Metric-Affine Gravity as an Effective Field Theory: Expectations vs. Reality

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Outline

Particles vs Fields

Quantum Mechanics (probabilities and Hilbert spaces) + Lorentz Invariance:

$$|p^\mu, i\rangle \xrightarrow{boost} |(\Lambda p)^\mu, i\rangle = U(\Lambda)_{i,j}\,|p^\mu, j\rangle$$

Unitary and infinite-dimension representations (counted and classified: Wigner).

Lorentz Covariant Dynamics . . .

$$\langle \text{future}|\,\exp\left(-i\,H\cdot T\right)|\text{past}\rangle$$

. . . + Locality + Cluster Decomposition

$$H = \int \mathcal{H}\, d^3x, \quad \text{with} \;\; \mathcal{H} = \partial_\mu\phi(x)\partial^\mu\phi(x) + \bar{\psi}(x)\partial\!\!\!/\psi(x) + A_\mu(x)\Box A^\nu(x) + \cdots$$

Non unitary and finite-dimension representations $\left[\phi(x), \psi(x), A_\mu(x), h_{\mu\nu}(x)\ldots\right]$

Particles vs Fields

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Non unitary and finite-dimension representations $\left[\phi(x), \psi(x), A_\mu(x), h_{\mu\nu}(x) \ldots\right]$

Particle states interpolated via local fields:

$$A_\mu(x)|\Omega\rangle \sim e^{ip\cdot x}\, \epsilon^i_\mu(p)\, |p, i\rangle$$ **d.o.f mismatch!**

Particles vs Fields

Particle states interpolated via local fields:

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Is this a problem? **Yes.**
The use of Lorentz Covariant fields generates Lorentz invariant scalar products (norms) for particle states

$$\langle\chi_\mu|\chi_\mu\rangle \sim \chi_1^2+\chi_2^2+\chi_3^2-\chi_0^2$$

Norm of different signatures $\longrightarrow$ no bounded probability, no QM!

Solution: EOM to achieve decoupling!

Outline

Rank-1 @LO

Solution: EOM to achieve decoupling!

For rank-1 fields B_μ, particle reps: $3+1$ (massive) or $2+1+1$ (massless + massless + massive).

Fundamental field	Symmetries	Decomposition in SO(3) irreps	Source
b_α	StrongGenSet[{}, GenSet[]]	$b^{\#1}_{1^-}{}_\alpha + b^{\#1}_{0^+}\, n_\alpha$	B_α

xAct, ***PSALTer*** output (more on this later!)

Let's consider the most generic quadratic setup

$$\mathcal{L} = \frac{1}{2} B_\mu \left(g^{\mu\nu} (a_1 \Box + a_2) + a_3\, \partial^\mu \partial^\nu \right) B_\nu$$

Rank-1 @LO

But which sector propagates?

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b_{α}	StrongGenSet[{}, GenSet[]]	$b^{\#1}_{1^-}{}_{\alpha} + b^{\#1}_{0^+}\, n_{\alpha}$	B_{α}

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The non-perturbative dynamics is defined by the inversion problem: **the propagator**.
This computation is trivial in terms of particle sectors (algebraic problem instead of tensor):

Dictionary: isolated poles $\sim$ propagation, pole residue $\sim$ (sign of) state norm

Rank-1 @LO

But which sector propagates? This computation is trivial in terms of particle sectors (algebraic problem instead of tensor):

Quadratic (free) action

$$S == \iiiint \left(\frac{1}{2}\left(a2\, b_{\mu}\, b^{\mu} + 2\, b^{\alpha}\, B_{\alpha} + b^{\mu}\left(a3\, \partial_{\nu}\partial_{\mu} b^{\nu} + a1\, \partial_{\nu}\partial^{\nu} b_{\mu}\right)\right)\right)[t, x, y, z]\, dz\, dy\, dx\, dt$$

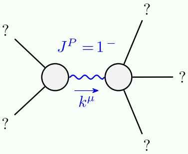
$$b^{\#1}_{0^+}{}^{\dagger}\ \boxed{\frac{1}{2}\left(a2 - (a1 + a3)\, k^2\right)}\ b^{\#1}_{0^+} \qquad b^{\#1}_{1^-}{}^{\dagger\,\alpha}\ \boxed{\frac{1}{2}\left(a2 - a1\, k^2\right)}\ b^{\#1}_{1^-\,\alpha}$$

$$B^{\#1}_{0^+}{}^{\dagger}\ \boxed{\frac{2}{a2-(a1+a3)\, k^2}}\ B^{\#1}_{0^+} \qquad B^{\#1}_{1^-}{}^{\dagger\,\alpha}\ \boxed{\frac{2}{a2-a1\, k^2}}\ B^{\#1}_{1^-\,\alpha}$$

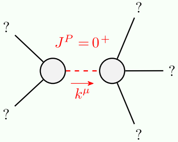
(No source constraints)

Can you spot the problem?

(No massless particles)



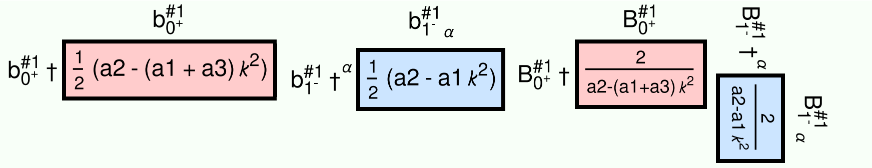
Massive particle	
Pole residue:	$\frac{2}{a1} > 0$
Polarisations:	3
Square mass:	$\frac{a2}{a1} > 0$
Spin:	1
Parity:	Odd



Massive particle	
Pole residue:	$-\frac{2}{a1+a3} > 0$
Polarisations:	1
Square mass:	$\frac{a2}{a1+a3} > 0$
Spin:	0
Parity:	Even

Rank-1 @LO

Can you spot the problem?



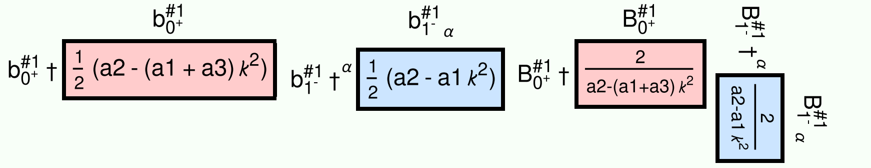
$a_1 = -a_3$ and no more ghosts:

$$\mathcal{L} = \frac{1}{2}\,B_\mu\left(g^{\mu\nu}(a_1\Box + a_2) + a_3\,\partial^\mu\partial^\nu\right)B_\nu =$$

$$\frac{1}{2}\,B_\mu\left(a_1(g^{\mu\nu}\Box - \partial^\mu\partial^\nu) + a_2 g^{\mu\nu}\right)B_\nu \sim -\frac{1}{4}F^2 + \frac{1}{2}M_B^2 B_\mu B^\mu \qquad \textbf{(Proca!)}$$

Rank-1 @LO (Foreshadowing NLO)

Ghost freedom + 3 polarization states = Proca theory.



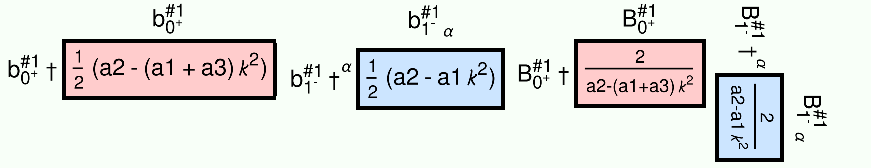
An explicit look at the propagator in coordinate space (and large momentum limit):

$$D_B(q, M_B) = \frac{-i}{q^2 - M_B^2}\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{M_B^2}\right)$$

In Proca, all particle components are present, only one propagates.

Rank-1 @LO

What about the massless case (Photon)?



$$a_1 = -a_3 \text{ and } a_2 = 0$$

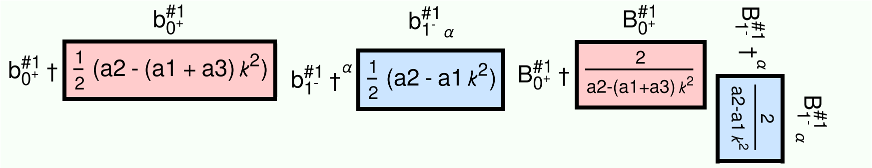
No Trivial Inversion - Gauge Symmetry (absence of 0^+ sector implies $B_\mu \sim B_\mu + \partial_\mu \phi$)

$$\mathcal{L} = \frac{1}{2} B_\mu \left(g^{\mu\nu}(a_1 \Box + a_2) + a_3\, \partial^\mu \partial^\nu \right) B_\nu$$

$$\frac{a_1}{2} B_\mu \left(g^{\mu\nu} \Box - \partial^\mu \partial^\nu \right) B_\nu \sim -\frac{1}{4} F^2$$ **(Maxwell bottom up!)**

Rank-1 @LO (Foreshadowing NLO)

Ghost freedom + 2 helicity states = Maxwell theory.



An explicit look at the propagator in coordinate space (and large momentum limit):

$$D_B(q) = \frac{-i}{q^2}\left(g_{\mu\nu} - (1-\xi)\frac{q_\mu q_\nu}{q^2}\right)$$

Differently from Proca, the 0^+ sector, $q^\mu q^\nu$, is a gauge artefact.

Outline

Rank-2 @LO

Next step, symmetric rank-2 $h_{\mu\nu}$ (systematic SPO use begins).

ON GHOST-FREE TENSOR LAGRANGIANS AND LINEARIZED GRAVITATION

P. van NIEUWENHUIZEN*
*Laboratoire de Physique Théorique et Hautes Energies, Orsay, France***

Received 22 February 1973
(Revised 28 May 1973)

Lagrangian Theory for Neutral Massive Spin-2 Fields (*).

R. J. RIVERS
Department of Physics, Imperial College - London

(ricevuto il 4 Maggio 1964)

Generic (massless) quadratic action:

$$\mathcal{L} = a_1\, \partial^\sigma h_\mu{}^\mu \partial_\sigma h_\nu{}^\nu + a_2\, \partial_\mu h^{\mu\nu} \partial^\sigma h_{\mu\sigma} + a_3\, \partial_\nu h_\mu{}^\mu \partial_\sigma h^{\sigma\nu} + a_4\, \partial_\sigma h_{\mu\nu} \partial^\sigma h^{\mu\nu}$$

Many particle sectors:

Fundamental field	Symmetries	Decomposition in SO(3) irreps	Source
$h_{\alpha\beta}$	StrongGenSet[{1, 2}, GenSet[(1,2)]]	$\frac{1}{3}\,\eta_{\alpha\beta}\, h^{\#1}_{0^+} + h^{\#1}_{2^+}{}_{\alpha\beta} + h^{\#1}_{1^-}{}_{\beta}\, n_\alpha + h^{\#1}_{1^-}{}_{\alpha}\, n_\beta - \frac{1}{3}\, h^{\#1}_{0^+}\, n_\alpha\, n_\beta + h^{\#2}_{0^+}\, n_\alpha\, n_\beta$	$\sigma_{\alpha\beta}$

Rank-2 @LO

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The use of SPOs allows a "straightforward" computational implementation

Particle spectrum for any tensor Lagrangian (PSALTer) - By W. Barker wb263@cam.ac.uk `github.com/wevbarker/PSALTer`.

...a software for cheaply computing the mass and energy of the particle spectrum for any (e.g. higher-rank) tensor field theory in the Wolfram Language... *PSALTer* automatically computes the spin-projection operators, saturated propagator, bare masses, residues of massive and massless poles and overall unitarity conditions in terms of the coupling constants. The constraints on the source currents and total number of gauge symmetries are produced as a by-product.

Manual and worked examples in forthcoming (next week) paper by W. Barker, C.M., C. Rigouzzo.

Rank-2 @LO

Many particle sectors (and convoluted conditions for unitarity):

	$\sigma^{\#1}_{0^+}$	$\sigma^{\#2}_{0^+}$
$\sigma^{\#1}_{0^+}\,\dagger$	$\frac{4(a_1+a_2+a_3+a_4)}{(-3a_3{}^2+4a_4(a_2+a_3+a_4)+4a_1(3a_2+4a_4))k^2}$	$-\frac{2\sqrt{3}(2a_1+a_3)}{(-3a_3{}^2+4a_4(a_2+a_3+a_4)+4a_1(3a_2+4a_4))k^2}$
$\sigma^{\#2}_{0^+}\,\dagger$	$\frac{2\sqrt{3}(2a_1+a_3)}{(3a_3{}^2-4a_4(a_2+a_3+a_4)-4a_1(3a_2+4a_4))k^2}$	$\frac{4(3a_1+a_4)}{(-3a_3{}^2+4a_4(a_2+a_3+a_4)+4a_1(3a_2+4a_4))k^2}$

	$h^{\#1}_{0^+}\,\dagger$	$h^{\#2}_{0^+}\,\dagger$
$h^{\#1}_{0^+}$	$(3a_1+a_4)\kappa^2$	$\frac{1}{2}\sqrt{3}(2a_1+a_3)\kappa^2$
$h^{\#2}_{0^+}$	$\frac{1}{2}\sqrt{3}(2a_1+a_3)\kappa^2$	$(a_1+a_2+a_3+a_4)\kappa^2$

	$h^{\#1}_{1^-}\,\dagger^{\alpha}$
$h^{\#1}_{1^-\,\alpha}$	$\frac{1}{2}(a_2+2a_4)\kappa^2$

(No source constraints)

	$h^{\#1}_{2^+}\,\dagger^{\alpha\beta}$
$h^{\#1}_{2^+\,\alpha\beta}$	$a_4\kappa^2$

	$\sigma^{\#1}_{1^-\,\alpha}$
$\sigma^{\#1}_{1^-}\,\dagger^{\alpha}$	$\frac{2}{(a_2+2a_4)k^2}$

	$\sigma^{\#1}_{2^+\,\alpha\beta}$
$\sigma^{\#1}_{2^+}\,\dagger^{\alpha\beta}$	$\frac{1}{a_4 k^2}$

Quadratic (free) action

$$S == \iiiint (h^{\alpha\beta}\ \sigma_{\alpha\beta} + a_1\,\partial_\beta h^{\chi}{}_{\chi}\,\partial^\beta h^{\alpha}{}_{\alpha} + a_2\,\partial_\alpha h^{\alpha\beta}\,\partial_\chi h_\beta{}^{\chi} + a_3\,\partial^\beta h^{\alpha}{}_{\alpha}\,\partial_\chi h_\beta{}^{\chi} + a_4\,\partial_\chi h_{\alpha\beta}\,\partial^\chi h^{\alpha\beta})[t,x,y,z]\,dz\,dy\,dx\,dt$$

Rank-2 @LO

Again, the game of ghost/higher poles suppressions reveals a narrow set of surviving linear theories

$$\mathcal{L} = a_1\,\partial^\sigma h_\mu^{\ \mu}\partial_\sigma h_\nu^{\ \nu} + a_2\,\partial_\mu h^{\mu\nu}\partial^\sigma h_{\mu\sigma} + a_3\,\partial_\nu h_\mu^{\ \mu}\partial_\sigma h^{\sigma\nu} + a_4\,\partial_\sigma h_{\mu\nu}\partial^\sigma h^{\mu\nu}$$

No Hexic/Quartic poles: $a_2 = -2\,a_4$

Removes the 1^- sector: **emergent gauge symmetry**

- $\delta h_{\mu\nu} = \partial_\mu \xi_\nu^T + \partial_\nu \xi_\mu^T$ with $\partial^\nu\,\xi_\nu^T = 0$ - Massless spin-2^+ + spin-0^+ propagating.
- Removing the extra scalar: $\delta h_{\mu\nu} = \partial_\mu \xi_\nu + \partial_\nu \xi_\mu$ - Only massless spin-2^+ - Linear Einstein.

A lesson from lower-rank spectrography: Unitarity and Causality (superluminal tachyons) is a powerful constraint. Only a few candidate theories survive. Organizing symmetries emerge.

Rank-2 @LO

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Emergent gauge symmetry

- Removing the extra scalar: $\delta h_{\mu\nu} = \partial_\mu \xi_\nu + \partial_\nu \xi_\mu$ - Only massless spin-2^+ - Linear Einstein.

Folklore theorems (Deser, Feynman, Gupta):
"Unique" non-linear deformation of Lagrangian and gauge symmetry generate Einstein and full diffeomorphism invariance.

(All order protection from ghostly 1^- state guaranteed)

$$\mathcal{L}(h) = \mathcal{L}_2(h) + \mathcal{L}_3(h) + \mathcal{L}_4(h) + \dots$$
$$\delta h_{\mu\nu} = \partial_\mu \xi_\nu + \partial_\nu \xi_\mu + \delta^1_{\mu\nu}(h,\xi) + \delta^2_{\mu\nu}(h,\xi) + \dots$$

$$\mathcal{L}(h) = \mathcal{L}(\delta_{\mu\nu} + k h_{\mu\nu}) \to \frac{2}{k^2} R(\delta_{\mu\nu} + k\, h_{\mu\nu})$$

Outline

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Use of SPO algebra and the success of the lower-rank cases triggered similar analyses for Rank-3 theories: **mostly top/bottom**

Gravity Lagrangian with ghost-free curvature-squared terms

Donald E. Neville
Physics Department, Temple University, Philadelphia, Pennsylvania 19122
(Received 2 December 1977; revised manuscript received 2 June 1978)

We construct a gravity theory using a set of Yang-Mills type gauge fields rather than the usual 10 metric fields $g_{\mu\nu}$. This formalism, first proposed by Utiyama and Kibble, allows us to construct a gravity Lagrangian containing six invariants bilinear in the curvature tensor (as well as the usual invariant linear in R). The new terms are of interest since they may absorb some of the divergences which result when gravity is renormalized in the presence of matter. We study the graviton propagator in this theory. Usually, when curvature-squared terms are added to the Lagrangian, ghosts appear in the graviton propagator, and/or its high-energy behavior worsens. In the present case, we find we must drop four of the six invariants in order to avoid such difficulties. One of the surviving invariants predicts the existence of an extremely massive pseudoscalar particle. Neither surviving invariant is of the proper form to absorb renormalization divergences. The present investigation does not fully test the potentialities of the Kibble-Utiyama formalism, since the torsion tensor is nonzero in their framework, and our Lagrangian included no terms constructed from this tensor.

Gravity theories with propagating torsion

Donald E. Neville
Physics Department, Temple University, Philadelphia, Pennsylvania 19122
(Received 19 March 1979; revised manuscript received 22 June 1979)

We study the propagators for a large class of gravity theories having a nonzero, metric-compatible torsion. The theories are derivable from a Lagrangian containing all possible invariants quadratic or less in the torsion and Riemann curvature tensors, except that invariants are dropped if they do not contribute to the propagator in the linearized limit. Therefore, the torsion in these theories is, in general, a propagating field rather than one which vanishes outside matter. We study the constraints imposed on the propagator by the requirement that the theory have no ghosts or tachyons. In particular, we find that the addition of a spin-2^+ torsion multiplet does not remove the spin-2^+ ghost contributed by higher-derivative terms (Riemann curvature-squared terms). We discuss the phenomenology of theories with propagating torsion. The torsion must couple to spins with coupling constants much smaller than the electromagnetic fine-structure constant, or the force between two macroscopic ferromagnets, due to torsion exchange, would be huge, far greater than the familiar magnetic force due to photon exchange. We briefly discuss the phenomenology of propagating torsion "potentials." Theories involving such potentials have been proposed recently by several authors.

PHYSICAL REVIEW D VOLUME 21, NUMBER 12 15 JUNE 1980

New ghost-free gravity Lagrangians with propagating torsion

E. Sezgin and P. van Nieuwenhuizen
Institute for Theoretical Physics, State University of New York, Stony Brook, New York 11794
(Received 20 November 1979; revised manuscript received 8 February 1980)

A new class of R^2 type of actions without ghosts and tachyons is found in which both the vierbein field $e_\mu{}^a$ and the spin connection $\omega_\mu{}^{ab}$ are independent and propagating fields. A complete set of spin-projection operators for $e_\mu{}^a$ and $\omega_\mu{}^{ab}$ is constructed. The particle content of these actions and other R^2 theories in the literature is given. The relation of this work to the interesting work of Neville is discussed.

Class of ghost-free gravity Lagrangians with massive or massless propagating torsion

E. Sezgin
*Institute for Theoretical Physics, University of California, Santa Barbara, California 93106**
(Received 17 October 1980)

A class of six-parameter ghost-free gravity Lagrangians are found, which propagate massive tordions with $J^P \leq 2^+$. One particular three-parameter ghost-free gravity Lagrangian is found which propagates a massless tordion with $J^P = 1^-$.

Rank-3 @LO

Metric Affine Gravity (MAG): a complete non-linear Lagrangian based on general covariance is the starting point.
A (deceptive) parallel with QED/YM: covariant derivatives define independent field with appropriate shifting symmetry.

QED:	MAG:
Compensating independent field:	
$D_\mu \Sigma = \partial_\mu \Sigma - i g B_\mu \Sigma$	$D_\mu \Sigma_\nu = \partial_\mu \Sigma_\nu - A_\mu{}^\rho{}_\nu \Sigma_\rho$
With shifting symmetry:	
$\delta B_\mu = \frac{1}{g} \partial_\mu \phi$	$\delta A_\mu{}^\rho{}_\nu = \partial_\mu \partial_\nu \xi^\rho$ (same $\xi^\rho(x)$ defining $\delta h_{\mu\nu}$)
Defining covariant building blocks:	
$F_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$	$F_{\mu\nu}{}^\rho{}_\sigma \equiv 2\left(\partial_{[\mu} A_{\nu]}{}^\rho{}_\sigma + A_{[\mu|}{}^\rho{}_\alpha A_{|\nu]}{}^\alpha{}_\sigma\right)$ $T_\mu{}^\alpha{}_\nu = A_\mu{}^\rho{}_\nu - A_\nu{}^\rho{}_\mu$ $Q_{\rho\mu\nu} = -\partial_\rho g_{\mu\nu} + A_\rho{}^\sigma{}_\mu g_{\sigma\nu} + A_\rho{}^\sigma{}_\nu g_{\sigma\mu}$

Rank-3 @LO

Tensors F, T and Q (no-shift) have a nice geometrical interpretation. We do not care.

Lengthy MAG actions (at least) quadratic in $A_\rho{}^\sigma{}_\nu$ are now at the theorist disposal.

$$
\begin{aligned}
S(g,A) = -\frac{1}{2}\int d^4x\,\sqrt{-g}\,\Big[&-a_0F + F^{\mu\nu\rho\sigma}\big(c_1F_{\mu\nu\rho\sigma} + c_2F_{\mu\nu\sigma\rho} + c_3F_{\rho\sigma\mu\nu} + c_4F_{\mu\rho\nu\sigma} +\\
&+ c_5F_{\mu\sigma\nu\rho} + c_6F_{\mu\sigma\rho\nu}\big) + F^{(13)\mu\nu}\big(c_7F^{(13)}_{\mu\nu} + c_8F^{(13)}_{\nu\mu}\big) + F^{(14)\mu\nu}\big(c_9F^{(14)}_{\mu\nu} + c_{10}F^{(14)}_{\nu\mu}\big) +\\
&+ F^{(14)\mu\nu}\big(c_{11}F^{(13)}_{\mu\nu} + c_{12}F^{(13)}_{\nu\mu}\big) + F^{\mu\nu}\big(c_{13}F_{\mu\nu} + c_{14}F^{(13)}_{\mu\nu} + c_{15}F^{(14)}_{\mu\nu}\big) + c_{16}F^2 +\\
&+ T^{\mu\rho\nu}\big(a_1T_{\mu\rho\nu} + a_2T_{\mu\nu\rho}\big) + a_3T^\mu T_\mu + Q^{\rho\mu\nu}\big(a_4Q_{\rho\mu\nu} + a_5Q_{\nu\mu\rho}\big) +\\
&+ a_6Q^\mu Q_\mu + a_7\widetilde{Q}^\mu\widetilde{Q}_\mu + a_8Q^\mu\widetilde{Q}_\mu + a_9T^{\mu\rho\nu}Q_{\mu\rho\nu} + T^\mu\left(a_{10}Q_\mu + a_{11}\widetilde{Q}_\mu\right)\Big] + \ldots,
\end{aligned}
$$

Percacci, Sezgin 1912.01023

Rank-3 @LO

KBFI NICPB

Decomposition in SO(3) irreps

$$
\begin{aligned}
&-\frac{1}{2}\eta_{\alpha\chi}\mathcal{A}^{\#1}_{1^{-}\,\beta}+\frac{1}{2}\eta_{\alpha\beta}\mathcal{A}^{\#1}_{1^{-}\,\chi}+\frac{4}{3}\mathcal{A}^{\#1}_{2^{-}\,\beta\chi\alpha}+\frac{1}{2}\mathcal{A}^{\#2}_{2^{-}\,\alpha\beta\chi}+\frac{1}{2}\mathcal{A}^{\#2}_{2^{-}\,\alpha\chi\beta}+\mathcal{A}^{\#1}_{3^{-}\,\alpha\beta\chi}+\frac{1}{3}\eta_{\beta\chi}\mathcal{A}^{\#6}_{1^{-}\,\alpha}-\frac{1}{6}\eta_{\alpha\chi}\mathcal{A}^{\#6}_{1^{-}\,\beta}-\frac{1}{6}\eta_{\alpha\beta}\mathcal{A}^{\#6}_{1^{-}\,\chi}+\frac{1}{15}\eta_{\beta\chi}\mathcal{A}^{\#4}_{1^{-}\,\alpha}+\\
&\frac{1}{15}\eta_{\alpha\chi}\mathcal{A}^{\#4}_{1^{-}\,\beta}+\frac{1}{15}\eta_{\alpha\beta}\mathcal{A}^{\#4}_{1^{-}\,\chi}+\mathcal{A}^{\#2}_{1^{+}\,\beta\chi}n_{\alpha}+\frac{1}{9}\eta_{\beta\chi}\mathcal{A}^{\#3}_{0^{+}}n_{\alpha}+\frac{1}{3}\mathcal{A}^{\#2}_{2^{+}\,\beta\chi}n_{\alpha}+\frac{2}{3}\mathcal{A}^{\#3}_{2^{+}\,\beta\chi}n_{\alpha}+\frac{2}{9}\eta_{\beta\chi}\mathcal{A}^{\#4}_{0^{+}}n_{\alpha}+\frac{1}{3}\eta_{\alpha\chi}\mathcal{A}^{\#1}_{0^{+}}n_{\beta}-\mathcal{A}^{\#1}_{1^{+}\,\alpha\chi}n_{\beta}+\\
&\mathcal{A}^{\#1}_{2^{+}\,\alpha\chi}n_{\beta}+\frac{1}{9}\eta_{\alpha\chi}\mathcal{A}^{\#3}_{0^{+}}n_{\beta}-\frac{1}{2}\mathcal{A}^{\#3}_{1^{+}\,\alpha\chi}n_{\beta}+\frac{1}{3}\mathcal{A}^{\#2}_{2^{+}\,\alpha\chi}n_{\beta}-\frac{1}{3}\mathcal{A}^{\#3}_{2^{+}\,\alpha\chi}n_{\beta}-\frac{1}{9}\eta_{\alpha\chi}\mathcal{A}^{\#4}_{0^{+}}n_{\beta}-\frac{1}{2}\mathcal{A}^{\#1}_{1^{-}\,\chi}n_{\alpha}n_{\beta}-\mathcal{A}^{\#2}_{1^{-}\,\chi}n_{\alpha}n_{\beta}+\frac{1}{6}\mathcal{A}^{\#6}_{1^{-}\,\chi}n_{\alpha}n_{\beta}-\\
&\frac{1}{15}\mathcal{A}^{\#4}_{1^{-}\,\chi}n_{\alpha}n_{\beta}-\frac{1}{3}\mathcal{A}^{\#5}_{1^{-}\,\chi}n_{\alpha}n_{\beta}+\frac{1}{3}\mathcal{A}^{\#3}_{1^{-}\,\chi}n_{\alpha}n_{\beta}-\frac{1}{3}\eta_{\alpha\beta}\mathcal{A}^{\#1}_{0^{+}}n_{\chi}+\mathcal{A}^{\#1}_{1^{+}\,\alpha\beta}n_{\chi}-\mathcal{A}^{\#1}_{2^{+}\,\alpha\beta}n_{\chi}+\frac{1}{9}\eta_{\alpha\beta}\mathcal{A}^{\#3}_{0^{+}}n_{\chi}-\frac{1}{2}\mathcal{A}^{\#3}_{1^{+}\,\alpha\beta}n_{\chi}+\\
&\frac{1}{3}\mathcal{A}^{\#2}_{2^{+}\,\alpha\beta}n_{\chi}-\frac{1}{3}\mathcal{A}^{\#3}_{2^{+}\,\alpha\beta}n_{\chi}-\frac{1}{9}\eta_{\alpha\beta}\mathcal{A}^{\#4}_{0^{+}}n_{\chi}+\frac{1}{2}\mathcal{A}^{\#1}_{1^{-}\,\beta}n_{\alpha}n_{\chi}+\mathcal{A}^{\#2}_{1^{-}\,\beta}n_{\alpha}n_{\chi}+\frac{1}{6}\mathcal{A}^{\#6}_{1^{-}\,\beta}n_{\alpha}n_{\chi}-\frac{1}{15}\mathcal{A}^{\#4}_{1^{-}\,\beta}n_{\alpha}n_{\chi}-\frac{1}{3}\mathcal{A}^{\#5}_{1^{-}\,\beta}n_{\alpha}n_{\chi}+\\
&\frac{1}{3}\mathcal{A}^{\#3}_{1^{-}\,\beta}n_{\alpha}n_{\chi}-\frac{1}{3}\mathcal{A}^{\#6}_{1^{-}\,\alpha}n_{\beta}n_{\chi}-\frac{1}{15}\mathcal{A}^{\#4}_{1^{-}\,\alpha}n_{\beta}n_{\chi}+\frac{2}{3}\mathcal{A}^{\#5}_{1^{-}\,\alpha}n_{\beta}n_{\chi}+\frac{1}{3}\mathcal{A}^{\#3}_{1^{-}\,\alpha}n_{\beta}n_{\chi}-\frac{1}{3}\mathcal{A}^{\#3}_{0^{+}}n_{\alpha}n_{\beta}n_{\chi}+\mathcal{A}^{\#2}_{0^{+}}n_{\alpha}n_{\beta}n_{\chi}-\frac{1}{6}\epsilon\eta_{\alpha\beta\chi\delta}\mathcal{A}^{\#1}_{0^{-}}n^{\delta}
\end{aligned}
$$

Problem # 1

Invariants of the Affine Connection are non-linear: $F_{\mu\nu}{}^{\rho}{}_{\sigma} \equiv 2\left(\partial_{[\mu}A_{\nu]}{}^{\rho}{}_{\sigma} + A_{[\mu|}{}^{\rho}{}_{\alpha}A_{|\nu]}{}^{\alpha}{}_{\sigma}\right)$

Problem # 2 Kinetic terms define $O(A^4)$ interactions (as in YM) .

Rank-3 @LO

Problem # 2 Kinetic terms define $O(A^4)$ interactions (as in YM) .

This link to YM is purely cosmetic. The shape of

$$F^i_{\mu\nu} \equiv 2\partial_{[\mu} B^i_{\nu]} - g f^{ijk} B^j_\mu B^j_\nu$$

is a consistent deformation of the gauge symmetry $\delta B^i_\mu(x) = \partial_\mu \epsilon^i(x)$, decoupling the (ghost-like) longitudinal state from the spectrum (**more on this later**).

The shift symmetry $\delta A_\mu{}^\rho{}_\sigma(x) = \partial_\mu \partial_\sigma \xi^\rho(x)$ *is not* connected to any decoupling in $A_\mu{}^\rho{}_\sigma$.

Geometrical Properties are fully detached from Quantum ones.

The (automatically generated) interactions expected to violently alter the LO properties.

Rank-3 @LO

What use of MAG?

- Define a viable linear spectrum (no ghosts, tachyons). 99% of dedicated literature.
- Assess dynamics/quantum corrections. Will the ghosts resurge at NLO? Is there a subset of MAG that is (modern) renormalizable? **Mostly uncharted territory**

Study of the linear spectrum and selection of healthy **free** theories: now trivial.

Computing radiative corrections in high-rank QFT: challenging. What to expect?

Rank-3 @LO

Seeking a LO ghost-free MAG, a little example (Torsionless subset $T_{\mu\ \nu}^{\ \alpha} = A_{\mu\ \nu}^{\ \rho} - A_{\nu\ \mu}^{\ \rho} = 0$):

$$S_2[g, A] = -\frac{1}{2}\int \mathrm{d}^4x\sqrt{-g}\Big[- a_0 F + F^{\mu\nu\rho\sigma}\Big(h_1 F_{\mu\nu\rho\sigma} + h_2 F_{\mu\nu\sigma\rho} + h_3 F_{\rho\sigma\mu\nu} + h_4 F_{\mu\rho\nu\sigma}\Big) +$$

$$+ F^{13\mu\nu}\Big(h_7 F^{13}{}_{\mu\nu} + h_8 F^{13}{}_{\nu\mu}\Big) + F^{14\mu\nu}\Big(h_9 F^{14}{}_{\mu\nu} + h_{10} F^{14}{}_{\nu\mu}\Big) + F^{14\mu\nu}\Big(h_{11} F^{13}{}_{\mu\nu} + h_{12} F^{13}{}_{\nu\mu}\Big)\Big]$$

MAG can be conveniently mapped into Quadratic Gravity + $K_{\sigma\ \mu}^{\ \rho}$ ($\mathbf{G^2K}$) via

$$A_{\sigma\ \mu}^{\ \rho} = \Gamma^{\nu}{}_{\sigma\mu} + K_{\sigma\ \mu}^{\ \rho}$$

and $S_2[g, A]$ maps into

$$\mathcal{S}_{G^2K} = \mathcal{S}_g + \mathcal{S}_{\nabla^2} + \mathcal{S}_{K^2} + \mathcal{S}_{R\nabla K} + \mathcal{S}_{K^3} + \mathcal{S}_{RK^2} + \mathcal{S}_{K^4} + \cdots$$

Rank-3 @LO

With the "quadratic" terms (defining propagation) being

$$\mathcal{S}_g = \int d^4x\sqrt{-g}\left[\alpha_0 R + \beta_1 R^2 + \beta_2 R_{\mu\nu}R^{\mu\nu}\right], \ (R \text{ usual curvature})$$

A glimpse of the typical G^2K *Kinetic* terms

$$\mathcal{S}_{R\nabla K} = \int d^4x\sqrt{-g}\left[\eta_1 \cdot R^{\alpha\mu}\nabla_\mu K_\alpha{}^\beta{}_\beta + \eta_3 \cdot R\nabla_\mu K^{\alpha\mu}{}_\alpha + \eta_5 \cdot R^{\alpha\mu}\nabla_\beta K_{\alpha\mu}{}^\beta + \eta_6 \cdot R^{\alpha\mu}\nabla_\beta K_\alpha{}^\beta{}_\mu\right]$$

$$\mathcal{S}_{K^2} = \int d^4x\sqrt{-g}\left[\lambda_1 \cdot K_{\alpha\mu\beta}K^{\alpha\mu\beta} + \lambda_2 \cdot K_{\alpha\beta\mu}K^{\alpha\mu\beta} + \lambda_3 \cdot K^\alpha{}_\alpha{}^\mu K_\mu{}^\beta{}_\beta + \lambda_4 \cdot K^\alpha{}_\alpha{}^\mu K^\beta{}_{\mu\beta} + \dots\right]$$

$$\mathcal{S}_{\nabla^2} = \int d^4x\sqrt{-g}\left[\zeta_1 \cdot \nabla_\mu K_\beta{}^\nu{}_\nu \nabla^\beta K^\alpha{}_\alpha{}^\mu + \zeta_2 \cdot \nabla_\mu K^\nu{}_{\beta\nu}\nabla^\beta K^\alpha{}_\alpha{}^\mu + \zeta_3 \cdot \nabla_\beta K_\mu{}^\nu{}_\nu \nabla^\beta K^\alpha{}_\alpha{}^\mu + \dots\right]$$

Rank-3 @LO

Larger freedom in the selection of ghost-free (linear) theories.

In general, fix some broad/arbitrary requirements and then start the usual algorithm by finding poles and residues (nothing new, only more involved).

- Graviton propagation must be preserved.
- No spin-3 or extra spin-2 (Simplifying assumptions) $\to a^{\{3-\}}_{1,1} \propto q^2(h_3 - 2h_2) - \frac{a_0}{2}$.
- I want only a ~~scalar~~/vector to propagate.
- No ghosts, no tachyons.

$$h_4 = h_3 = h_2 = 0\,,\;\; h_7 = -h_8\,,\; h_{12} = -\frac{1}{3}\left(3+\sqrt{15}\right)h_8\,,\; h_{12} = -\frac{1}{6}\left(4+\sqrt{15}\right)h_8$$

Cumbersome constraints over the coupling, but code-friendly!

Outline

MAG as a Phenomenological Lagrangian?

Finding linear healthy theories is getting easier, but that is not the end. Whether they provide a predictive framework is questionable and needs study.

What is the final use/vision for these theories?

The dream is Yang-Mills (Dyson renormalization)

The hope is EFT (Modern renormalization)

In general, we get none of the above

MAG as a Phenomenological Lagrangian?

Predictions from QFT, not a given:

PHENOMENOLOGICAL LAGRANGIANS*

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Are Nonrenormalizable Gauge Theories Renormalizable?

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Need for "symmetric" interactions and structural constraints to achieve predictivity . . .

Phenomenological Lagrangians

Renormalization of UV-divergences shapes the Lagrangian

- An initial finite set of operators in the Lagrangian $\hat{O}_i$ is closed under renormalization. Renormalizable theory.

$$\mathcal{L}_{int} = \sum_i c_i \hat{O}_i \xrightarrow{renormalization} \sum_i \tilde{c}_i \hat{O}_i, \quad (\tilde{c}_i \sim \alpha_i/\epsilon_{UV} + C_i)$$

Finite (in number) set of parameters stays finite.
Predictions in terms of finite set of small couplings C_i.
Experiment growth in precision: same parameters, more loops.

Phenomenological Lagrangians

Renormalization of UV-divergences shapes the Lagrangian

- Renormalization via an initial finite set of operators $\hat{O}_i$ requires the introduction of a new set of higher-dimensionality *ad infinitum*. Non renormalizable theory...in Dyson's sense!

$$\mathcal{L}_{int} = \sum_i c_i \hat{O}_i \to \sum_i c_i \hat{O}_i + \sum_i \frac{\tilde{c}_i^1}{\Lambda} \hat{O}_i^1 \to \sum_i c_i \hat{O}_i + \sum_i \frac{\tilde{c}_i^1}{\Lambda} \hat{O}_i^1 + \sum_i \frac{\tilde{c}_i^2}{\Lambda^2} \hat{O}_i^2 + \dots$$

Can be renormalizable in the **modern sense:**
Structural constraints are needed to ensure that counterterms exist for all the divergences to be, at each order, absorbed by renormalizing amplitudes at a given scale.

Paradigmatic examples ChPT, Einstein Gravity.

Phenomenological Lagrangians

Are non-renormalizable theories renormalizable?

$$\mathcal{L}_{int} = \sum_i c_i \hat{O}_i \to \sum_i c_i \hat{O}_i + \sum_i \frac{\tilde{c}_i^1}{\Lambda} \hat{O}_i^1 \to \sum_i c_i \hat{O}_i + \sum_i \frac{\tilde{c}_i^1}{\Lambda} \hat{O}_i^1 + \sum_i \frac{\tilde{c}_i^2}{\Lambda^2} \hat{O}_i^2 + \dots$$

An organizing/controlling symmetry exists, ensuring that, at a given order in $1/\Lambda$, no new operators are created.

Relevant for high-rank QFT/MAG: no detuning of kinetic term due to radiative corrections.
Explicit example below!

The organizing symmetry is relevant for us to "believe" the EFT predictions.
Low-energy footprint of unknown high-energy symmetric theory:
Universal Low-Energy theorems.

Renormalizable EFTs come with a large cut-off scale Λ pointing to their demise:...

Phenomenological Lagrangians

Renormalizable EFTs come with a large cut-off scale Λ pointing to their demise:

- For a fixed energy $E < \Lambda$, at a fixed precision $L \sim (E/\Lambda)^n$ only a finite set of operators $\sim \Lambda^{-n}$ are needed.
  EFT is predictive (but needs increasing experimental input with increased precision).
- Kinetic Detuning: higher-order operators (R^2 in Einstein, for instance) appear to introduce higher derivative kinetic terms: *dipole ghosts.*

EFT scaling takes care of it:

$$\partial^n \to \frac{\partial^n}{\Lambda^n}$$

Multiderivative ghosts have masses $\sim \Lambda$, pushed outside the EFT validity range.

Outline

Massive vector @NLO, Rank-1

Before computing NLO corrections to MAG vector models, something simpler.

A study of kinetic detuning with a careless $A_\mu(x)$

We have seen at the starting slides how Proca theory ...

$$\mathcal{S}_2 = -\frac{1}{2}\int \frac{d^4p}{(2\pi)^4}\, A^\mu(p)\left(P^{1^-}_{\mu\nu}\,(p^2 - m_V^2) - m_V^2 P^{0^+}_{\mu\nu}\right) A^\nu(-p)$$
$$= \frac{1}{2}\int d^4x A^\mu(x)\left(g_{\mu\nu}(\Box + m_V^2) - \partial_\mu\partial_\nu\right) A^\nu(x)\,, \qquad \left(\text{with} \quad P^{1^-}_{\mu\nu} = g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\,,\ P^{0^+}_{\mu\nu} = \frac{p_\mu p_\nu}{p^2}\right)$$

...emerges from the simplest inclusion of unitarity and tachyon-freedom for a rank-1 field $A_\mu(x)$.

No gauge symmetries, all particle components used, but only one propagates.

Massive vector @NLO, Rank-1

No gauge symmetries, all particle components used, but only one propagates.

For this reason, self-interacting Proca theories have often welcomed terms polynomial in $A^2 = A_\mu A^\mu$.

No Dyson renormalizable: large-momentum behaviour of the Proca propagator

$$D_{\mu\nu} = \frac{-i}{q^2 - m_V^2}\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{m_V^2}\right) = -i\left(\frac{P^{1^-}_{\mu\nu}}{q^2 - m_V^2} - \frac{P^{0^+}_{\mu\nu}}{m_V^2}\right)$$

Maybe we can build a predictive EFT?

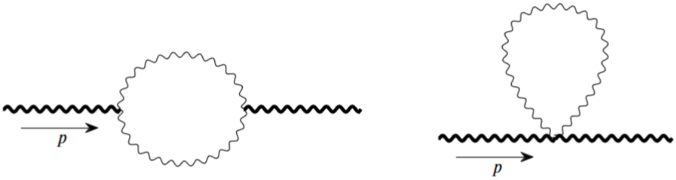
Let's consider the addendum:

$$\mathcal{S}_i = -\int d^4x \left(\frac{g_3}{4} A_\nu(x) A^\nu(x) \partial_\mu A^\mu(x) + \frac{g_4}{4}\left(A_\nu(x) A^\nu(x)\right)^2\right)$$

After all, we have no gauge symmetries to respect!

Massive vector @NLO, Rank-1

Assessing the 2-point functions is already quite revealing:



Action is deformed by radiative corrections:

$$\rightarrow \frac{1}{2}\int d^4x A^\mu(x)\Bigg[\left(Z_T^0 + Z_T^2\frac{\Box}{m_V^2} + Z_T^4\frac{\Box^2}{m_V^4} + \cdots\right)(g_{\mu\nu}\Box - \partial_\mu\partial_\nu) +$$

$$+Z_m\, m_V^2\, g_{\mu\nu} + \left(Z_L^0 + Z_L^2\frac{\Box}{m_V^2} + Z_L^4\frac{\Box^2}{m_V^4} + \cdots\right)\partial_\mu\partial_\nu\Bigg]A^\nu(x), \quad \left[Z_i^j = \frac{1}{(4\pi)^2\epsilon}z_i^j + \tilde{Z}_i^j\right]$$

Massive vector @NLO, Rank-1

Action is deformed by radiative corrections:

$$\rightarrow \frac{1}{2}\int d^4x A^\mu(x)\Bigg[\left(Z_T^0 + Z_T^2\frac{\Box}{m_V^2} + Z_T^4\frac{\Box^2}{m_V^4} + \cdots\right)(g_{\mu\nu}\Box - \partial_\mu\partial_\nu) +$$
$$+ Z_m\, m_V^2\, g_{\mu\nu} + \left(Z_L^0 + Z_L^2\frac{\Box}{m_V^2} + Z_L^4\frac{\Box^2}{m_V^4} + \cdots\right)\partial_\mu\partial_\nu\Bigg]A^\nu(x), \quad \left[Z_i^j = \frac{1}{(4\pi)^2\epsilon}z_i^j + \tilde{Z}_i^j\right]$$

The direct computation gives, in a scale-less renormalization, the following values for the singular parts

$$z_m = \frac{3}{16}\left(g_3^2 + 24 g_4^2\right)\,, \quad z_T^0 = -\frac{3}{16}g_3^2\,, \quad z_T^2 = z_T^4 = 0\,, \quad z_L^0 = \frac{9}{16}g_3^2\,, \quad z_L^2 = \frac{3}{16}g_3^2\,, \; z_L^4 = \frac{1}{32}g_3^2$$

Not a deformation of the starting Lagrangian!

Massive vector @NLO, Rank-1

$$\to \frac{1}{2}\int d^4x A^\mu(x)\Bigg[\left(Z_T^0 + Z_T^2\frac{\Box}{m_V^2} + Z_T^4\frac{\Box^2}{m_V^4} + \cdots\right)(g_{\mu\nu}\Box - \partial_\mu\partial_\nu) +$$

$$+Z_m\, m_V^2\, g_{\mu\nu} + \left(Z_L^0 + Z_L^2\frac{\Box}{m_V^2} + Z_L^4\frac{\Box^2}{m_V^4} + \cdots\right)\partial_\mu\partial_\nu\Bigg]A^\nu(x), \quad \left[Z_i^j = \frac{1}{(4\pi)^2\epsilon}z_i^j + \tilde{Z}_i^j\right]$$

Not a deformation of the starting Lagrangian!

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- Longitudinal components have momentum dependence: ghost resurrected (no controlling symmetry).
- New dipole ghosts: dangerous higher-order operators not dampened by large cut-off mass. **Scale accordingly with m_V.** Assumed not big!
- All operators are generated, no control, no predictions.

Massive vector @NLO, Rank-1

As known, a predictive framework is at hand using $U(1)$ to decouple the longitudinal state:

$$\mathcal{S}_i = -\int d^4x \left(\frac{c_3}{4} \frac{(F^2)^2}{m_\Lambda^4} + \frac{c_4}{4} \frac{Tr[F^4]}{m_\Lambda^4} + \sum_i^{\infty} \lambda_i \, \omega_i \, [F, \partial/m_\Lambda] \right)$$

Infinite sum of polynomials, of increasing dimensionality, built from U(1)-invariant form $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$. Decoupling guaranteed!

The large mass $m_\Lambda \gg m_V$ introduced to justify the neglecting of higher-order operators. Enters naturally in the definition of the dimensionful couplings.

Massive vector @NLO, Rank-1

KBFI NICPB

Compute radiative corrections:

$$\to \frac{1}{2}\int d^4x A^\mu(x)\Bigg[\left(Z_T^0 + Z_T^2\frac{\Box}{{m_\Lambda}^2} + Z_T^4\frac{\Box^2}{{m_\Lambda}^4} + \cdots\right)(g_{\mu\nu}\Box - \partial_\mu\partial_\nu) +$$

$$+Z_m\, m_V^2\, g_{\mu\nu} + \left(Z_L^0 + Z_L^2\frac{\Box}{{m_\Lambda}^2} + Z_L^4\frac{\Box^2}{{m_\Lambda}^4} + \cdots\right)\partial_\mu\partial_\nu\Bigg]A^\nu(x), \quad \left[Z_i^j = \frac{1}{(4\pi)^2\epsilon}z_i^j + \tilde{Z}_i^j\right]$$

but now:

$$z_m = 0\,, \qquad z_T^0 = -\frac{m_V^4}{m_\Lambda^4}(7c_3 + 16c_4) \tag{1}$$

$Z_L^i \equiv 0$. No m_V but m_Λ in defining higher-order corrections.

Predictive and no detune of kinetic term.

Outline

KBFI NICPB

Massive vector @NLO, MAG

KBFI NICPB

Main lesson from rank-1 experience: **predictive program is vulnerable if interactions are detached from the spectrum.**

We left MAG after (successful) spectral analysis of the torsionless subset:

$$S_2[g,A] = -\frac{1}{2}\int \mathrm{d}^4x\sqrt{-g}\Big[-a_0 F + F^{\mu\nu\rho\sigma}\Big(h_1 F_{\mu\nu\rho\sigma} + h_2 F_{\mu\nu\sigma\rho} + h_3 F_{\rho\sigma\mu\nu} + h_4 F_{\mu\rho\nu\sigma}\Big) +$$

$$+ F^{13\mu\nu}\Big(h_7 F^{13}{}_{\mu\nu} + h_8 F^{13}{}_{\nu\mu}\Big) + F^{14\mu\nu}\Big(h_9 F^{14}{}_{\mu\nu} + h_{10} F^{14}{}_{\nu\mu}\Big) + F^{14\mu\nu}\Big(h_{11} F^{13}{}_{\mu\nu} + h_{12} F^{13}{}_{\nu\mu}\Big)\Big]$$

Reminder: geometrically induced link between quadratic and non-linear part:
$F_{\mu\nu}{}^{\rho}{}_{\sigma} \equiv 2\left(\partial_{[\mu} A_{\nu]}{}^{\rho}{}_{\sigma} + A_{[\mu|}{}^{\rho}{}_{\alpha} A_{|\nu]}{}^{\alpha}{}_{\sigma}\right)$

MAG can be conveniently mapped into Quadratic Gravity + $K_{\sigma}{}^{\rho}{}_{\mu}$ ($\mathbf{G^2K}$) via

$$A_{\sigma}{}^{\rho}{}_{\mu} = \Gamma^{\nu}{}_{\sigma\mu} + K_{\sigma}{}^{\rho}{}_{\mu}$$

Massive vector @NLO, MAG

$S_2[g, A]$ maps into

$$\mathcal{S}_{G^2K} = \mathcal{S}_g + \mathcal{S}_{\nabla^2} + \mathcal{S}_{K^2} + \mathcal{S}_{R\nabla K} + \mathcal{S}_{K^3} + \mathcal{S}_{RK^2} + \mathcal{S}_{K^4} + \cdots$$

We fixed $\mathcal{S}_g + \mathcal{S}_{\nabla^2} + \mathcal{S}_{K^2} + \mathcal{S}_{R\nabla K}$, so to have a healthy linear spectrum:

Graviton + Massive Vector $m_V^2 = -3(4 + \sqrt{15})a_0/h_8$

Automatically, we also have non-zero $\mathcal{S}_{K^3} + \mathcal{S}_{RK^2} + \mathcal{S}_{K^4}$ ...

Massive vector @NLO, MAG

Cubic self-interactions:

$$\mathcal{S}_{K^3} = \frac{g_K}{6} \int d^4x \sqrt{-g} \Big[$$

$$(-3+\sqrt{15})\, K_{\mu\rho}{}^{\beta} K^{\mu\nu\rho} \nabla_\beta K_\nu{}^\sigma{}_\sigma + (3-\sqrt{15})\, K^{\mu\nu}{}_\mu K_\nu{}^{\rho\beta} \nabla_\beta K_\rho{}^\sigma{}_\sigma - K_{\mu\rho}{}^{\beta} K^{\mu\nu\rho} \nabla_\beta K^\sigma{}_{\nu\sigma} +$$

$$+ K^{\mu\nu}{}_\mu K_\nu{}^{\rho\beta} \nabla_\beta K^\sigma{}_{\rho\sigma} + (3-\sqrt{15})\, K_{\mu\rho}{}^{\beta} K^{\mu\nu\rho} \nabla_\nu K_\beta{}^\sigma{}_\sigma + K_{\mu\rho}{}^{\beta} K^{\mu\nu\rho} \nabla_\nu K^\sigma{}_{\beta\sigma} + (-3+\sqrt{15})\, K^{\mu\nu}{}_\mu K_\nu{}^{\rho\beta} \nabla_\rho K_\beta{}^\sigma{}_\sigma$$

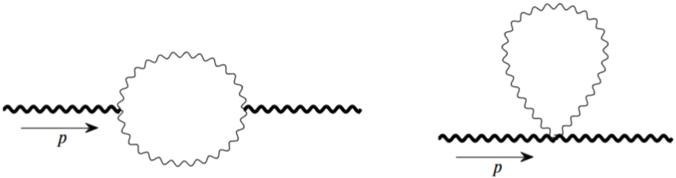
$$- K^{\mu\nu}{}_\mu K_\nu{}^{\rho\beta} \nabla_\rho K^\sigma{}_{\beta\sigma} + K_{\mu\rho}{}^{\beta} K^{\mu\nu\rho} \nabla_\sigma K_{\beta\nu}{}^\sigma - K^{\mu\nu}{}_\mu K_\nu{}^{\rho\beta} \nabla_\sigma K_{\beta\rho}{}^\sigma - K_{\mu\rho}{}^{\beta} K^{\mu\nu\rho} \nabla_\sigma K_{\nu\beta}{}^\sigma + K^{\mu\nu}{}_\mu K_\nu{}^{\rho\beta} \nabla_\sigma K_{\rho\beta}{}^\sigma \Big]$$

and quartic self-interactions

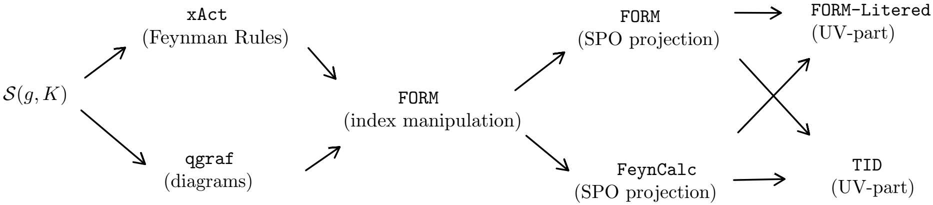
$$\mathcal{S}_{K^4} = \frac{g_K^2}{12} \int d^4x \sqrt{-g} \Big[K^f{}_{\sigma f} K_{\beta\rho}{}^\sigma K^{\mu\nu}{}_\mu K_\nu{}^{\rho\beta} - K^f{}_{\sigma f} K^{\mu\nu}{}_\mu K_\nu{}^{\rho\beta} K_{\rho\beta}{}^\sigma - K_{\mu\rho}{}^\beta K^{\mu\nu\rho} K_\nu{}^{\sigma f} K_{\sigma\beta f} +$$

$$+ 2\, K^{\mu\nu}{}_\mu K_\nu{}^{\rho\beta} K_\rho{}^{\sigma f} K_{\sigma\beta f} + K_\beta{}^{\sigma f} K_{\mu\rho}{}^\beta K^{\mu\nu\rho} K_{\sigma\nu f} - 2\, K_\beta{}^{\sigma f} K^{\mu\nu}{}_\mu K_\nu{}^{\rho\beta} K_{\sigma\rho f} \Big],$$

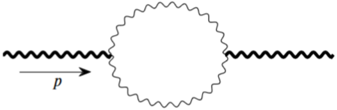
Massive vector @NLO, MAG

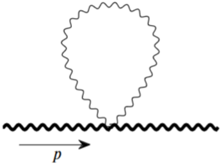


Topologies are the same of the rank-1 case, computational complexity isn't: (many extenal tools helped)



Massive vector @NLO, MAG





Analogously to rank-1 case, consider the deformations induced by renormalizing the two-point function.

The following tree-level operators are renormalized.

$$\mathcal{S}_{R\nabla K} = \int d^4x\sqrt{-g}\left[\eta_1 \cdot R^{\alpha\mu}\nabla_\mu K_\alpha{}^\beta{}_\beta + \eta_3 \cdot R\nabla_\mu K^{\alpha\mu}{}_\alpha + \eta_5 \cdot R^{\alpha\mu}\nabla_\beta K_{\alpha\mu}{}^\beta + \eta_6 \cdot R^{\alpha\mu}\nabla_\beta K_\alpha{}^\beta{}_\mu\right]$$

$$\mathcal{S}_{K^2} = \int d^4x\sqrt{-g}\left[\lambda_1 \cdot K_{\alpha\mu\beta}K^{\alpha\mu\beta} + \lambda_2 \cdot K_{\alpha\beta\mu}K^{\alpha\mu\beta} + \lambda_3 \cdot K^\alpha{}_\alpha{}^\mu K_\mu{}^\beta{}_\beta + \lambda_4 \cdot K^\alpha{}_\alpha{}^\mu K^\beta{}_{\mu\beta} + \dots\right]$$

Massive vector @NLO, MAG

Defining the parameter finite/UV splitting as:

$$\zeta_i = \frac{1}{(4\pi)^2\epsilon}\zeta_i^\epsilon + \zeta_i^0, \qquad \lambda_i = \frac{1}{(4\pi)^2\epsilon}\lambda_i^\epsilon + \lambda_i^0,$$

we mutate the LO values

$$\zeta_1^0 = \frac{1}{2}\left(\sqrt{15} - 4\right), \quad \zeta_2^0 = \frac{1}{6}\left(\sqrt{15} - 3\right), \quad \zeta_3^0 = -\zeta_1^0, \quad \zeta_4^0 = -\zeta_2^0, \quad \zeta_5^0 = -\frac{1}{12},$$

$$\zeta_6^0 = -\zeta_5^0, \quad \zeta_7^0 = 0, \quad \zeta_8^0 = \zeta_5^0, \quad \zeta_9^0 = -\zeta_2^0, \quad \zeta_{10}^0 = \frac{1}{6}, \quad \zeta_{11}^0 = 0, \quad \zeta_{14}^0 = \frac{1}{12},$$

$$\zeta_{15}^0 = \zeta_2^0, \quad \zeta_{16}^0 = -\frac{1}{6}, \quad \zeta_{24}^0 = 0, \quad \zeta_{25}^0 = 0,$$

$$\lambda_1 = 0, \quad \lambda_2 = \frac{m_V^2}{6}, \quad \lambda_3 = 0, \quad \lambda_4 = -\lambda_2, \quad \lambda_5 = 0 \ldots$$

Massive vector @NLO, MAG

...to the NLO values (the second power of the unique expansion parameter g_K implicit, overall)

$$\zeta_1^\epsilon = \frac{1}{192}\left(78793 - 20357\sqrt{15}\right), \quad \zeta_2^\epsilon = \frac{1}{192}\left(9641 - 2533\sqrt{15}\right), \quad \zeta_3^\epsilon = \frac{1}{768}\left(81916\sqrt{15} - 316919\right),$$

$$\zeta_4^\epsilon = \frac{1}{256}\left(3300\sqrt{15} - 13039\right), \quad \zeta_5^\epsilon = \frac{3}{16}\left(1 + 3\sqrt{15}\right), \quad \zeta_6^\epsilon = \frac{19217 - 4408\sqrt{15}}{3072},$$

$$\zeta_7^\epsilon = \frac{1}{128}\left(40\sqrt{15} - 181\right), \quad \zeta_8^\epsilon = \frac{1}{32}\left(386\sqrt{15} - 1521\right), \quad \zeta_9^\epsilon = \frac{1}{192}\left(32315 - 8383\sqrt{15}\right),$$

$$\zeta_{10}^\epsilon = \frac{1}{384}\left(17249 - 4408\sqrt{15}\right), \quad \zeta_{11}^\epsilon = \frac{1}{16}\left(2\sqrt{15} - 11\right), \quad \zeta_{14}^\epsilon = \frac{1}{16}\left(829 - 209\sqrt{15}\right),$$

$$\zeta_{15}^\epsilon = \frac{1}{384}\left(16502\sqrt{15} - 63337\right), \quad \zeta_{16}^\epsilon = \frac{1}{768}\left(10280\sqrt{15} - 41287\right), \quad \zeta_{24}^\epsilon = -\frac{5}{512}\left(88\sqrt{15} - 325\right),$$

$$\zeta_{25}^\epsilon = \frac{1}{256}\left(136\sqrt{15} - 487\right),$$

$$\lambda_1^\epsilon = \frac{1}{64}\left(180\sqrt{15} - 697\right) m_V^2, \quad \lambda_2^\epsilon = \frac{1}{64}\left(649 - 180\sqrt{15}\right) m_V^2,$$

$$\lambda_3^\epsilon = \frac{5}{192}\left(133 - 36\sqrt{15}\right) m_V^2, \quad \lambda_4^\epsilon = \frac{5}{96}\left(36\sqrt{15} - 109\right) m_V^2, \; \lambda_5^\epsilon = \frac{1}{192}\left(389 - 180\sqrt{15}\right) m_V^2 .$$

Massive vector @NLO, MAG

Not a renormalization of the starting Lagrangian.

- No large mass (as expected by the dimensionality of terms investigated) comes to the rescue.
- One example for all: spin-3 is reintroduced

$$a_{1,1}^{\{3,-\}} = -\frac{g_K^2}{(4\pi)^2\,\epsilon}\left[\frac{3}{2}m_V^2 + \frac{9}{128}\left(8\sqrt{15}-31\right)\left(\frac{7}{6}q^2 + \frac{q^4}{m_V^2}\right)\right], \quad (2)$$

No chance to build a sensible EFT in this model.

Maybe I have been unlucky with this particular model?
This behaviour is common in high-rank QED, more different examples in *2403.15003*.

Outline

Hopes for an EFT for MAG: Work in progress

Interactions disconnected from the spectrum give the model no chance.

Obvious solution: **Symmetry First:**

Over the generic $\mathcal{S}_{G^2K}$

$$\mathcal{S}_{G^2K} = \mathcal{S}_g + \mathcal{S}_{\nabla^2} + \mathcal{S}_{K^2} + \mathcal{S}_{R\nabla K} + \mathcal{S}_{K^3} + \mathcal{S}_{RK^2} + \mathcal{S}_{K^4} + \cdots$$

we impose invariance under some guessed gauge transformation.
For instance

$$\delta A_{\mu}{}^{\rho}{}_{\nu} = x\, g_{\mu\nu}\partial_\rho\phi + y\, g_{\mu\nu}\xi_\rho + z\, \partial^\rho\Omega_{\mu\nu} \ldots$$

and hope to get ghost freedom for free.

In this way: presence of ghost connected with the hard-breaking of the symmetry.

Hopes for an EFT for MAG: Work in progress

Some (dull) advance in this regard:

The shift symmetry

$$\delta A_{\mu}{}^{\rho}{}_{\nu} = z\,\partial^{\rho}\Omega_{\mu\nu}$$

suppresses all quadratic operators FF, QQ in the (torsionless) MAG action but these:

$$S_2[g,A] = -\frac{1}{2}\int \mathrm{d}^4x\sqrt{-g}\Big[-a_0 F + h_7 F^{13\mu\nu}\Big(F^{13}{}_{\mu\nu} - F^{13}{}_{\nu\mu}\Big)\Big]$$

- Only graviton and a massless vector propagates.
- Spectrum radiatively stable (1L checked).
- in $F^{13}{}_{\mu\nu} - F^{13}{}_{\nu\mu}$ interactions cancel out. Theory is free (modulo gravitational interaction).

Hopes for an EFT for MAG: Work in progress

The symmetric, radiative stable theory

$$S_2[g,A] = -\frac{1}{2}\int \mathrm{d}^4x\sqrt{-g}\Big[-a_0 F + h_7 F^{13\mu\nu}\Big(F^{13}{}_{\mu\nu} - F^{13}{}_{\nu\mu}\Big)\Big],$$

once redefined via $K_\sigma{}^\rho{}_\mu$, **it is the theory of the abelian vector in** $g^{\mu\nu}K_\mu{}^\rho{}_\nu$

A perfecly fine EFT (or better) as previously analyzed for the rank-1 case.
But (probably) dynamically identical to the lower-rank representation.

A BEGINNING, BUT A LOT OF WORK AHEAD!

Outline

Conclusions

- High-rank field theory, an exciting arena for new ideas!
- Indices bring complexity, computational methods are mandatotory (*PSALTer*, *FORM*).
- Is MAG an EFT? Geometrical features seem to work against it.
- Spectra and radiative corrections are interconnected. Our spectral assessments are empty if not respected by interactions.
- Therefore: **Symmetry first, spectrum later.** Ghost-free spectra shaped by symmetry stand a chance to survive. Only hope for high-rank field theory.

Thank you for your attention!

Q&A