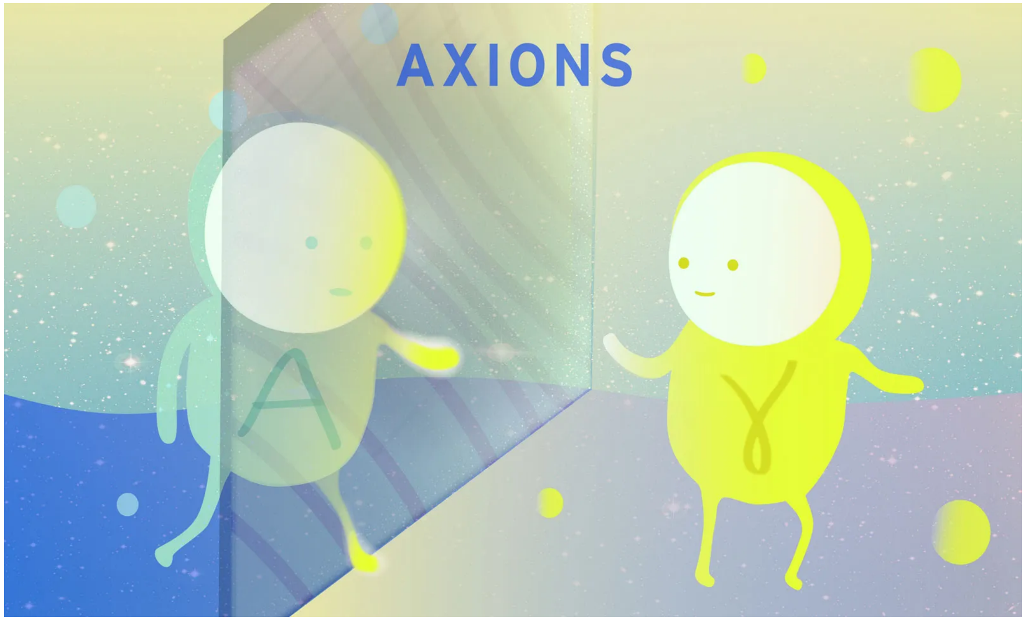
Revisited axion contribution to dark radiation using momentum-dependent evolution

Maxim Laletin

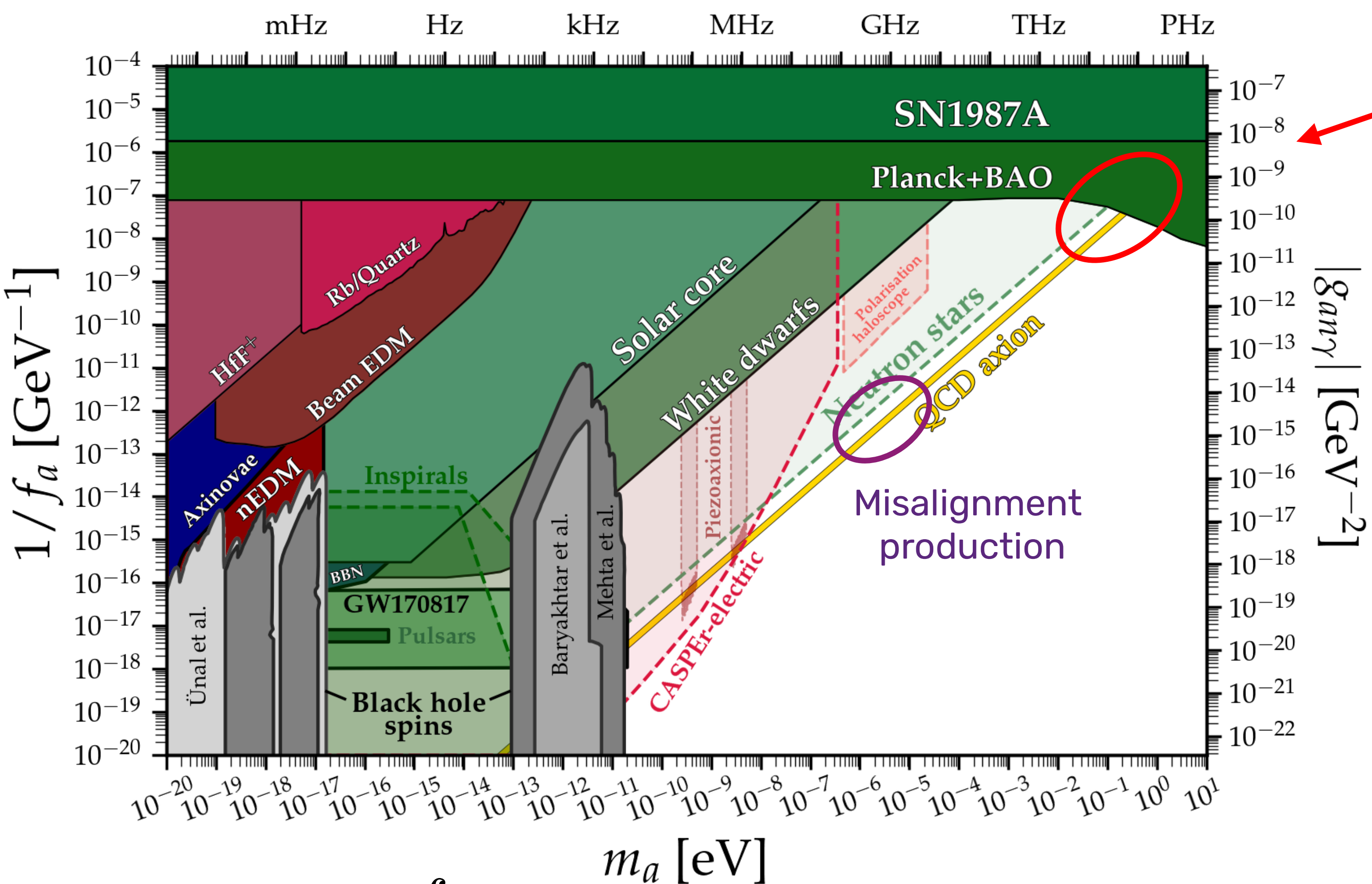


Based on **2410.18186**
with **M. Badziak**

Seminar @ FUW

5/12/2024

QCD axion and thermal production



Efficient production from the interactions in the early Universe plasma (thermal)

$$\Gamma \propto \frac{1}{f_a^2}$$

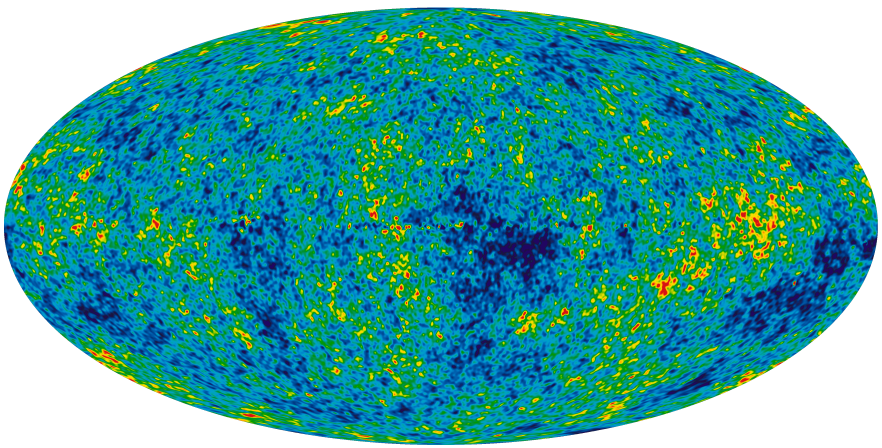
Such axions can’t be DM because they are too hot

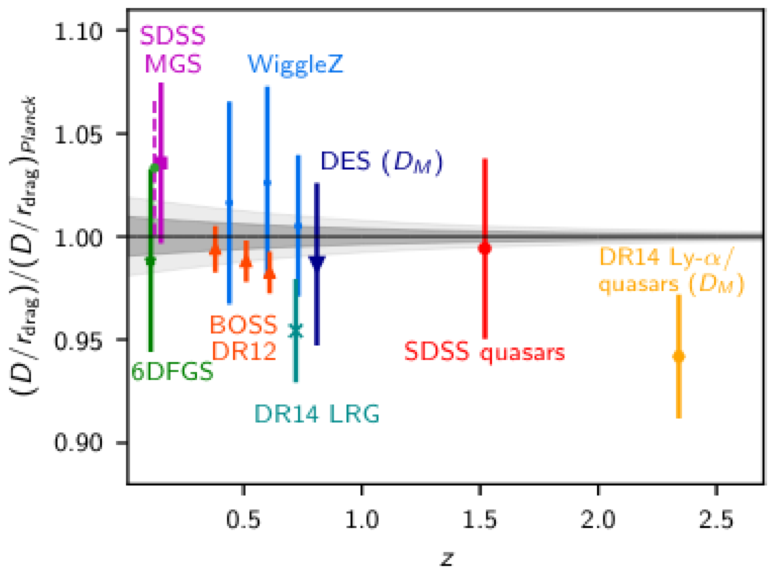
But their abundance can put constraints on the models

$$m_a = \frac{m_\pi f_\pi}{2 f_a}$$

CMB and baryon acoustic oscillations

Additional radiation energy density affects the CMB spectrum (especially low-l mode polarization) and baryon acoustic oscillation (BAO) geometry



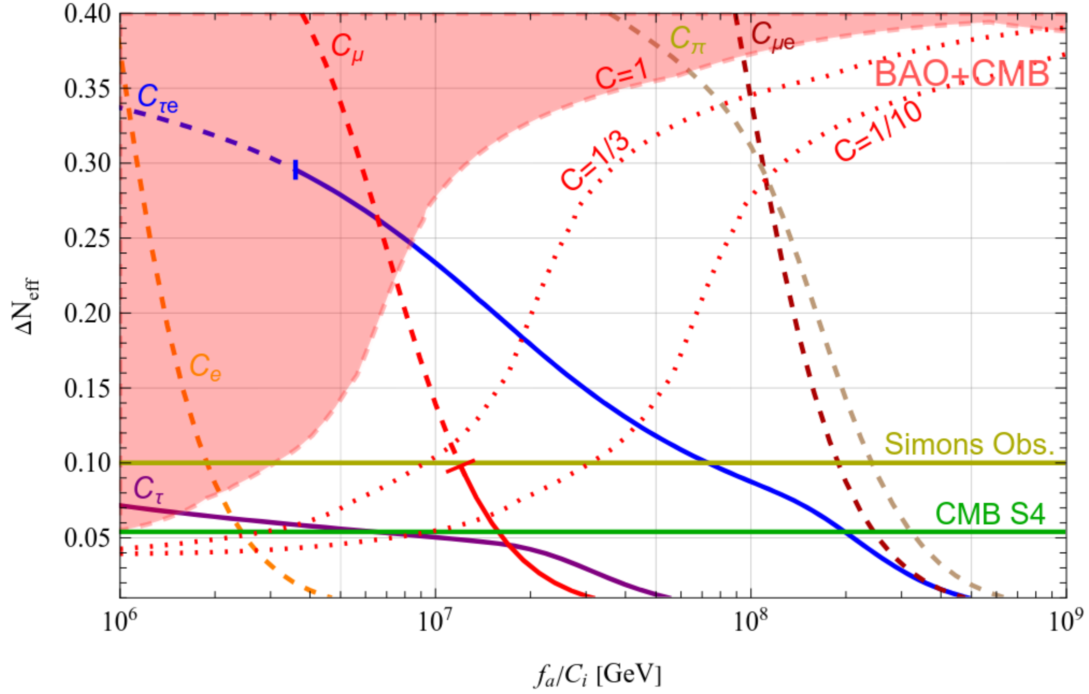


Can be effectively represented as $\Delta N_{\rm eff} \propto \rho_a$ $\Delta N_{\rm eff} \propto n_a^{4/3}$

Planck constraint $\Delta N_{\rm eff} \leq 0.3 \; (95\% \text{ C.L.})$

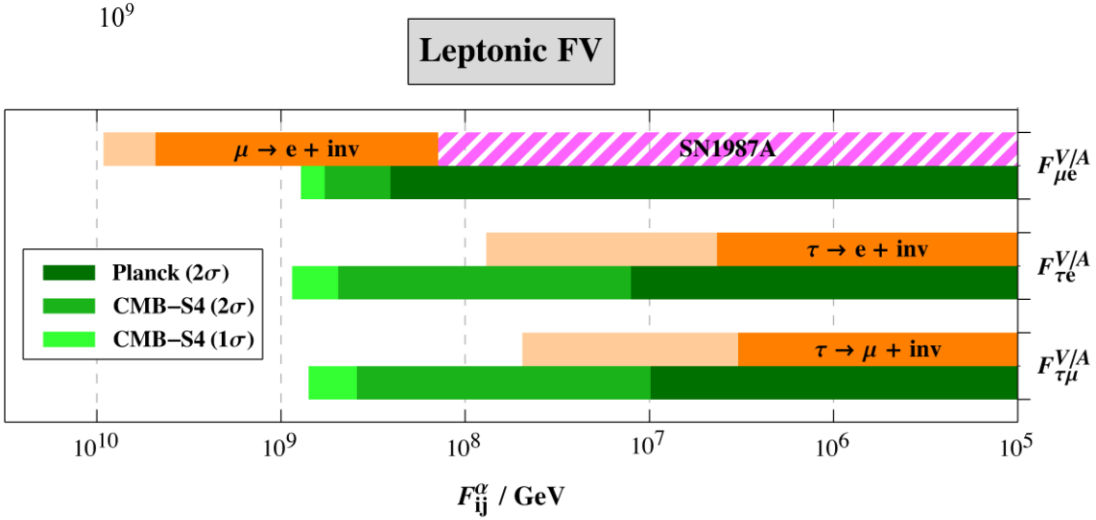
PLANCK, 1807.06209

Constraints on axion interactions with leptons



The abundance of axions is calculated using the Boltzmann equation for the axion number density

Badziak+, 2403.05621



D'Eramo+, 2111.12108

Standard approach

Solving the Boltzmann equation for the density of axions in the expanding Universe

$$\frac{dn}{dt} + 3Hn = \Gamma_+ - \Gamma_-$$

$$\begin{cases} \Gamma_+ = n_{\rm eq}^2 \langle \sigma v \rangle & \text{annihilations} \\ \Gamma_+ = n_{\rm eq}/\tau & \text{decays} \end{cases}$$

The inverse rates Γ_- are functions of *n*

$$x = \frac{m}{T}$$

$$Y = \frac{n}{s}$$

$$\Rightarrow$$

$$\frac{dY}{dx} = \frac{(1+\tilde{g})}{sHx}\left[\Gamma_+ - \Gamma_-(Y)\right]$$

$$\tilde{g} = \frac{1}{3}\frac{d\ln h_s}{d\ln x}$$

Axion interactions

At tree-level axions have interaction vortices with the SM particles with just one branch

$$\mathcal{L}_{\rm int}^{(a)} = \frac{1}{2f}\,\partial_\mu a\, J_a^\mu + \frac{a}{f}\sum_X \frac{\alpha_X}{8\pi} C_{XX} X_{\mu\nu}\tilde{X}^{\mu\nu}$$

Lagrangian has to satisfy the shift symmetry $a \to a + 2\pi$

Thus, the rates of all the reactions involving two axions are supressed by $(1/f_a)^2$ factor

Boltzmann equation for axions

Thus, we can write the Boltzmann equation for axions in the early Universe in the following form

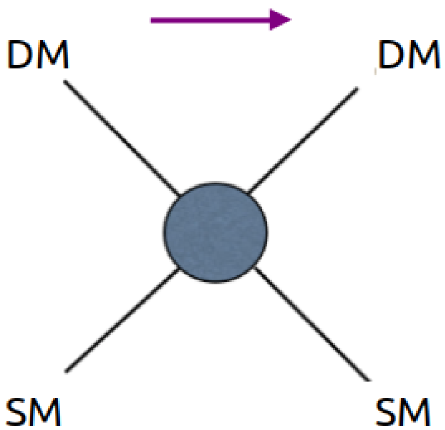
$$\frac{dY}{dx} = \frac{(1+\tilde{g})}{sHx} \sum_i \gamma_i \left[1 - \frac{Y}{Y_{\rm eq}} \right] \begin{cases} \gamma_i = n_{\rm eq}^{(i)} / \tau & \text{Decay} \\ \gamma_i = n_{\rm eq}^{(i)} n_{\rm eq}^{(j)} \langle \sigma v \rangle & \text{(co)annihilation/ scattering} \end{cases}$$

However, this approach is based on the assumptions:

1. Axions are in kinetic equilibrium with the SM
2. Equilibrium distributions have the Maxwell-Boltzmann shape

Kinetic equilibrium

Scatterings on the SM plasma particles maintain kinetic equilibrium



$$f_\chi = \frac{n_\chi}{n_\chi^{\rm eq}} \exp\left(-E/T_{\rm SM}\right)$$

For WIMPs:

Decoupling from chemical equilibrium (freeze-out) x ~ 25
Decoupling from kinetic equilibrium x >> 100

However, axion elastic scatterings are suppressed by $(1/f_a)^2$

The *shape* of axion energy distribution is <u>determined by other processes</u>

Full Boltzmann equation

To take everything into account we need to solve the Boltzmann equation for the distribution function

$$2E_i\left(\partial_t - H\,p\partial_p\right) f_i(p) = C\left[f_i\right]$$

If all the particles in reactions have equilibrium-shaped distributions we reproduce the standard Boltzmann equation for the density by integrating over $g_i \int \frac{d^3p_i}{(2\pi)^3}$

$$\frac{g_i}{s}\int \frac{d^3p_i}{(2\pi)^3} f_i = Y \qquad \frac{dY}{dx} = \frac{(1+\tilde{g})}{sHx}\sum_i \gamma_i\left[1 - \frac{Y}{Y_{\rm eq}}\right]$$

Integral of the collision term C[f]

Contribution to dark radiation (ΔN_{eff})

Hot axions at late stages can cause deviations in the CMB data. Can be parameterized via the effective number of relativistic degrees of freedom ΔN_{eff}

$$\Delta N_{\mathrm{eff}} = \frac{8}{7}\left(\frac{11}{4}\right)^{4/3}\frac{\rho_a}{\rho_\gamma} \qquad \rho_a = \frac{g_a}{2\pi^2}\int dE_a\, E_a^3\, f_a(E_a)$$

Simple formula, assuming axions in thermal equilibrium

$$\Delta N_{\mathrm{eff}} = \frac{4}{7}\left(\frac{11}{4}\cdot\frac{2\pi^4 h_s(x)}{45\zeta(3)}Y_a\right)^{4/3} \approx 74.85\, Y_a^{4/3}$$

Contribution to dark radiation (ΔN_{eff})

The simple formula misses the fact that even if axions have a thermal shape of the distribution, their abundance is not equilibrium

$$f_a = \frac{A}{\exp(E/T) - 1} \qquad A \equiv n_a / n_a^{\rm eq}$$ (normalization factor)

$$n_a = A \cdot g_a \frac{\zeta(3)}{\pi^2} T^3 \,, \qquad \rho_a = A \cdot g_a \frac{\pi^2}{30} T^4$$

$$\rho_a = \frac{g_a}{A^{1/3}} \cdot \frac{\pi^2}{30} \left(\frac{\pi^2 n_a}{\zeta(3) g_a} \right)^{4/3}$$

Simple formula underestimates the ΔN_{eff} by a factor $A^{1/3}$

Lepton-flavour violating decays

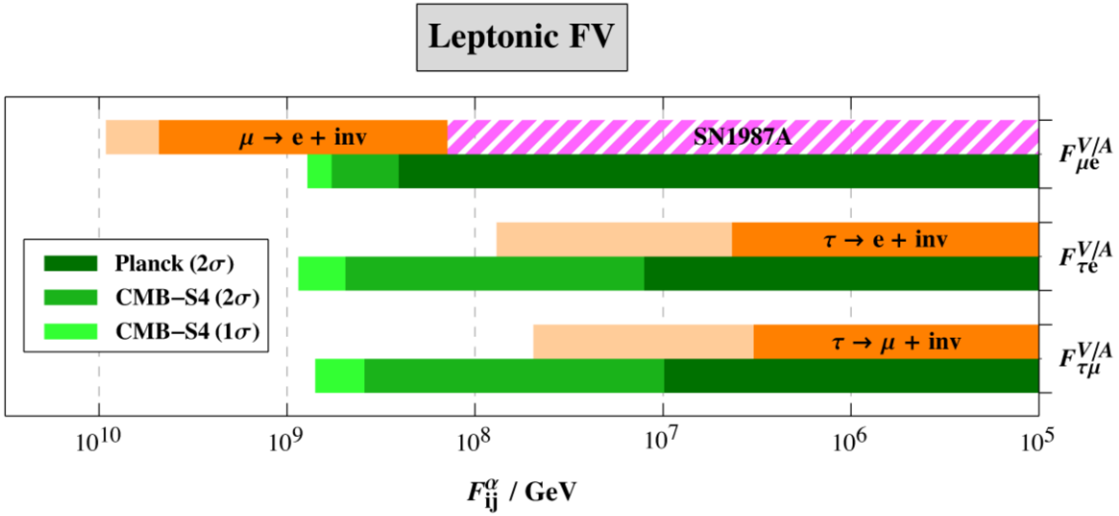
Axions can be produced via LFV decays

$$\mathcal{L}_{\text{eff}} = \frac{\partial_\mu a}{2 f_a} \bar{f}_i \gamma^\mu \left(C^V_{f_i f_j} + C^A_{f_i f_j} \gamma_5 \right) f_j - \frac{m_a^2}{2} a^2$$

Causing $l_i^\pm \to l_j^\pm + a$

D'Eramo+, 2111.12108

We concentrate on tau decays as muon decays are severely constrained by laboratory measurements and SN1987A



Collision term for decay (j → i+a)

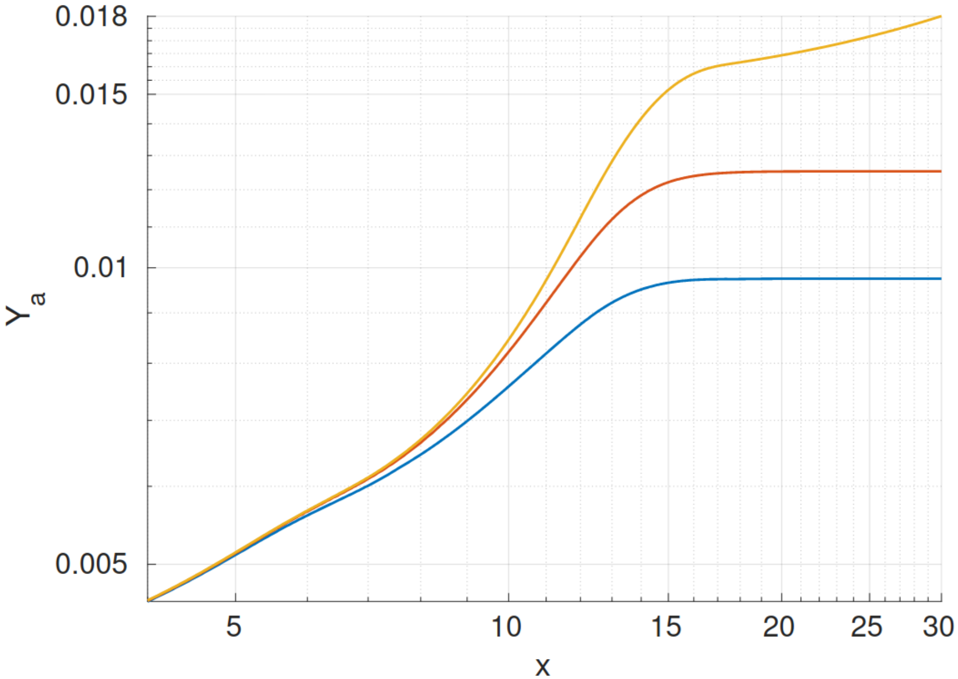
$$C[f_a] = \frac{1}{2g_a E_a}\Big[\int \frac{d^3p_j}{(2\pi)^3 2E_j}\int \frac{d^3p_i}{(2\pi)^3 2E_i}(2\pi)^4\delta^{(4)}(\mathcal{P}_j-\mathcal{P}_i-\mathcal{P}_a)\,|M|^2_{j\to ia}\, f_j(1\pm f_a)(1\pm f_i)$$
$$- \frac{d^3p_j}{(2\pi)^3 2E_j}\int \frac{d^3p_i}{(2\pi)^3 2E_i}(2\pi)^4\delta^{(4)}(\mathcal{P}_j-\mathcal{P}_i-\mathcal{P}_a)\,|M|^2_{j\leftarrow ia}\, f_i f_a(1\pm f_j)\Big]$$

Without any assumptions, this expression simplifies to an analytical formula (due to the simplicity of the amplitude squared and the kinematics of the 2-body decay)

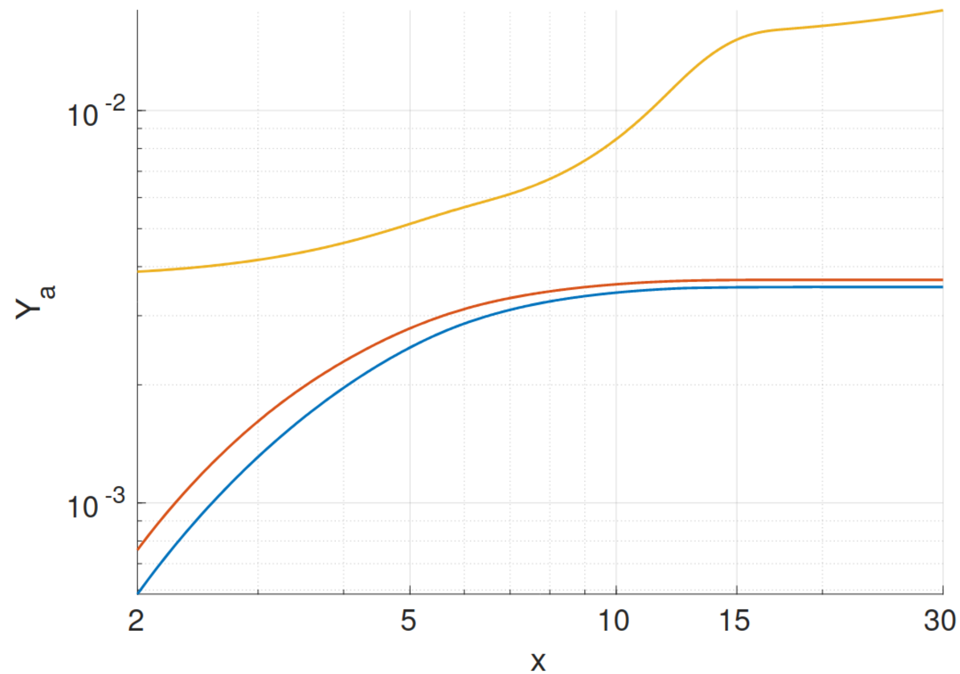
$$C[f_a] = \frac{x|\mathcal{M}|^2}{8\pi q^2}\log\left[\frac{1+\exp(-\epsilon_1(x,q))}{1+\exp(-\epsilon_2(x,q))}\right](f_a - f_a^{\mathrm{eq}})$$

$$x = m_\tau/T \qquad q = p/T$$

Results for the abundance Y



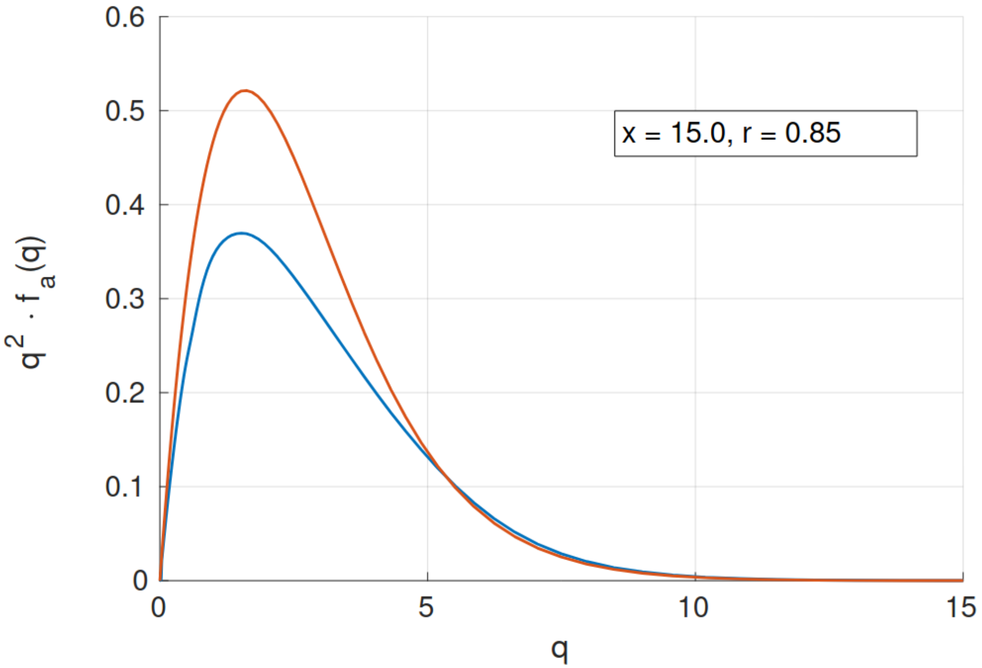
$$f/C_{\tau\mu} = 10^7\,\mathrm{GeV}$$



$$f/C_{\tau\mu} = 2 \cdot 10^8\,\mathrm{GeV}$$

nBE solution
fBE solution
Equilibrium abundance

Axion distribution functions



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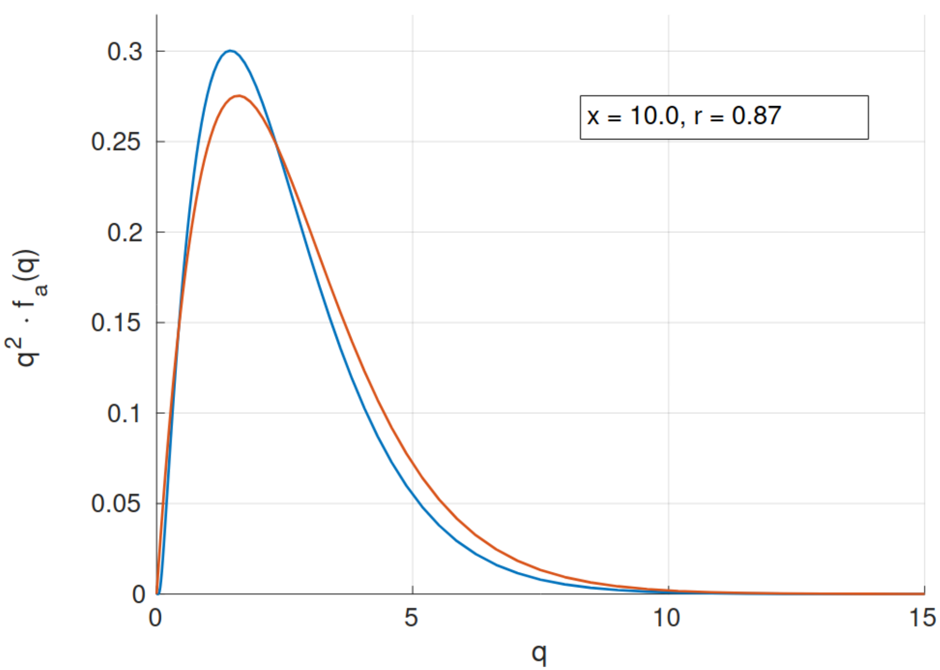

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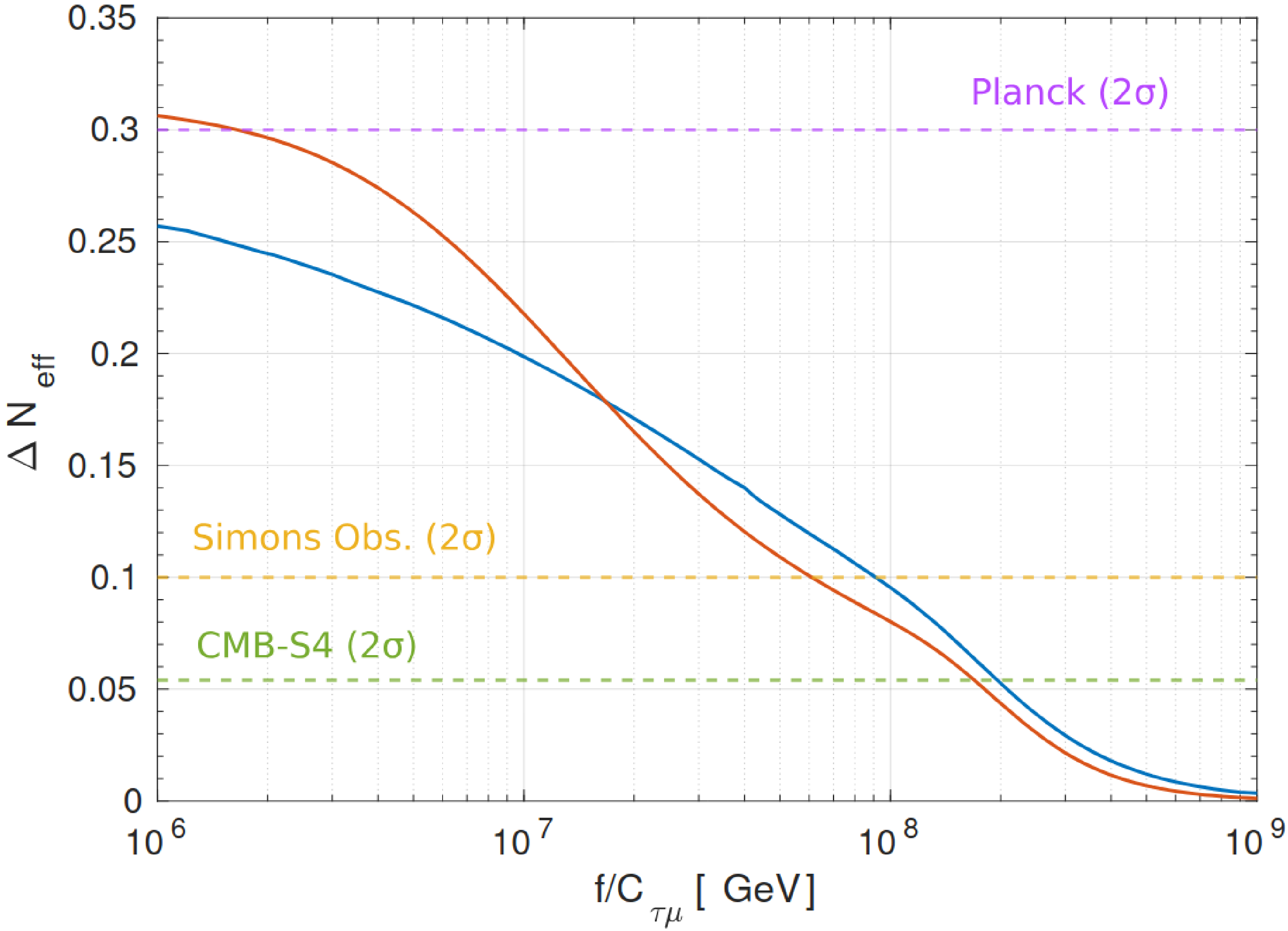
Equilibrium shape (Bose-Einstein)
Realistic shape

Difference in the ΔN_{eff}



Diagonal interactions with muons

$$\mu^+\mu^- \to \gamma a$$

Annihilation of leptons into axion

$$\mu^\pm\gamma \to \mu^\pm a$$

Primakoff scattering

These processes in the early Universe can be *a probe of the axion coupling* to muons.

- Electron coupling is tightly constrained by XENONnT and white dwarf luminosity function XENON, 2207.11330 Bertolami+, 1406.7712
- Muon (and tau) couplings are less constrained (by SN1987A)

Caputo+, 2109.03244

$$\frac{f_a}{|C_\mu|} \gtrsim 1.2 \times 10^7 \,\mathrm{GeV}$$

Collision term for i + j → k + a

$$C[f_a] = \frac{1}{2g_a E_a}\Big[\int d\Pi_k d\Pi_i d\Pi_j\, (2\pi)^4 \delta^{(4)}(\mathcal{P}_i + \mathcal{P}_j - \mathcal{P}_a - \mathcal{P}_k)\, |M|^2_{ij\to ak}\, f_i f_j (1+f_a)(1\pm f_k)$$
$$- \int d\Pi_k d\Pi_i d\Pi_j\, (2\pi)^4 \delta^{(4)}(\mathcal{P}_i + \mathcal{P}_j - \mathcal{P}_a - \mathcal{P}_k)\, |M|^2_{ij\leftarrow ak}\, f_a f_k (1-f_j)(1-f_i) \Big]$$

With equilibrium condition

$$f_a f_k^{\rm eq}(1\pm f_i^{\rm eq})(1\pm f_j^{\rm eq}) = \frac{f_a}{f_a^{\rm eq}} f_i^{\rm eq} f_j^{\rm eq}(1+f_a^{\rm eq})(1\pm f_k^{\rm eq})$$

Simplifies to $$C[f_a] = \frac{1}{2g_a E_a}\left(1 - \frac{f_a}{f_a^{\rm eq}}\right)\gamma_{\rm ann}$$

$$\gamma_{\rm ann} = \int d\Pi_k d\Pi_i d\Pi_j\, (2\pi)^4 \delta^{(4)}(\mathcal{P}_i + \mathcal{P}_j - \mathcal{P}_a - \mathcal{P}_k)\, |M|^2_{ij\to ak}\, f_i^{\rm eq} f_j^{\rm eq}(1\pm f_k^{\rm eq})$$

Collision term for i + j → k + a

General expression for the differential rate

$$\gamma_{ij\to ak} = \frac{1}{p_a} \int dE_k \, \frac{(1 \pm f_k(E_k))}{16\,(2\pi)^4} \int \frac{ds}{p_k^* \sqrt{s}} \int dt \, |\mathcal{M}|^2 \int d\cos\phi \, \frac{f_i^* \cdot f_j^*}{\sqrt{1 - \cos\phi^2}}$$

Can be simplified by using Maxwell-Boltzmann distributions

$$\gamma_{ij\to ak} = \frac{\exp(-q)}{(2\pi)^2} \int dE_k \; E_k \; f_k(E_k) \int ds \; \sigma_{ak\to ij}(s) \; v_{\mathrm{Mol}}$$

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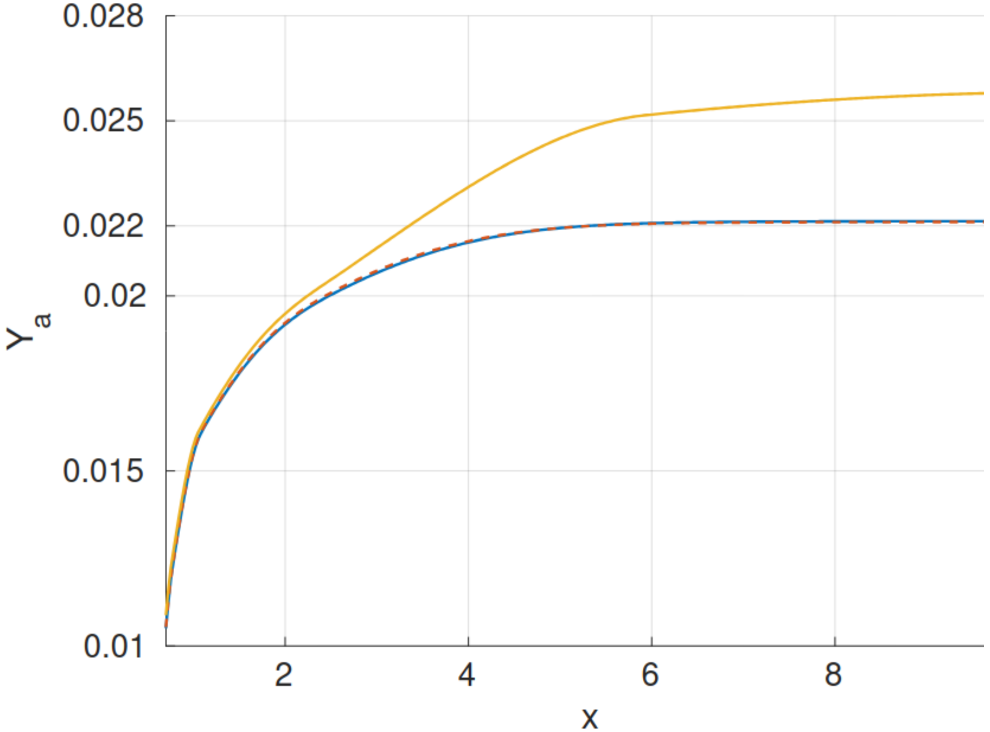

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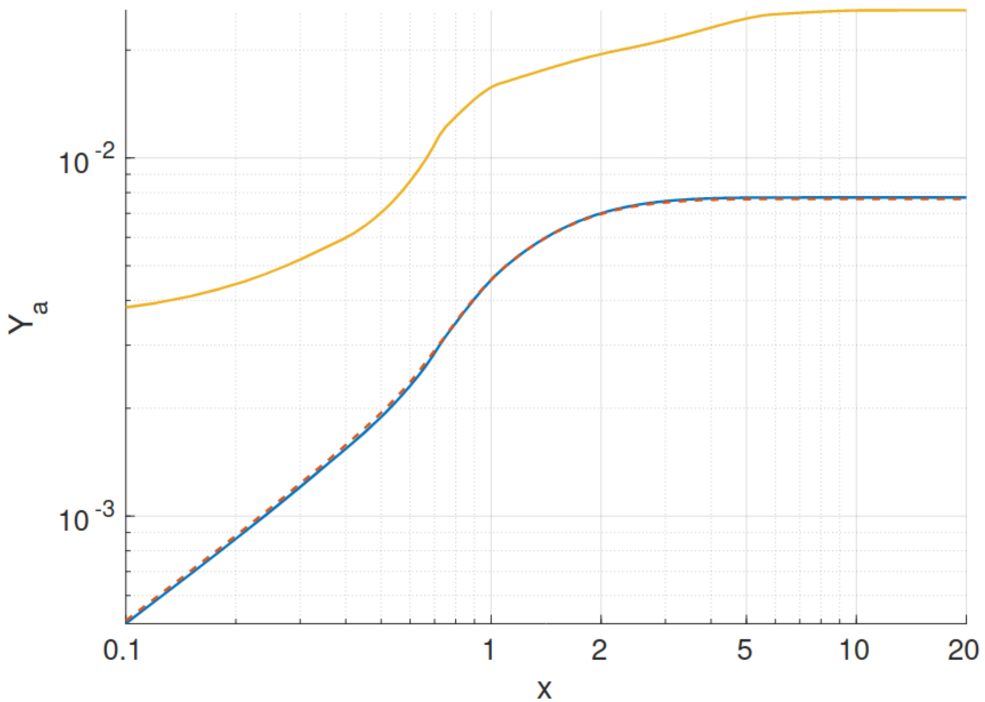

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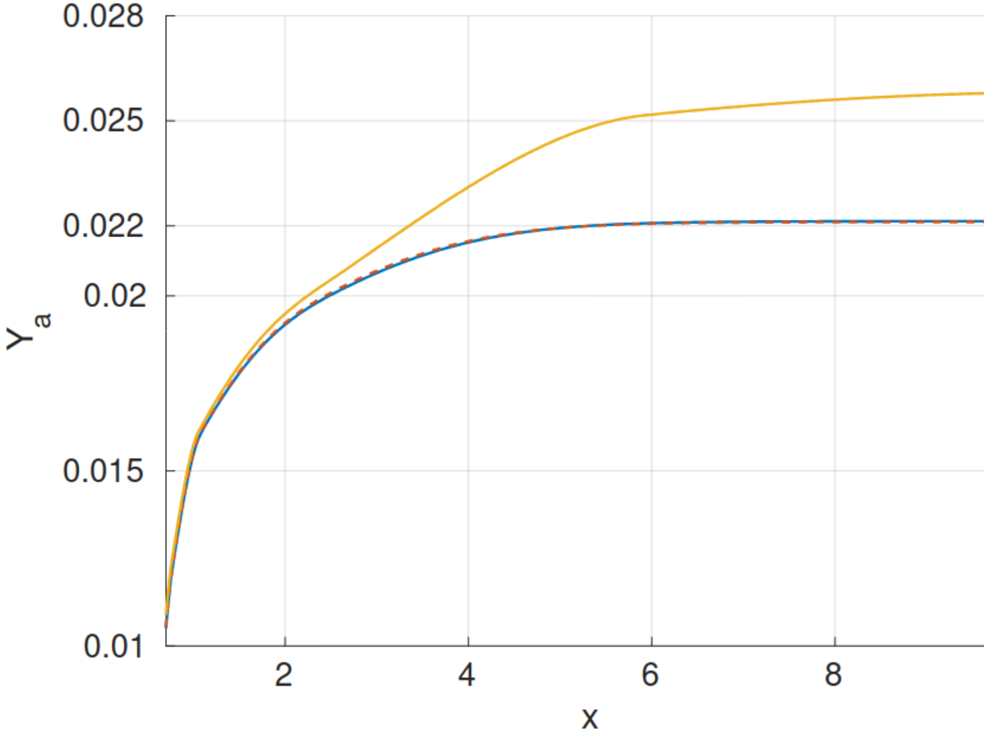

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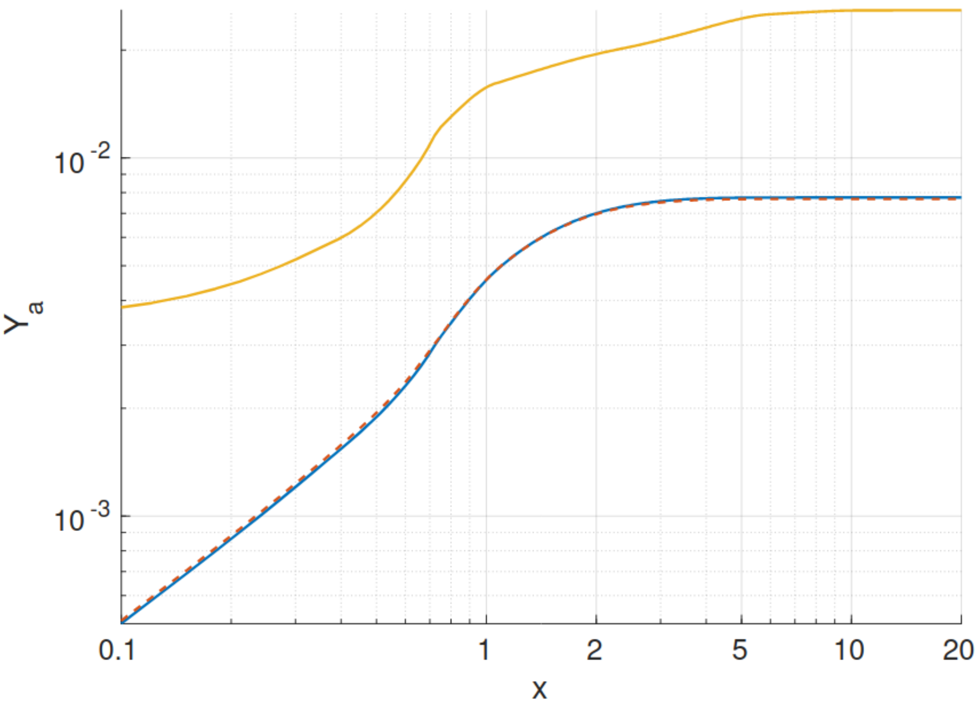

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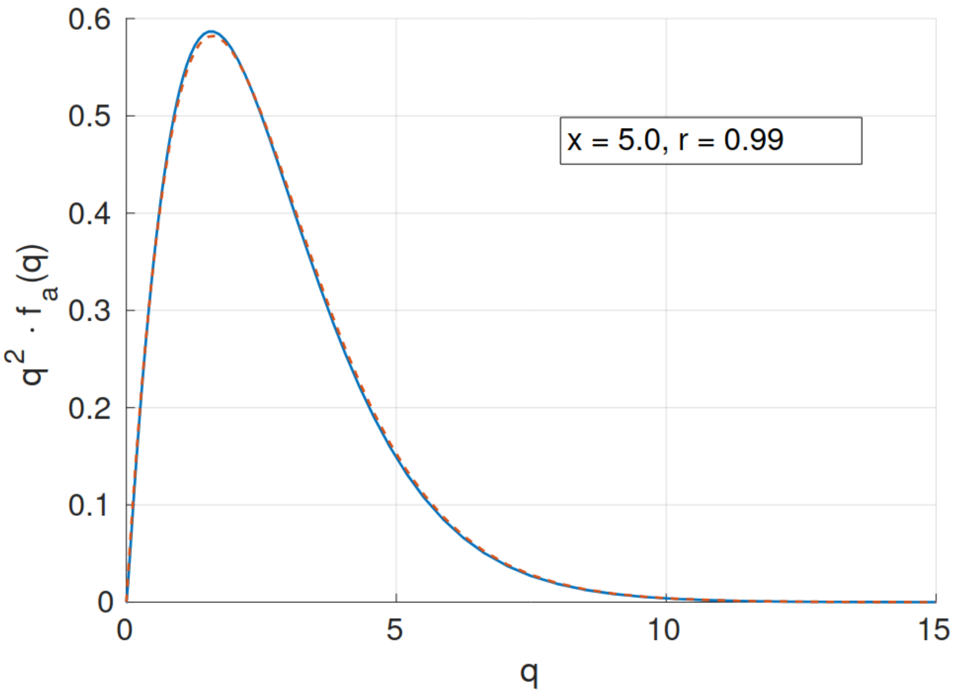

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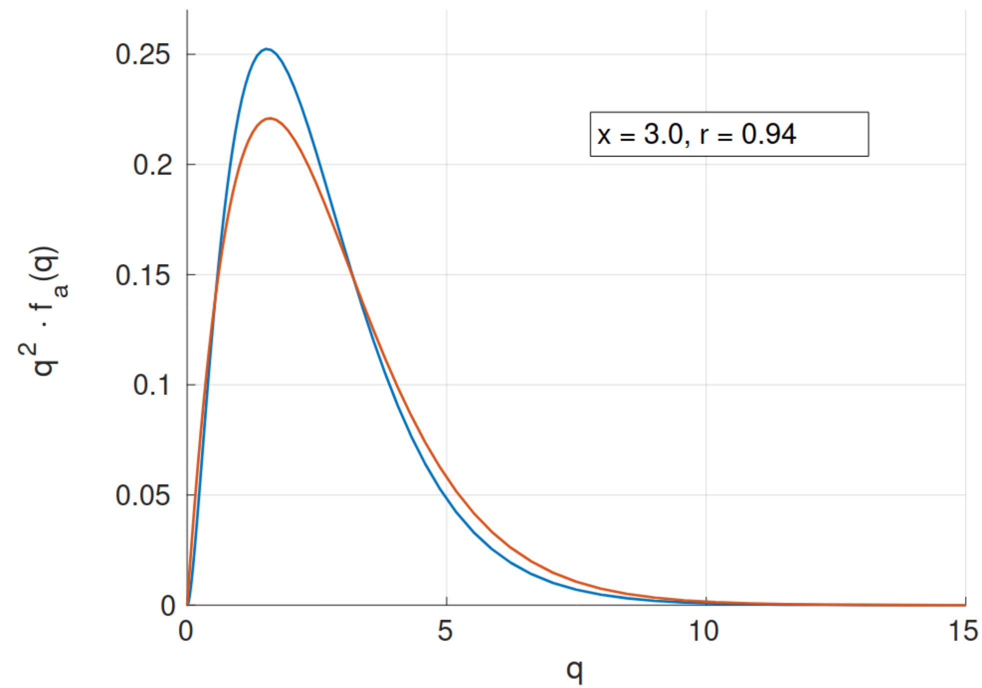

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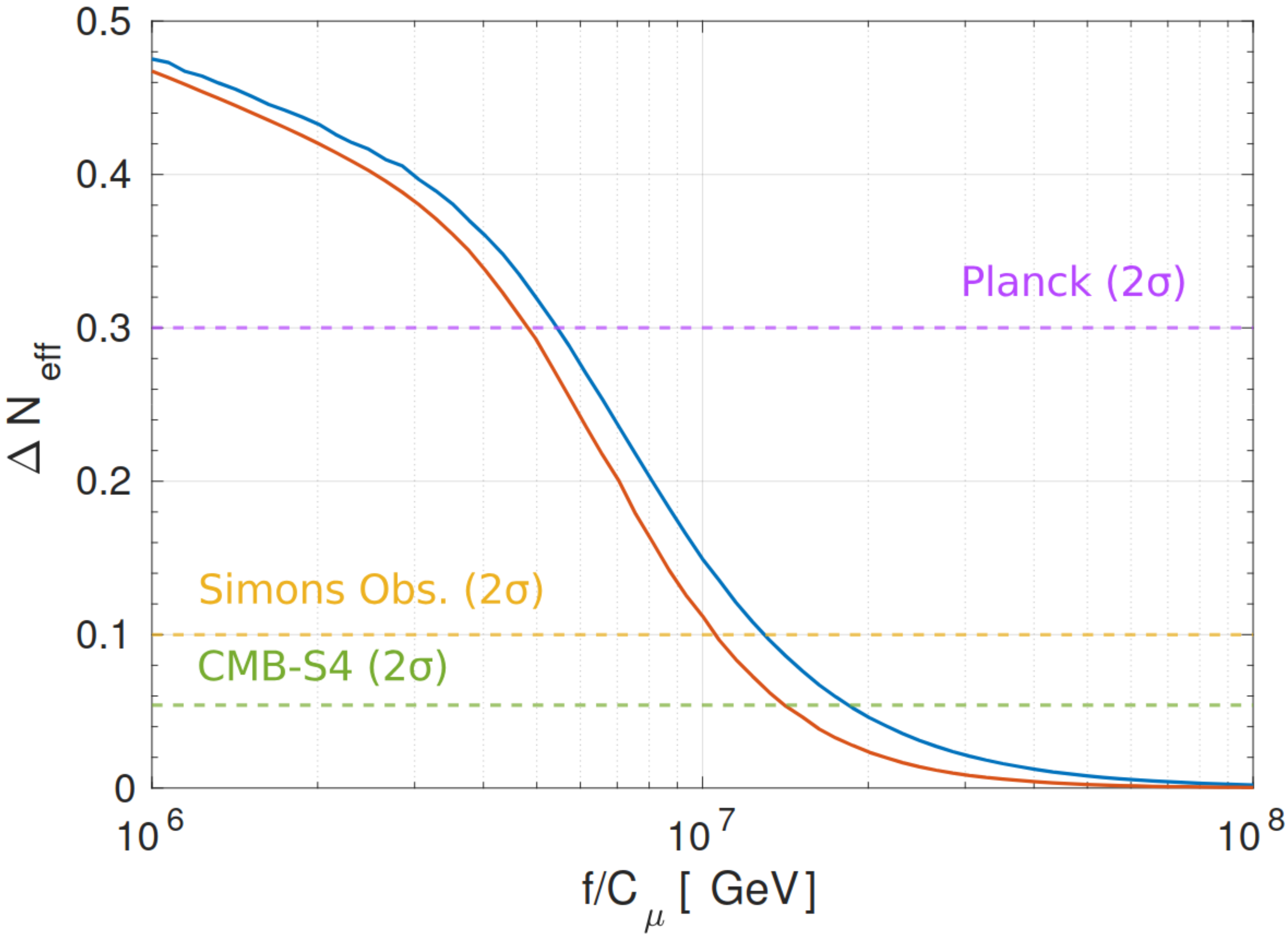
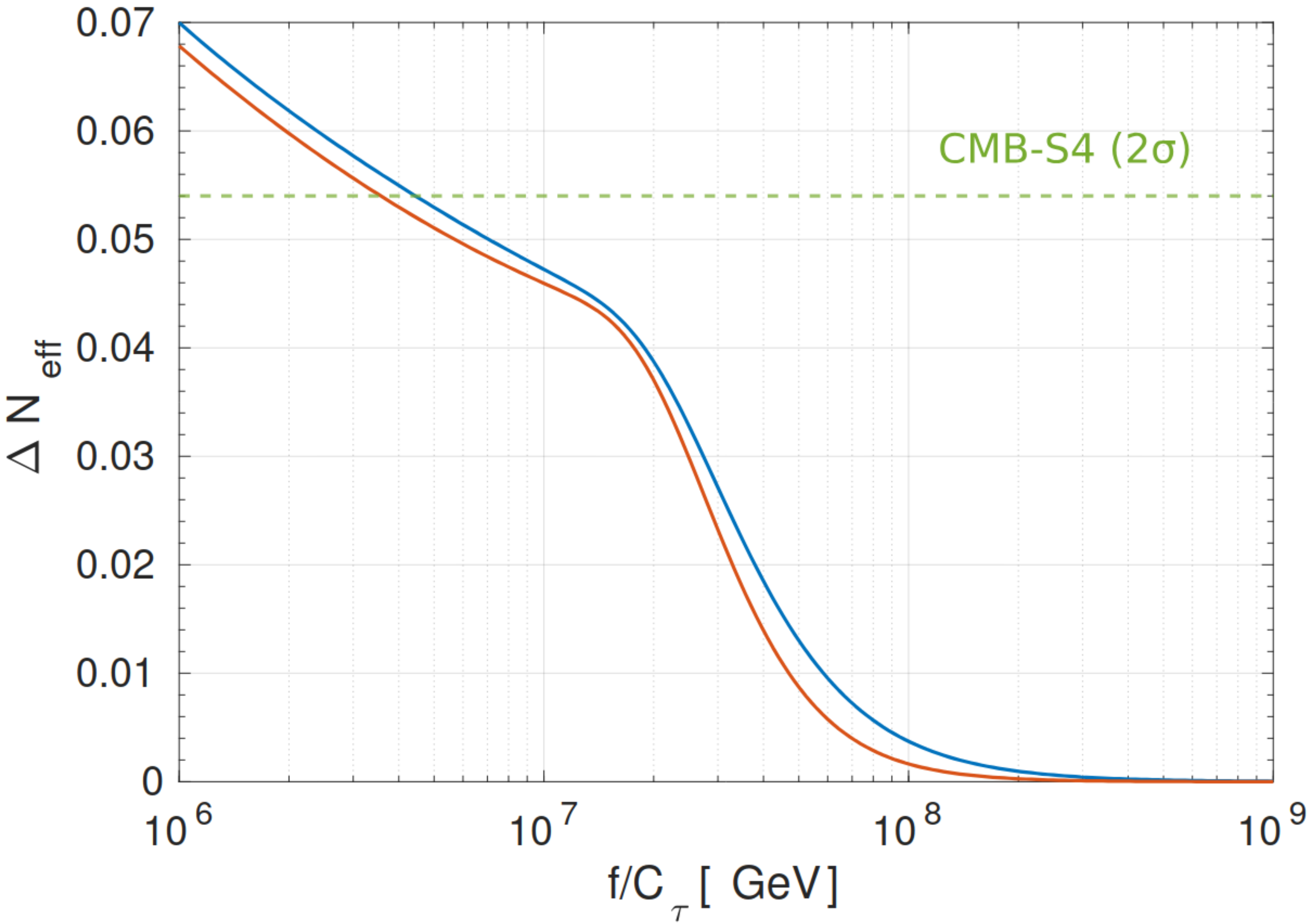
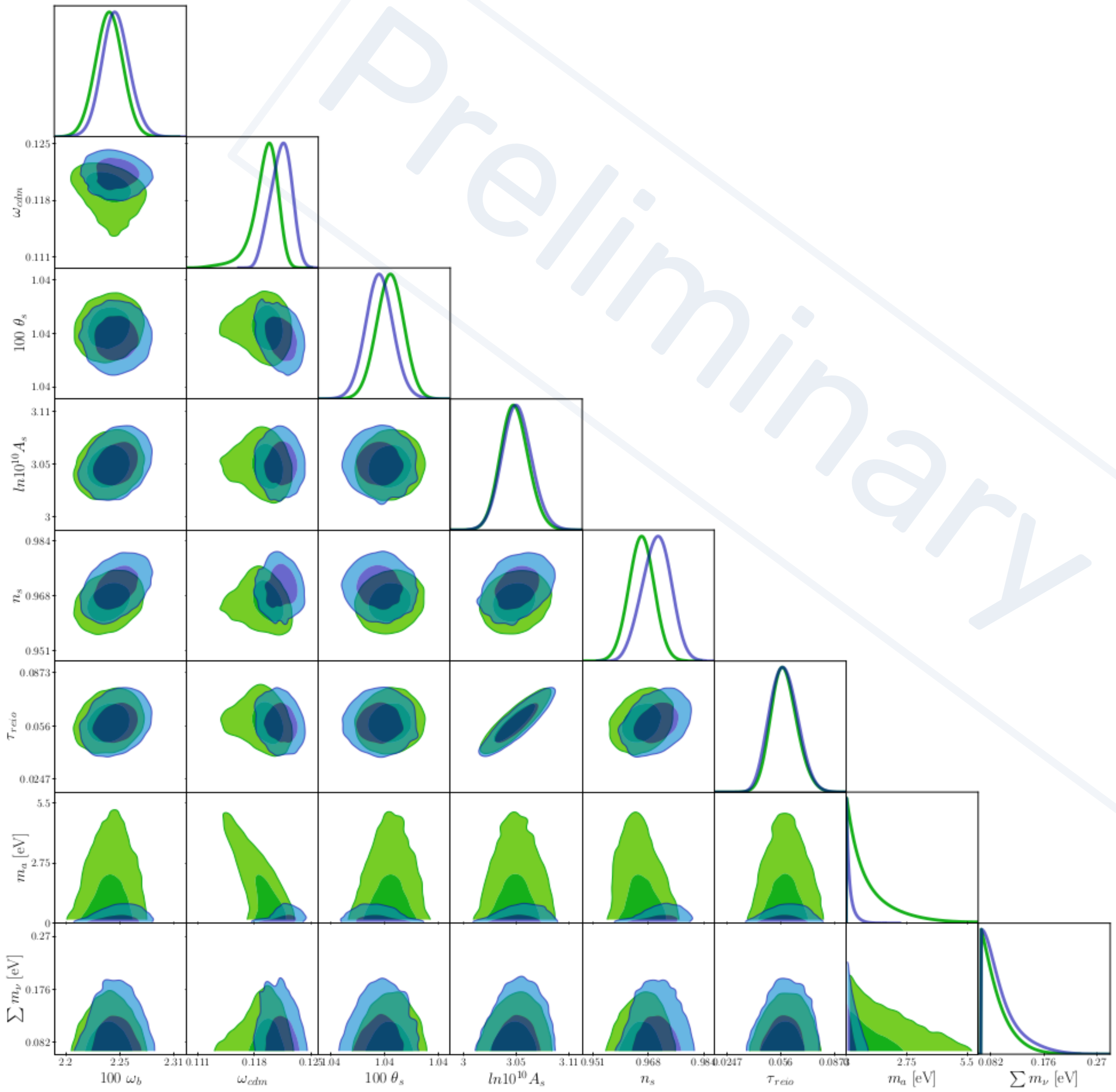


Difference in the ΔN_{eff} (tau scatterings)



Future prospects



Full cosmological scan of axion models using CLASS/Monte-Python

- Includes the impact of axion mass around recombination
- More precise constraints

1D and 2D posterior distributions for 8 cosmological parameters for tau scatterings (green) and tau decays (blue)

From master thesis of A. Gomulka (2024)

Similar studies

- A. Notari, F. Rompineve, and G. Villadoro, "*Improved Hot Dark Matter Bound on the QCD Axion*", Phys. Rev. Lett. 131 (2023), no. 1 011004, 2211.03799

  The impact of axion-pion scatterings on ΔN_{eff} using momentum-dependent calculation

- K. Bouzoud and J. Ghiglieri, "*Thermal axion production at hard and soft momenta*", 2404.06113

  Axion-gluon scatterings above QCD transition

- F. D'Eramo and A. Lenoci, "*Back to the phase space: thermal axion dark radiation via couplings to standard model fermions*", 2410.21253

  Axion-lepton and axion-quark scatterings (LF conserving)

Numerical packages

We are actively developing numerical packages to solve the full phase-space Boltzmann equation (fBE) in the early Universe

- **PyBolt** (with A. Gomulka and M. Lukawski)

  Solves fBE and nBE for a given model and interaction processes (in Python)

  https://github.com/Maxim-Laletin/PyBolt

- **CollCalc** (with K. Szafranski)

  Calculates collision integrals in full generality for annihilation and co-annihilation processes (in C++)

  https://github.com/Maxim-Laletin/CollCalc

Conclusion

- Axion lepton-flavour conserving and violating interactions can be **probed** and constrained by cosmological observations, competing againts the laboratory and astrophysical tests

- We revisit the axion production using a more general momentum-dependent approach to **recalculate the constraints** on the models with axion interactions

- The approach we consider is important for the studies of thermal axions and other BSM particles