Spinning Primordial Black Holes from First Order Phase Transitions

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Based on *Spinning Primordial Black Holes from First Order Phase Transition*, **IKB**, U. K. Dey, JHEP 07 (2024) 006, arXiv: 2311.03406

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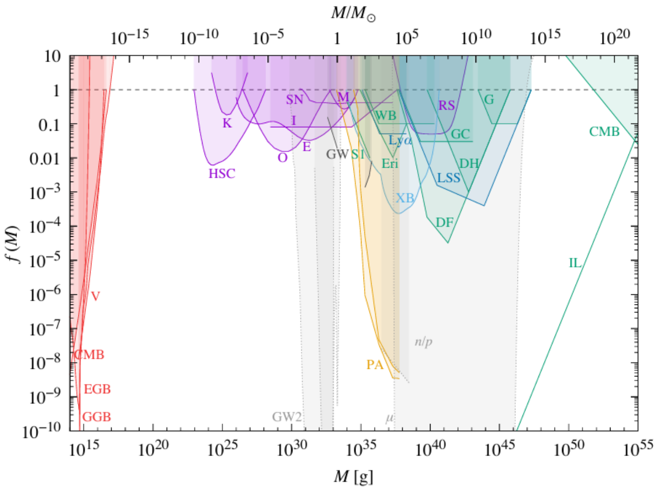
- Many theoretical predictions, such as Hawking evaporation or superradiant instability, can be verified from PBHs.
- PBHs can originate from inflation, cosmic strings, first-order phase transitions (FOPT), etc
- Can be expressed by its mass, spin and charge.

Constraints



(taken from arXiv: 2002.12778)

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- In order to have a FOPT, there must be a barrier between the two minima, which must be crossed.
- Physically, this corresponds to nucleation of true vacuum bubbles.

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  3. Nucleation Temperature:
     T_n: $\quad (\Gamma/V)|_{T=T_n} \sim \mathcal{O}(1)$

1. Introduction
2. Creation Mechanism
3. Generation of PBH Spin: Broad PS
4. Dependence on FOPT Parameters
5. Future Scope

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- These overdense regions are generated from curvature (or density) perturbations.
- Inflation, FOPT, etc can lead to these curvature perturbations.
- Curvature perturbations from FOPTs can arise through the difference in nucleation time of the true vacuum bubble in different Hubble patches.

Curvature Perturbation from FOPT

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- For strong and slow FOPTs ($\alpha \sim \mathcal{O}(1)$) (*2208.14086*),

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- For super-strong (supercooled) and slow FOPTs ($\alpha \gg 100$) (*2402.04158*),

$$A = f(\beta/H) \propto (\beta/H)^{-5/2}$$

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- PBH spin is calculated from the angular momentum of the region which collapses.

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  3. Turn Around Point
  4. Reference and RMS Spin

Spectral Moments

- The spectral index of the density perturbation can be expressed as

$$\sigma_n^2 = \frac{4}{9}\eta_{\rm init}^4 \int_{k_{\rm max}/r_k}^{k_{\rm max}} \frac{dk}{k} k^{2n+4}\mathcal{P}(k),$$

  where $r_k = k_{\rm max}/k_{\rm min}$.

- For our power spectrum, the spectral moments take the form,

$$\sigma_n^2 = \frac{4}{9}\eta_{\rm init}^4 A^2 k_{\rm max}^{4+2n} \frac{1 - r_k^{-7-2n}}{7+2n}.$$

- Furthermore, if we consider $k_{\rm min} = 0$,

$$\sigma_n^2 = \frac{4}{9}\eta_{\rm init}^4 A^2 k_{\rm max}^{4+2n} \frac{1}{7+2n}.$$

Profile Shape

- The density perturbation,

$$\delta_{\rm CMC}(\eta, \mathbf{r}) = \delta_{pk} g_\delta(r; k_\delta).$$

- The density profile for the case of the broad power spectrum,

$$g_\delta(r; k_\delta) = a_\delta^* - b_\delta^* (k_{\rm max} r)^2,$$

where

$$a_\delta^* \approx 13.76 - 16.5\alpha_\delta,$$
$$b_\delta^* \approx 3.21\alpha_\delta - 2.33,$$

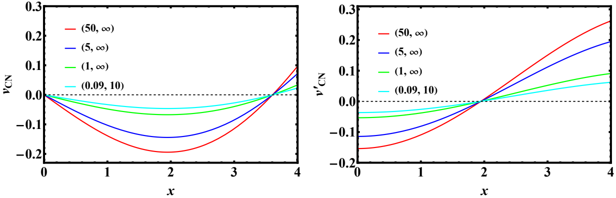
and $\alpha_\delta = k_\delta^2 / k_{\rm max}^2$.

- Density perturbation at $k_\delta = k_{c\delta} = \sigma_1/\sigma_0$,

$$g_\delta(r; k_{c\delta}) = \psi_\delta(r) = 1 - \frac{7}{54}(k_{\rm max} r)^2 = 1 - \frac{(r k_{c\delta})^2}{6}.$$

Turn Around Point

- The point in time when the overdense region decouples from the background and the process of the collapse start.
- In case of radiation domination, irrespective of the dependence on k, $x_{\text{ta}} = 1.95$.



Dependence of the velocity and the change in velocity on $x = k\eta$. The different curves represent different values of (j, r_k) where $\mathcal{P}_\zeta \propto k^j$.

Reference and RMS Spin

- The reference spin of the overdense region at turn around point,

$$A_{\rm ref}(\eta_{\rm ta}) = \frac{\frac{4}{3}\left[a^4\rho_b g_{\rm CN}\right]_{\eta=\eta_{\rm ta}}(1-f)^{5/2}R_*^5}{GM_{ta}^2}.$$

where,

$$R_* = \sqrt{3}\frac{\sigma_1}{\sigma_2},$$

$$g_{\rm CN} = \frac{2}{3}Ak_{\rm max}G,$$

$$G^2 = \int_0^1 dx x T_{v_{\rm CN}}^2(x)x^3,$$

$$T_{v_{\rm CN}}^2(x) = \frac{\sqrt{3}}{8}\frac{((x/\sqrt{3})^2-2)\sin(x/\sqrt{3})+2(x/\sqrt{3})\cos(x/\sqrt{3})}{(x/\sqrt{3})^2}.$$

Reference and RMS Spin

- Simplifying this for our case we find,

$$A_{\mathrm{ref}}(\eta_{\mathrm{ta}}) = 0.00286A(1-f)^{-1/2}.$$

- The RMS spin of the region can be expressed as,

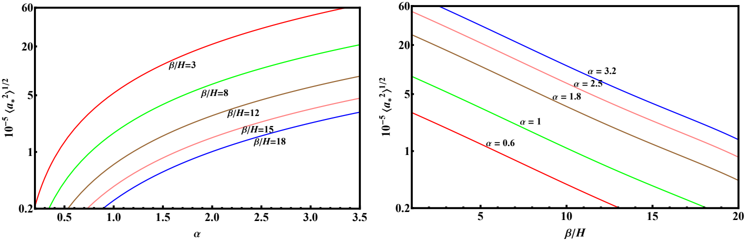
$$\sqrt{\langle a_*^2\rangle} = A_{\mathrm{ref}} \times 5.96 \times \frac{\sqrt{1-\gamma^2}}{\gamma^6\nu},$$

where $\nu = 1.92/\sigma_0$, and $\gamma = \sigma_1^2/(\sigma_0\sigma_2)$.

- Finally, the RMS spin can be expressed as,

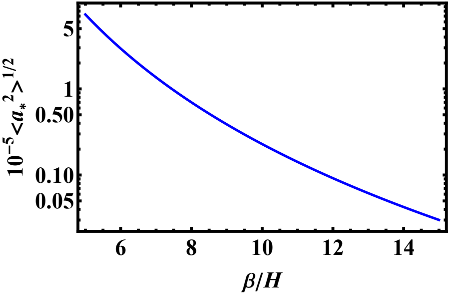
$$\sqrt{\langle a_*^2\rangle} = 3.4 \times 10^{-4} \left(\frac{M_{\mathrm{PBH}}}{M_H}\right)^{-1/3} A^2$$

Case-I: $\alpha \sim \mathcal{O}(1)$



Dependence of the RMS spin of a PBH population on α and β/H.

Case-II: $\alpha \gg 1$



Dependence of the RMS spin of a PBH population on β/H for $\alpha \gg 1$.

Future Scope

- Some studies have shown that the value of j may vary depending on the FOPT parameters and value of k. This scheme of calculation can be modified to account for those cases as well.
- FOPTs in Non-standard cosmology, such as some early matter-dominated era, may give rise to PBH with high initial spin, which has implications in Hawking evaporation superradiant instability, gravitational waves, etc.

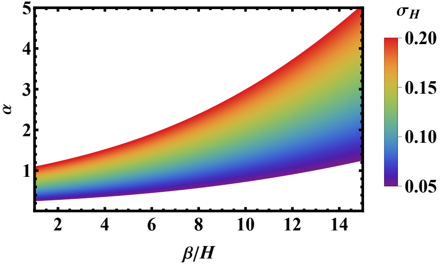
Thanks!

Behaviour of $\mathcal{P}_\zeta$ for $\alpha \sim \mathcal{O}(1)$

The form of the curvature perturbation in this case can be expressed as,

$$\mathcal{P}_\zeta(k) = 34.5[\sigma_H(\alpha, \beta/H)]^2 (kR_{\mathcal{H}})^3,$$

where the behaviour of the function $\sigma_H(\alpha, \beta/H)$ can be expressed as,



Solution from Cosmological Linear Perturbation Theory I

The gauge invariant quantities corresponding to the density and the velocity perturbations can be expressed as,

$$\Delta(x) = D\sqrt{3}\left(\frac{\sin z}{z} - \cos z\right),$$
$$V(x) = D\left[\frac{3}{4}\left(\frac{2}{z^2} - 1\right)\sin z - \frac{3}{2}\frac{\cos z}{z}\right],$$

where D is an arbitrary constant, whose value depends on the shape of perturbation, $z = x/\sqrt{3}$, and $x = k\eta$. For the CMC

Solution from Cosmological Linear Perturbation Theory II

gauge, the density perturbation and the velocity of the region can be expressed as

$$\delta_{\rm CMC} = D\frac{\sqrt{3}z^2}{z^2+2}\left(2\frac{\sin z}{z} - \cos z\right),$$
$$v_{\rm CMC} = -\frac{3}{4}D\frac{(z^2-2)\sin z + 2z\cos z}{z^2+2}.$$

For the conformal Newtonian gauge, the quantities take the form

$$\delta_{\rm CN} = \sqrt{3}D\frac{2(z^2-1)\sin z + (2-z^2)z\cos z}{z^4},$$
$$v_{\rm CN} = \frac{3}{4}D\frac{(2-z^2)\sin z - 2z\cos z}{z^2}.$$