Hunting for a G_2 snake

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Inspiration

- **Bronisław Jakubczyk**, Professor of Mathematics, Mathematical Institute, Polish Academy of Sciences, Warszawa
- **Krzysztof Tchoń**, Professor of Control Engineering and Robotics, Institute of Computer Engineering, Control and Robotics, Wrocław University of Technology
- **Masato Ishikawa**, Dr.Eng., Associate Professor Department of Mechanical Engineering, Graduate School of Engineering, Osaka University

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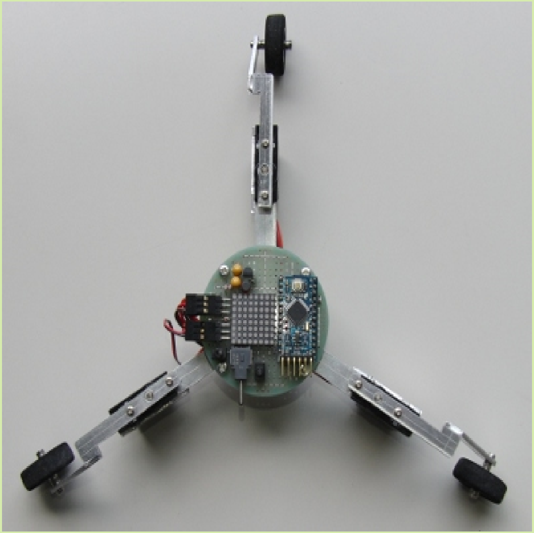
Trident snake



Trident snake - an animation

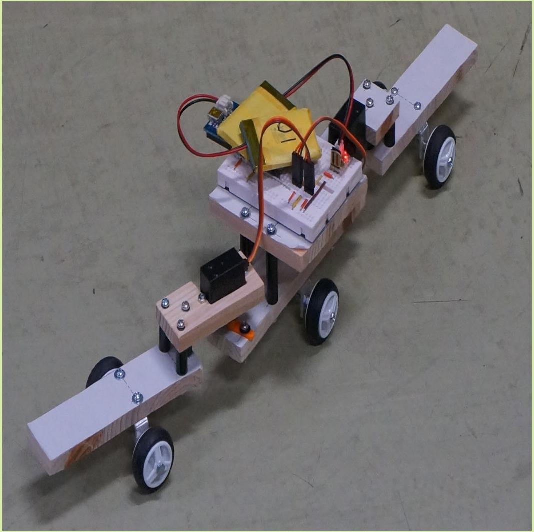
Trident snake - translational control

Trident snake - rotational control

Almost real snake robot

Even better almost real snake robot

Simplest real snake robot



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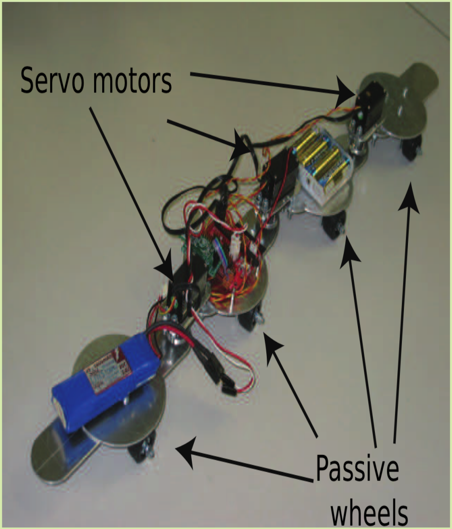

Simplest snake's animation

Simplest snake animation - movement in the orthogonal direction

Parametrizing M the configuration space of the trident snake

Coordinates: $(x, y, \phi_1, \phi_2, \phi_3, \alpha)$

Parametrizing M the configuration space of the tri-segment snake

Coordinates: $(x, y, \phi_1, \phi_2, \alpha)$

Nonholonomic constraints

- Movement of each wheel is constrained by the condition that the wheel can NOT move in the direction perpendicular to it.
- Consider a bar with end points $(x_i, y_i) = \mathbf{r}_i$ and $(x_j, y_j) = \mathbf{r}_j$ and the wheel attached at a point $(\bar{x}, \bar{y}) = \bar{\mathbf{r}}$ on the bar.
- The above mentioned constraint means that:

$$\frac{\mathrm{d}\mathbf{r}}{\mathrm{d}t} \times (\mathbf{r}_i - \mathbf{r}_j) = 0,$$

or, simpler:

$$\omega_{ij} := \mathrm{d}\mathbf{r} \times (\mathbf{r}_i - \mathbf{r}_j) = 0.$$

I emphasize that the 1-form ω_{ij} is a scalar form! Explicitly:

$$\omega_{ij} = (y_i - y_j)\mathrm{d}\bar{x} - (x_i - x_j)\mathrm{d}\bar{y} = 0.$$

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Possible movements

- A movement of a snake in the plane corresponds to a curve $\gamma(t) = (x(t), y(t), \phi_i(t), \alpha(t))$ in the configuration space M.
- Velocity of a snake at time t is $\dot{\gamma}(t)$. This is a vector tangent to the curve $\gamma(t)$, and in turn tangent to the configuration space M at point $\gamma(t)$.
- The six (five) - dimensional space $\mathrm{T}_{\gamma(t)}M$ of possible velocities of a snake is constrained by THREE constraints enforced by the three wheels.
- Indeed, the velocity of a snake at point $\gamma(t)$ has to satisfy

$$\dot{\gamma}(t) \lrcorner \omega_{ij} = 0. \qquad (*)$$

And we have THREE ω_{ij}s.

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Nonholonomic distributions

- In case of a trident snake, the configuration space *M* is **6**-dimensional and the space of velocities at each point is restricted from dimension 6, by **three** linear conditions (*), to a vector space of dimension 6-3=**three.** This defines a **rank three distribution in dimension six.**
- In case of a tri-segment snake, the configuration space *M* is **5**-dimensional and the space of velocities at each point is restricted from dimension 5, by **three** linear conditions (*), to a vector space of dimension 5-3=**two.** This defines a **rank two distribution in dimension five.**
- ARE THESE DISTRIBUTIONS GENERIC?

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Generalities on distributions

- A rank r **distribution** $\mathcal{D}$ on a manifold M of dimension n is a smooth assignment $x \mapsto D_x$ of vector subspaces $D_x \subset \mathrm{T}_x M$ of dimension r to each point x of M.
- Given a rank r distribution $\mathcal{D}$ one constructs spaces:

$$\mathcal{D}^{-1} = \mathcal{D}, \quad \mathcal{D}^{-2} = [\mathcal{D}^{-1}, \mathcal{D}^{-1}] + \mathcal{D}^{-1}, \dots \mathcal{D}^{k-1} = [\mathcal{D}^{0}, \mathcal{D}^{k}] + \mathcal{D}^{k}.$$

- These, at each point $x \in M$, define a sequence of integers $N(x) = (n_{-1}, n_{-2}, \dots, n_p, \dots)$, called the **growth vector**, which are the dimensions of vector spaces $D_x^{-s} = \mathcal{D}^{-s}(x)$. We will only consider $\mathcal{D}$ such that $N(x) = \mathrm{const}$.
- Note that if $\mathcal{D}^{-2} = \mathcal{D}^{-1}$ the distribution $\mathcal{D}$ is **integrable**.
- On the other extreme, the distribution $\mathcal{D}$ is **bracket generating** if there exists an integer $p < 0$ such that $n_p = n = \dim M$.

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Symbol of a distribution

- From now on: only **bracket generating distributions.**
- A **symbol algebra** of a distribution $\mathcal{D}$ at $x \in M$ is a **nilpotent Lie algebra** $\mathfrak{g}_-(x)$ defined as a direct sum:

$$\mathfrak{g}_-(x) = \mathfrak{g}_p(x) \oplus \cdots \oplus \mathfrak{g}_{-2}(x) \oplus \mathfrak{g}_{-1}(x),$$

where $\mathfrak{g}_{-1}(x) = D_x^{-1}$ and

$$\mathfrak{g}_{-2}(x) = D_x^{-2}/D_x^{-1}, \quad \ldots, \quad \mathfrak{g}_p(x) = D_x^p/D_x^{p+1}.$$

- The commutator in $\mathfrak{g}_-(x)$ is defined in such a way that $\mathfrak{g}_{k-1} = [\mathfrak{g}_{-1}(x), \mathfrak{g}_k(x)]$ and $[\mathfrak{g}_{-1}(x), \mathfrak{g}_p(x)] = \{0\}$.
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Remarks

- In low dimensions the growth vector N of a distribution $\mathcal{D}$ may determine its symbol. This happens e.g. in the case of distributions with growth vectors $N = (\mathbf{2}, 3, \mathbf{5})$ and $N = (\mathbf{3}, \mathbf{6})$. It follows that the distributions with such growth vectors are **generic** among, respectively, all rank **2** distributions in dimension **5**, and rank **3** distributions in dimension **6**.
- Returning to **snakes** I repeat the question: ARE THE DISTRIBUTIONS CORRESPONDING TO THE POSSIBLE VELOCITY SPACES OF **TRIDENT SNAKE** AND **TRI-SEGMNET SNAKE**, RESPECTIVELY, **(3,6)** and **(2,3,5)**?

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- Is the growth vector N of the **velocity distribution** of the **trident snake** $(3,6)$? If so, can one arrange a **geometry** of this snake (by changing lengths of the sides of the triangle, and changing the lengths of the legs) to **get a snake having velocity distribution** with $\mathfrak{so}(3,4)$ symmetry?
- More interestingly:
- Is the growth vector N of the **velocity distribution** of the **tri-segment snake** $(2,3,5)$? If so, can one arrange a **geometry** of this snake to get a **snake having velocity distribution** with $\mathfrak{g}_2$ symmetry?
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What is simple and what is difficult?

- It is very **easy** to see that the **growth vectors** of velocity distributions of the trident snake and the tri-segment snake **are**, respectively, $(3, 6)$ and $(2, 3, 5)$. This is **indpenendent of the particular design** of the snakes!
- What is **difficult**, is to **calculate invariants** of the velocity distributions for these snakes. **Finding solutions for the symmetry equations** for these distributions **is** equally **difficult.**
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Hunting - introducing nice coordinates

- Four points $\mathbf{r}_i = (x_i, y_i), = 1, 2, 3, 4$, on the plane $(x0y)$ corresponding to the ends of the segments of the snake. **Dimension count**: $4 \times 2 = 8$
- **Holonomic constraints** - the lengths of the segments are equal, say, to a, b, c, which gives **three** constraints:

$$|\mathbf{r}_1 - \mathbf{r}_2|^2 = a^2, \quad |\mathbf{r}_2 - \mathbf{r}_2|^2 = b^2, \quad |\mathbf{r}_3 - \mathbf{r}_4|^2 = c^2.$$

Dimension count: 8-3=5. This is the dimension of the configuration psace for the snake.

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Hunting - nonholonomic constraints

- Wheels are placed at the end points $\mathbf{r}_1$ and $\mathbf{r}_2$ of the snake, as well somwhere at the middle segment, at a point $\mathbf{r} = (1 - s)\mathbf{r}_2 + s\mathbf{r}_3$, say.
- **Nonholonomic constraints**:

$$(\mathbf{r}_1 - \mathbf{r}_2) || \mathrm{d}\mathbf{r}_1, \quad \& \quad (\mathbf{r}_4 - \mathbf{r}_3) || \mathrm{d}\mathbf{r}_4, \quad \&$$

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Hunting - equations for the nonholonomic system

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Task: solve an equivalence problem for the so defined $(2, 3, 5)$ distribution. **In particular**: calculate the Cartan quartic as a function of the design parameters (a, b, c, s). **Find** (a, b, c, s) for which Cartan quartic is zero.

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$$(x_1 - x_2)^2 + (y_1 - y_2)^2 = a^2$$
$$(x_2 - x_3)^2 + (y_2 - y_3)^2 = b^2$$
$$(x_3 - x_4)^2 + (y_3 - y_4)^2 = c^2.$$

Nonholonomic constraints:

$$\omega_1 = (x_1 - x_2)\mathrm{d}y_1 - (y_1 - y_2)\mathrm{d}x_1$$
$$\omega_2 = (x_2 - x_3)((1 - s)\mathrm{d}y_2 + s\mathrm{d}y_3) - (y_2 - y_3)((1 - s)\mathrm{d}x_2 + s\mathrm{d}x_3)$$
$$\omega_3 = (x_4 - x_3)\mathrm{d}y_4 - (y_4 - y_3)\mathrm{d}x_4.$$

Distribution: Anihilator of $(\omega_1, \omega_2, \omega_3)$ restricted from $\mathbf{R}^8$ to a leaf of the foliation given by the holonomic constraints.
Task: solve an equivalence problem for the so defined $(2, 3, 5)$ distribution. **In particular**: calculate the Cartan quartic as a function of the design parameters (a, b, c, s). **Find** (a, b, c, s) for which Cartan quartic is zero.

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Hunting - techniques to catch a G_2 snake

- A 5-manifold M with a $(2, 3, 5)$ distribution $\mathcal{D}$ on defines an exceptional **parabolic geometry** of type (G_2, P) where P is a 9-dimensional parabolic subgroup of split real form of the exceptional Lie group G_2 corresponding to a cross at the first root of the Dynkin diagram.
- Such geometry can be described by $\mathfrak{g}_2$ Cartan connection Ω on the corresponding **14**-dimensional Cartan bundle $P \to \mathcal{G} \to M$.
- The **curvature** of this connection $\mathcal{R} = \mathrm{d}\Omega + \Omega \wedge \Omega$ vanishes if and only if the distribution $\mathcal{D}$ on M has symmetry Lie algebra isomorphic to $\mathfrak{g}_2$.
- The curvature of Ω, in general, has 24-independent components, but 19 of them are expressible in terms of **five fundamental ones** (call them $(A_1, A_2, A_3, A_4, A_5)$) as derivatives of the A_is.
- The A_is can be collected to a tensorial object called **Cartan quartic**, whose vanishing is neccessary and sufficient for the full curvature $\mathcal{R}$ to vanish.

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- There exists an algorithm of calculating $[g]$ for a given $(2,3,5)$ distribution $\mathcal{D}$.
- It therefore 'suffices' to take the tri-segment snake distribution $\mathcal{D}$, as expressed in terms of $(\omega_1,\omega_2,\omega_3)$, to calculate the associated conformal class $[g]$, to calculate its Weyl tensor, and to equate it to zero.
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- So we have $M \subset \mathbf{R}^8$ and we have a 2-distribution $\mathcal{D}$ tangent to the submanifold M. Smells like a **CR structure**, doesn't it?
- Question: **Is there a** constant **linear map** $J_{\mathcal{D}} : \mathbf{R}^8 \to \mathbf{R}^8$, **such that** $J_{\mathcal{D}}^2 = -id$, **and such that** $J_{\mathcal{D}}(TM) \cap TM = \mathcal{D}$?
- Answer (**a big surprise for me!**): Such a **constant** $J_{\mathcal{D}}$ **exists** if and only if $s = 1/2$, i.e. when **the wheel at snake's middle segment is located precisely in the center**! Moreover, if $s = 1/2$ such $J_{\mathcal{D}}$ is **unique.**
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- **New point of view**: A tri-segment snake with $s = 1/2$ is a 5-dimensional CR manifold of real codimension **three** and complex dimension **one** embedded in $\mathbf{C}^4$.
- **New approach**: Consider only tri-segment snakes with $s = 1/2$ and find lengths (a, b, c) for which the resulting $(3, 1)$-CR-snake is the simplest.
- Need theory of real codimension 3, complex dimension 1, CR manifolds.
- Since I did not find such theory in the literature, I had to made it myself.
- I solved the local equivalence problem for such CR manifolds. Although it is **not** a parabolic geometry, Cartan equivaence method quickly leads to a construction of the full system of its local differential invariants.
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- **New approach**: Consider only tri-segment snakes with $s = 1/2$ and find lengths (a, b, c) for which the resulting $(3, 1)$-CR-snake is the simplest.
- Need theory of real codimension 3, complex dimension 1, CR manifolds.
- Since I did not find such theory in the literature, I had to made it myself.
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- **New idea**: for the $s = 1/2$ snake adapt the coordinates to the corresponding CR geometry; Then use Cartan equivalence method for the corresponding CR geometry to bring the coframe defining the $(2,3,5)$ distribution to the CR simplest form; Then use the resulting G_2 freedom to calculate the conformal metric $[g]$ associated with the distribution;
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- They do! After **six** weeks of **constant** struggle I was eventually able to calculate the conformal class $[g]$ corresponding to the snake velocity distribution! I was also able to calculate the Cartan quartic as a function of (a, b, c). And...
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- Together with **Gil Bor** we are now trying to assign snakes to **every parabolic geometry** with symbol algebra having step $p \geq 2$.
- What we definitely can do up to now is to **design a 'snake'** or, better to say, 'planar robot', **whose configuration space M has a given dimension n and whose velocity distribution $\mathcal{D}$ has rank r**. In particular we now know that **given r and n there can be many 'topologically nonequivalent snakes' with M of dimension n and $\mathcal{D}$ of dimension r**. This is simply governed by **Euler's formula relating numbers of vertices, edges and faces of a planar figure.**
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