

Gravitational waves from low-scale cosmic strings

Based on: Kai Schmitz & Tobias Schröder, Phys. Rev. D 110 (2024) 6, 063549, 2405.10937.

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Gravitational waves from the early Universe

Gravitational waves from cosmic strings

Low-scale cosmic strings

From VOS to BOS

Conclusions

Gravitational-wave echo from the Big Bang



[National Astronomical Observatory of Japan, gwpo.nao.ac.jp]

Primordial gravitational waves (GWs): Chance to peek behind the veil of the CMB

- Probe cosmology of the primordial Universe at very early times
- Probe particle physics at extremely high energies → New physics!?

1 Inflationary tensor perturbations

- · Accelerated expansion before the Hot Big Bang
- Complementarity: GWs + CMB observations



Abbrevations: GW: gravitational wave; CMB: cosmic microwave background; QFT: quantum field theory; EW: electroweak; QCD: quantum chromodynamics; PBH: primordial black hole; GUT: grand unified theory

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Osmological phase transition

- · First-order transition in the QFT vacuum structure
- Complementarity: GWs + EW / QCD / dark sector



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③ Enhanced scalar perturbations

- · Overdensities that emit GWs and collapse to PBHs
- Complementarity: GWs + primordial black holes



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Osmic defects

- · Phase transition remnants preserving the old vacuum
- Complementarity: GWs + grand unified theories



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- Topological defects after spontaneous symmetry breaking, s.t. π₁ (*M*) nontrivial
- For instance, breaking of global / local U(1); symmetry restored at string cores
- Condensed matter: Magnetic field vortices (quantum vortices) in a superconductor

Relevant parameters

- $G\mu$: String tension = energy per unit length, in units of $G = 1/M_{\rm P}^2$
- α : Size of string loops at time of formation, in units of the horizon $d_h \sim t \sim H^{-1}$

Cosmic strings in grand unified theories



Cosmic-string tension: Controlled by energy scale of spontaneous symmetry breaking

$$\mu\sim 2\pi v^2\,,\qquad G\mu\sim 4\times 10^{-8}\left(\frac{v}{10^{15}\,{\rm GeV}}\right)^2$$

Interesting possibilities

$$v \sim \Lambda_{
m GUT} \sim 10^{15\cdots 16}\,{
m GeV}\,, \qquad v \sim \Lambda_{
m intermediate} \sim 10^{9\cdots 10}\,{
m GeV}$$

[Allen, Martins, Shellard: ctc.cam.ac.uk/outreach]



Infinitely long strings and string loops; scaling regime: $\rho_{\rm cs} \propto \rho_{\rm crit} \propto H^2$



Gravitational waves from

- Cusps
- Kinks
- Kink–kink collisions

- Nambu–Goto strings: Infinitely thin, particle emission irrelevant at late times
- Abelian-Higgs strings: Short-lived loops, decay into massive particles

[Vachaspati, Vilenkin: PRD 31 (1985) 3052] [LISA Cosmology Working Group, Auclair et al.: 1909.00819]



Broadband signal

- Reflects scaling regime, GW emission during radiation and matter domination
- Interesting target for future GW experiments. Source of the PTA signal?



[NANOGrav: 2306.16219]

[NANOGrav: 2306.16213]



Recent PTA results

- Evidence for nHz GWB signal
- Stable cosmic strings do not yield a good fit (spectrum too flat)
- Alternatives doing a better job: metastable strings, superstrings

PTA upper limit on the tension of stable Nambu-Goto strings

 $G\mu \lesssim 10^{-10} \quad \longleftrightarrow \quad v \lesssim 5 imes 10^{13} \, {
m GeV}$



Different models

- GW emission dominated by cusps (c), kinks (k), fundamental mode (m); numerical result (n)
- GWs from cosmic strings only or in combination with GWs from supermassive BH binaries

[LISA Cosmology Working Group: 2405.03740]



Expected sensitivity: $G\mu \sim 10^{-(16\cdots 17)} \iff v \sim \text{few} \times 10^{10} \text{ GeV}$

- GW signal from cosmic strings competes with galactic and extragalactic foregrounds

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Phenomenology at low string tension



• Shrink because of GW emission

 $\frac{d\ell}{dt} = -\Gamma G \mu \,, \qquad \Gamma \simeq 50$

Characteristic length at birth

$$\ell_* = 2\alpha t_* , \qquad \alpha \simeq 0.05$$



Phenomenology at low string tension



Loop length decreases linearly in time between birth and today

$$\ell(t_0) = \ell_* - \Gamma G \mu(t_0 - t_*), \qquad t_* \in [t_{\text{ini}}, t_0]$$

Computation of GW signal only valid starting from some early initial time t_{ini}

Phenomenology at low string tension



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Computation of GW signal only valid starting from some early initial time t_{ini}

Observation: For late t_{ini} and small enough $G\mu$, no loop ever reaches zero length! Shortest loops = earliest loops today. Length: $\ell_{min} = \ell_* (t_{ini}) - \Gamma G\mu (t_0 - t_{ini})$

Sharp cutoff frequency

Present-day frequencies of GWs emitted by strings

$$f = \frac{a(t)}{a_0} \frac{2k}{\ell(t)}$$

 $\begin{array}{ll} 2k/\ell(t) & \mbox{Frequency at emission} \\ \ell(t) & \mbox{Loop length at emission} \\ k & \mbox{Mode number } (k=1,2,\cdots) \\ a(t)/a_0 & \mbox{Cosmological redshift factor} \end{array}$

 $G\mu = 10^{-13}$

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$a(t)/a_0$	Cosmological redshift factor	

Minimal length ℓ_{\min} implies frequency cutoff

- Focus on fundamental mode (k = 1) for now
- Shortest loops today: minimal length, minimal redshift \rightarrow highest possible frequency

$$f_{\rm cut} = \frac{2}{\ell_{\rm min}} = \frac{2}{2\alpha t_{\rm ini} - \Gamma G \mu \left(t_0 - t_{\rm ini}\right)}$$

 $G\mu = 10^{-13}$



 $G\mu = 10^{-19}$



$$t_{\rm ini} > t_{\rm cut} \equiv \frac{\Gamma G \mu}{2\alpha} t_0 \simeq 2.2 \, {\rm s} \left(\frac{G \mu}{10^{-20}}\right) \,, \qquad T_{\rm ini} < T_{\rm cut} \simeq 330 \, {\rm keV} \left(\frac{10^{-20}}{G \mu}\right)^{1/2}$$

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Problem: tini is model-dependent and typically not well known

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Problem: tini is model-dependent and typically not well known

Four reasonable options

1 $t_{ini} = t_{form} Network formation$

$$ho_{
m tot} = 3H^2 M_{
m Pl}^2 \sim \mu^2$$

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0	$t_{ m ini} = t_{ m form}$	Network formation	$ ho_{ m tot} = 3H^2 M_{ m Pl}^2 \sim \mu^2$
0	$t_{ m ini} = t_{ m fric}$	End of friction regime	$eta T^3/\mu \sim 2H$

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8	$t_{ m ini} = t_{ m kink}$	Particles from kinks subdominant	$P_{ m kink} \sim rac{N_k \mu^{1/2}}{\ell} \sim \Gamma G \mu^2$

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Four reasonable options

0	$t_{ m ini} = t_{ m form}$	Network formation	$ ho_{ m tot}=3H^2M_{ m Pl}^2\sim\mu^2$
2	$t_{ m ini} = t_{ m fric}$	End of friction regime	$eta T^3/\mu \sim 2H$
3	$t_{ m ini} = t_{ m kink}$	Particles from kinks subdominant	$P_{ m kink} \sim rac{N_k \mu^{1/2}}{\ell} \sim \Gamma G \mu^2$
4	$t_{ m ini} = t_{ m cusp}$	Particles from cusps subdominant	$P_{ m cusp} \sim rac{N_c \mu^{3/4}}{\ell^{1/2}} \sim \Gamma G \mu^2$



- $G\mu$ and T_{ini} values resulting in a cutoff frequency in the k = 1 GWB spectrum
- Hierarchy of temperature scales for $G\mu \sim 10^{-20}$: $T_{\rm cusp} \ll T_{\rm fric} \ll T_{\rm kink} \ll T_{\rm form}$

Earlier results in the literature



What's new?

$$\Omega_{\rm GW}\left(f\right) = \frac{8\pi}{3H_0^2} \left(G\mu\right)^2 \sum_{k=1}^{k_{\rm max}} \frac{\Gamma}{H_{k_{\rm max}}^q} \frac{1}{k^q} \frac{2k}{f} \int_{t_{\rm ini}}^{t_0} dt \left(\frac{a\left(t\right)}{a_0}\right)^5 n\left(\frac{2k}{f} \frac{a\left(t\right)}{a_0}, t\right)$$

Earlier results in the literature



What's new? Nothing! Our $\Omega_{\rm GW}$ is standard, but we do not integrate from $t_{\rm ini}=0$

$$\Omega_{\rm GW}\left(f\right) = \frac{8\pi}{3H_0^2} \left(G\mu\right)^2 \sum_{k=1}^{k_{\rm max}} \frac{\Gamma}{H_{k_{\rm max}}^q} \frac{1}{k^q} \frac{2k}{f} \int_{t_{\rm ini}}^{t_0} dt \left(\frac{a\left(t\right)}{a_0}\right)^5 n\left(\frac{2k}{f} \frac{a\left(t\right)}{a_0}, t\right)$$

Example spectra

Numerical spectra based on VOS loop

number densities



	$\log_{10}\left(G\mu ight)$	$\log_{10}\left({{{{T}_{\mathrm{ini}}}}/{\mathrm{GeV}}}\right)$	
1	-20.0	-5.1	$(T_{\rm cusp})$
2	-19.0	-3.9	(T_{cusp})
3	-23.4	-3.3	$(T_{\rm fric})$
4	-22.4	-2.3	$(T_{\rm fric})$

- No fine-tuning required
- Sweet spot where signal even observable by BBO and DECIGO
- Power-law behavior can be understood analytically

Challenge: Subtraction of galactic and extragalactic foregrounds. Impossible?

Summation of oscillation modes



Total GWB spectrum

$$\Omega_{\rm GW}(f) = \sum_{k=1}^{k_{\rm max}} \frac{\Omega_{\rm GW}^{(1)}(f/k)}{H_{k_{\rm max}}^q k^q}$$

can be written in terms of $\Omega_{\rm GW}^{(1)},$ i.e., spectrum from the fundamental mode

Simple approximation for $\Omega_{\rm GW}^{(1)}$

$$\Omega_{\mathrm{GW}}^{(1)} \approx \Theta \left(f_{\mathrm{cut}} - f \right) \mathcal{A} f^{3/2}$$

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Power-law behavior of the total GWB spectrum at low and high frequencies

$$h^2\Omega_{
m low} \propto f^{3/2}\,, \qquad h^2\Omega_{
m high} \propto rac{1}{q+^{1/2}}\left(rac{f_{
m cut}}{f}
ight)^{q-1}$$

Features in the GW spectrum



Novel features in the spectrum: Series of peaks and dips at integer multiples of f_{cut} on top of a broken power law $(f^{3/2} \rightarrow f^{-1/3}) \rightarrow$ Clear target for GW experiments

Assumption: $G\mu$ is so low that no loop has fully decayed yet because of GW emission Consequence: All loops produced since t_{ini} still exist in our present Universe Assumption: $G\mu$ is so low that no loop has fully decayed yet because of GW emission Consequence: All loops produced since t_{ini} still exist in our present Universe

Present-day loop number density

$$N(t_0) \approx \int_0^\infty d\ell \, n_{\rm RM}\left(\ell, t_0\right) \sim \frac{50}{\rm kpc^3} \left(\frac{10^2\,{\rm s}}{t_{\rm ini}}\right)^{3/2}$$

Present-day loop energy density

$$h^2\Omega\left(t_0\right) \approx \frac{1}{\rho_{\rm crit}} \int_0^\infty d\ell\,\mu\,\ell\,n_{\rm RM}\left(\ell,t\right) \simeq 10^{-13} \left(\frac{G\mu}{10^{-19}}\right) \left(\frac{10^2\,{\rm s}}{t_{\rm ini}}\right)^{1/2} \label{eq:hamiltonian}$$

- Cosmologically harmless
- Signatures from nearby loops? Microlensing, GW bursts?

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Sharp spectral features follow from the assumption of a unique initial loop length, $\ell_* \simeq 2\alpha t_*$ with $\alpha \simeq 0.05$. But initial loop length deviates from perfect delta peak.

Loop number density in terms of the loop production function

$$n(\ell, t) = \int_{t_{\text{ini}}}^{t} dt' f(\ell', t') \left(\frac{a(t')}{a(t)}\right)^{3}, \quad \ell' = \ell + \Gamma G \mu \left(t - t'\right)$$

Standard choice in the velocity-dependent one-scale (VOS) model

$$f(\ell, t) = \frac{\mathcal{F}C_r}{2\alpha t^4} \,\delta\left(\ell - 2\alpha t\right) \,, \quad \alpha \simeq 0.05$$

Numerical simulations by Blanco-Pillado, Olum, and Shlaer (BOS) better described by

$$f(\ell, t) = \frac{A_r}{\sqrt{2\pi} \,\sigma \ell^5 \, t^{5/2}} \, \exp\left[-\frac{1}{2\sigma^2} \left(\ln\left(\frac{\ell}{2t}\right) - \nu\right)^2\right], \quad \nu \simeq -3.0, \quad \sigma \simeq 0.14$$

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New loop number densities providing a better description of the BOS results

$$n_{\rm RR}\left(\ell,t\right) \approx \frac{A_r ({\rm erf}_{t_{\rm ini}} - {\rm erf}_t)/2}{t^{3/2} \left(\ell + \Gamma G \mu t\right)^{5/2}}, \quad n_{\rm RM}\left(\ell,t\right) \approx \left(\frac{a_{\rm eq}}{a\left(t\right)}\right)^3 \frac{A_r \left({\rm erf}_{t_{\rm ini}} - {\rm erf}_{t_{\rm eq}}\right)/2}{t_{\rm eq}^{3/2} \left(\ell + \Gamma G \mu t\right)^{5/2}}$$

Smeared GW spectrum



Series of peaks and dips washed out for broader distributions of initial loop lengths Still, even for broad distributions, oscillations in the index n_t may remain detectable

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 $G\mu \sim 10^{-33} \cdots 10^{-19} \qquad \longleftrightarrow \qquad v \sim 10^2 \, {\rm GeV} \cdots 10^9 \, {\rm GeV}$

Initial time $t_{ini} \neq 0$: Loop production no longer impeded by thermal friction, GW emission from loops no longer subdominant to particle emission from cusps and kinks,

$$\Omega_{\mathrm{GW}} = rac{16\pi}{3H_0^2} \left(G\mu\right)^2 \sum_{k=1}^{k_{\mathrm{max}}} rac{kP_k}{f} \int_{t_{\mathrm{ini}}}^{t_0} dt \; [\cdots]$$

- No loop produced at $t \ge t_{\rm ini}$ ever shrinks to zero length ightarrow microlensing, bursts?
- Sharp frequency cutoff in k=1 GWB spectrum, series of peaks and dips in $\Omega_{
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Next steps

- Nonscaling models where particle emission occurs whenever $\ell \leq \ell_{\rm crit}$
- Model building: Cosmic strings $\nu \sim 10^9 \, {\rm GeV},$ GWs from phase transition?

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Stay tuned! Thanks a lot for your attention