et al - 2018, Clesse et al. - 2017]

Constraints on PBH abundances

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  e) **gravitational wave (GW) production** associated to PBHs

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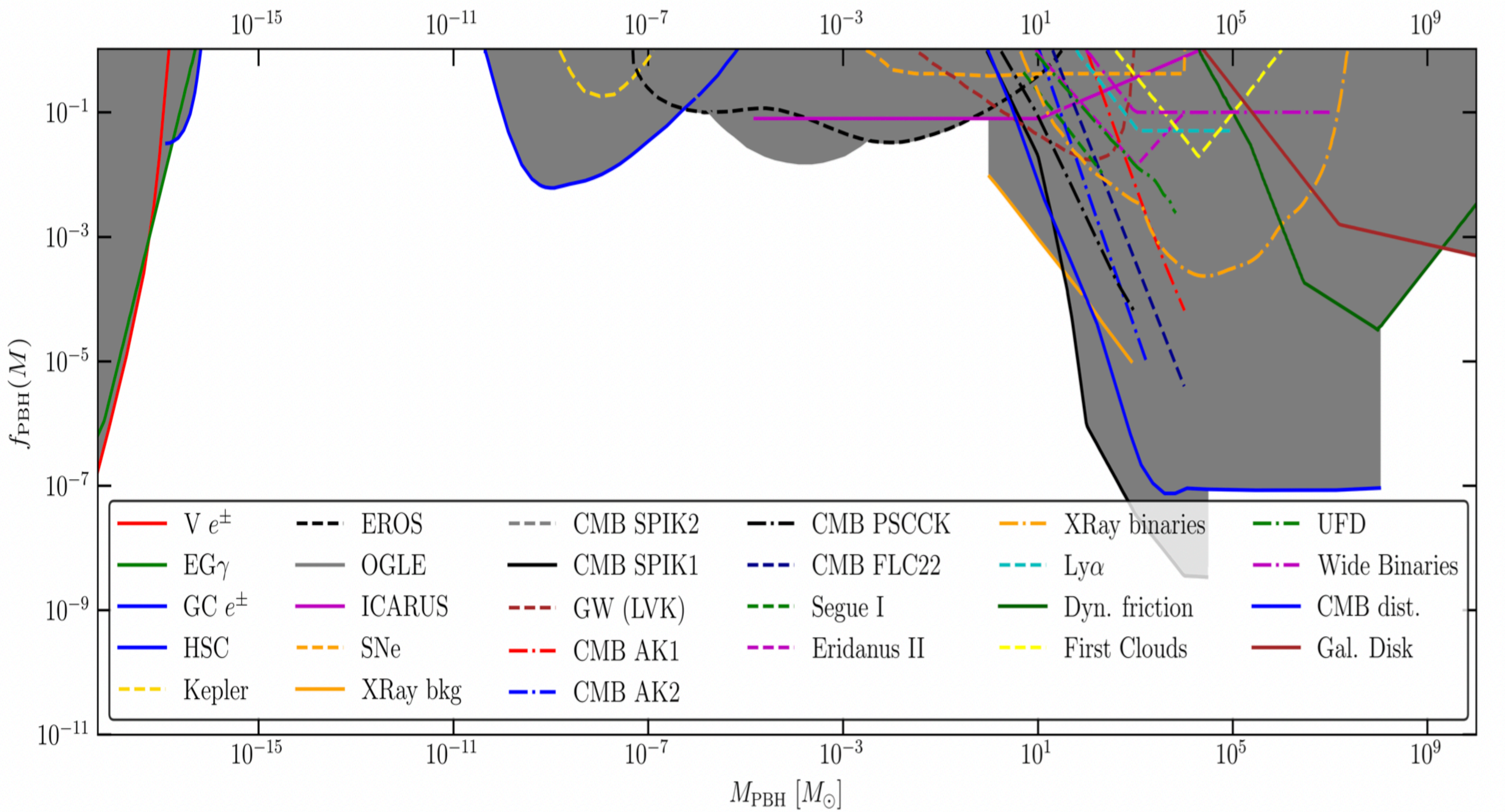
$$f_{\rm PBH}(M) \equiv \frac{\Omega_{\rm PBH,0}}{\Omega_{\rm DM,0}}$$



Open Issues in PBH Physics

- PBH formation process [e.g. non spherical collapse, non standard w, shape of the collapsing overdensity]
- Modelling/Computation of PBH abundances (Peak theory vs Press-Schechter formalism)
- Clustering properties of PBHs
- Merger rates of PBHs
- etc.

PBHs and GWs

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- 1) **Primordial induced GWs** generated through second order gravitational effects: $\mathscr{L}^{(3)}_{\Phi,h} \ni h\Phi^2$, [Bugaev - 2009, Kohri & Terada - 2018].

  $h \rightarrow$ GWs $\Phi^2 \rightarrow$ PBHs

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- 4) **GWs induced** at second order **by PBH number density fluctuations** [Papanikolaou et al. - 2020], abundantly produced during a PBH-dominated era.

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- 1) **Primordial scalar-induced GWs (SIGWs)** generated through second order gravitational effects: $\mathscr{L}^{(3)}_{\Phi,h} \ni h\Phi^2$, [Bugaev - 2009, Kohri & Terada - 2018].

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Scalar Induced Gravitational Waves

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$$\mathscr{L}^{(3)}_{\Phi,h} \ni h\Phi^2, \qquad \mathrm{d}s^2 = a^2(\eta)\left\{-(1+2\Phi)\mathrm{d}\eta^2 + \left[(1-2\Phi)\delta_{ij} + \frac{h_{ij}}{2}\right]\mathrm{d}x^i\mathrm{d}x^j\right\}.$$

Scalar Induced Gravitational Waves

- The equation of motion for the Fourier modes, $h_{\vec{k}}$, read as:

$$\mathscr{L}^{(3)}_{\Phi,h} \ni h\Phi^2 \Rightarrow h^{s,''}_{\vec{k}} + 2\mathscr{H} h^{s,'}_{\vec{k}} + k^2 h^{s}_{\vec{k}} = 4S^{s}_{\vec{k}}\,.$$

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$$S^s_{\vec{k}} = \int \frac{\mathrm{d}^3\vec{q}}{(2\pi)^{3/2}} e^s_{ij}(\vec{k}) q_i q_j \left[2\Phi_{\vec{q}}\Phi_{\vec{k}-\vec{q}} + \frac{4}{3(1+w)}(\mathscr{H}^{-1}\Phi'_{\vec{q}} + \Phi_{\vec{q}})(\mathscr{H}^{-1}\Phi'_{\vec{k}-\vec{q}} + \Phi_{\vec{k}-\vec{q}})\right]\,.$$

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$$\Downarrow$$

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Focusing on the ultra-light PBHs

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$$\Omega_{\rm PBH} = \rho_{\rm PBH}/\rho_{\rm tot} \propto a^{-3}/a^{-4} \propto a$$

PBH-dominated era phenomenology

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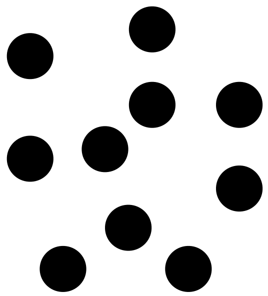
- **GWs induced by PBH number density fluctuations can interpret** in a very good agreement **the recently released PTA GW data** [Lewicki et al. - 2023, Basilakos et al. - 2023]

Gravitational waves from PBH number density fluctuations

[**T. Papanikolaou**, V. Vennin, D. Langlois, **JCAP 03 (2021) 053**]

The PBH Matter Field



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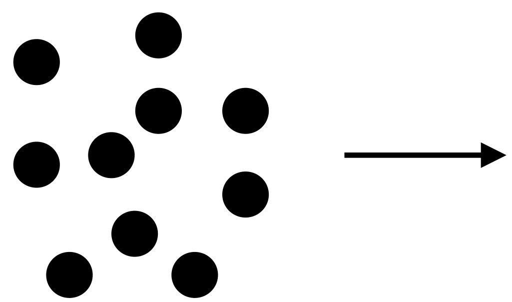

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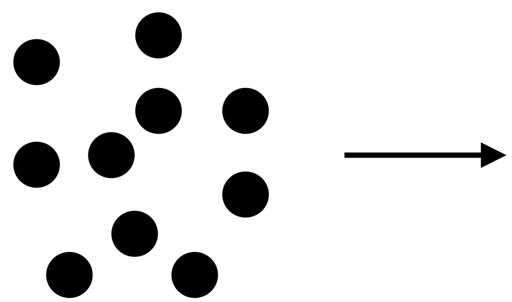

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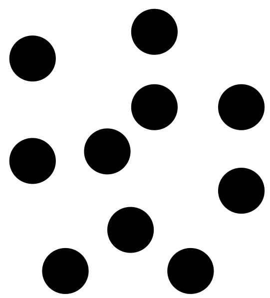

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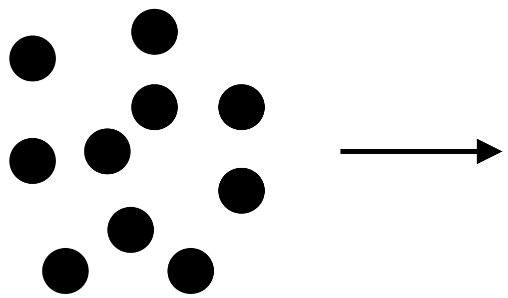

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$$S_{\rm f} \simeq \delta_{\rm PBH} \equiv \frac{\delta\rho_{\rm PBH}}{\rho_{\rm PBH}} \simeq \frac{\delta n_{\rm PBH}}{n_{\rm PBH}}$$

[Isocurvature perturbation]

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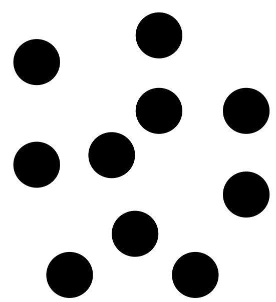

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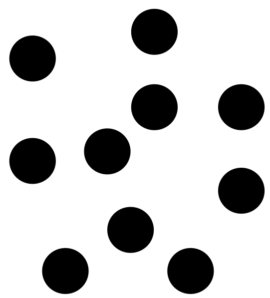
$$P_{\delta_{\rm PBH}}(k) \equiv \langle | \delta_k^{\rm PBH} |^2 \rangle = \frac{4\pi}{3}\left(\frac{\bar{r}}{a}\right)^3 = \frac{4\pi}{3k_{\rm UV}^3}, \quad \text{where} \quad k < k_{\rm UV} = \frac{a}{\bar{r}}$$

This **isocurvature perturbation,** $\delta_{\rm PBH}$ generated during the RD era **will convert** during the PBHD era **to a curvature perturbation** $\zeta_{\rm PBH}$, associated to a PBH gravitational potential Φ.

The PBH Matter Field



→ { **Poisson Statistics** [Desjacques & Riotto - 2018, Ali-Haimoud - 2018]

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↓

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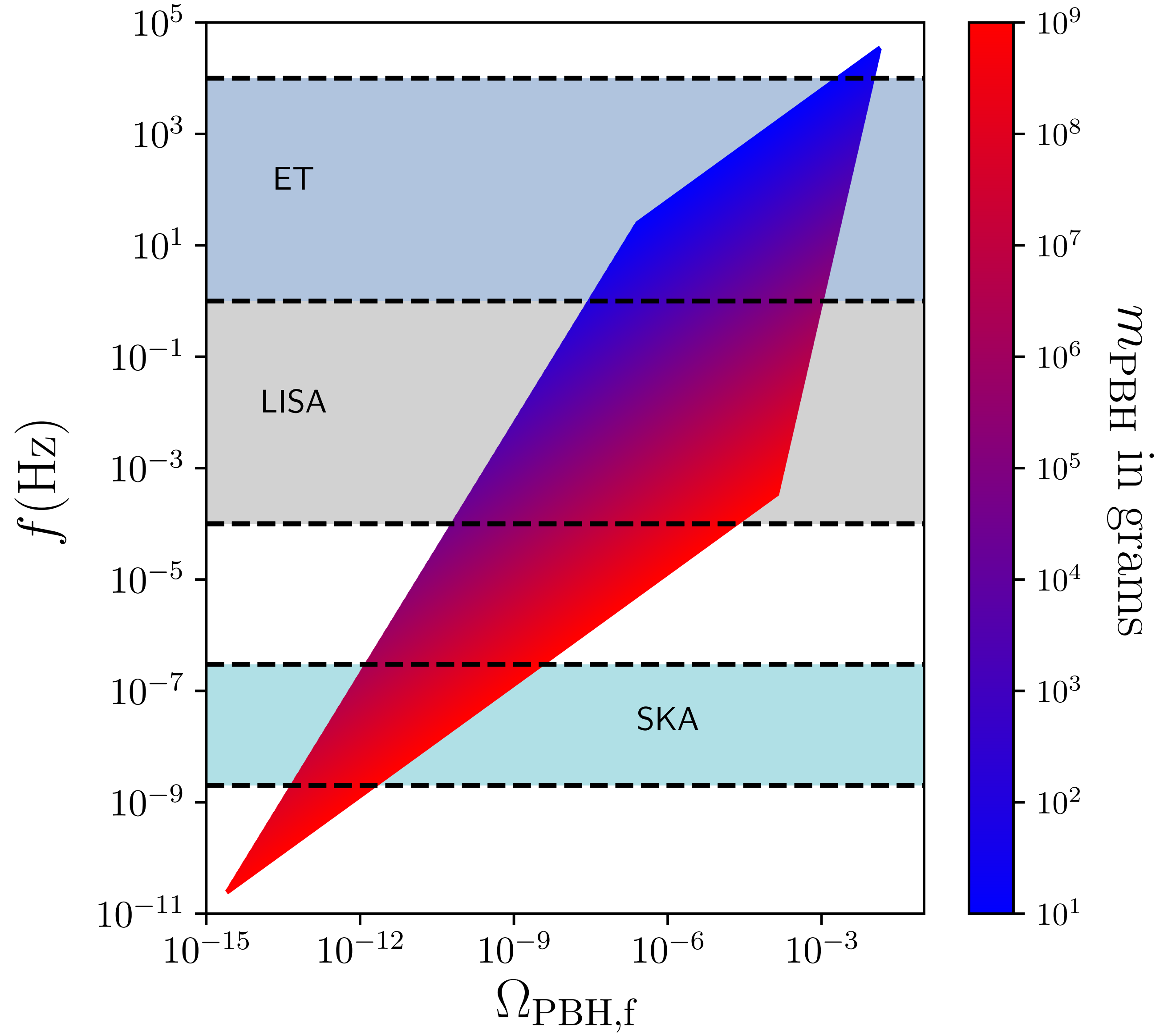
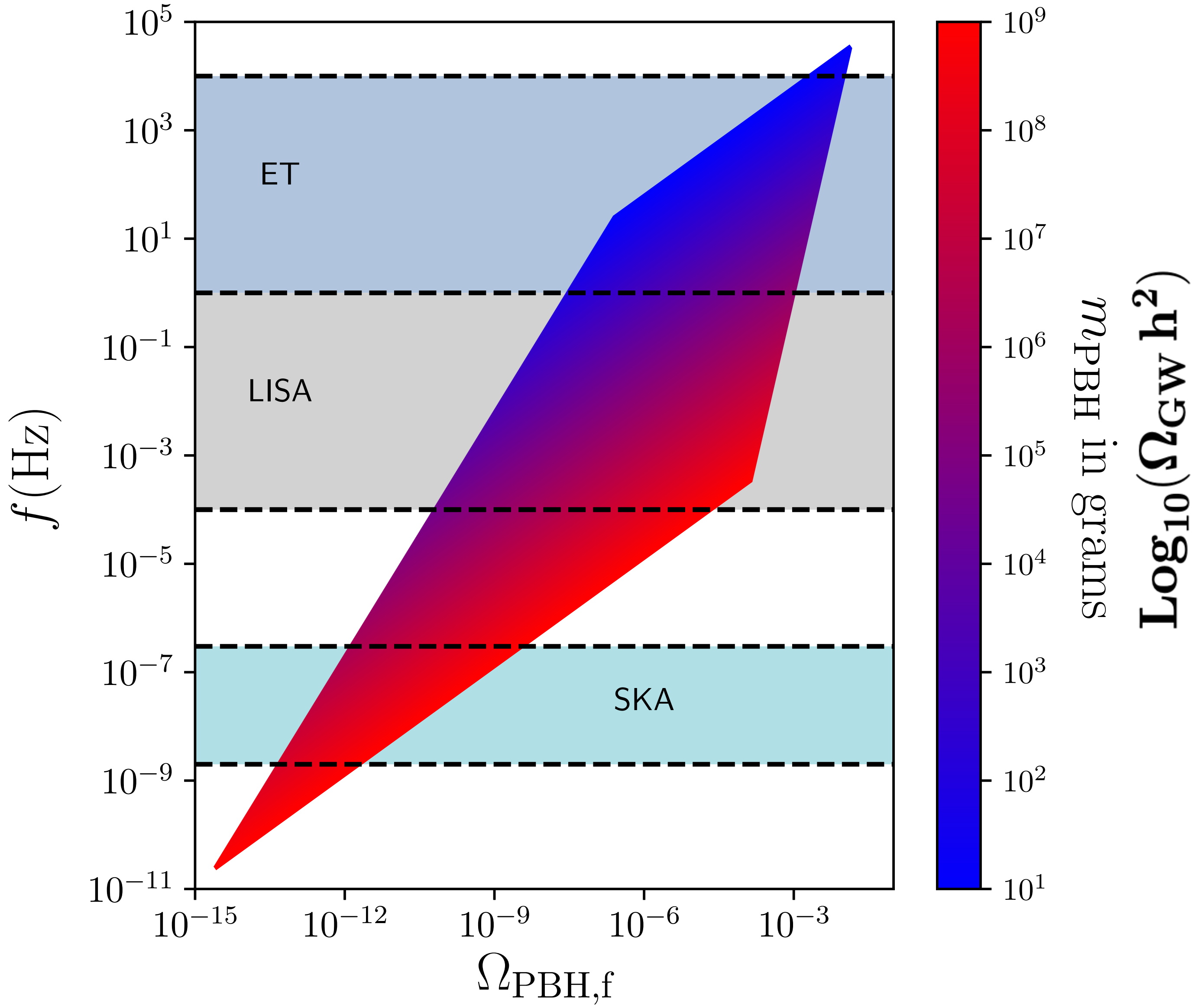
$$\mathscr{P}_\Phi(k) = S_\Phi^2(k)\frac{2}{3\pi}\left(\frac{k}{k_{\rm UV}}\right)^3\left(5+\frac{4}{9}\frac{k^2}{k_{\rm d}^2}\right)^{-2}, \text{ with } S_\Phi(k) \equiv \left(\sqrt{\frac{2}{3}}\frac{k}{k_{\rm evap}}\right)^{-1/3}$$

GW Detectability

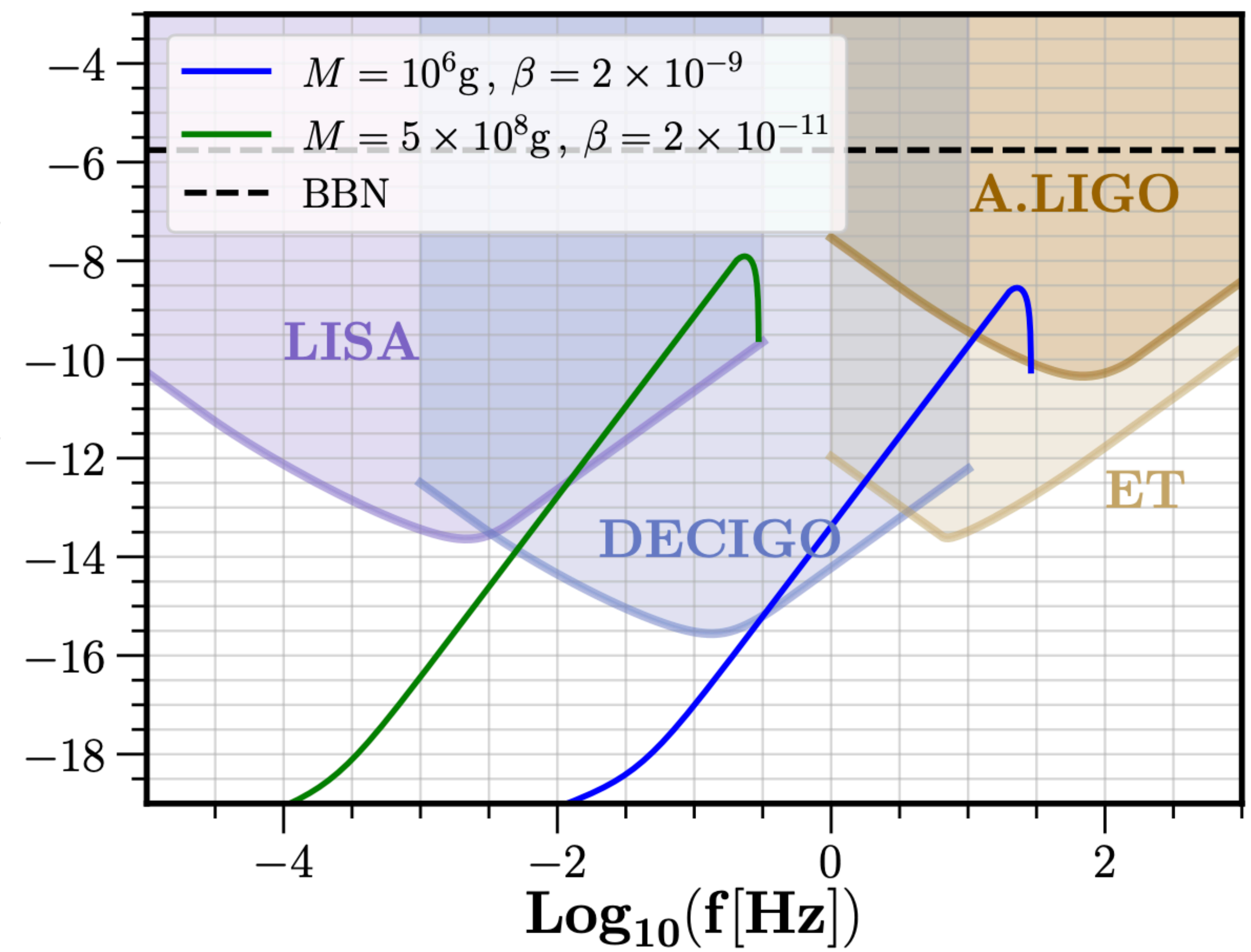


[Papanikolaou et al. - 2020]

GW Detectability



[Papanikolaou et al. - 2020]



[Domenech et al. - 2020]

- **Peaked GW signal at around** k_{UV} due **to the suddenness of the transition to the IRD era** for **PBH monochromatic mass distributions** [Domenech et al. - 2020].

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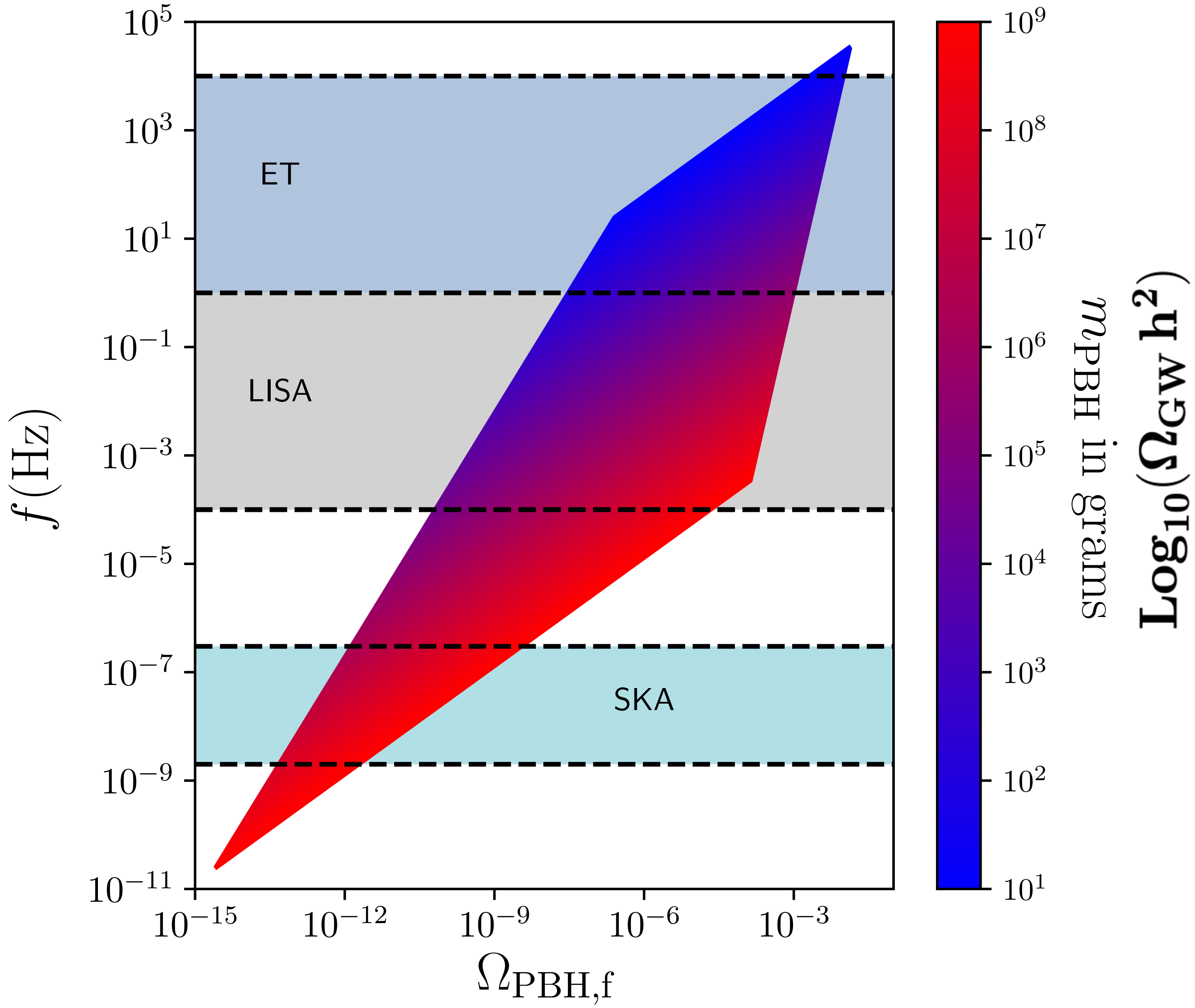

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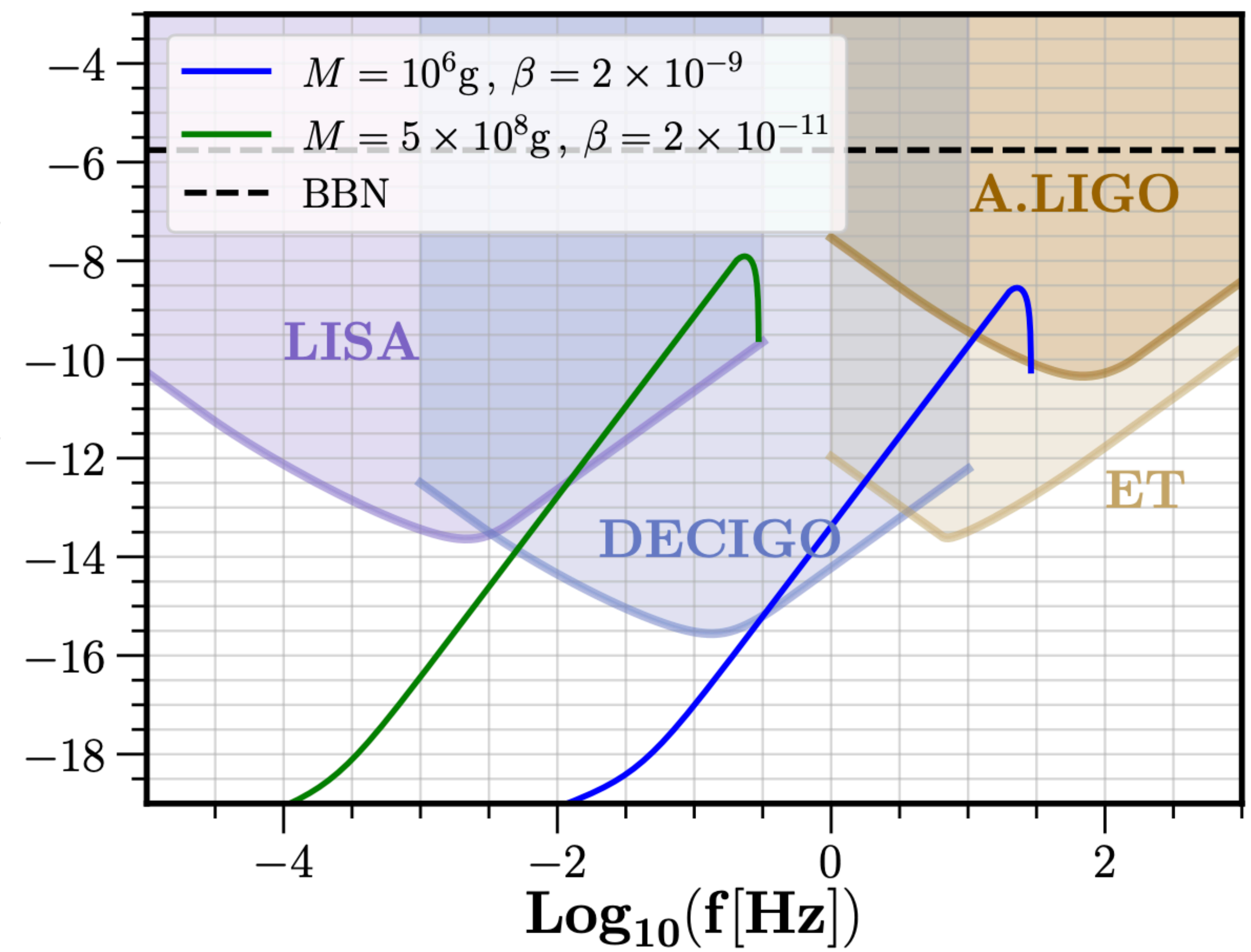

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- **Peaked GW signal at around** k_{UV} due **to the suddenness of the transition to the IRD era** for **PBH monochromatic mass distributions** [Domenech et al. - 2020].

- During the transition, Φ' **goes very quickly from** $\Phi' = 0$ (since in a MD era Φ = constant) **to** $\Phi' \neq 0$. This entails **a resonantly enhanced production of GWs sourced mainly by the** $\mathscr{H}^{-2}\Phi'^2$ **term in** $S_{\vec{k}}$.

Gravitational waves from PBH number density fluctuations: The effect of an extended PBH mass distribution

[T. Papanikolaou, JCAP 10 (2022) 089]

The PBH mass function and the PBH abundance

$$\mathscr{P}_\zeta(k) = A_\zeta \left(k/k_0\right)^{n_\mathrm{s}(k)-1},$$

$$\text{with } n_\mathrm{s}(k) = n_{\mathrm{s},0} + \frac{\alpha_\mathrm{s}}{2!} \ln\left(\frac{k}{k_0}\right)$$

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$$\beta(M) \equiv \frac{1}{\rho_\mathrm{tot}}\frac{\mathrm{d}\rho_\mathrm{PBH}}{\mathrm{d}\ln M} \text{ within peak theory}$$

$$\Omega_\mathrm{PBH}(t) = \int_{M_\mathrm{min}}^{M_\mathrm{max}} \bar{\beta}\,(M,t)\left\{1 - \frac{t - t_\mathrm{ini}}{\Delta t_\mathrm{evap}(M_\mathrm{f})}\right\}^{1/3} \mathrm{d}\ln M$$

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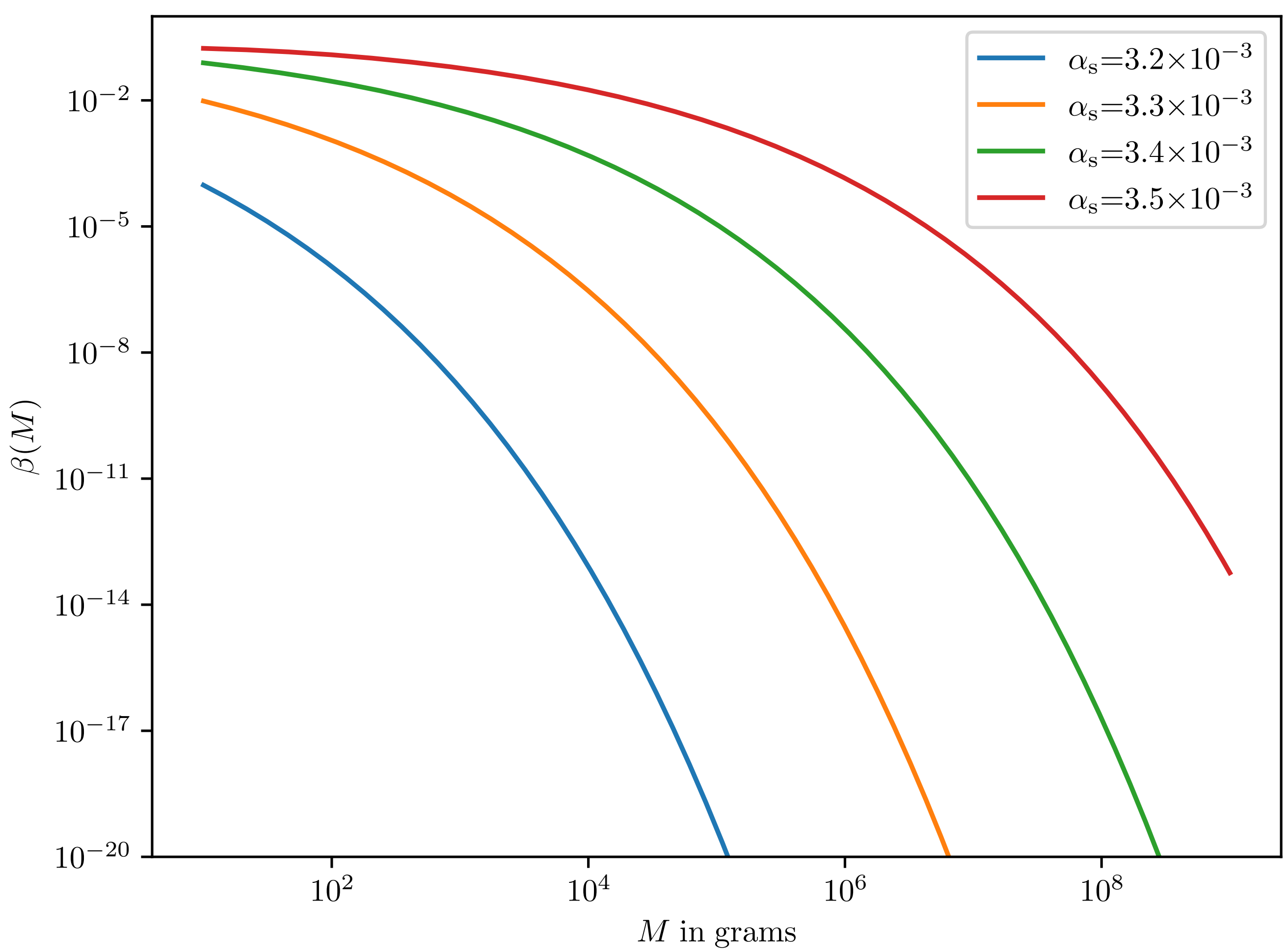

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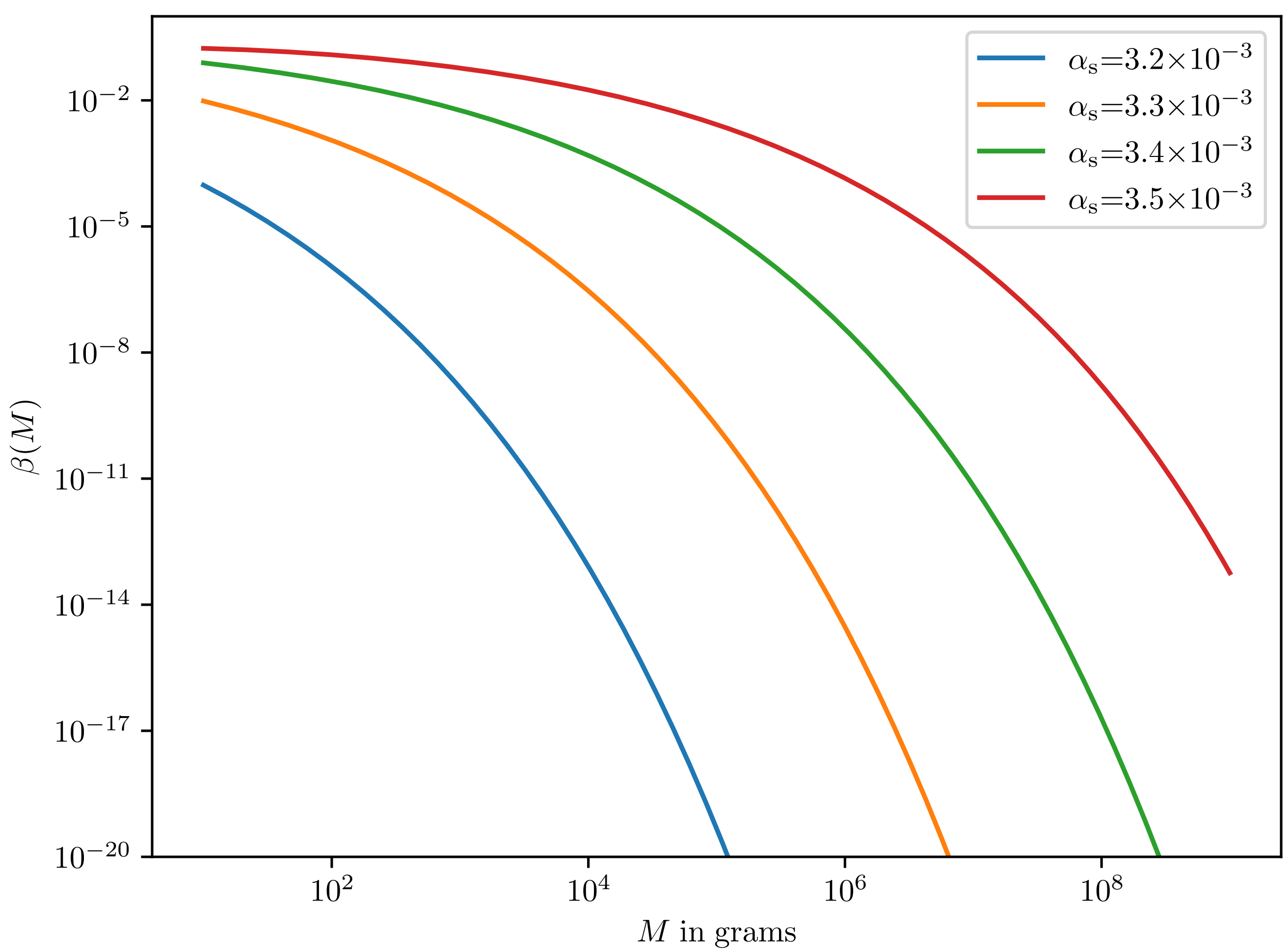

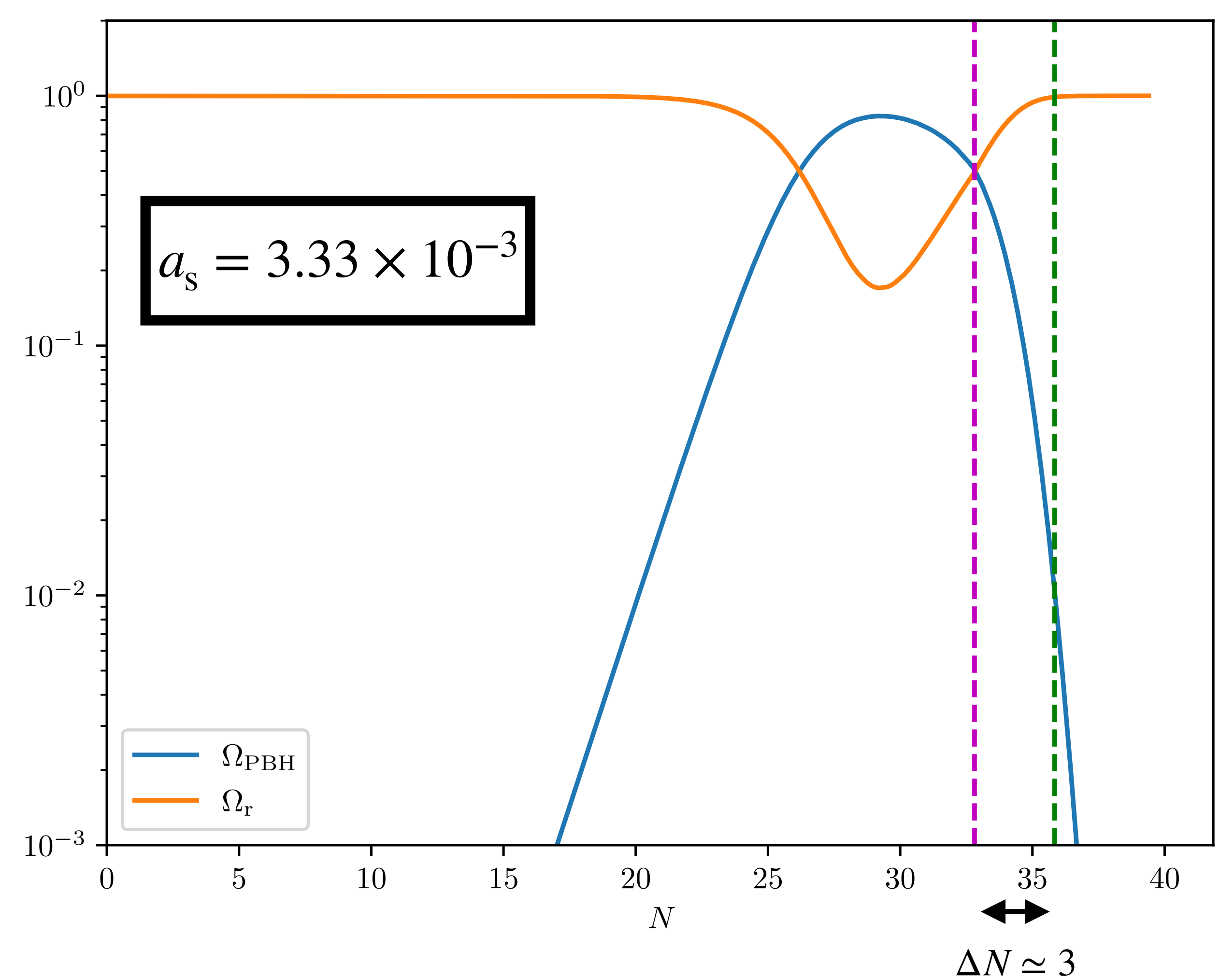

Gradual Transition

Evolving the PBH gravitational potential

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Evolving the PBH gravitational potential

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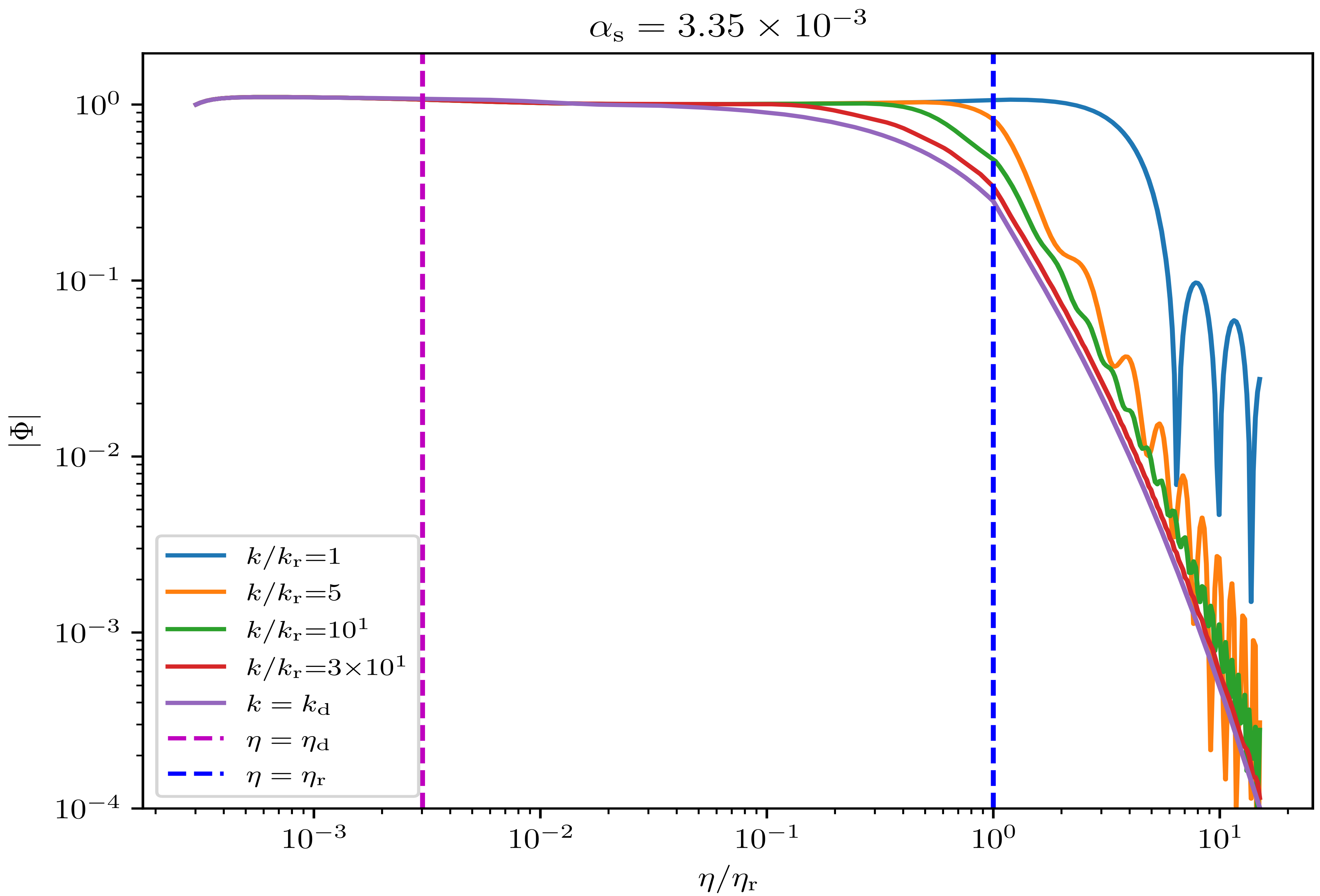
$$\delta'_{\rm PBH} = -\theta_{\rm PBH} + 3\Phi' - a\Gamma\Phi$$
$$\theta'_{\rm PBH} = -\mathscr{H}\theta_{\rm PBH} + k^2\Phi$$
$$\delta'_{\rm r} = -\frac{4}{3}(\theta_{\rm r} - 3\Phi') + a\Gamma\frac{\rho_{\rm PBH}}{\rho_{\rm r}}(\delta_{\rm PBH} - \delta_{\rm r} + \Phi)$$
$$\theta'_{\rm r} = \frac{k^2}{4}\delta_{\rm r} + k^2\Phi - a\Gamma\frac{3\rho_{\rm PBH}}{4\rho_{\rm r}}\left(\frac{4}{3}\theta_{\rm r} - \theta_{\rm PBH}\right)$$
$$\Phi' = -\frac{k^2\Phi + 3\mathscr{H}^2\Phi + \frac{3}{2}\mathscr{H}^2\left(\frac{\rho_{\rm PBH}}{\rho_{\rm tot}}\delta_{\rm PBH} + \frac{\rho_{\rm r}}{\rho_{\rm tot}}\delta_{\rm r}\right)}{3\mathscr{H}}$$

$$\delta_\alpha \equiv (\rho_\alpha - \rho_{\rm tot})/\rho_{\rm tot},$$
$$\theta \equiv \partial v_i/\partial x_i$$
$$' \equiv \frac{\rm d}{{\rm d}\eta}, \text{ with } {\rm d}\eta \equiv {\rm d}t/a$$

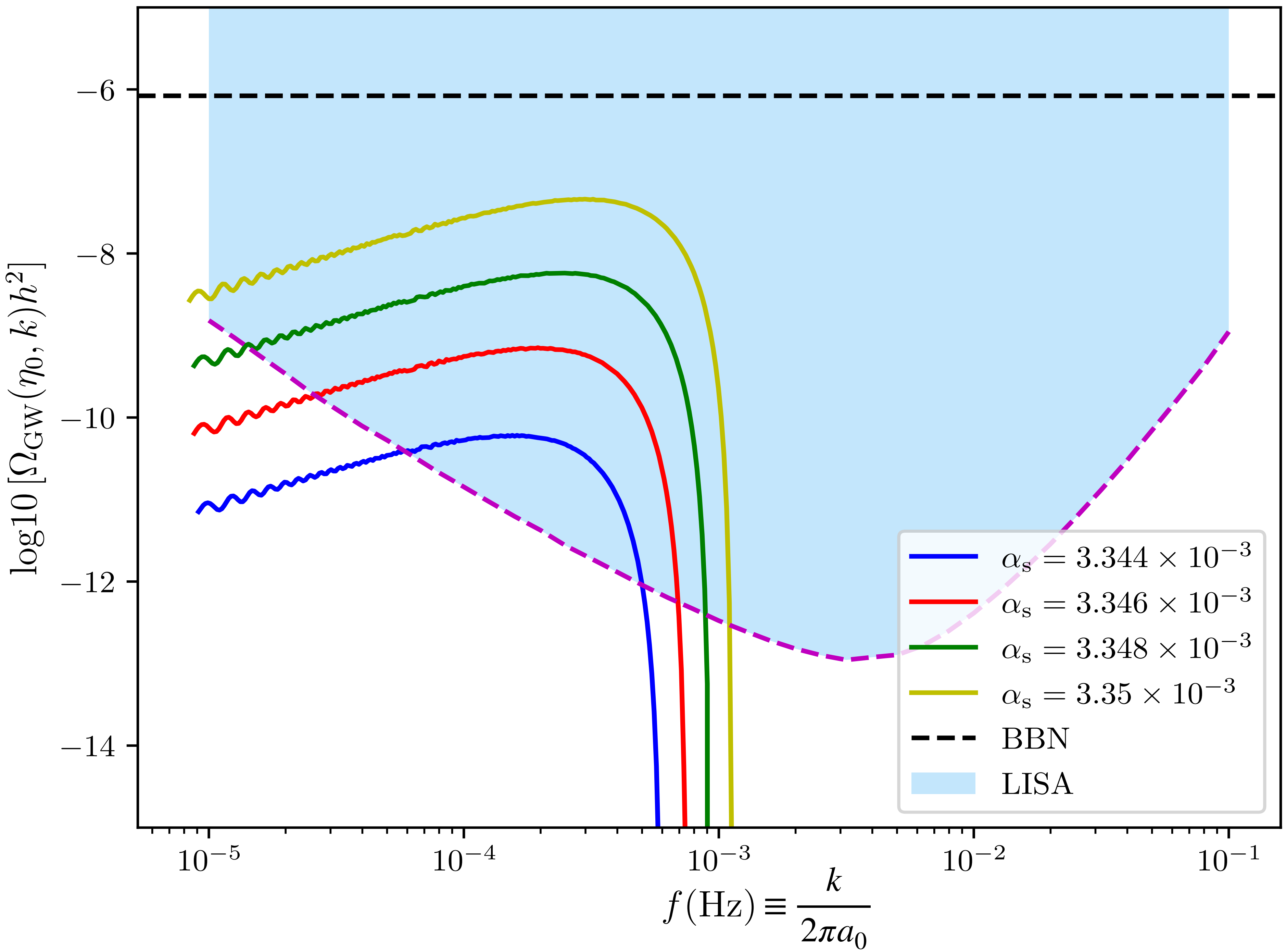
$$\langle\Gamma\rangle(t) = \frac{\int_{t_{\rm evap,min}}^{t_{\rm evap,max}} \beta(t_{\rm evap})\Gamma_M(t_{\rm evap}, t){\rm d}\ln t_{\rm evap}}{\int_{t_{\rm evap,min}}^{t_{\rm evap,max}} \beta(t_{\rm evap}){\rm d}\ln t_{\rm evap}}, \text{ with } \Gamma_M(t_{\rm evap}, t) \equiv -\frac{1}{M}\frac{{\rm d}M}{{\rm d}t} = \frac{1}{3(t_{\rm evap} - t)}$$

Adiabatic initial conditions : $\delta_{\rm PBH,ini} = -2\Phi_{\rm ini}, \quad \delta_{\rm r,ini} = \frac{4}{3}\delta_{\rm PBH,ini}, \quad \theta_{\rm PBH,ini} = \theta_{\rm r,ini} = 0, \quad \Phi_{\rm ini} = 1$

The gravitational potential Φ



The GW spectrum



$$\left.\begin{array}{r} \eta_r > \eta_d \Rightarrow \alpha_s > 3.316 \times 10^{-3} \\ \Omega_{\rm GW,BBN} < 0.05 \Rightarrow \alpha_s < 3.355 \times 10^{-3} \end{array}\right\} \Rightarrow \boxed{3.316 \times 10^{-3} < \alpha_s < 3.355 \times 10^{-3}}$$

Gravitational waves from PBH number density fluctuations: The effect of primordial non-Gaussianities

[**T. Papanikolaou**, X.C He. X.H. Ma, Y.F. Cai, E.N. Saridakis, M. Sasaki,
Phys. Lett. B 857 (2024) 138997]

Primordial non-Gaussianities of local type

$$
\begin{aligned}
\langle \mathcal{R}(\boldsymbol{k_1})\mathcal{R}(\boldsymbol{k_2})\rangle &\equiv (2\pi)^3\delta^{(3)}(\boldsymbol{k_1}+\boldsymbol{k_2})P_{\mathcal{R}}(k) \\
\langle \mathcal{R}(\boldsymbol{k_1})\mathcal{R}(\boldsymbol{k_2})\mathcal{R}(\boldsymbol{k_3})\rangle &\equiv (2\pi)^3\delta^{(3)}(\boldsymbol{k_1}+\boldsymbol{k_2}+\boldsymbol{k_3}) \\
&\quad \times \frac{6}{5} f_{\mathrm{NL}} \left[P_{\mathcal{R}}(k_1)P_{\mathcal{R}}(k_2) + 2\ \mathrm{perms}\right] \\
\langle \mathcal{R}(\boldsymbol{k_1})\mathcal{R}(\boldsymbol{k_2})\mathcal{R}(\boldsymbol{k_3})\mathcal{R}(\boldsymbol{k_4})\rangle &\equiv (2\pi)^3\delta^{(3)}(\boldsymbol{k_1}+\boldsymbol{k_2}+\boldsymbol{k_3}+\boldsymbol{k_4}) \\
&\quad \times \left\{ \frac{54}{25} g_{\mathrm{NL}} \left[P_{\mathcal{R}}(k_1)P_{\mathcal{R}}(k_2)P_{\mathcal{R}}(k_3) + 3\ \mathrm{perms}\right] \right. \\
&\qquad \left. + \tau_{\mathrm{NL}} \left[P_{\mathcal{R}}(k_1)P_{\mathcal{R}}(k_2)P_{\mathcal{R}}(|\boldsymbol{k_1}+\boldsymbol{k_3}|) + 11\ \mathrm{perms}\right] \right\}
\end{aligned}
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$$\langle \mathcal{R}(\boldsymbol{k_1})\mathcal{R}(\boldsymbol{k_2})\rangle \equiv (2\pi)^3\delta^{(3)}(\boldsymbol{k_1}+\boldsymbol{k_2})P_{\mathcal{R}}(k)$$

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[Path integral formalism for n-point correlation functions (galaxy halo bias)]

[S. Matarrese et al. - 1986, S. Matarresse and L. Verde - 2008]

$$\xi_{\rm PBH}(\mathbf{x}_1,\mathbf{x}_2) \equiv \langle \delta_{\rm PBH}(\mathbf{x}_1)\delta_{\rm PBH}(\mathbf{x}_2)\rangle = \int \mathscr{P}_{\rm PBH}(k)e^{\mathbf{k}\cdot(\mathbf{x}_1-\mathbf{x}_2)}\mathrm{d}\ln k$$

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$kR \ll 1$

$R \sim 1/k_{\rm f}$

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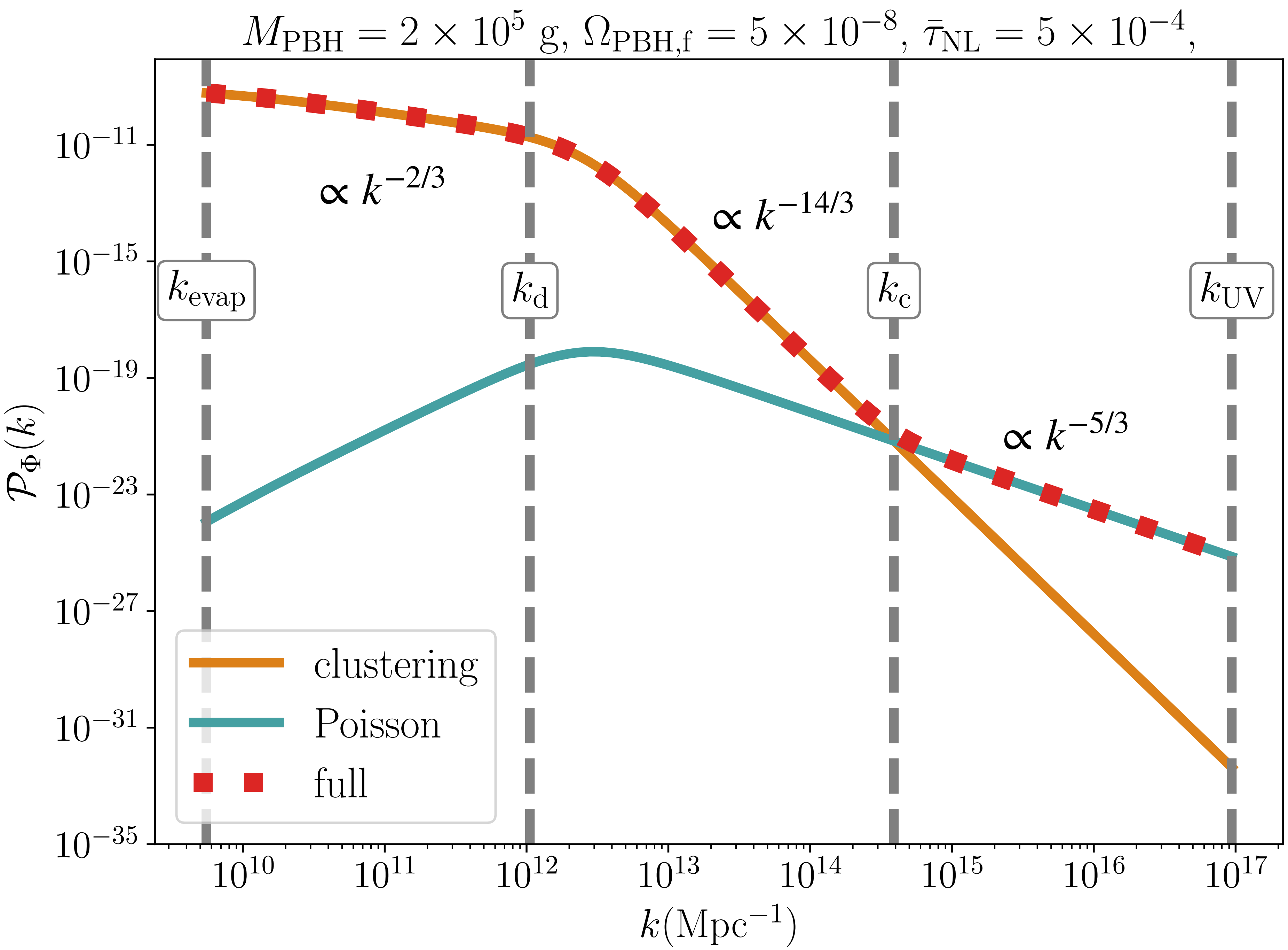
The non-Gaussian PBH matter power spectrum

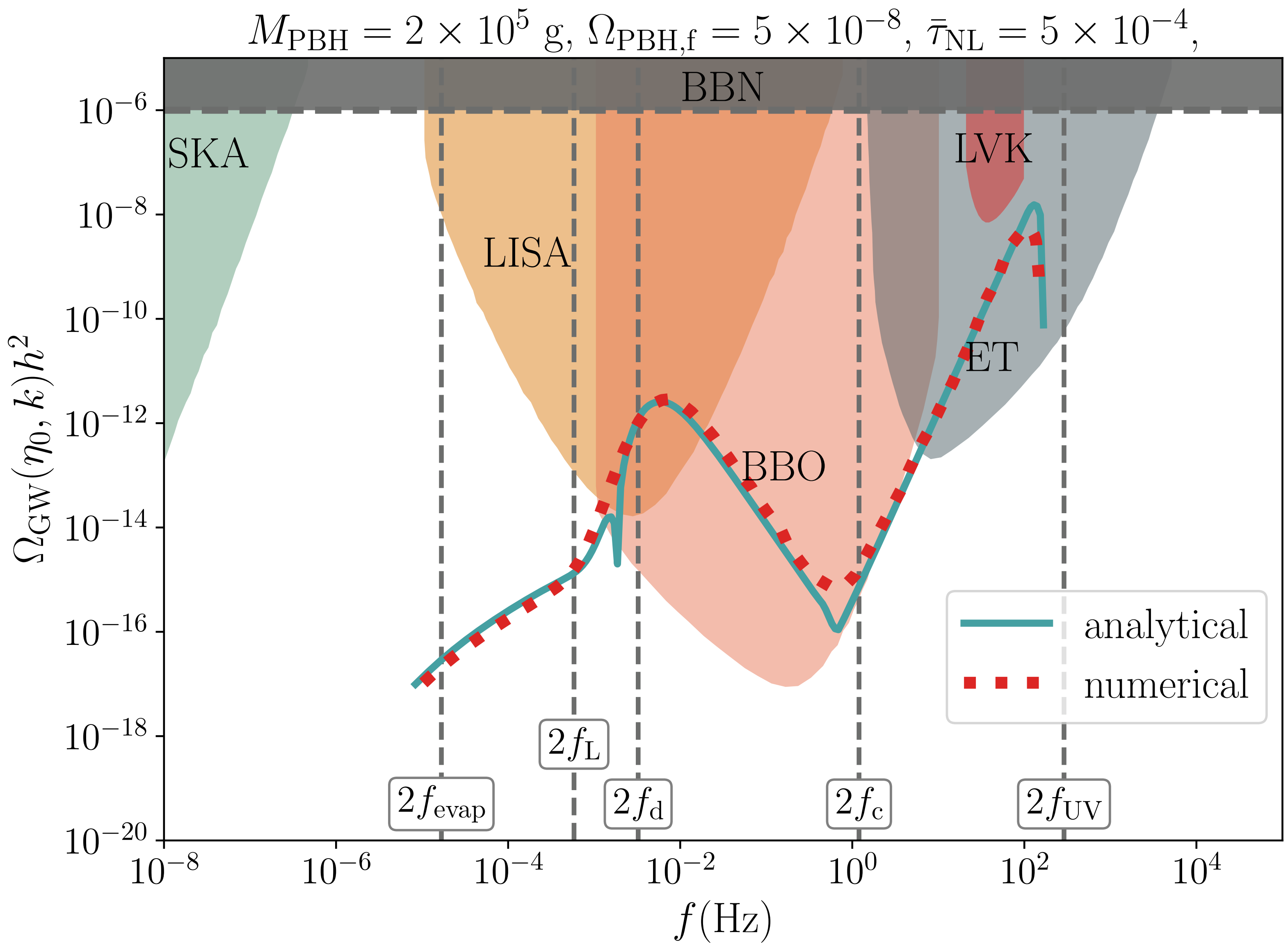
$M_{\rm PBH} = 2 \times 10^5{\rm g},\ \Omega_{\rm PBH,f} = 5 \times 10^{-8},\ \bar{\tau}_{\rm NL} = 5 \times 10^{-4}$

The non-Gaussian PBH matter power spectrum



Scale Hierarchy : $10^5\mathrm{Mpc}^{-1} < k_{\rm evap} < k_{\rm d} < k_{\rm c} < k_{\rm UV} \ll k_{\rm f} \sim 1/R$

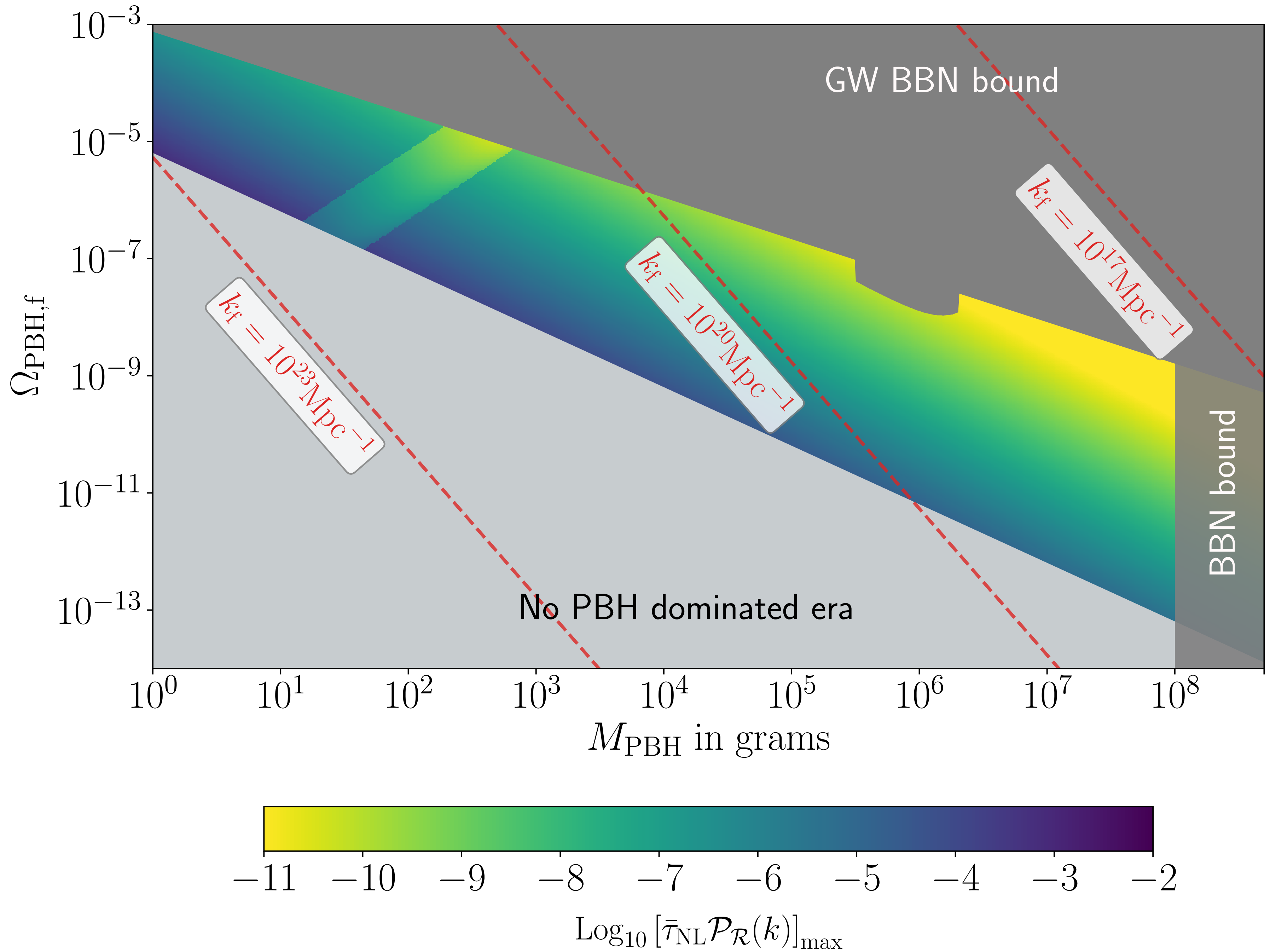
Non-Gaussian Induced GWs



$$
\Omega_{\rm GW}(\eta_0,k)h^2 \simeq \begin{cases}
3\times 10^{-80}\left(\frac{k}{10^4{\rm Mpc}^{-1}}\right)^{11/3}\left(\frac{M_{\rm PBH}}{10^4{\rm g}}\right)^{41/6}\left(\frac{\Omega_{\rm PBH,f}}{10^{-10}}\right)^{16/3} & 2k_{\rm c} < k < 2k_{\rm UV} \\
5\times 10^{-13}\left(\frac{k}{10^4{\rm Mpc}^{-1}}\right)^{-7/3}\left(\frac{M_{\rm PBH}}{10^4{\rm g}}\right)^{11/6}\left(\frac{\Omega_{\rm PBH,f}}{10^{-10}}\right)^{16/3}\left(\frac{\bar{\tau}_{\rm NL}\mathcal{P}_{\mathcal{R}}}{10^{-15}}\right)^{2} & 2k_{\rm d} < k < 2k_{\rm c} \\
4\times 10^{-75}\left(\frac{k}{10^4{\rm Mpc}^{-1}}\right)^{17/3}\left(\frac{M_{\rm PBH}}{10^4{\rm g}}\right)^{17/2}\left(\frac{\bar{\tau}_{\rm NL}\mathcal{P}_{\mathcal{R}}}{10^{-15}}\right)^{2} & \\
\quad + 10^{-39}\left(\frac{k}{10^4{\rm Mpc}^{-1}}\right)\left(\frac{M_{\rm PBH}}{10^4{\rm g}}\right)^{4}\left(\frac{\Omega_{\rm PBH,f}}{10^{-10}}\right)^{22/9}\left(\frac{\bar{\tau}_{\rm NL}\mathcal{P}_{\mathcal{R}}}{10^{-15}}\right)^{2} & 2k_{\rm evap} < k < 2k_{\rm d}
\end{cases}
$$

Constraining non-Gausianities

$$\Omega_{\rm GW}(2k_{\rm d},\eta_0) \leq 10^{-6} \Rightarrow \bar{\tau}_{\rm NL}\mathscr{P}_{\mathscr{R}}(k) \leq 4\times 10^{-20}\Omega_{\rm PBH,f}^{-17/9}\left(\frac{M_{\rm PBH}}{10^4 {\rm g}}\right)^{-17/9}$$



Testing alternative gravity theories with PBHs and induced GWs:

The case of non-singular bouncing cosmology

[**T. Papanikolaou**, S. Banerjee, Y.F. Cai, S. Capozziello, E.N. Saridakis, **JCAP 06 (2024) 066**]

The non-singular bouncing paradigm

Motivation

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- It is **free of the initial singularity** problem.

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- With **matter contracting phases**, one can easily give rise to **scale-invariant curvature power spectra**, observed on the CMB scales.

The non-singular bouncing paradigm

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- With **matter contracting phases**, one can easily give rise to **scale-invariant curvature power spectra**, observed on the CMB scales.

"Caveats"

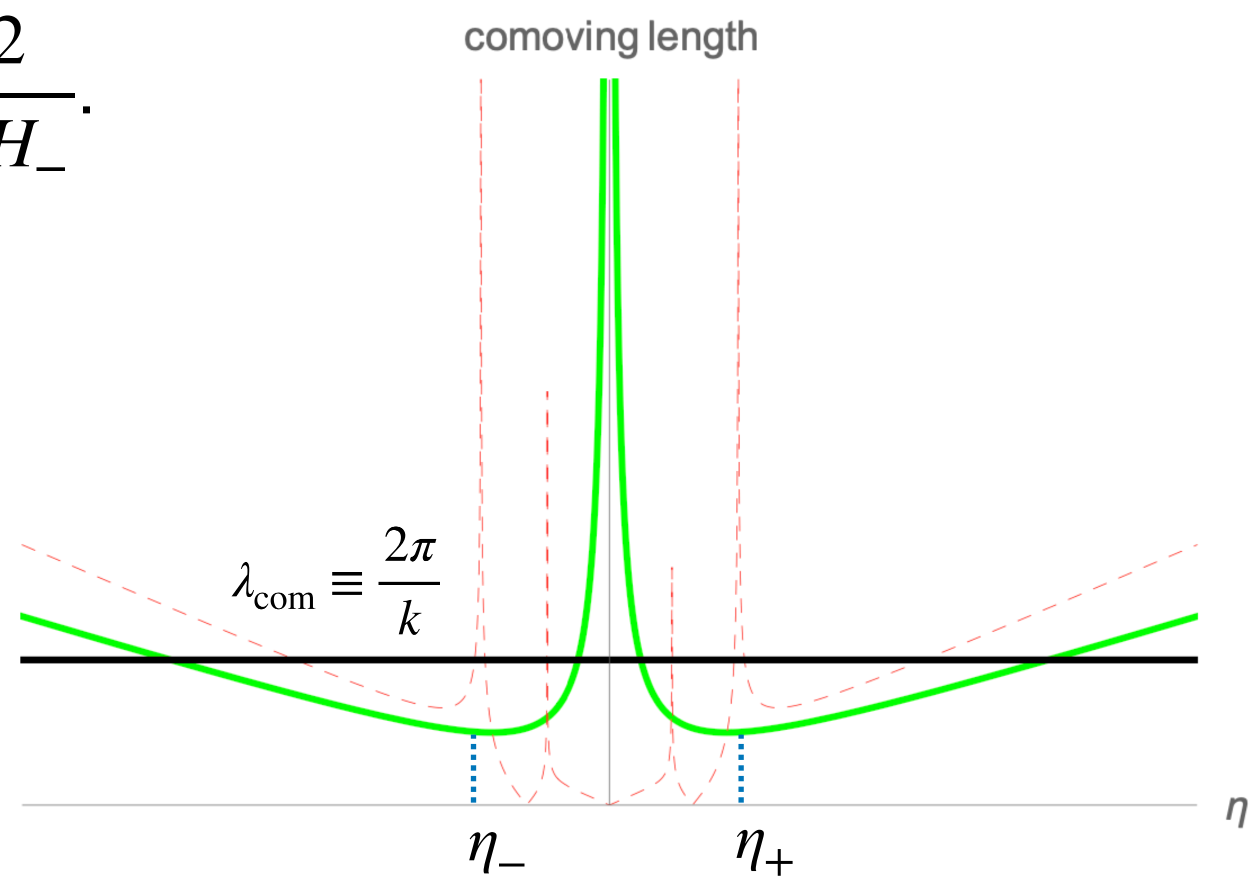
- Effective v**iolation of the null energy condition** for a short period of time, $T_{\mu\nu}k^{\mu}k^{\nu} < 0 \Rightarrow \rho + 3p < 0$.

Non-singular matter bouncing cosmology: The background evolution

Background dynamics

A. *Matter contracting phase*

$$a(t) = a_- \left(\frac{t - \tilde{t}_-}{t_- - \tilde{t}_-} \right)^{2/3}, \text{ with } t_- - \tilde{t}_- = \frac{2}{3H_-}.$$



B. *Bouncing phase*

$$a(t) = a_b e^{\frac{\Upsilon t^2}{2}}, \text{ with } H(t) = \Upsilon t.$$

C. *HBB expanding phase*

$$a(t) = a_+ \left(\frac{t - \tilde{t}_+}{t_+ - \tilde{t}_+} \right)^{1/2}, \text{ with } t_+ - \tilde{t}_+ = \frac{1}{2H_+}.$$

The dynamics of primordial curvature perturbations

$$v_k'' + \left(c_\mathrm{s}^2 k^2 - \frac{z''}{z}\right) v_k = 0, \text{ with } v_k \equiv z\mathscr{R}_k \text{ and } z \equiv \frac{a\sqrt{\rho + p}}{c_s H M_\mathrm{Pl}}.$$

[Mukhanov-Sasaki (MS) equation]

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- Considering Bunch Davies vacuum as our initial conditions on sub-horizon scales, i.e. $v_k = e^{-ik\eta}/\sqrt{2k}$ for $k \gg aH$, one can solve analytically the MS equation.

- Remarkably, one finds an analytic approximation for the curvature power spectrum $\mathscr{P}_{\mathscr{R}}(k)$ reading as

$$\mathscr{P}_{\mathscr{R}}(k) \equiv \frac{k^3 |\mathscr{R}_k|^2}{2\pi^2} \simeq \begin{cases} \frac{a_-^3 H_-^2}{48\pi^2 c_s M_{\rm Pl}^2 a^3} & \text{for } c_s k \ll |aH| \\ \frac{a_-^3 H_-^2}{12\pi^2 c_s M_{\rm Pl}^2 a^3} \left(\frac{c_s k}{aH}\right)^2 & \text{for } c_s k \gg |aH|. \end{cases}$$

The dynamics of primordial curvature perturbations

$$v_k'' + \left(c_s^2 k^2 - \frac{z''}{z}\right) v_k = 0, \text{ with } v_k \equiv z\mathscr{R}_k \text{ and } z \equiv \frac{a\sqrt{\rho + p}}{c_s H M_{\rm Pl}}.$$

[Mukhanov-Sasaki (MS) equation]

- c_s **is the curvature perturbation sound speed** depending on the details of the **underlying gravity theory**. For GR, $c_s = 1$.
- Considering Bunch Davies vacuum as our initial conditions on sub-horizon scales, i.e. $v_k = e^{-ik\eta}/\sqrt{2k}$ for $k \gg aH$, we solve analytically the MS equation.
- One then can find an analytic approximation for the curvature power spectrum $\mathscr{P}_{\mathscr{R}}(k)$ during the contracting phase reading as

$$\mathscr{P}_{\mathscr{R}}(k) \equiv \frac{k^3 |h_k|^2}{2\pi^2} \simeq \begin{cases} \dfrac{a_-^3 H_-^2}{48\pi^2 c_s M_{\rm Pl}^2 a^3} \text{ for } c_s k \ll |aH| \\ \dfrac{a_-^3 H_-^2}{12\pi^2 c_s M_{\rm Pl}^2 a^3} \left(\dfrac{c_s k}{aH}\right)^2 \text{ for } c_s k \gg |aH|. \end{cases}$$

- **$\mathscr{P}_{\mathscr{R}}(k)$ increases on super-horizon scales during the contracting phase!**

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- Since $\mathscr{R}$ **is conserved on super-horizon scales** during **the HBB era** one can show that at horizon-crossing time, i.e. $k = aH$ **(PBH formation time)** that

$$\mathscr{P}_{\mathscr{R}}(k) \simeq \frac{0.7\Upsilon^8 \cos^2 A^2}{c_{\rm s,m}^3 H_-^4 H_+^2 \pi^2 (H_+^2 + 2\Upsilon)^4} - \frac{1.4B^2\sqrt{c_{\rm s,m}}\Upsilon^{17/2}\cos A \sin A\sqrt{k}}{c_{s,m}^3 H_-^4 H_+^2 \pi^2 (H_+^2 + 2\Upsilon)^4}$$
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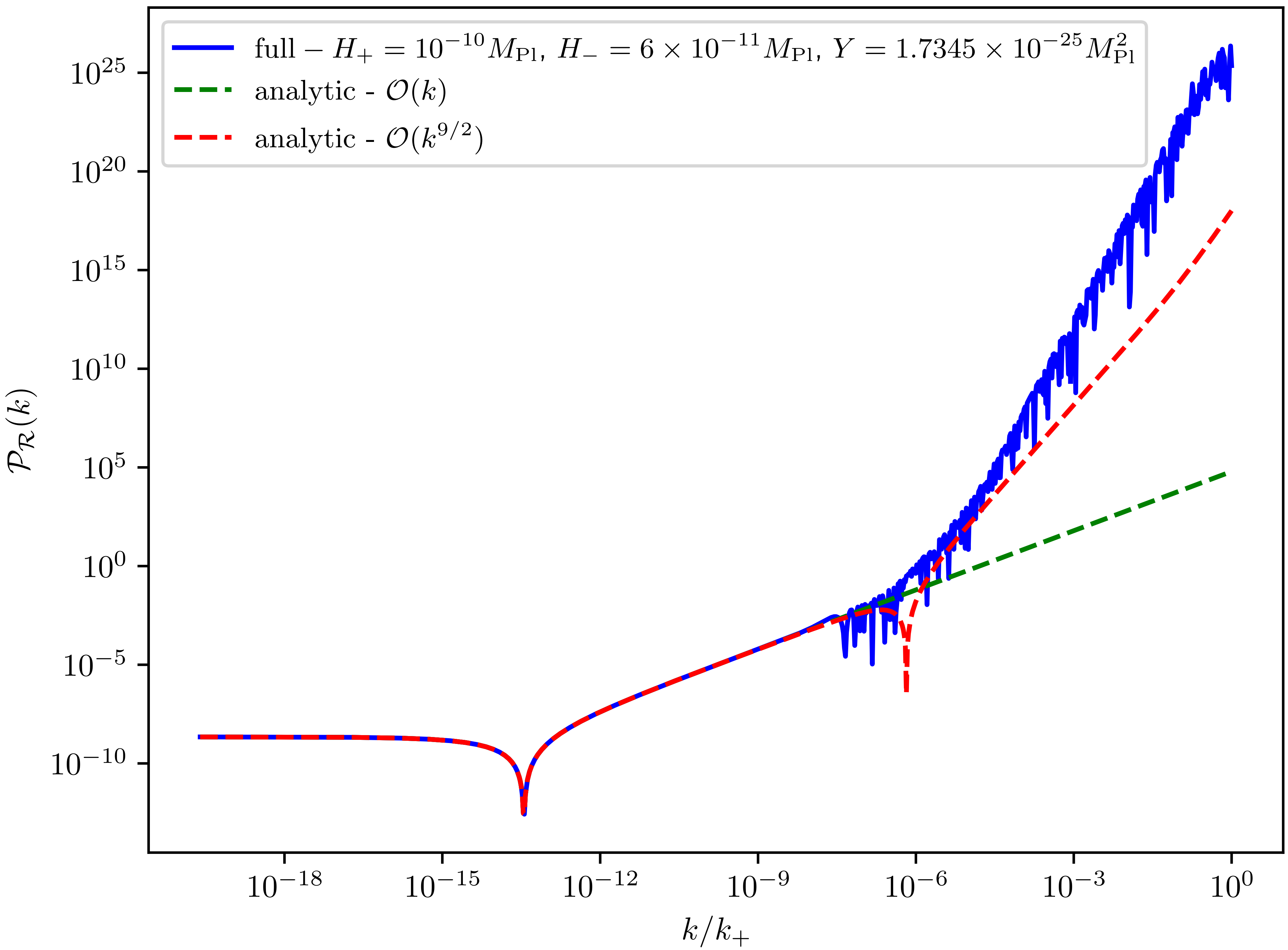
The curvature power spectrum



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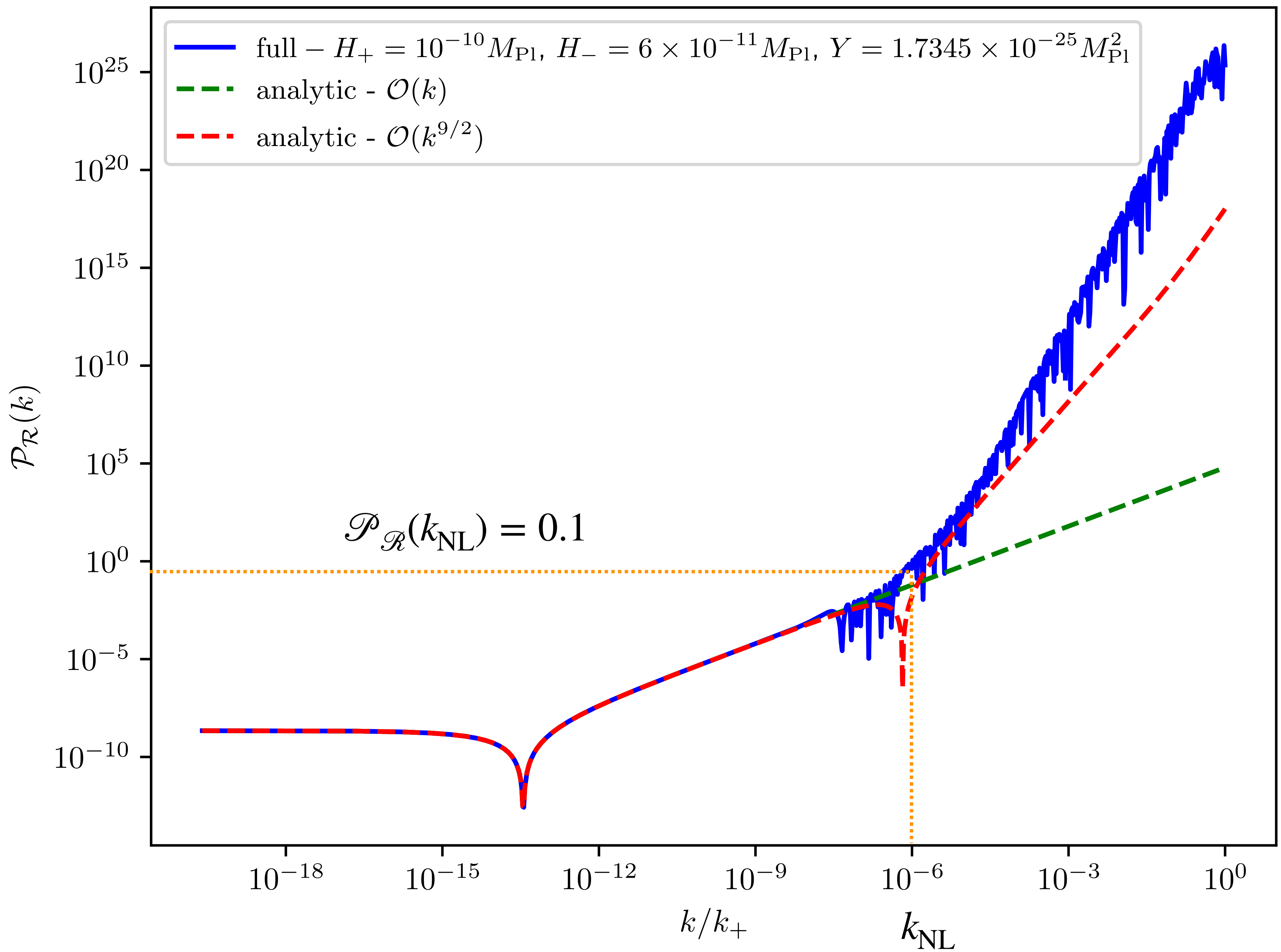

The PBH abundance

$$\beta(M) \equiv \frac{1}{\rho_{\rm tot}} \frac{{\rm d}\rho_{\rm PBH}}{{\rm d}\ln M} = \int_{\delta_{\rm c}}^{1} {\rm PDF}(\delta){\rm d}\delta \text{ within peak theory}$$

$$\Rightarrow f_{\rm PBH} \equiv \frac{\Omega_{\rm PBH,0}}{\Omega_{\rm DM,0}} = \left(\frac{\beta(M)}{3.27\times 10^{-8}}\right)\left(\frac{106.75}{g_{*,{\rm f}}}\right)^{1/4}\left(\frac{M}{M_\odot}\right)^{-1/2}.$$

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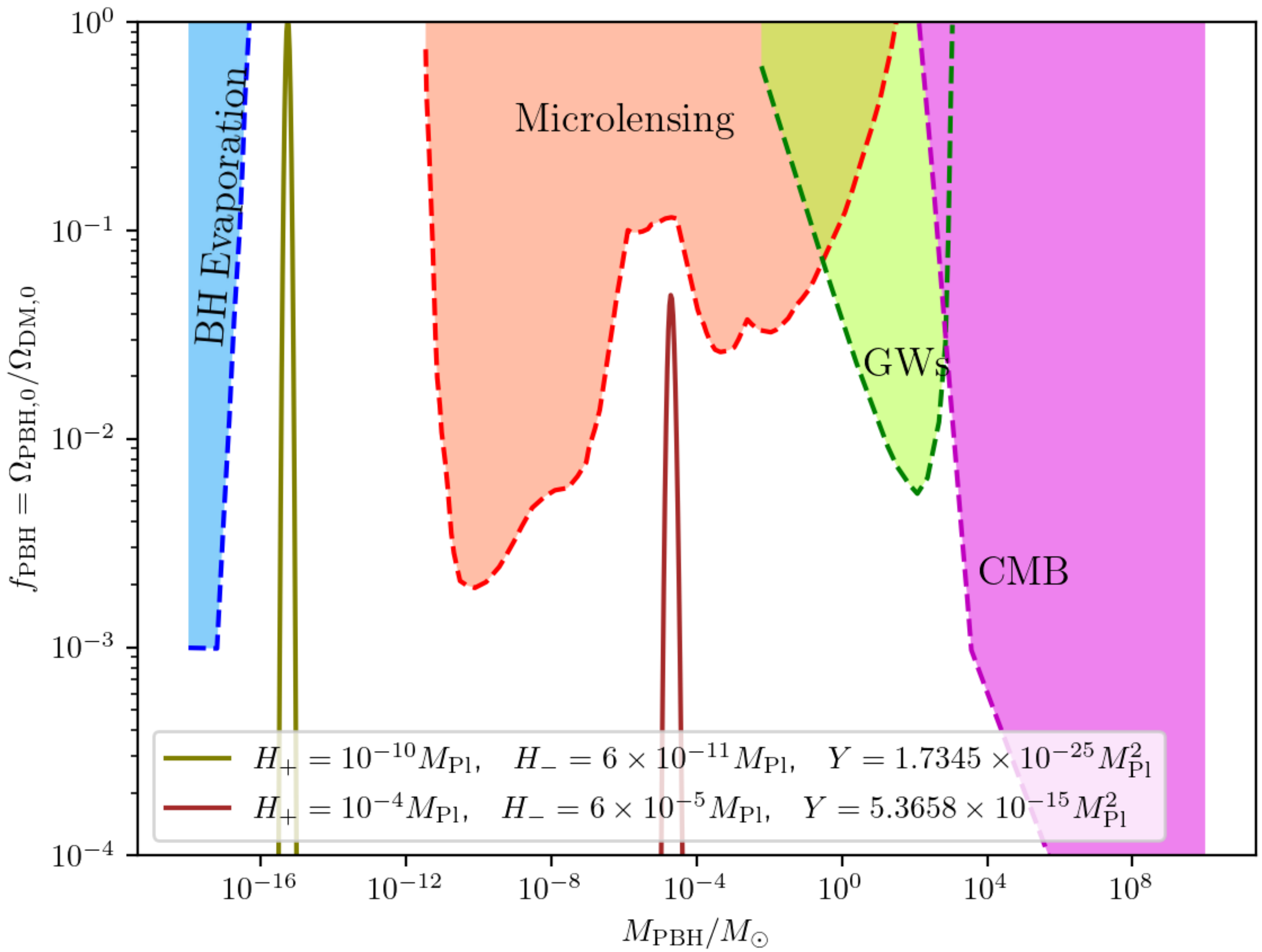
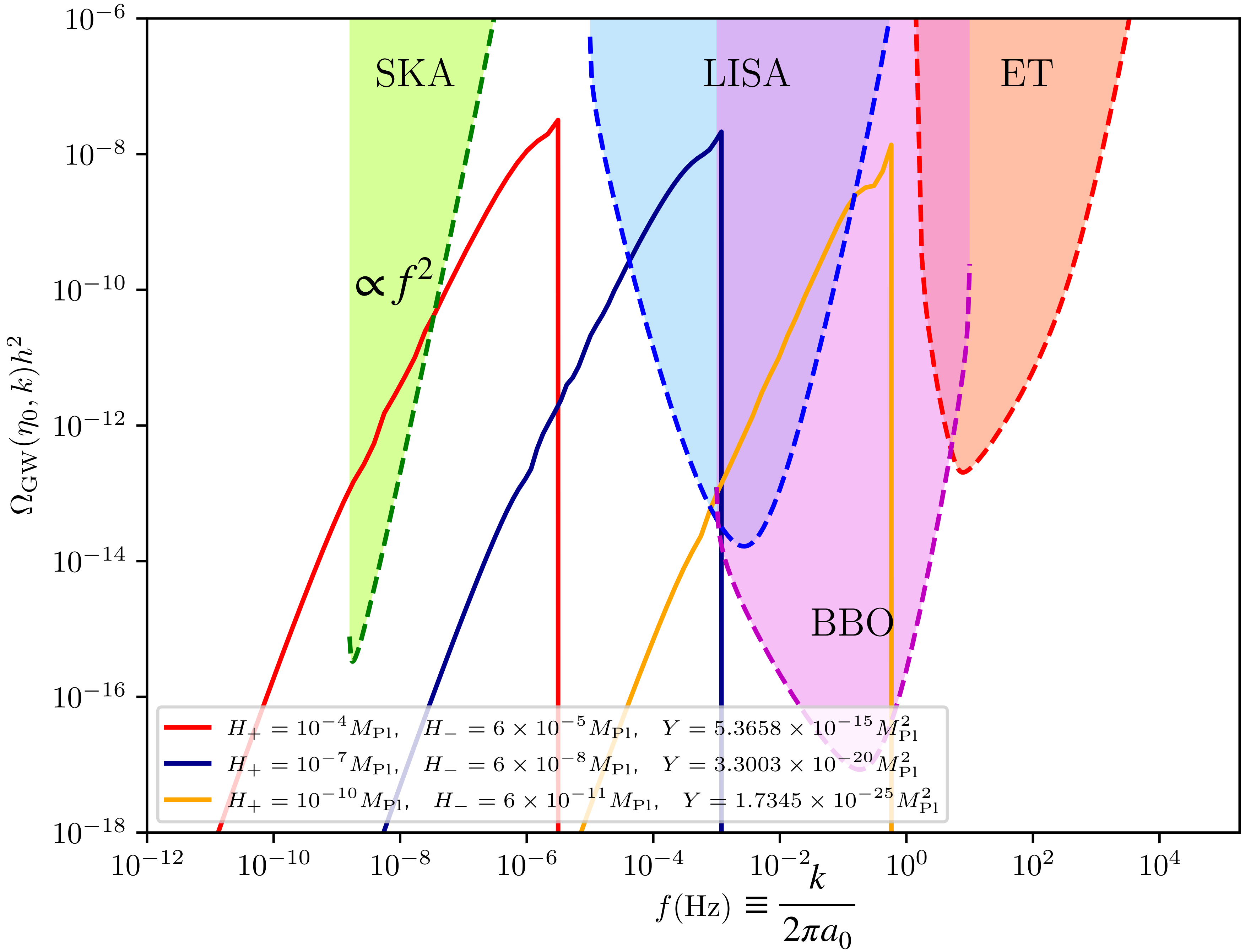
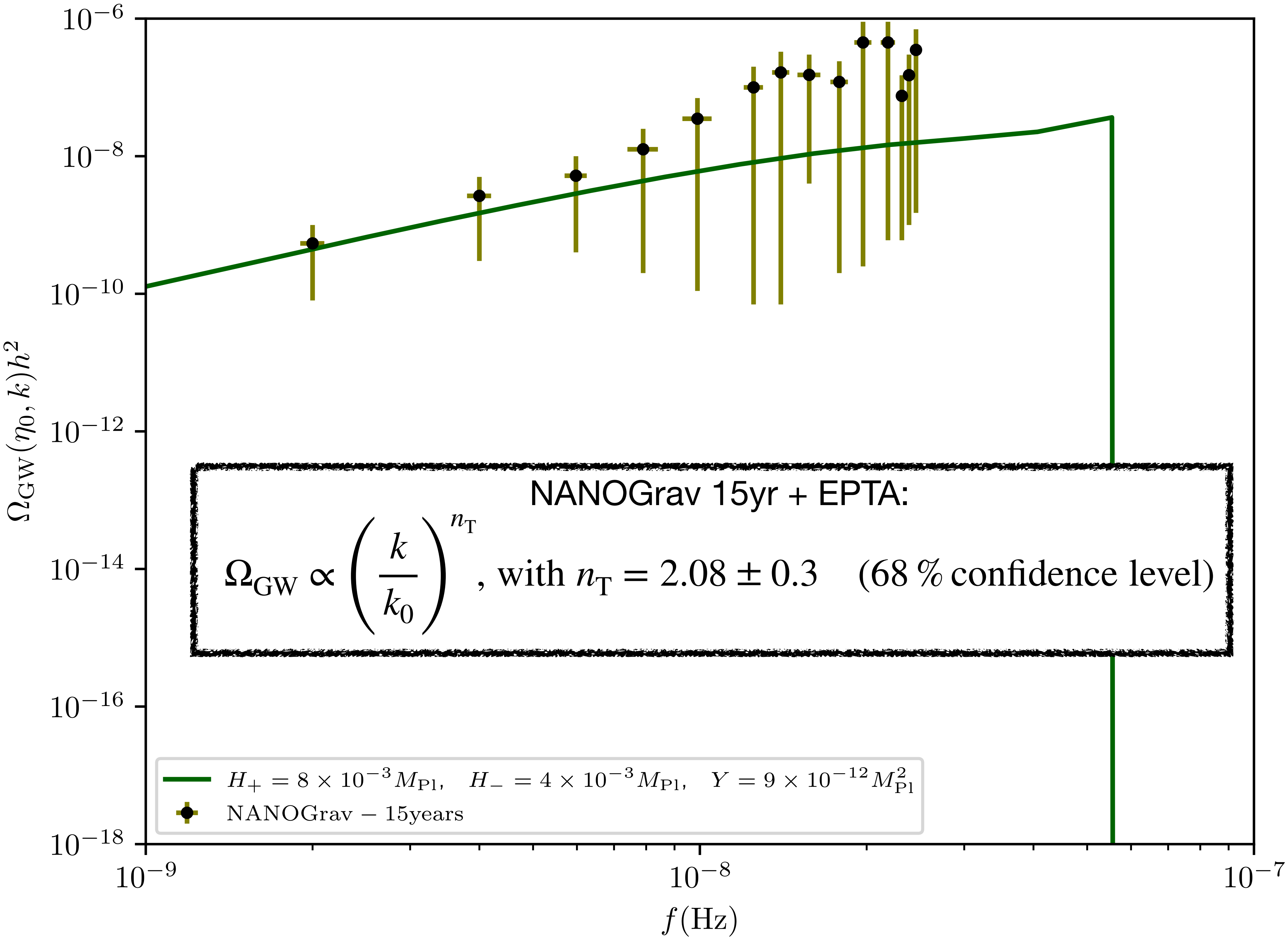


The scalar-induced GW signal



$$\Omega_{\rm GW} \propto \mathscr{P}_h(k) \propto \iint \mathrm{d}u\,\mathrm{d}v\, f^2(u,v,\eta)\mathscr{P}_{\mathscr{R}}(ku,\eta)\mathscr{P}_{\mathscr{R}}(kv,\eta) \propto \mathscr{P}_{\mathscr{R}}^2 \propto k^2 \propto f^2$$

Induced GW signal at nHz



Testing alternative gravity theories with PBHs and induced GWs

- **Scalar induced gravitational waves from primordial black hole Poisson fluctuations in f(R) gravity**, T. Papanikolaou C. Tzerefos, S. Basilakos and E. N. Saridakis, **JCAP 10 (2022) 013** • e-Print: 2112.15059 [astro-ph.CO]

- **No constraints for f(T) gravity from gravitational waves induced from primordial black hole fluctuations,** T.Papanikolaou, C. Tzerefos, S. Basilakos and E. N. Saridakis, **Eur. Phys. J. C 83 (2023) 1, 31** • e-Print: 2205.06094 [gr-qc]

- **Constraining F(R) bouncing cosmologies with primordial black holes**, S. Banerjee, T. Papanikolaou, E. N. Saridakis, **Phys. Rev. D 106 (2022) 12, 124012** • e-Print: 2206.01150 [gr-qc]

- **Scalar induced gravitational waves in modified teleparallel gravity theories**, C. Tzerefos, T. Papanikolaou, S. Basilakos and E. N. Saridakis, **Phys. Rev. D 107 (2023) 12, 124019** • e-Print: 2303.16695 [gr-qc]

- **Primordial black holes in loop quantum cosmology: the effect on the threshold**, T. Papanikolaou, **Class. Quant. Grav. 40 (2023) 13, 134001** • e-Print: 2301.11439 [gr-qc]

- **Gravitational-wave signatures of gravito-electromagnetic couplings**, T. Papanikolaou, C. Tzerefos, S. Capozziello, G. Lambiase, e-Print: 2408.17259 [astro-ph.CO]

Probing fundamental high energy physics theories with PBHs and induced GWs:

The case of no-scale supergravity

[S. Basilakos, D.V. Nanopoulos, **T. Papanikolaou**, E. N. Saridakis, C. Tzerefos
Phys. Lett. B 850 (2024) 138507]

No-Scale Supergravity

- **Supergravity (SUGRA)** [D. Z. Freedman et al. - 1976, S. Deser and B. Zumino - 1976] is the **low-energy effective field theory (EFT)** a higher dimensional **Superstring theory,** combining the principles of **supersymmetry (SUSY) and general relativity**.

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- The simplest representation of SUGRA is characterized by 2 functions. The **Kahler potential** K and the **superpotential W**. At the end, one can write the SUGRA action and the effective inflationary potential as

$$S = \int \mathrm{d}^4x\sqrt{-g}\left(K_{i\bar{j}}\partial_\mu\Phi^i\partial^\mu\bar{\Phi}^{\bar{j}} - V\right), \text{ with } V(\Phi,\bar{\Phi}) = e^K(K^{i\bar{j}}\mathscr{D}_iW\mathscr{D}_{\bar{j}}\bar{W} - 3\,|\,W\,|^2),$$

$$\text{where}\quad K^{i\bar{j}}(\Phi,\bar{\Phi}) \equiv \frac{\partial^2 K}{\partial\Phi^i\partial\bar{\Phi}^{\bar{j}}},\quad \mathscr{D}_iW \equiv \partial_iW + K_iW\,.$$

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- The simplest representation of SUGRA is characterized by 2 functions. The **Kahler potential** K and the **superpotential W**.

- For $K = -3\ln\left(T + \bar{T} - \frac{\phi\bar{\phi}}{3}\right)$ and $\phi = \sqrt{3c}\tanh\left(\frac{\chi}{\sqrt{3}}\right)$ one gets Starobinsky inflation [J. R. Ellis, D. V. Nanopoulos & K. Olive - PRL 2013],

$$V(\chi) = \frac{\mu^2}{4}\left(1 - e^{-\sqrt{\frac{2}{3}}\chi}\right)^2 .$$

PBHs in no-scale SUGRA inflection-point inflation

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- Working within the simplest no-scale SUGRA model, namely the Wess-Zumino one, where $W = \frac{\mu}{2}\phi^2 - \frac{\lambda}{3}\phi^3$, one can **produce naturally (light and not only) PBHs** by introducing **non-perturbative deformations of** K [D.V. Nanopoulos, V. Spanos, and I. Stamou - 2020] which can be recast as

$$K = -3\ln\left(T + \bar{T} - \frac{\phi\bar{\phi}}{3} + ae^{-b(\phi+\bar{\phi})^4}(\phi+\bar{\phi})^4\right).$$

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- One then gets the following **inflection-point inflationary potential**:

$$V(\phi) = \frac{3e^{12b\phi^2}\phi^2(c\mu^2 - 2\sqrt{3c}\lambda\mu\phi + 3\lambda^2\phi^2)}{\left[-48a\phi^4 + e^{4b\phi^2}(-3c+\phi^2)\right]^2\left\{e^{4b\phi^2} - 24a\phi^2[6 + 4b\phi^2(-9 + 8b\phi^2)]\right\}}.$$

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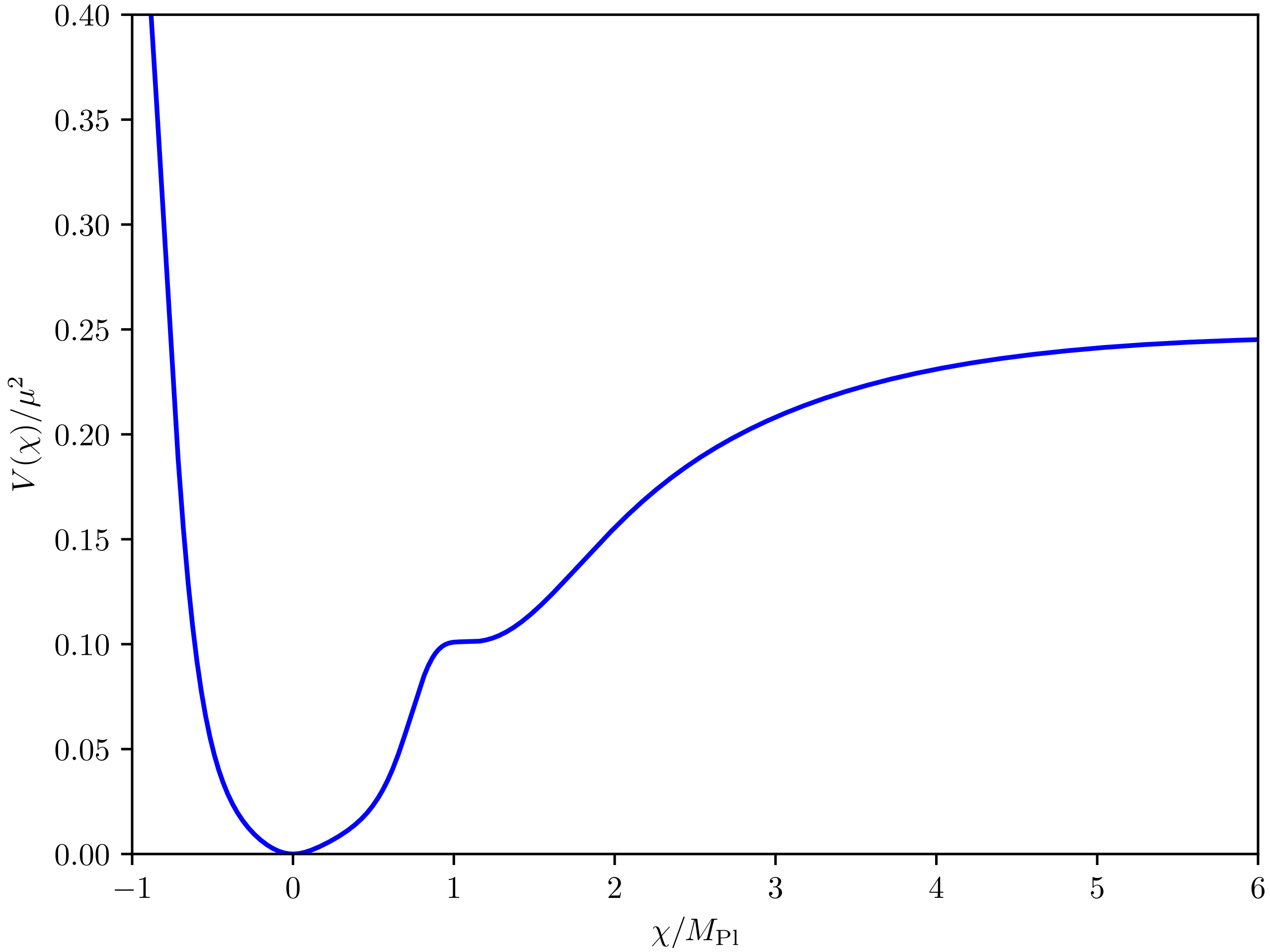
- In particular, **inflection-point inflation**, where $V''(\chi_{\rm inflection}) = V'(\chi_{\rm inflection}) \simeq 0$, one realises naturally an **ultra slow-roll (USR) phase**, during which the **non-constant mode of the curvature perturbation grows exponentially** leading to **PBH formation.**

Ultra-light PBHs in no-scale SUGRA



$a = -1,\ b = 22.35,\ c = 0.065,\ \mu = 2 \times 10^{-5}$

$\lambda/\mu = 0.3333449$ and $\chi_0 = 1.034 M_{\rm Pl}$

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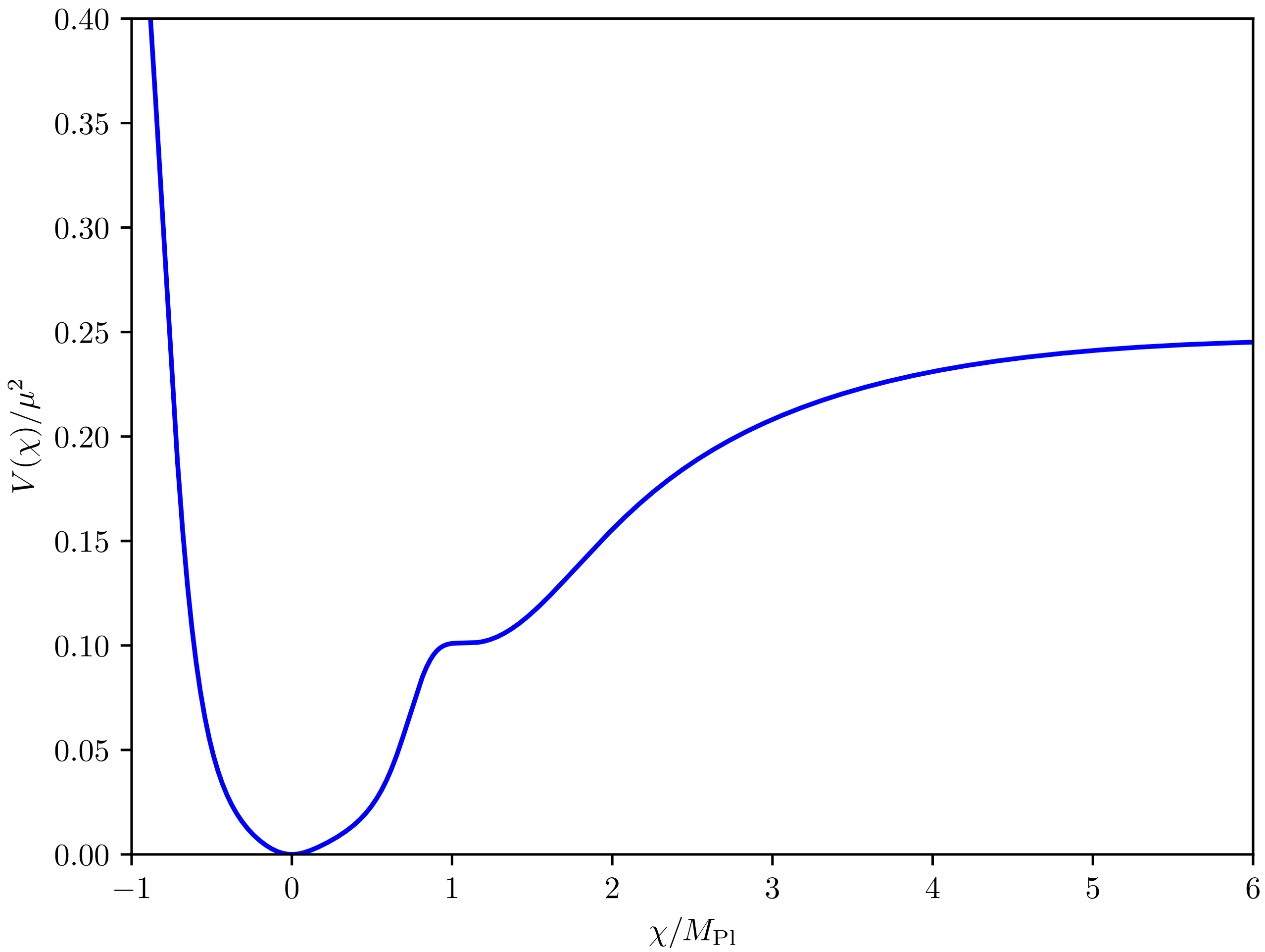

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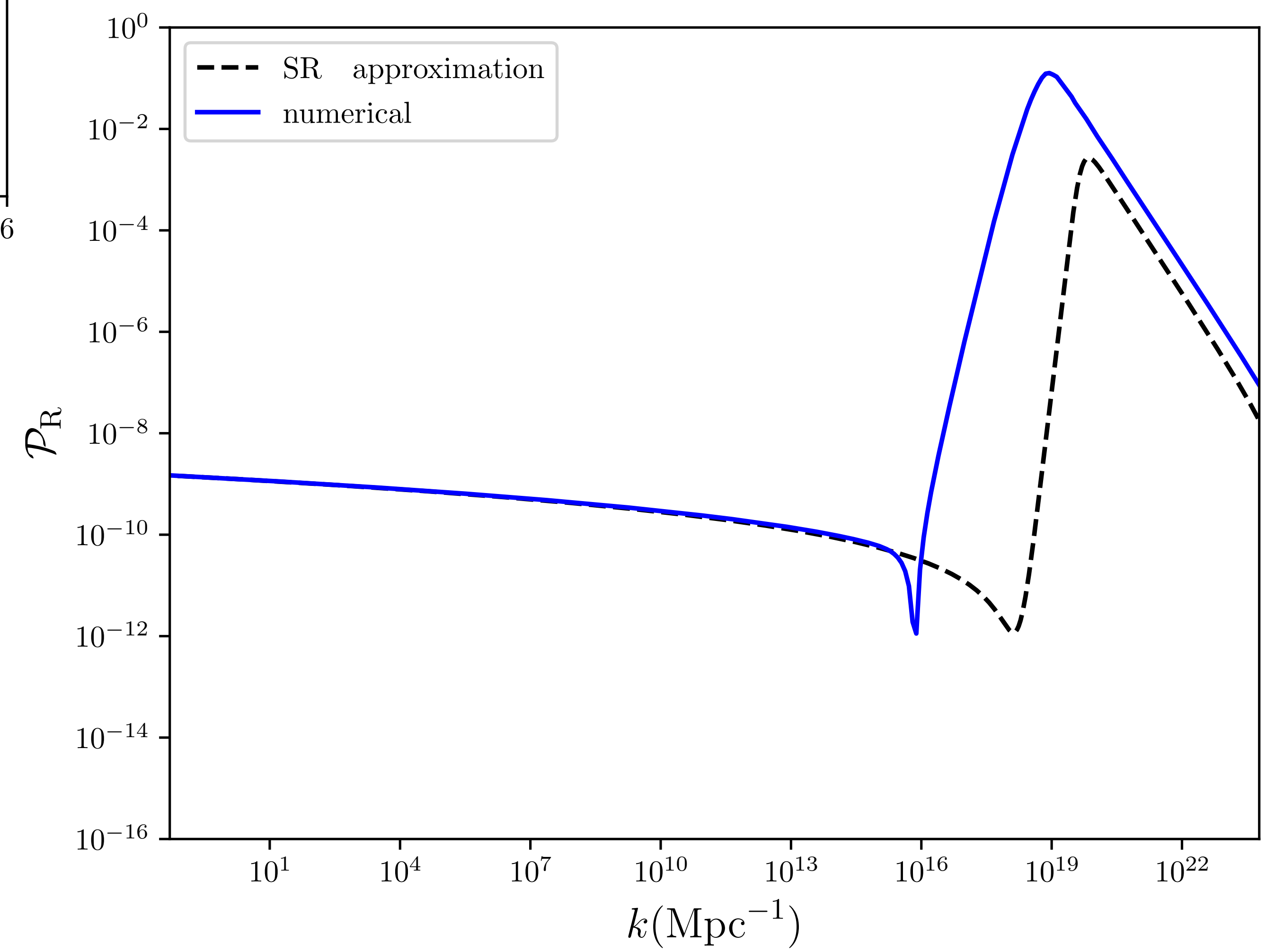

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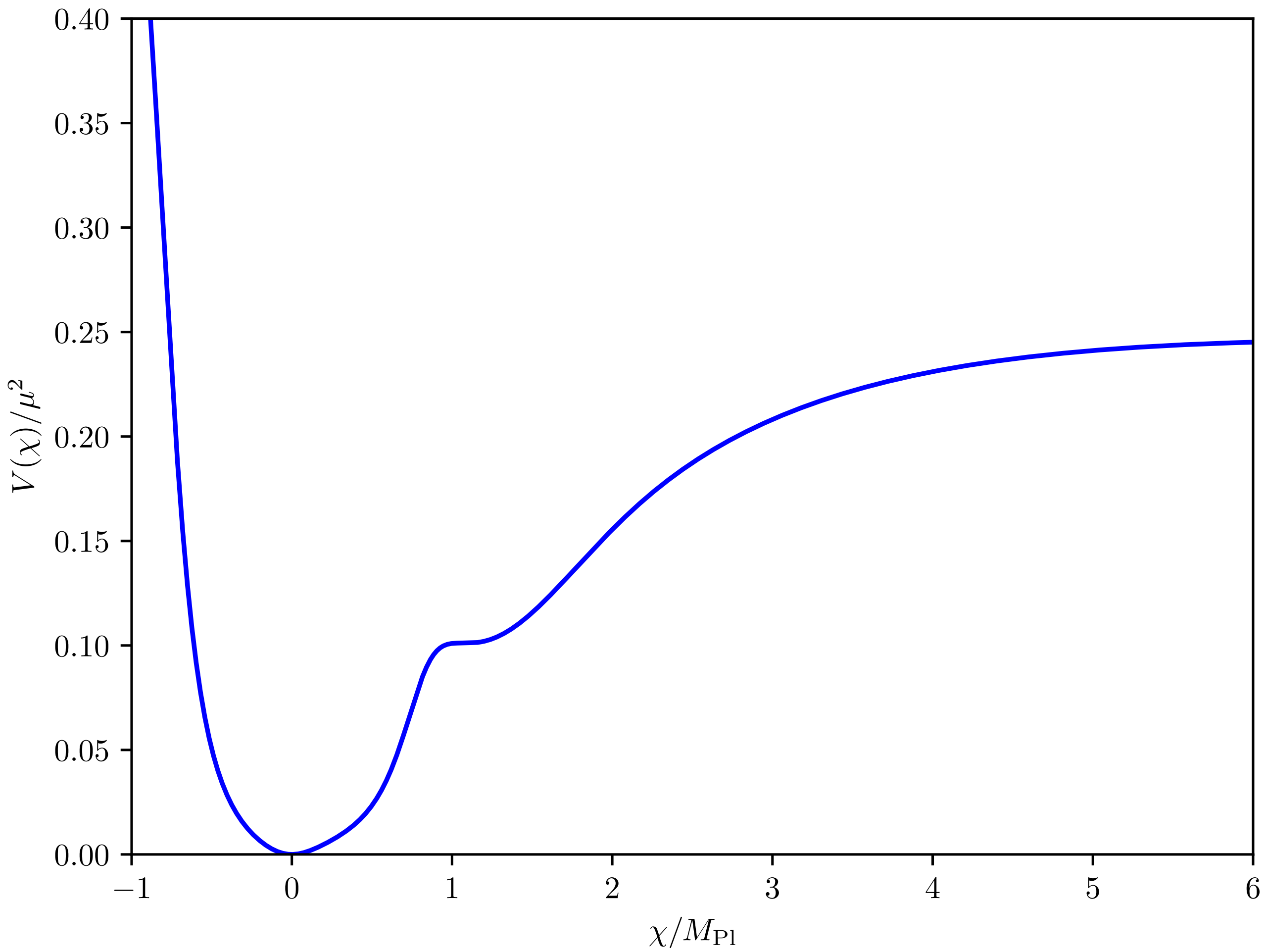

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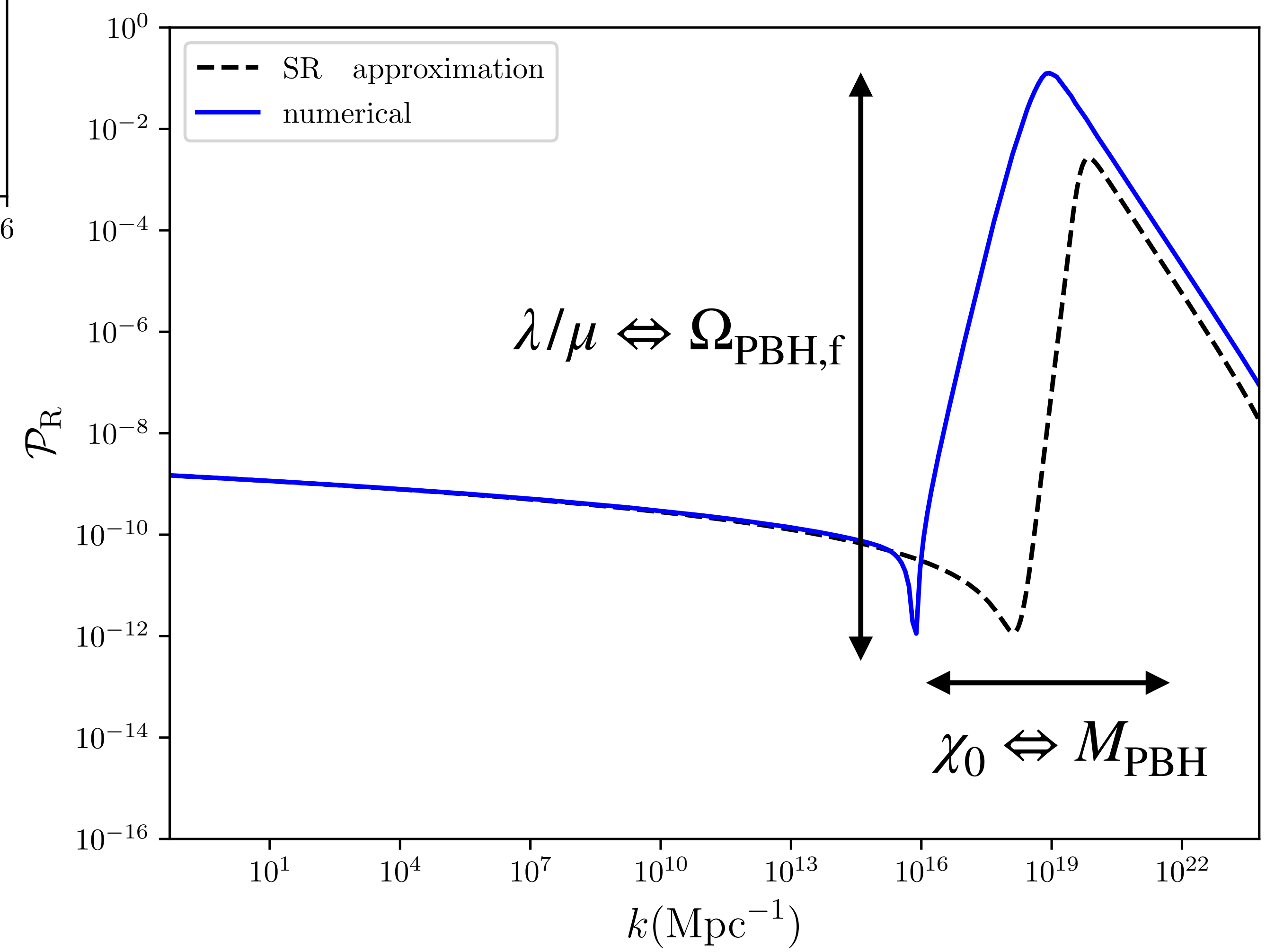

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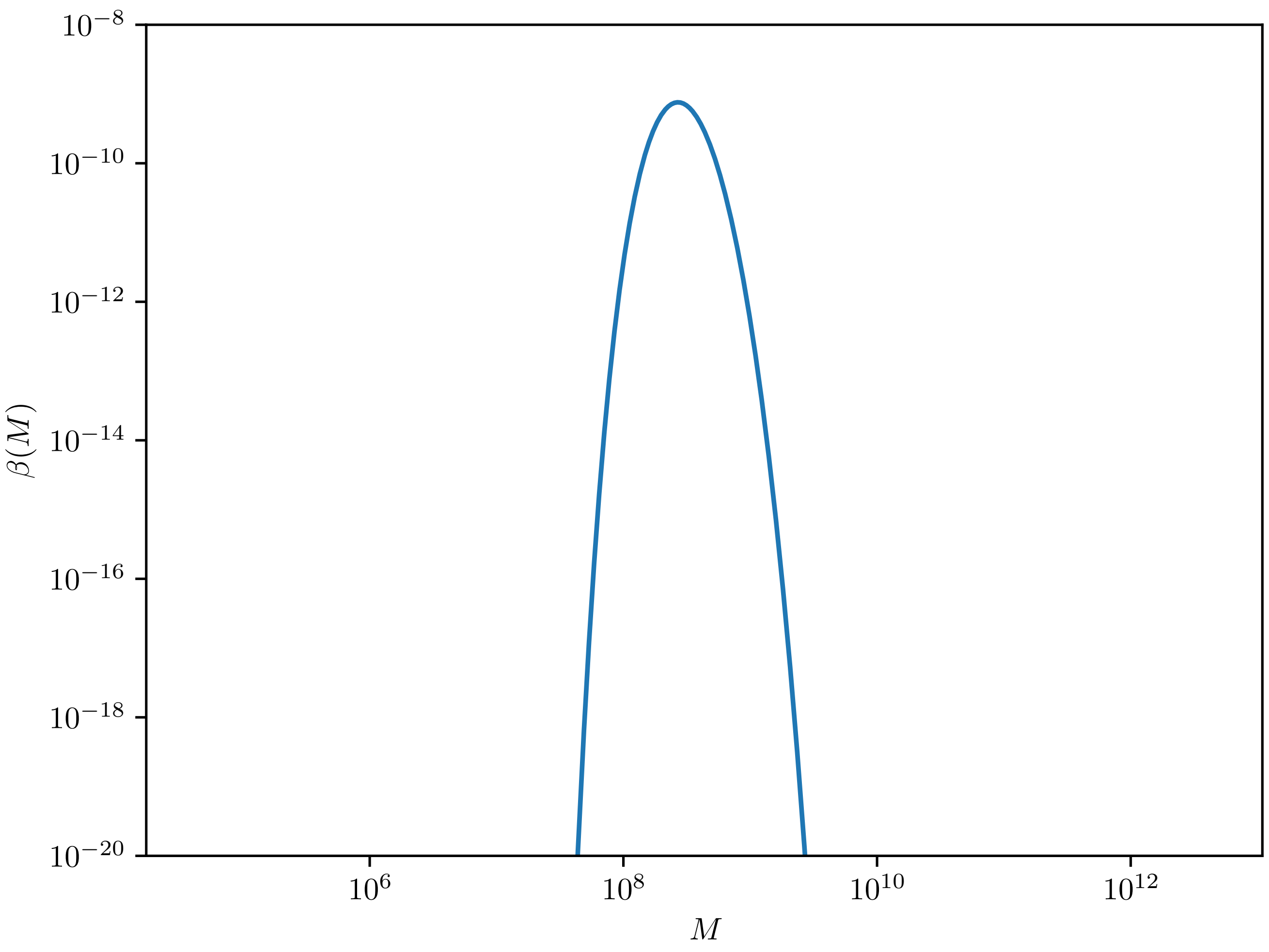
Ultra-light PBH domination



Working within peak theory, we derive the PBH mass function defined as

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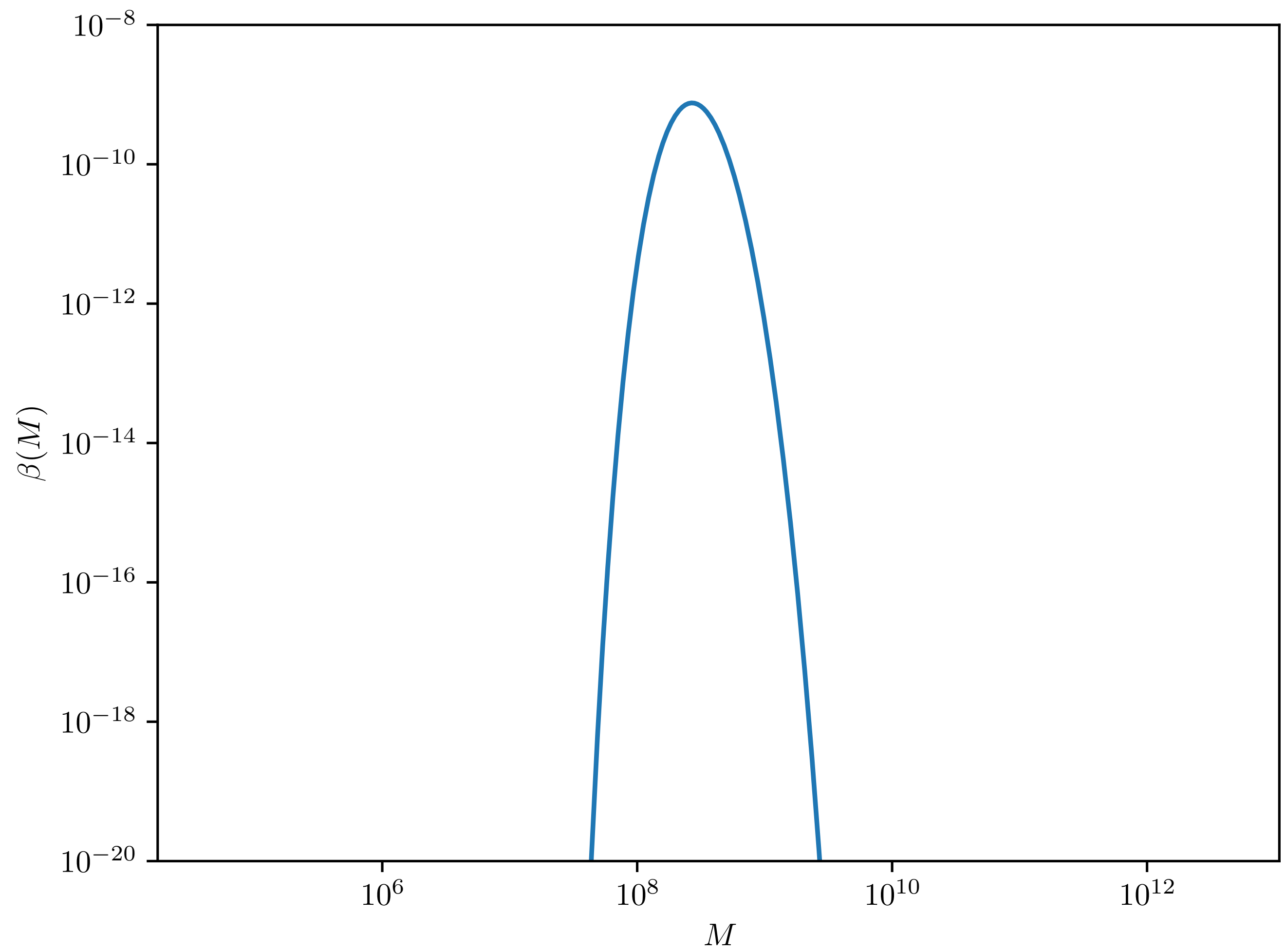

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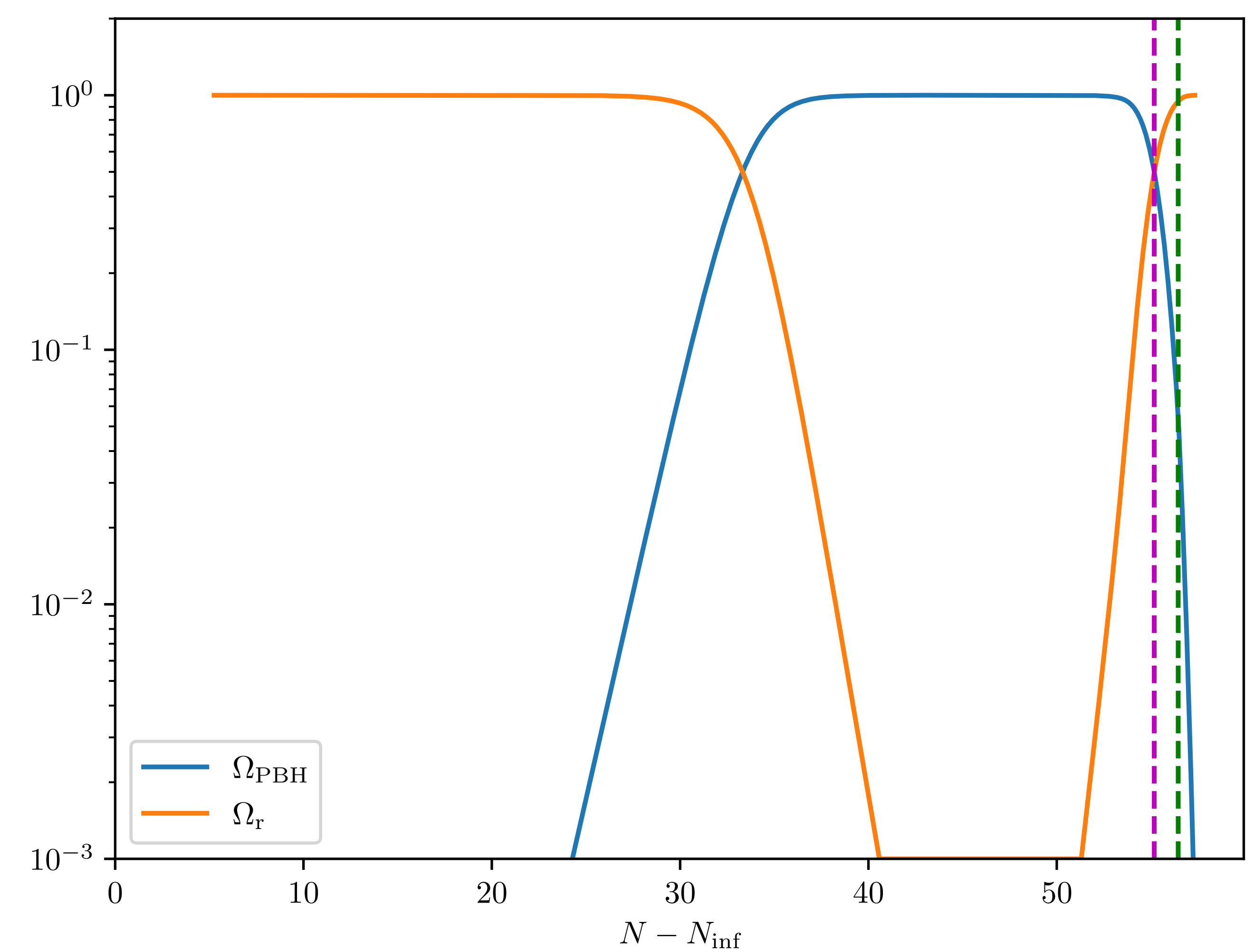
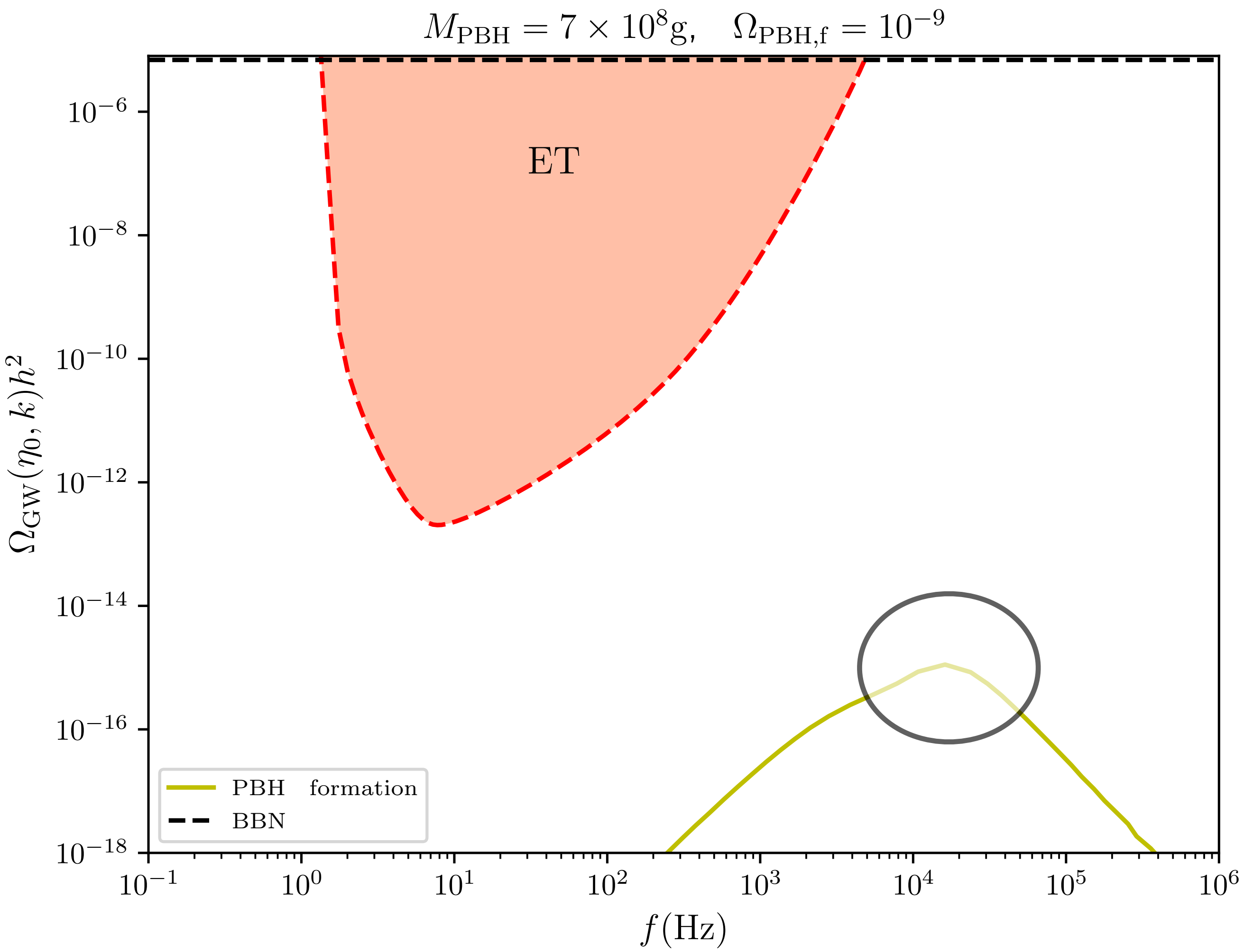
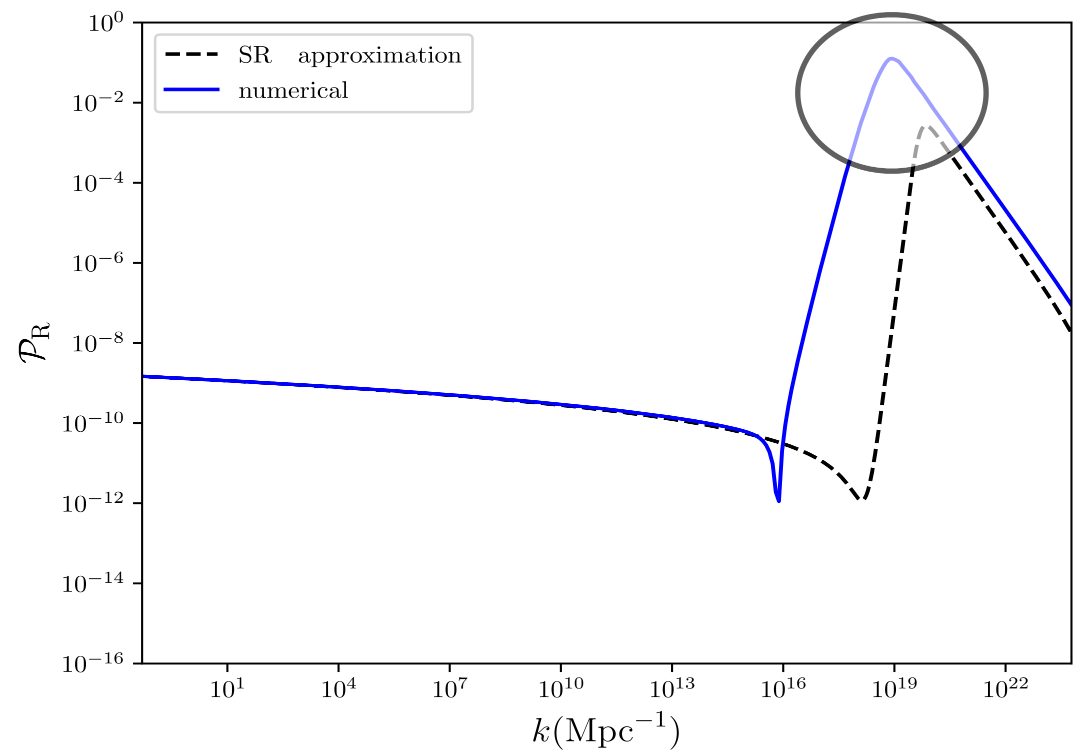
Accounting for the PBH Hawking evaporation the PBH abundance reads

$$\Omega_{\rm PBH}(t) = \int_{M_{\rm min}}^{M_{\rm max}} \bar{\beta}\,(M,t) \left\{1 - \frac{t - t_{\rm ini}}{\Delta t_{\rm evap}(M_{\rm f})}\right\}^{1/3} \mathrm{d}\ln M\,.$$



GWs induced by inflationary adiabatic perturbations.





$$\Omega_{\rm GW} \propto \mathcal{P}_h(k) \sim \iint \mathrm{d}u\mathrm{d}v f^2(u,v,\eta)\mathcal{P}_{\mathcal{R}}(ku,\eta)\mathcal{P}_{\mathcal{R}}(kv,\eta)$$

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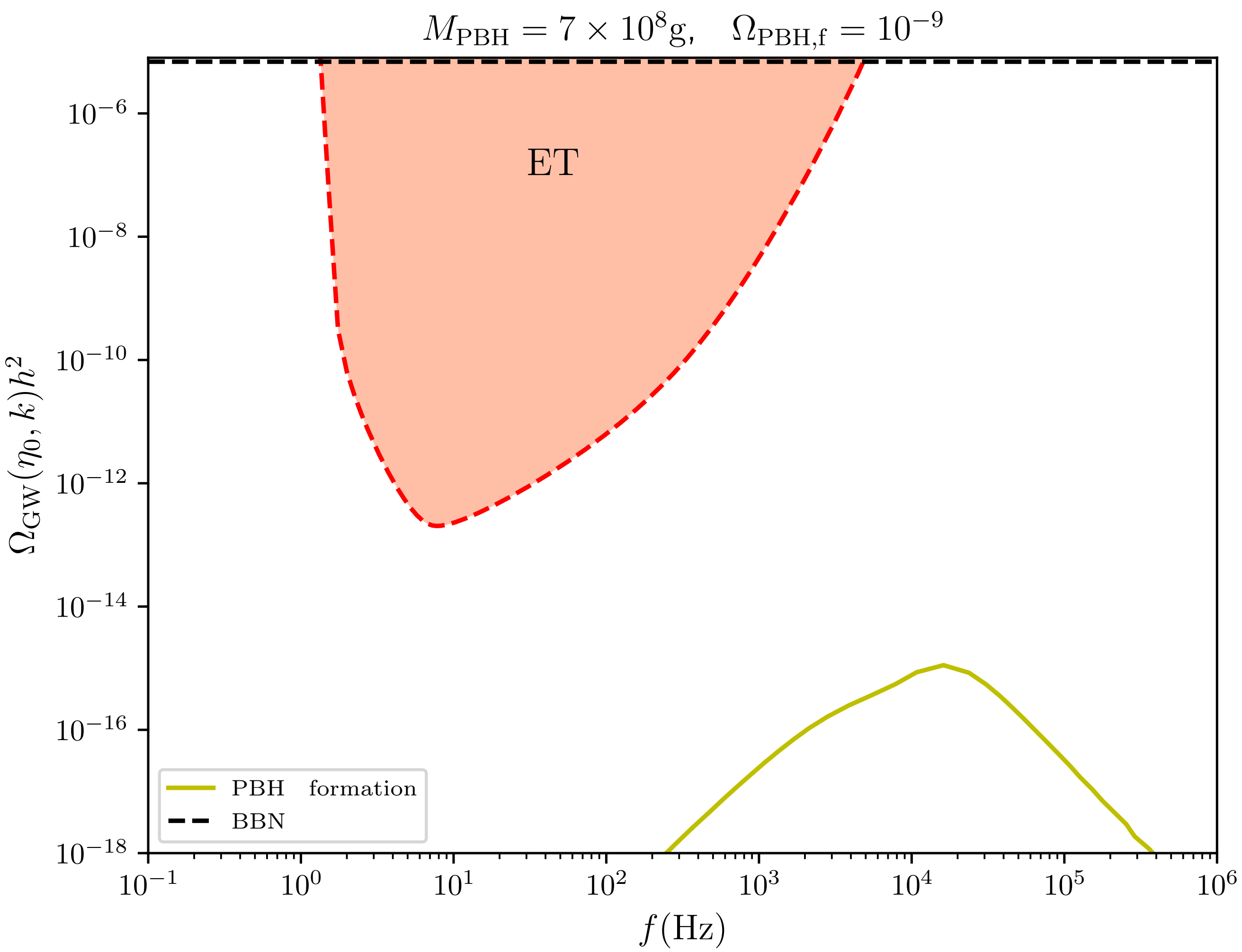

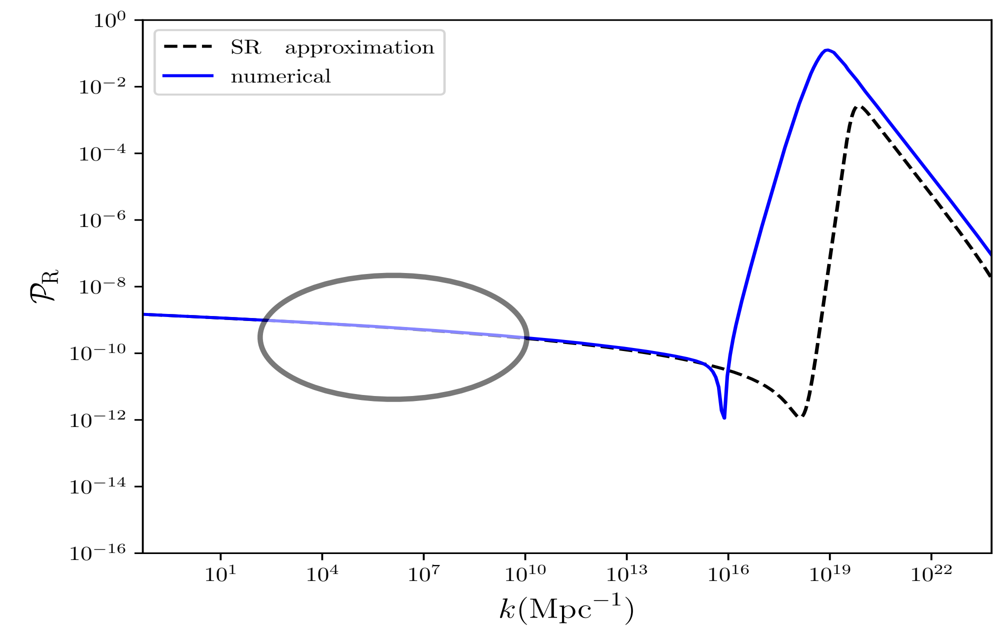

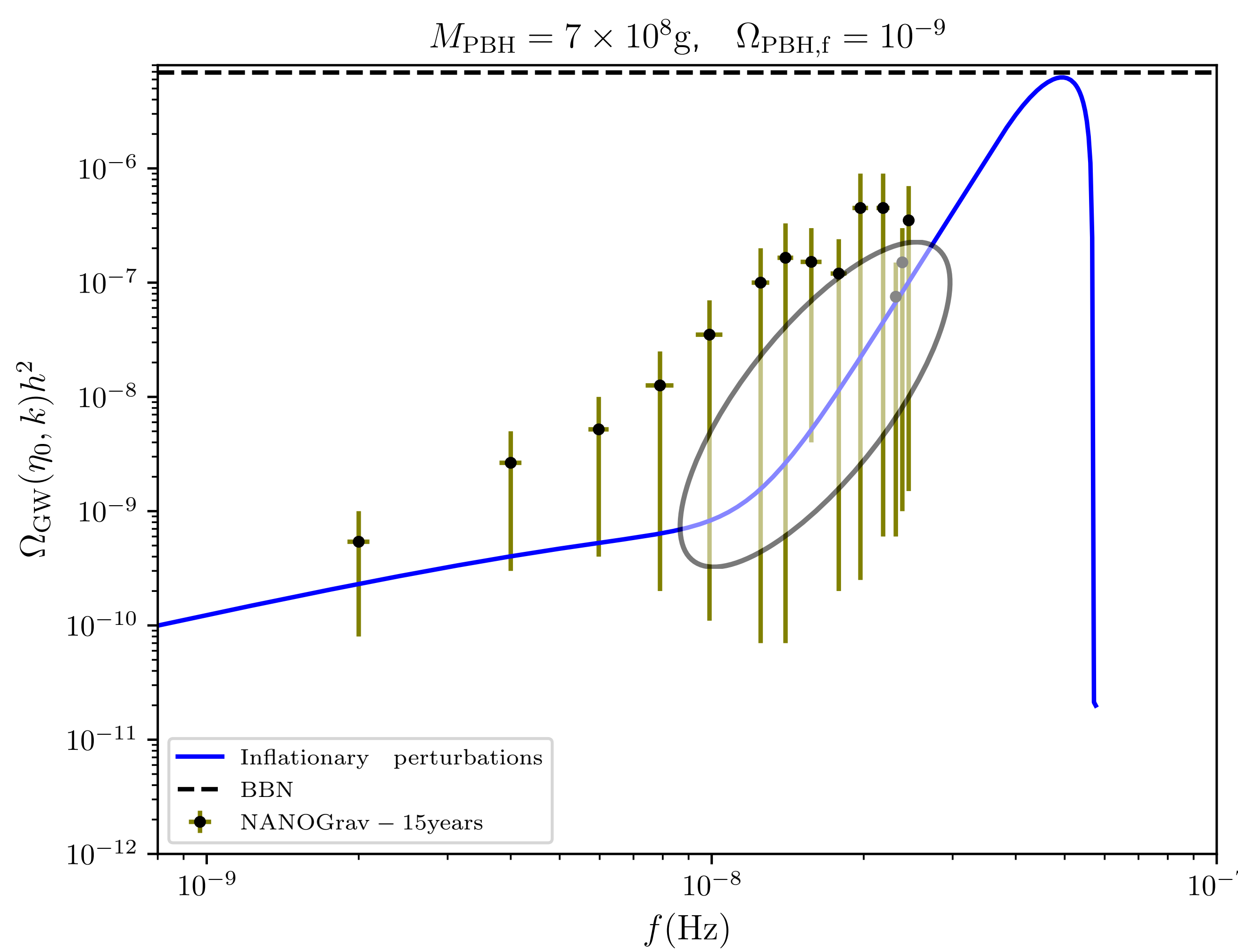

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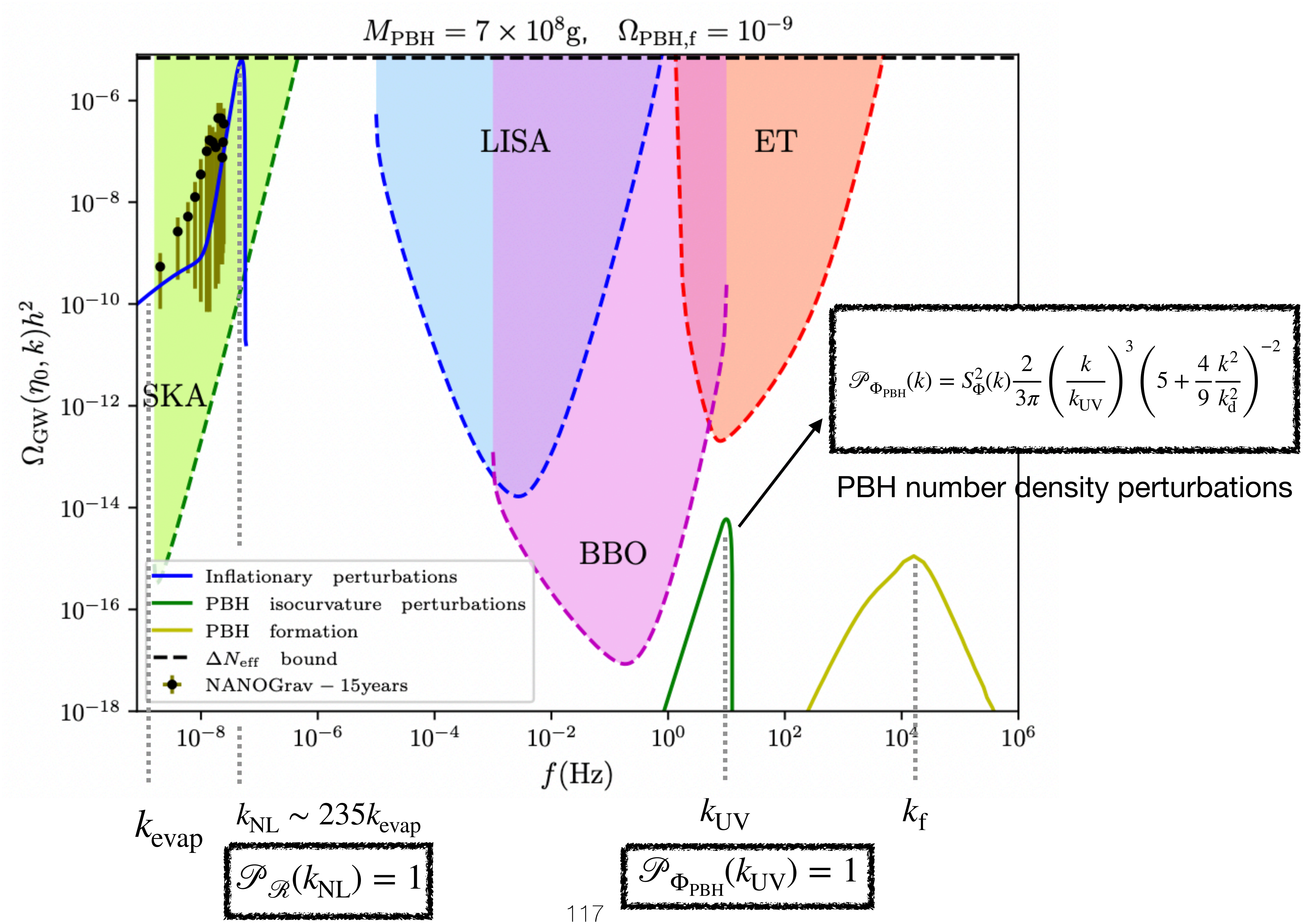
- **Resonantly enhanced GW production** sourced mainly by the $\mathscr{H}^{-2}\Phi'^2$ term in $S_{\vec{k}}$.

A distinctive three-peaked GW signal



Testing fundamental high energy physics theories with PBHs and induced GWs

- **Primordial black holes and gravitational waves from non-canonical inflation,** T. Papanikolaou, A. Lymperis, S. Lola, E.N. Saridakis, Published in: **JCAP 03 (2023) 003** • e-Print: 2211.14900 [astro-ph.CO]

- **Induced gravitational waves from flipped SU(5) superstring theory at nHz**, S. Basilakos, D. V. Nanopoulos, T. Papanikolaou, E.N. Saridakis, C. Tzerefos, **Phys. Lett. B 849 (2024) 138446**, e-Print: 2309.15820 [hep-th]

- **Revisiting string-inspired running-vacuum models under the lens of light primordial black holes**, T. Papanikolaou, C. Tzerefos, S. Basilakos, E.N. Saridakis, N. E. Mavromatos, **Phys. Rev. D 110 (2024) 2, 024055** • e-Print: 2402.19373 [gr-qc]

- **Observable Signatures of No-Scale Supergravity in NANOGrav**, S. Basilakos, D. V. Nanopoulos, T. Papanikolaou, E.N. Saridakis, C. Tzerefos, **JMPD (2024)** • e-Print: 2409.02936 [gr-qc]

- **Gravitational wave signatures from reheating in Cern-Simons running-vacuum cosmology**, S. Basilakos, C. Tzerefos, T. Papanikolaou, S. Basilakos, N.E. Mabromatos, e-Print: 2411.14223 [gr-qc]

Conclusions

- **PBHs** are associated with a **very rich phenomenology depending on their mass.**

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C. Fundamental High Energy Physics (HEP) theories

Thanks for your attention!

Appendix

Deriving $\mathscr{P}_\Phi(k)$

- The uniform energy density curvature perturbations $\zeta_{\rm PBH}$ and $\zeta_{\rm r}$:

$$\zeta_{\rm r} = -\Phi + \frac{1}{4}\delta_{\rm r}, \quad \zeta_{\rm PBH} = -\Phi + \frac{1}{3}\delta_{\rm PBH}$$

- The isocurvature perturbation

$$S = 3\left(\zeta_{\rm PBH} - \zeta_{\rm r}\right) = \delta_{\rm PBH} - \frac{3}{4}\delta_{\rm r}$$

- The comoving curvature perturbation $\mathscr{R}$

$$\mathscr{R} = \frac{2}{3}\frac{\Phi'/\mathscr{H} + \Phi}{1+w} + \Phi \simeq -\zeta \text{ for k} \ll \mathscr{H}$$

- When $w = 0$, $\Phi = \text{constant}$ and $\Phi' = 0$. Thus, for $k \ll \mathscr{H}$, $\mathscr{R} = -\zeta = \frac{5}{3}\Phi$.
- For $k \ll \mathscr{H}$, $\zeta = \zeta_{\rm PBH} = \zeta_{\rm r} + S/3 \simeq S/3 \simeq \delta_{PBH}(t_{\rm f})/3$. Thus, $\Phi = \delta_{\rm PBH}(t_{\rm f})/5$ for $k \ll \mathscr{H}$.
- For sub-horizon scales, i.e. $k \gg \mathscr{H}$,

$$\frac{d^2\delta_{\rm PBH}}{ds^2} + \frac{2+3s}{2s(s+1)}\frac{d\delta_{\rm PBH}}{ds} - \frac{3}{2s(s+1)}\delta_{\rm PBH} = 0 \Rightarrow \delta_{\rm PBH} = \frac{2+3s}{2+3s_{\rm f}}\delta_{\rm PBH}(t_{\rm f}) \text{ with s = a/a}_{\rm d}$$

- Knowing that $\delta_{\rm PBH} = -\frac{2}{3}\left(\frac{k}{aH}\right)^2\Phi$, one gets that $\Phi = -\frac{9}{4}\left(\frac{\mathscr{H}_{\rm d}}{k}\right)^2 \delta_{\rm PBH}(t_{\rm f}) \text{ for k} \gg \mathscr{H}$.

From the curvature power spectrum to the PBH mass function

$\mathscr{P}_\zeta(k)$ →

$$\sigma^2 = \frac{4(1+w)^2}{(5+3w)^2}\int_0^\infty \frac{\mathrm{d}k}{k}(kR)^4\tilde{W}^2(k,R)\mathscr{P}_\zeta(k)$$

$$\mu^2 = \frac{4(1+w)^2}{(5+3w)^2}\int_0^\infty \frac{\mathrm{d}k}{k}(kR)^4\tilde{W}^2(k,R)\mathscr{P}_\zeta(k)\left(\frac{k}{aH}\right)^2$$

$$\text{Peak Theory}: \mathscr{N}(\nu) = \frac{\mu^3}{4\pi^2}\frac{\nu^3}{\sigma^3}e^{-\nu^2/2},$$

$$\text{where } \nu \equiv \frac{\delta}{\sigma}$$

$$\beta_\nu = \frac{M_{\mathrm{PBH}}(\nu)}{M_{\mathrm{H}}}\mathscr{N}(\nu)\Theta(\nu-\nu_{\mathrm{c}})$$

$$M_{\mathrm{PBH}} = M_{\mathrm{H}}\mathscr{K}(\delta-\delta_{\mathrm{c}})^\gamma$$

[Niemeyer et al. - 1997]

$$\delta_{\mathrm{m}} = \delta_l - \frac{3}{8}\delta_l^2$$

[DeLuca et al., Young et al - 2019]

$$\beta(M) = \int_{\nu_{\mathrm{c}}}^{\frac{4}{3\sigma}} \mathrm{d}\nu \frac{\mathscr{K}}{4\pi^2}\left(\nu\sigma - \frac{3}{8}\nu^2\sigma^2 - \delta_{\mathrm{c}}\right)^\gamma \frac{\mu^3\nu^3}{\sigma^3}e^{-\nu^2/2},$$

where δ_{c} depending on the shape of $\mathscr{P}_\zeta(k)$

[Musco et al - 2020]

The PBH matter power spectrum

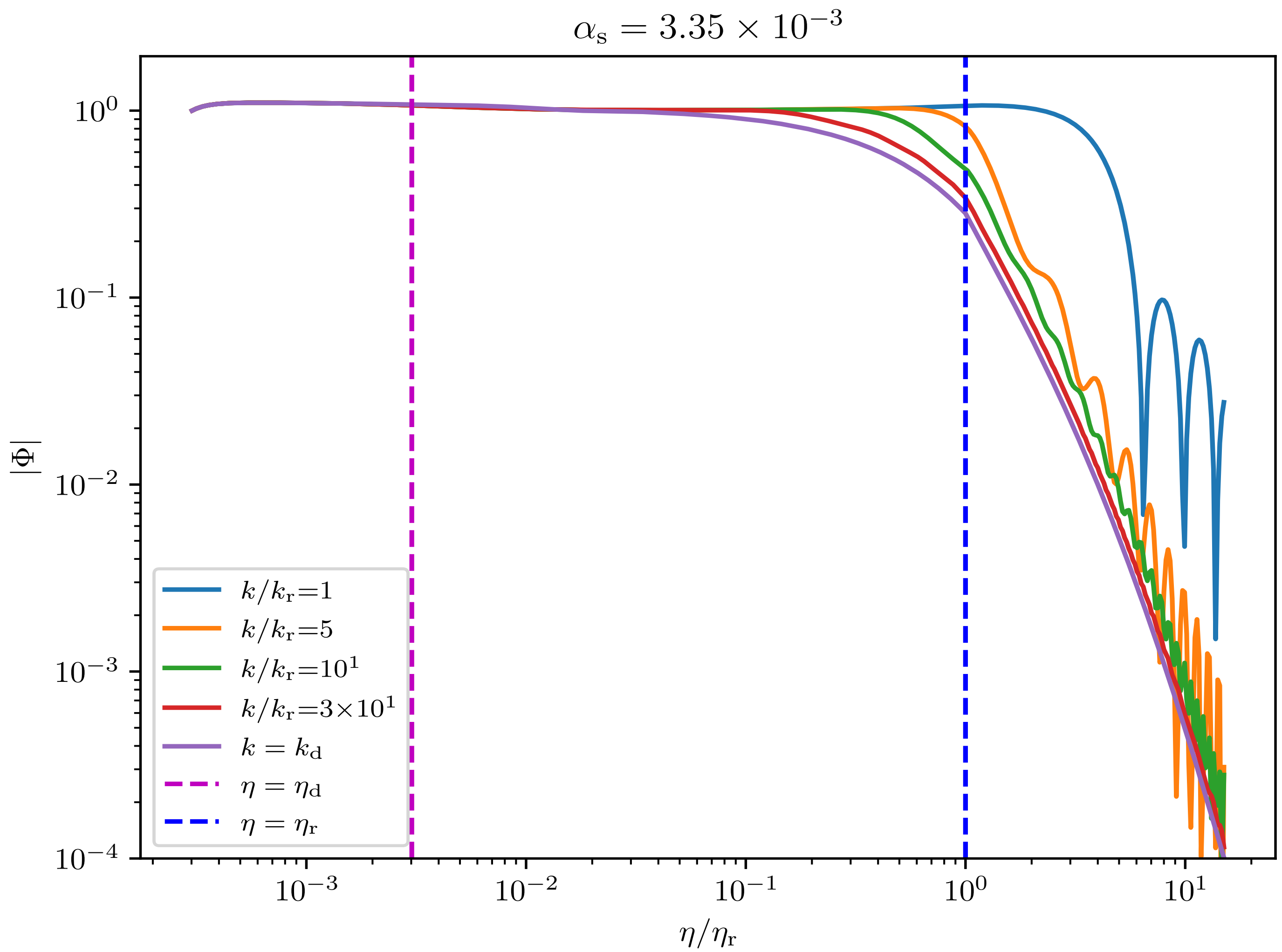
- In this case, we have a gas of **PBHs with different masses.** We should define a PBH mean separation scale accounting for the extended PBH mass distribution function.

$$\langle M\rangle(t) \equiv \frac{\int_{M_{\rm min}}^{M_{\rm max}} M\bar{\beta}(M,t)\left\{1-\frac{t-t_{\rm ini}}{\Delta t_{\rm evap}(M_{\rm f})}\right\}^{1/3}\mathrm{d}\ln M}{\int_{M_{\rm min}}^{M_{\rm max}}\bar{\beta}(M,t)\,\mathrm{d}\ln M} \Rightarrow \bar{r} = \left(\frac{3\langle M\rangle}{4\pi\rho_{\rm PBH}}\right)^{1/3}.$$

$$P_{\delta_{\rm PBH}}(k) \equiv \langle|\delta_k^{\rm PBH}|^2\rangle = \frac{4\pi}{3k_{\rm UV}^3},\quad \text{where}\quad k < k_{\rm UV} = \frac{a}{\bar{r}}$$

$$\mathscr{P}_\Phi(k) = S_\Phi^2(k)\frac{2}{3\pi}\left(\frac{k}{k_{\rm UV}}\right)^3\left(5+\frac{4}{9}\frac{k^2}{k_{\rm d}^2}\right)^{-2}$$

The gravitational potential Φ



The scales considered

$$a)\ \delta_{\mathrm{PBH},k} \propto a : \delta_{\mathrm{PBH},k_{\mathrm{NL}}}(\eta_r) = 1 \Rightarrow k_{\mathrm{NL}} = k_{\mathrm{UV}}^{3/7}\left(\frac{3\pi}{2}\right)^{1/7}\left(\frac{a_d}{a_r}\right)^{2/7}\left(\frac{4a_d^2}{9t_d^2}\right)^{2/7}$$

$b)$ Being quite conservative, we consider only modes $k \in [k_r, k_{\max}]$.

$$k_{\max} = \min[k_d, k_{\mathrm{NL}}] \Rightarrow \mathscr{P}_\Phi(k) = \frac{2}{75\pi}\left(\frac{k}{k_{\mathrm{UV}}}\right)^3$$

Primordial non-Gaussianities of local type

$$\xi_{\rm PBH}(\mathbf{x}_1, \mathbf{x}_2) \equiv \langle \delta_{\rm PBH}(\mathbf{x}_1)\delta_{\rm PBH}(\mathbf{x}_2)\rangle = \int \mathscr{P}_{\rm PBH}(k) e^{\mathbf{k}\cdot(\mathbf{x}_1-\mathbf{x}_2)} {\rm d}\ln k$$

$kR \ll 1$ $\quad$ $R \sim 1/k_{\rm f}$

$$\mathscr{P}_{\delta_{\rm PBH}}(k) \simeq \mathscr{P}_{\mathscr{R}}(k)\nu^4 \left(\frac{4}{9\sigma_R}\right)^4 \int \frac{{\rm d}^3p_1{\rm d}^3p_2}{(2\pi)^6} \tau_{\rm NL}(p_1,p_2,p_1,p_2) W^2_{\rm local}(p_1) W^2_{\rm local}(p_2) P_{\mathscr{R}}(p_1) P_{\mathscr{R}}(p_2)$$

$$+\frac{k^3}{2\pi^2}(k - \text{independent terms})$$

$\bar{\tau}_{\rm NL}$

$$\mathscr{P}_{\Phi}(k) = S^2_{\Phi}(k)\left(5 + \frac{4}{9}\frac{k^2}{k_{\rm d}^2}\right)^{-2}\left[\left(\frac{4\nu}{9\sigma_R}\right)^4 \bar{\tau}_{\rm NL}\mathscr{P}_{\mathscr{R}}(k) + \mathscr{P}_{\delta_{\rm PBH},{\rm Poisson}}(k)\right]$$

The non-Gaussian PBH matter power spectrum

Ansatz 1 : $\mathscr{P}_{\mathscr{R}} = \mathscr{P}_{\mathscr{R}}(k_{\rm f}) e^{-\frac{1}{2\sigma^2}\ln^2\left(\frac{k}{k_{\rm f}}\right)} + 2.2 \times 10^{-9} \left(\frac{k}{0.05{\rm Mpc}^{-1}}\right)^{0.965-1}$, with $\mathscr{P}_{\mathscr{R}}(\rm k_f) \simeq 10^{-2}$

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$$\text{Ansatz 2}: \tau_\mathrm{NL}(k_1,k_2,k_3,k_4) = \frac{\tau_\mathrm{NL}(k_\mathrm{f})}{6}\left[e^{-\frac{1}{2\sigma_\tau^2}\left(\ln^2\frac{k_1}{k_\mathrm{f}}+\ln^2\frac{k_2}{k_\mathrm{f}}\right)} + \text{ 5 perms}\right]$$

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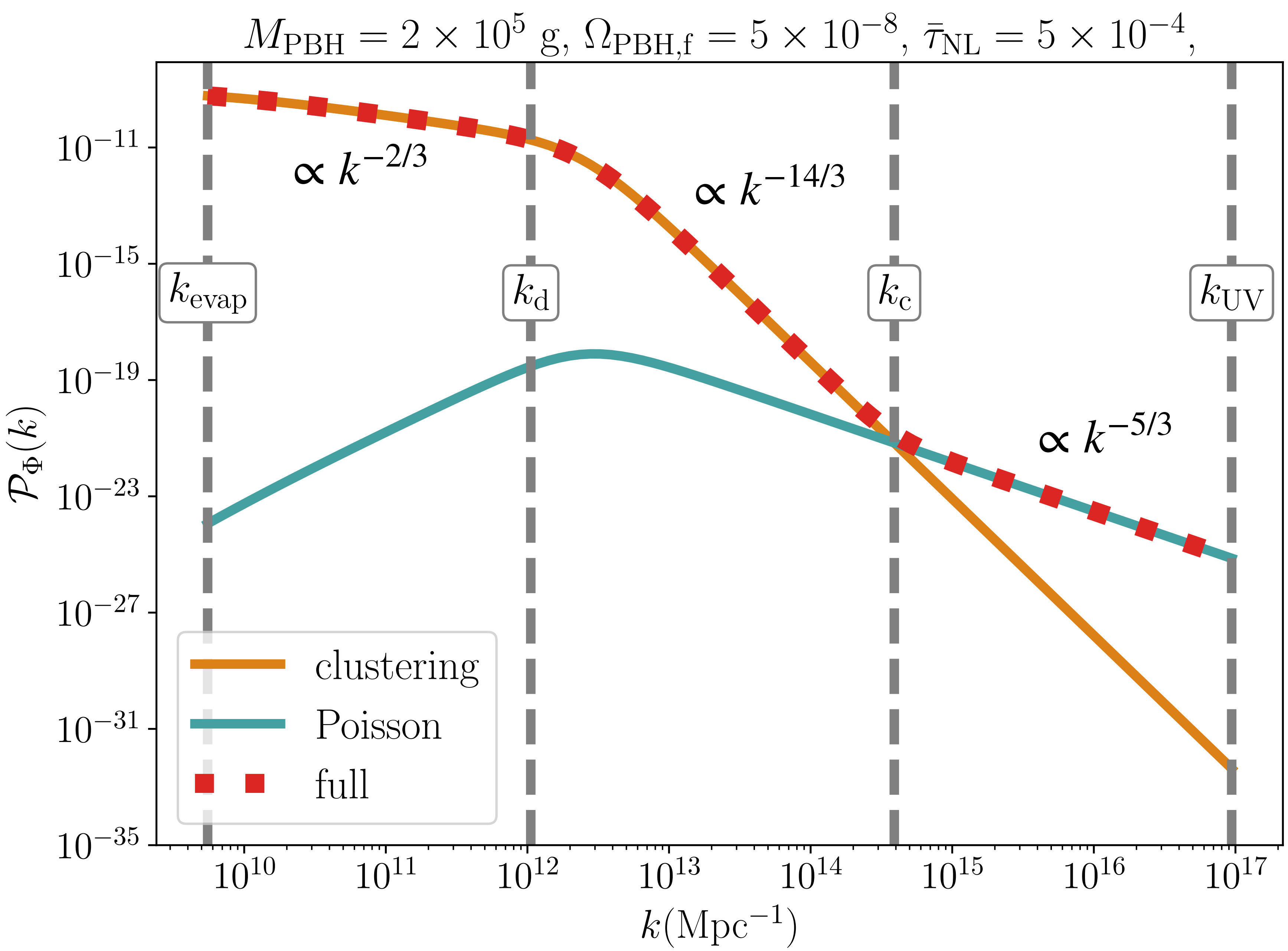

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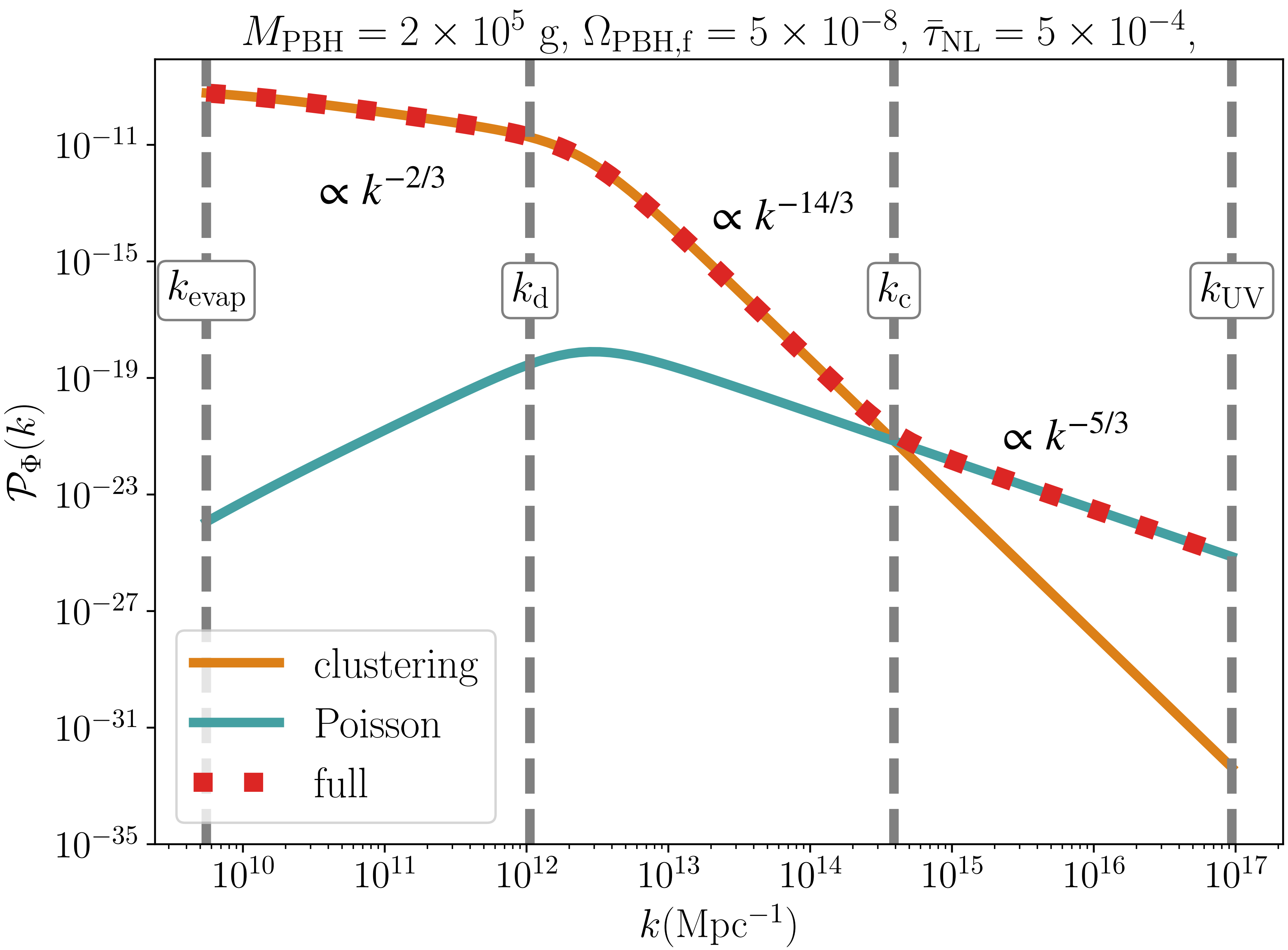

Scale Hierarchy : $10^5\mathrm{Mpc}^{-1} < k_{\rm evap} < k_{\rm d} < k_{\rm c} < k_{\rm UV} \ll k_{\rm f} \sim 1/R$

Hierarchy of scales

$$\frac{k_{\rm UV}}{k_{\rm f}} = \left(\frac{\Omega_{\rm PBH,f}}{\gamma}\right)^{1/3}, \frac{k_{\rm d}}{k_{\rm f}} = \sqrt{2}\Omega_{\rm PBH,f},$$

$$\frac{k_{\rm evap}}{k_{\rm f}} = \left(\frac{3.8 g_* \Omega_{\rm PBH,f}}{960\gamma}\right)^{1/3} \left(\frac{M_{\rm PBH}}{M_{\rm Pl}}\right)^{-2/3}.$$

$$\Omega_{\rm GW}(\eta_0, k_{\rm UV}) < 10^{-6} \text{ (BBN GW bound) : } \Omega_{\rm PBH,f} \lesssim 10^{-6} \left(\frac{M_{\rm PBH}}{10^4 {\rm g}}\right)^{-17/24}$$

$$\text{Early PBH domination : } \Omega_{\rm PBH,f} > 6 \times 10^{-10} \frac{10^4 {\rm g}}{M_{\rm PBH}}$$

$$\text{PBH mass range : } 1{\rm g} < M_{\rm PBH} < 10^9 {\rm g}$$

↓

$$\text{Scale Hierarchy : } 10^5 {\rm Mpc}^{-1} < k_{\rm evap} < k_{\rm d} < k_{\rm c} < k_{\rm UV} \ll k_{\rm f} \sim 1/R$$

Cosmological motivation for no-scale SUGRA

No-scale SUGRA models have some very **attractive features**:

1. They naturally give rise to **Strarobinsky-like inflationary models,** favoured by Planck [J.R. Ellis et al. - 2013, C. Kounnas et al. - 2015].

2. The **inflationary energy scale being naturally much smaller than** $M_{\rm Pl}$ [J. R. Ellis, D. V. Nanopoulos, K. A. Olive and K. Tamvakis - (1982)] consistent with a very small $r < 0.004$.

3. The inflaton can be viewed as a singlet field in a see-saw mechanism responsible for the **generation of neutrino-masses** [Murayama et al. - 1993], providing as well an efficient scenario for **reheating and leptogenesis** [M. Fukugita and T. Yanagida - 1986].

4. One is naturally met with a **vanishing cosmological constant** at the tree level [E. Cremmer, S. Ferrara, C. Kounnas and D. V. Nanopoulos - 1983].