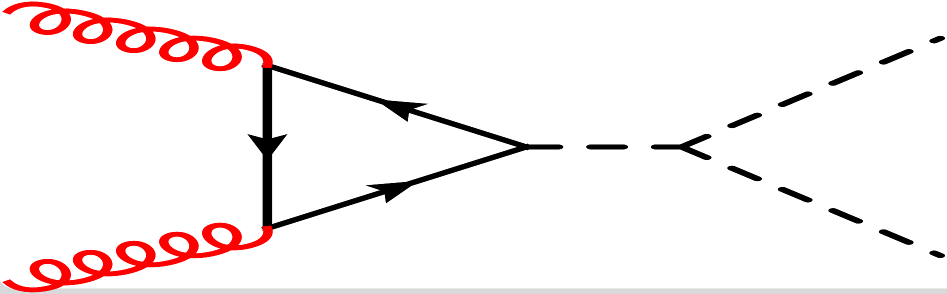
Karlsruhe Institute of Technology

Institute for Theoretical Particle Physics

Higgs Boson Pair Production at the LHC

Warsaw, May 23, 2024

Matthias Steinhauser | TTP KIT



I. Why double Higgs production?
II. Fast NLO QCD
III. Towards NNLO
IV. Electroweak corrections
V. Conclusions

SM couplings

gauge couplings

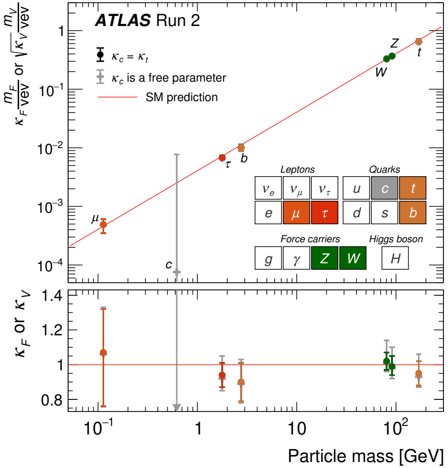
$\alpha_1, \alpha_2, \alpha_s$

Yukawa couplings

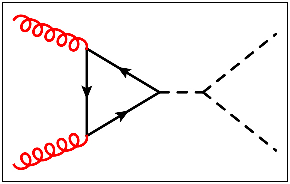
$y_t, y_b, \ldots$

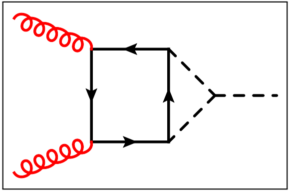
Self coupling

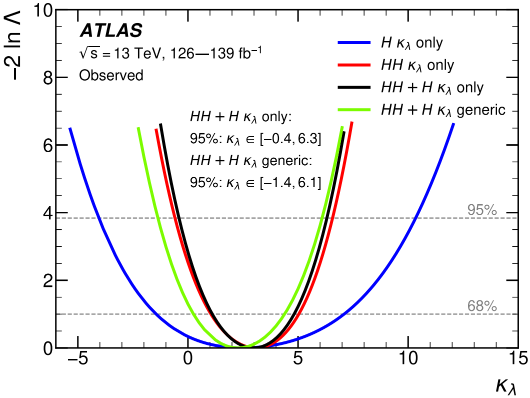
$\lambda = \frac{m_H^2}{2v^2} \approx 0.125$



λ from *H* and *HH* production



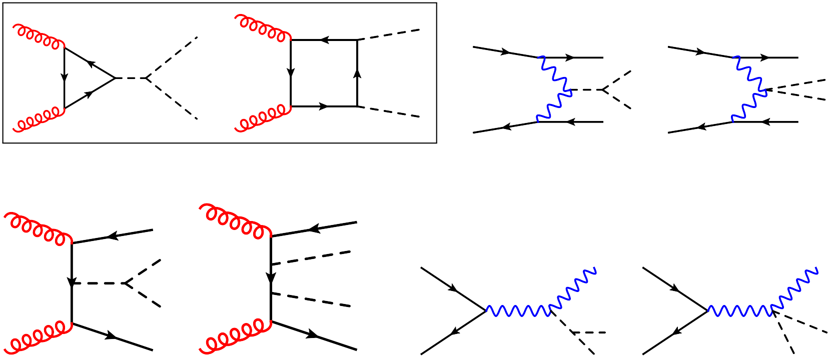




HL-LHC: $\mathcal{O}(50\%)$
FCC-hh: $\mathcal{O}(5\%)$

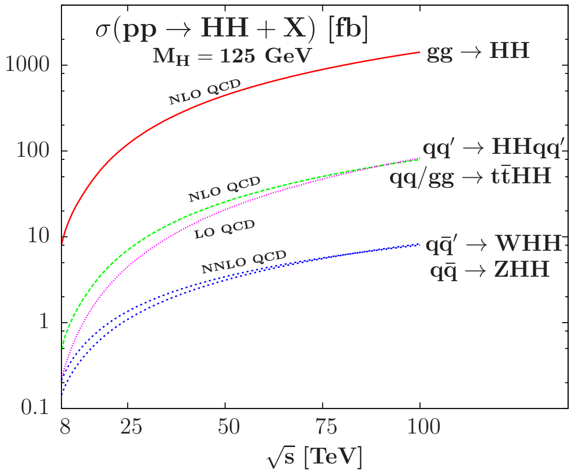
$$\kappa_\lambda = \frac{\lambda_{\rm HHH}}{\lambda^{\rm SM}_{\rm HHH}}$$

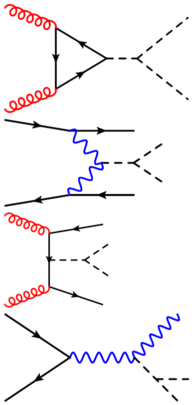
Double Higgs production in SM



Double Higgs production in SM (2)

[Baglio,Djouadi,Gröber,Mühlleitner,Quevillon,Spira'12]

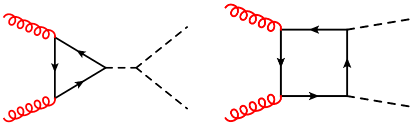


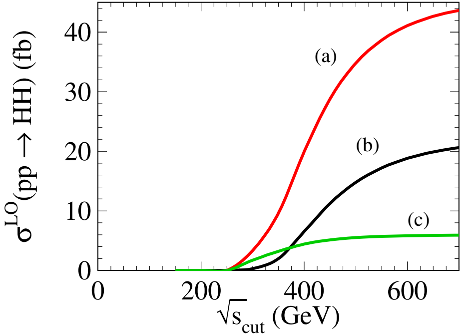


$K^{\rm NLO} \approx 1.9$

$K^{\rm NNLO} \approx 1.2$

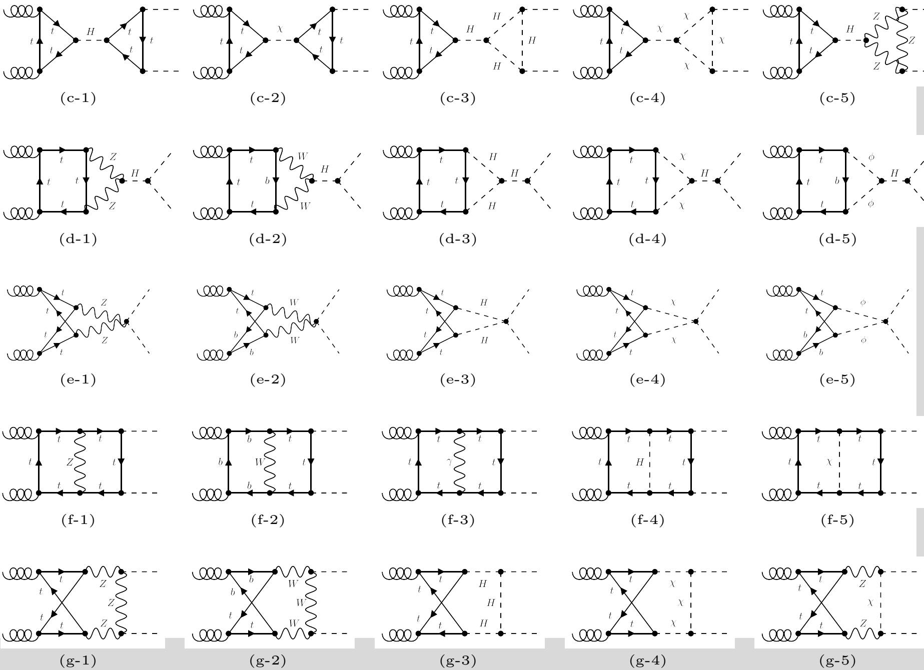
Double Higgs production in SM at LO





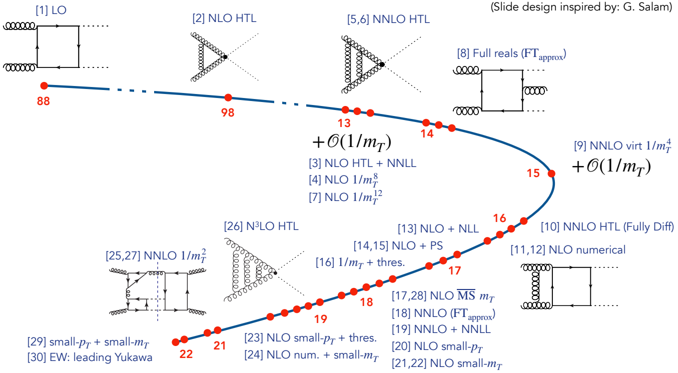
(a) box
(b) box+triangle
(c) triangle

$$\mathcal{A} \sim \left[-\frac{4}{3} + \frac{4m_H^2}{s-m_H^2} \right]$$


(c-1) (c-2) (c-3) (c-4) (c-5)
(d-1) (d-2) (d-3) (d-4) (d-5)
(e-1) (e-2) (e-3) (e-4) (e-5)
(f-1) (f-2) (f-3) (f-4) (f-5)
(g-1) (g-2) (g-3) (g-4) (g-5)

HH: Theory History

(Slide design inspired by: G. Salam)



[1] Glover, van der Bij 88; [2] Dawson, Dittmaier, Spira 98; [3] Shao, Li, Li, Wang 13; [4] Grigo, Hoff, Melnikov, Steinhauser 13; [5] de Florian, Mazzitelli 13; [6] Grigo, Melnikov, Steinhauser 14; [7] Grigo, Hoff 14; [8] Maltoni, Vryonidou, Zaro 14; [9] Grigo, Hoff, Steinhauser 15; [10] de Florian, Grazzini, Hanga, Kallweit, Lindert, Maierhöfer, Mazzitelli, Rathlev 16; [11] Borowka, Greiner, Heinrich, SPJ, Kerner, Schlenk, Schubert, Zirke 16; [12] Borowka, Greiner, Heinrich, SPJ, Kerner, Schlenk, Zirke 16; [13] Ferrera, Pires 16; [14] Heinrich, SPJ, Kerner, Luisoni, Vryonidou 17; [15] SPJ, Kuttimalai 17; [16] Gröber, Maier, Rauh 17; [17] Baglio, Campanario, Glaus, Mühlleitner, Spira, Streicher 18; [18] Grazzini, Heinrich, SPJ, Kallweit, Kerner, Lindert, Mazzitelli 18; [19] de Florian, Mazzitelli 18; [20] Bonciani, Degrassi, Giardino, Gröber 18; [21] Davies, Mishima, Steinhauser, Wellmann 18, 18; [22] Mishima 18; [23] Gröber, Maier, Rauh 19; [24] Davies, Heinrich, SPJ, Kerner, Mishima, Steinhauser, David Wellmann 19; [25] Davies, Steinhauser 19; [26] Chen, Li, Shao, Wang 19, 19; [27] Davies, Herren, Mishima, Steinhauser 19, 21; [28] Baglio, Campanario, Glaus, Mühlleitner, Ronca, Spira 21; [29] Bellafronte, Degrassi, Giardino, Gröber, Vitti 22; [30] Davies, Mishima, Schönwald, Steinhauser, Zhang 22;

[slide from Stephen Jones]

Glossary

- Integration-by-parts (IBP) reduction to master integrals (MIs):
  - Feynman amplitutude ➪ many [$\mathcal{O}(10^4 \ldots 10^5 \ldots)$] integrals
  - They are not independent.
  - IBP: Establish linear relations between the integrals.
  - MIs: List of independent elemens in linear system.
- Differential equations (DEs) for master integrals:
  - If the MIs depend of 2 dimensionful quantities, e.g. q^2 and m^2 one can establish a system of differential equations for the MIs by taking dervatives w.r.t. $x \equiv q^2/m^2$.
- Form factors for $gg \to HH$ ➪ virtual corrections

$$\mathcal{M} = \varepsilon_{1,\mu}\varepsilon_{2,\nu}\left(\mathcal{M}_1 A_1^{\mu\nu} + \mathcal{M}_2 A_2^{\mu\nu}\right)$$

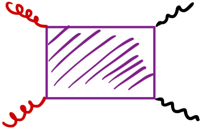
$$\mathcal{M}_1 \sim \frac{3m_H^2}{s-m_H^2} F_{\rm tri} + F_{\rm box1} \qquad \mathcal{M}_2 \sim F_{\rm box2}$$

- $2 \to 2$ kinematics

  Mandelstam variables: s, t, u $\qquad s + t + u = 2m_H^2$

  transverse momentum of the Higgs bosons: $p_T^2 = \frac{ut-m_H^4}{s}$

Analytic expansions



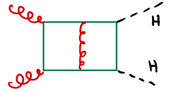
- Numeric calculations [pySecDec, Heinrich,...]
- Analytic calculations [Duhr,...,Tancredi,...]
- Analytic expansions:
  - large-mass expansion: $m_t^2 \gg s, t, \ldots$
    `exp` [Harlander,Seidensticker,Steinhauser'98]
    "simple": vacuum integrals and massless integrals
  - high energy: $m_t^2 \ll s, t, \ldots$
    involved asymptotic expansion
    complicated MIs
  - $t \to 0$
    (often) Taylor expansion

High energy expansion

High energy expansion

[Davies,Mishima,Steinhauser,Wellmann'18,. . . ,Davies,Mishima,Schönwald,Steinhauser,Zhang'22]

- Taylor expansion in m_H
- IBP reduction (s, t, m_t)
- differential equations in m_t^2/s
- Ansatz for $m_t^2 \ll s, t$ $\qquad I_n = \sum_{i,j,k} c_{ijk}^{(n)} \epsilon^i (m_t/\sqrt{s})^j \log^k(m_t^2/s)$
- system of linear equations for $c_{ijk}^{(n)}$
- BCs depend on t/s
  ➪ complicated box integrals; needed in the limit $m_t \to 0$
  involved asymptotic expansion
  (expansion-by-regions, Mellin-Barnes integrals, PSLQ, . . .)
- deep expansion: $(m_t^2/s)^{16} \ldots (m_t^2/s)^{50} \ldots$



Padé improvement

$$\sum_{k=0}^{N} c_k m_t^k \quad \to \quad \frac{a_0 + \cdots + a_r {m_t}^r}{1 + b_1 + \cdots + b_s m_t^s} \qquad r + s = N$$

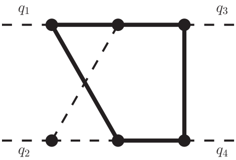
➪ For each phase-space point $(\sqrt{s}, p_T)$:
prediction of central value and corresponding uncertainty

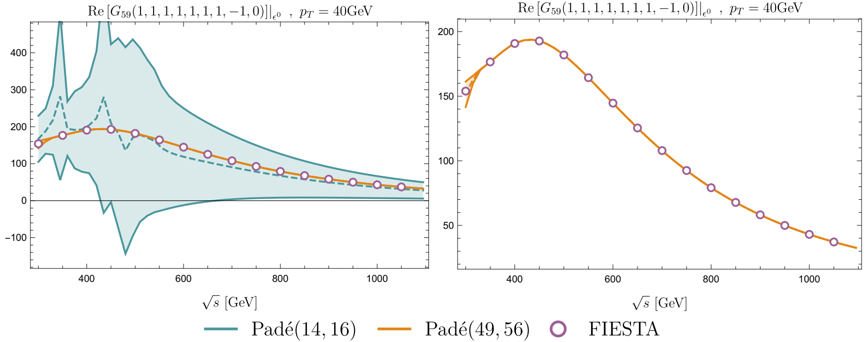
$$p_T^2 = (tu - m_H^4)/s, s + t + u = 2m_H^2$$

High energy expansion $\oplus$ PA

- expansion up to $(m_t^2)^{56}$
- construct PAs with input for $(N_{\min}, N_{\max})$

  (for each phase-space point)





High energy expansion ⊕ PA

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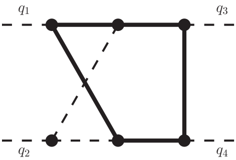
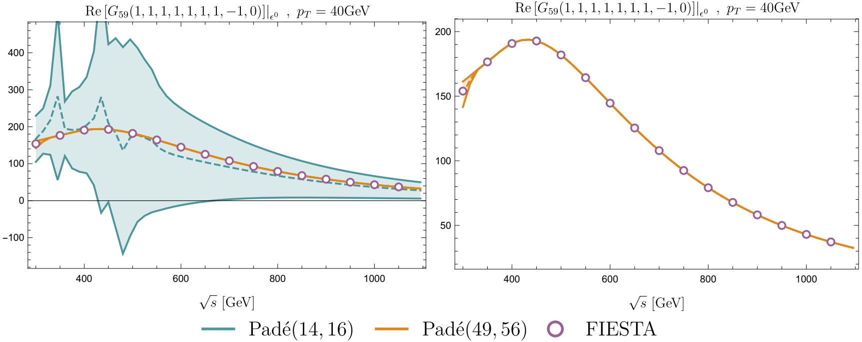
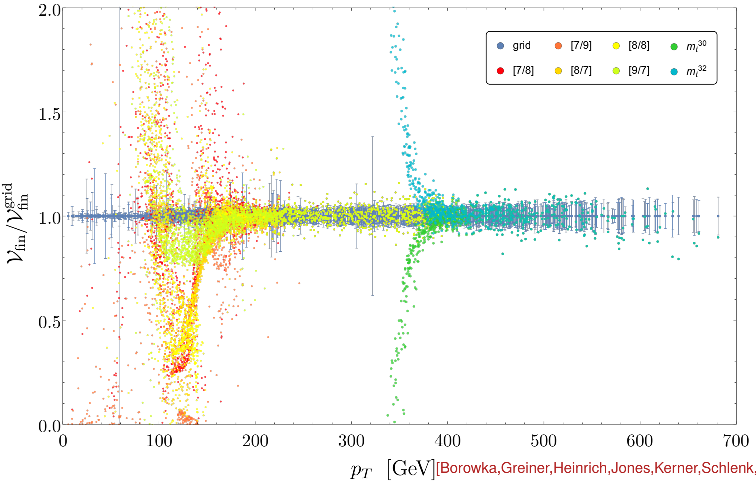


PA is a precision tool



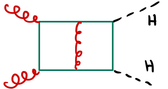
$\mathcal{V}_{\rm fin}$: virtual NLO corrections to $gg \to HH$



[Borowka,Greiner,Heinrich,Jones,Kerner,Schlenk,Schubert,Zirke'16]

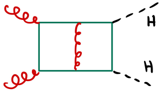
+ [Davies,Mishima,Steinhauser,Wellmann'18]

[Davies,Heinrich,Jones,Kerner,Mishima,Steinhauser,Wellmann'19]



Forward expansion $t \to 0$

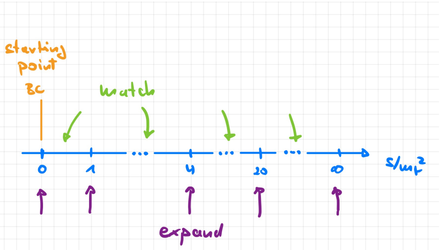
$t \to 0$ expansion



[Bonciani,Degrassi,Giardino,Gröber'18]
[Bellafronte,Degrassi,Giardino,Gröber,Vitti'22; ...]
[Davies,Mishima,Schönwald,Steinhauser'23]

- forward scattering kinematics
- Taylor expansion
- same differential equations as for high-energy expansion $\{m_t^2/s, t/s\}$
- construct for each MI expansion in t
- BC at $t = 0$: $f(s/m_t^2)$
- compute $f(s/m_t^2)$ with “expand and match” [Fael,Lange,Schönwald,Steinhauser'21'22]

“Expand and match” [Fael,Lange,Schönwald,Steinhauser’21’22]

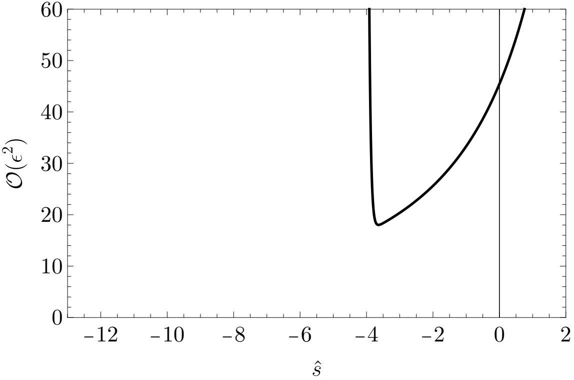


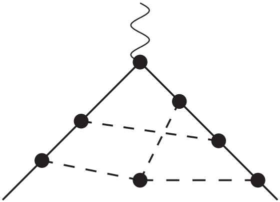
- semi-analytic results for $f(s/m_t^2)$
- differential equation for MIs in s/m_t^2
- (Power-log) ansatz for MIs
  - ➪ insert in differential equation
  - ➪ linear equations
- BCs for $s/m_t^2 \to 0$ (“simple”)
- move step-by-step to $s/m_t^2 \to \infty$
- thresholds are properly taken into account by the ansatz

Expansion of (unknown) function $f(s/m_t^2)$ around properly chosen s/m_t^2 values with precise numerical coefficients

Similar approaches: [Blümlein,Czakon,Hidding,Laporta,Lee,Liu,Smirnov,...]

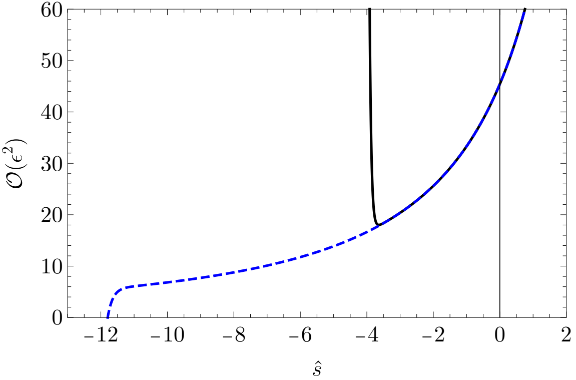
Sample MI

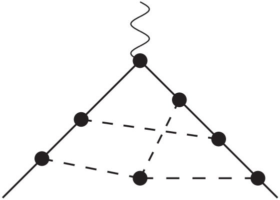




- Expansion around $\hat{s} \equiv s/m^2 = 0$

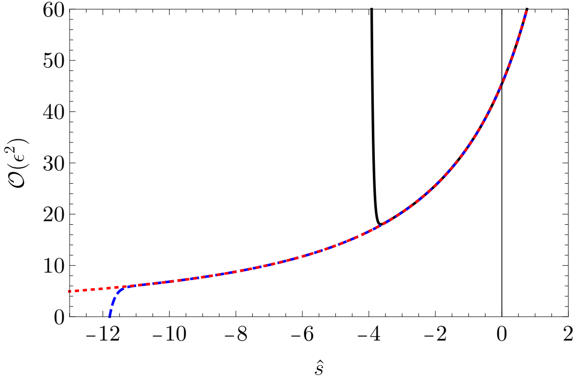
Sample MI

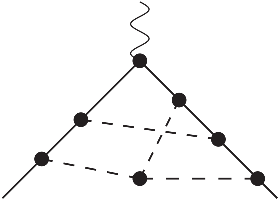




- Expansion around $\hat{s} \equiv s/m^2 = 0$
- Expansion around $\hat{s} = -4$, matched at $\hat{s} = -2$

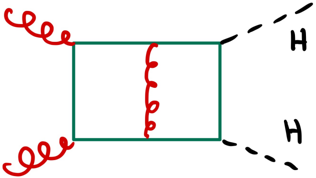
Sample MI





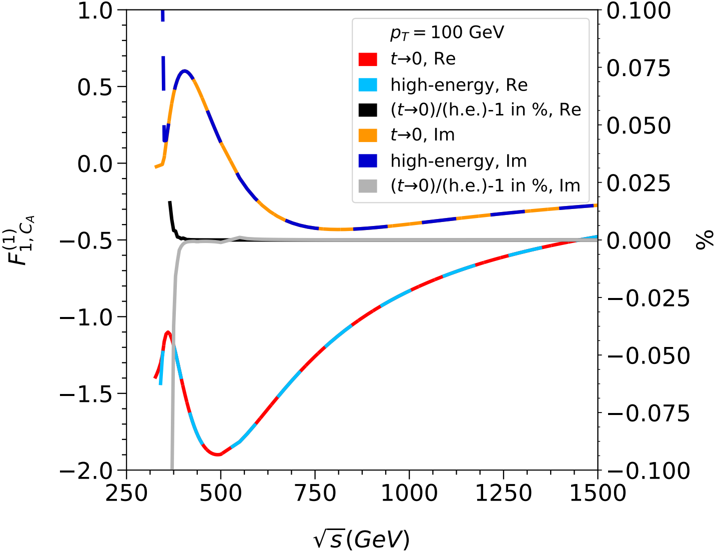
- Expansion around $\hat{s} \equiv s/m^2 = 0$
- Expansion around $\hat{s} = -4$, matched at $\hat{s} = -2$
- Expansion around $\hat{s} = -8$, matched at $\hat{s} = -6$

Back to $gg \to hh$ @ NLO



$$\begin{cases} \text{high-energy expansion} \\ t \to 0 \text{ expansion} \end{cases}$$

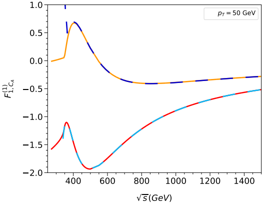
Combine: $t \to 0$ and h.e. at 2 loops



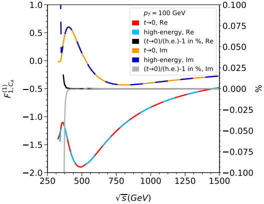
[Davies,Mishima,Schönwald,Steinhauser'23]

Combine: $t \to 0$ and h.e. at 2 loops

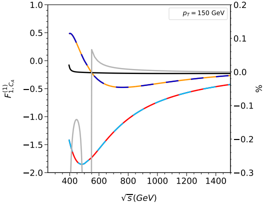
$p_T = 50$ GeV

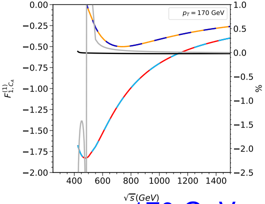


$p_T = 100$ GeV

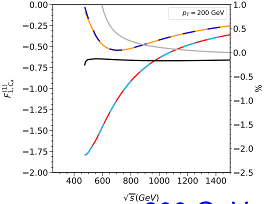


$p_T = 150$ GeV

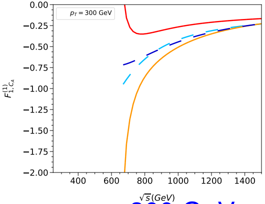




$p_T = 170$ GeV



$p_T = 200$ GeV



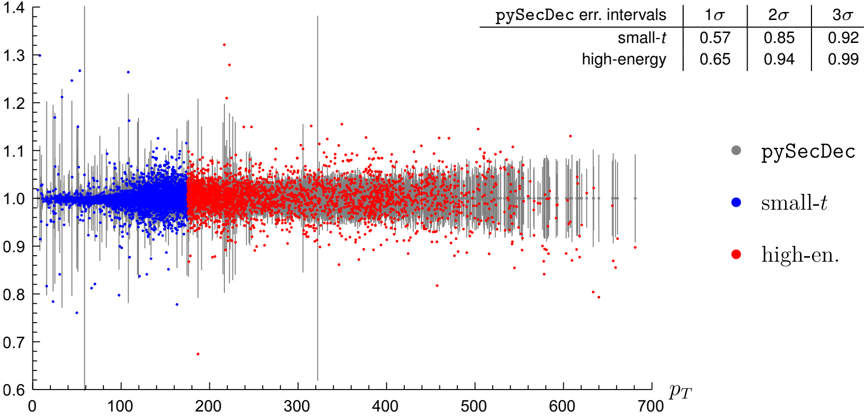
$p_T = 300$ GeV

[Davies,Mishima,Schönwald,Steinhauser'23]

$\mathcal{V}_{\text{fin}}$: virtual NLO QCD corrections

Comparison to “pySecDec”

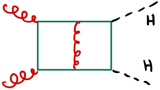
pySecDec err. intervals	1σ	2σ	3σ
small-t	0.57	0.85	0.92
high-energy	0.65	0.94	0.99



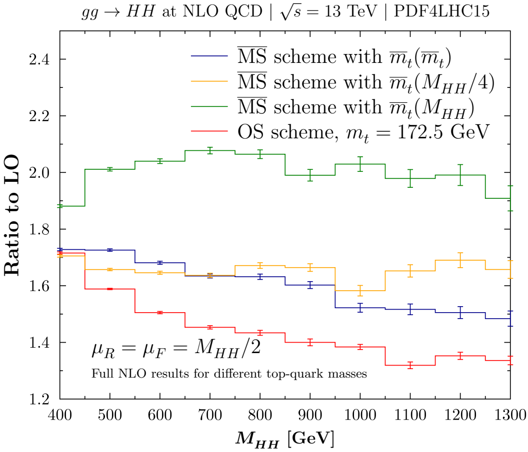
https://github.com/mppmu/hhgrid

[Borowka,Greiner,Heinrich,Jones,Kerner,Schlenk,Schubert,Zirke’16]

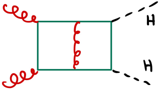
$gg \to HH$ at NLO



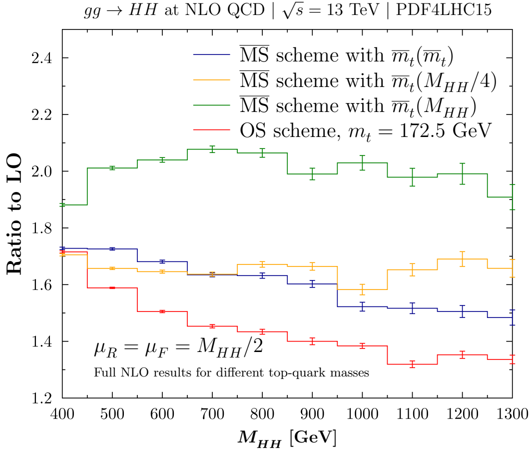
[Baglio,Campanario,Glaus,Mühlleitner,Ronca,Spira'20]
[Borowka,Greiner,Heinrich,Jones,Kerner,Schlenk,Schubert,Zirke'16]



$gg \to HH$ at NLO



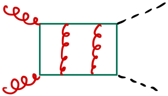
[Baglio,Campanario,Glaus,Mühlleitner,Ronca,Spira'20]
[Borowka,Greiner,Heinrich,Jones,Kerner,Schlenk,Schubert,Zirke'16]



➪ Need for NNLO

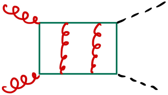
Can we go to 3 loops?

large-mass expansion: DONE [Davies,Steinhauser'19]
high-energy: NO

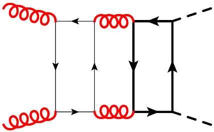
$t \to 0$: YES, if we can do the reduction for  $\{s, t, m_t^2\}$

➪ currently not possible

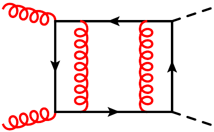
But: invert order:

1. expand in t ➪ no t dependence
2. reduce:  $\{s, m_t^2\}$ (still challenging)

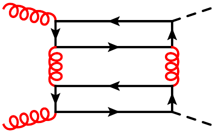
3-loop classification



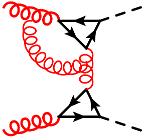
[Davies,Schönwald,Steinhauser'23]



in progress

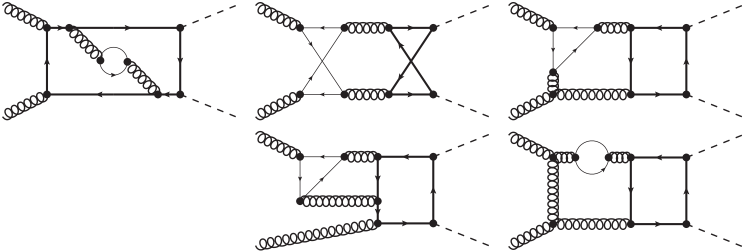


TODO



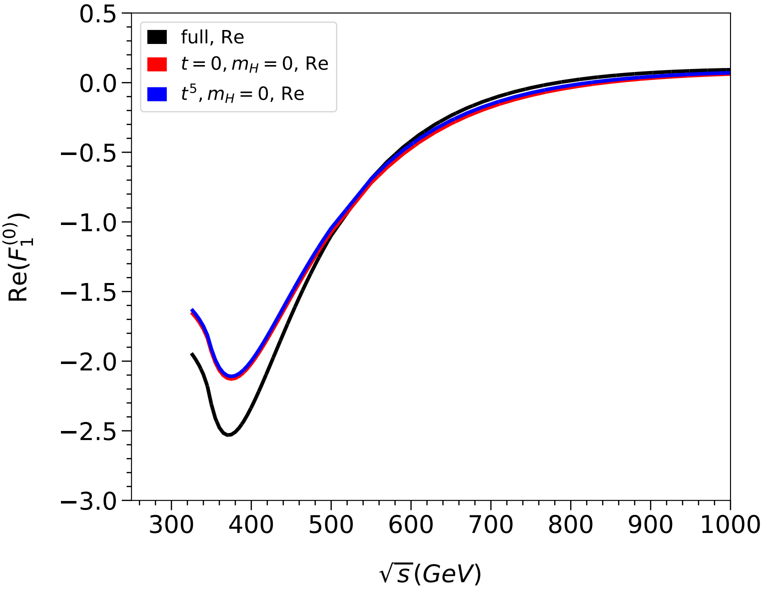
[Davies,Schönwald, Steinhauser,Vitti'soon]

Fermionic corrections



- $t = 0$, $m_H = 0$
- 31 integral families — tapir: [Gerlach,Herren,Lang'23]
- 176 MIs — Kira: [Klappert,Lange,Maierhöfer,Usovitsch'20]
- useful: `LiteRed` [Lee], `LIMIT` [Herren], `Feynson` [Magerya]
- reduction: about 1 week for most complicated family

1-loop result, $p_T = 100$ GeV



2-loop result, $p_T = 100$ GeV

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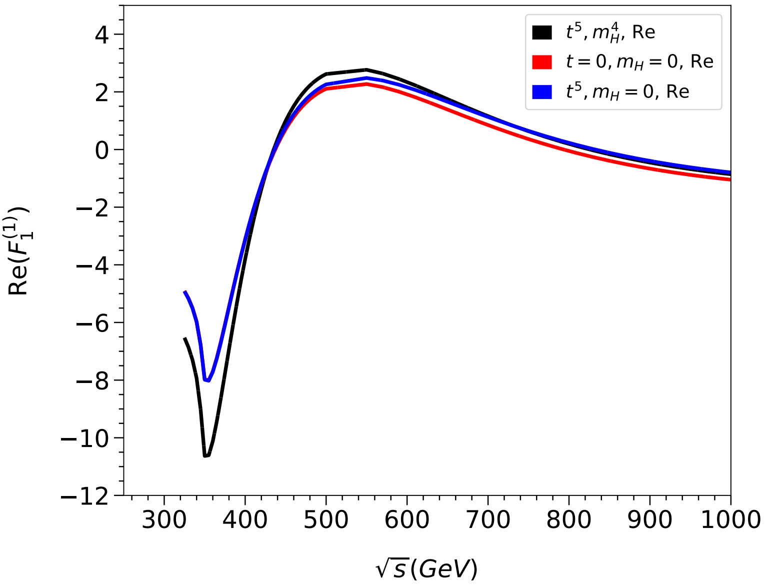
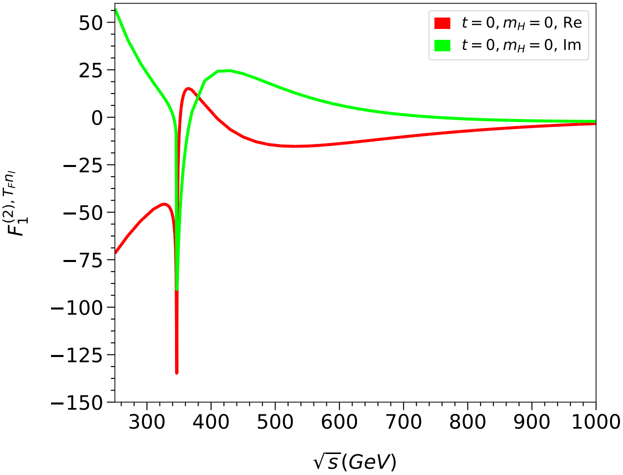
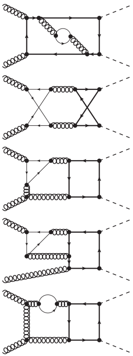
Karlsruhe Institute of Technology



3-loop n_l for $t = 0, m_H = 0$

[Davies,Schönwald,Steinhauser'23]





At threshold:

$v = \sqrt{1 - 4m_t^2/s}$

NLO: $v \log v$, $v^2 \log v$, $v^3 \log^2 v, \ldots$

NNLO: $v \log^2 v$, $v^2 \log^2 v$, $v^3 \log^3 v, \ldots$

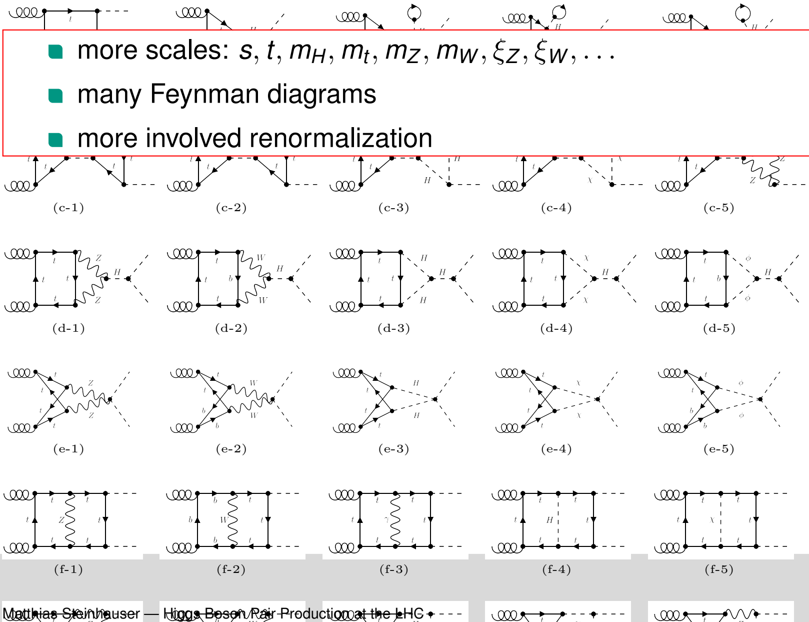
Challenges beyond the fermionic corrections

- 203 integral families
- IBP reduction
  - hardest job: 41 days, $>$ 2 TB RAM; took several attempts
  - 33.000 MIs accross all families
- Minimization of MIs
  - cannot be done with `Kira`
  - Apply `FIRE`'s `FindRule` to all 2.600.000 input integrals
    + additional test reductions with `FIRE`
    ➪ 1561 MIs
- Solve differential equation system
  BC: large-m_t limit: DONE

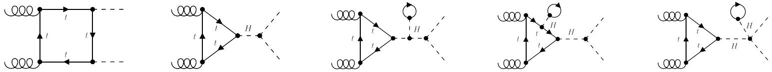
Electroweak corrections

Challenges

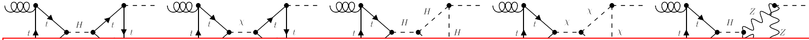
- more scales: $s, t, m_H, m_t, m_Z, m_W, \xi_Z, \xi_W, \ldots$
- many Feynman diagrams
- more involved renormalization



Challenges



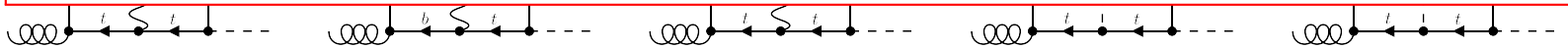
(a-1) (a-2) (b-1) (b-2) (b-3)



- more scales: $s, t, m_H, m_t, m_Z, m_W, \xi_Z, \xi_W, \ldots$
- many Feynman diagrams
- more involved renormalization
- Known results:
  - High-energy expansion for y_t^4 [Davies,Mishima,Schönwald,Steinhauser,Zhang'22]
  - Leading top quark mass corrections [Mühlleitner,Schlenk,Spira'22]
  - Full electroweak corrections to $gg \to HH$ in large-m_t limit

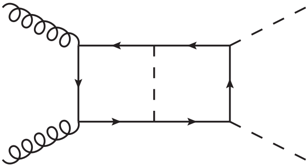
    [Davies,Mishima,Schönwald,Steinhauser,Zhang'23]
  - full numerical calculation [Bi,Huang,Huang,Ma,Yu'23]

    $\mathrm{d}\sigma/\mathrm{d}M_{HH}$: $+15\% \ldots -10\%$



(f-1) (f-2) (f-3) (f-4) (f-5)

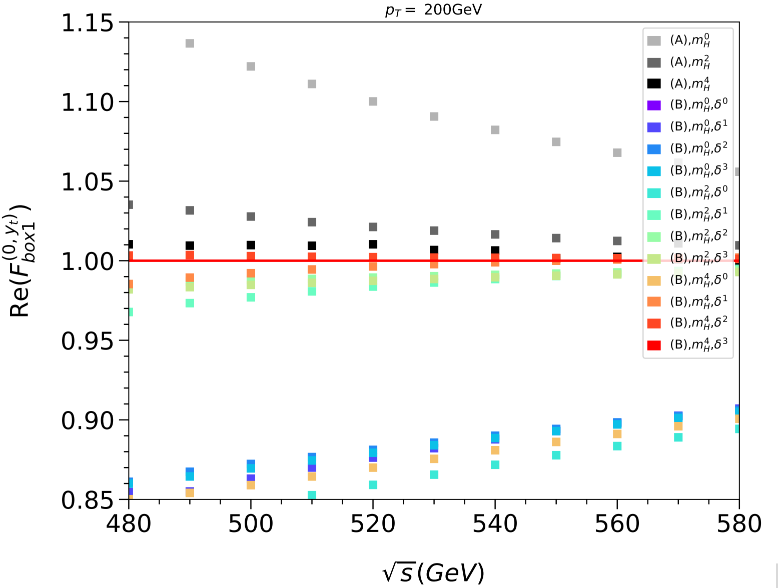
High energy expansion for $gg \to HH$ ("y_t^4")



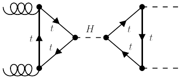
2 strategies

- A: $s, t \gg m_t^2 \gg (m_H^{\mathrm{int}})^2, (m_H^{\mathrm{ext}})^2$
- B: $s, t \gg m_t^2 \approx (m_H^{\mathrm{int}})^2 \gg (m_H^{\mathrm{ext}})^2$

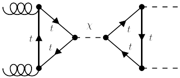
High energy expansion for $gg \to HH$ ("y_t^4")



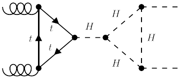
[Davies,Mishima,Schönwald,Steinhauser,Zhang'22]



(c-1)



(c-2)



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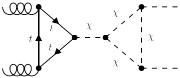

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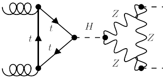
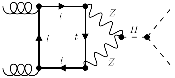


(c-5)



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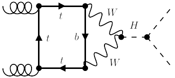

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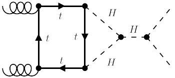

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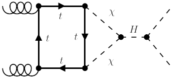

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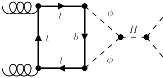

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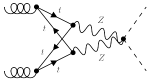
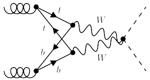


(e-1)



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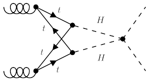

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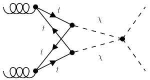

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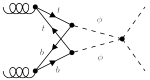

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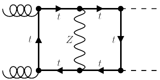

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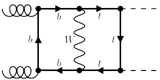

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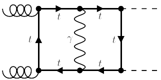

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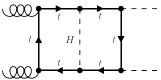

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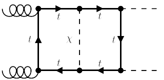

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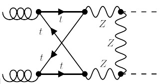

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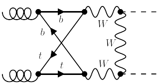

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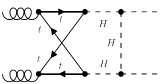

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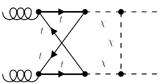

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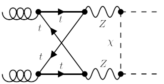


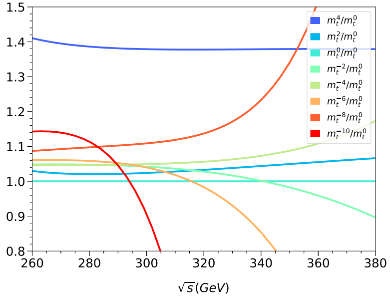
(g-5)

Full electroweak corrections to $gg \to HH$ in large-m_t limit

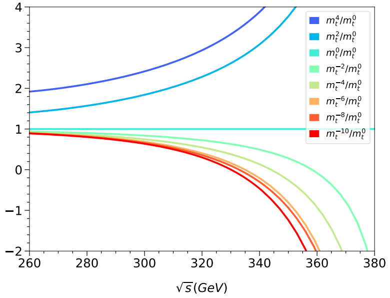
[Davies,Mishima,Schönwald,Steinhauser,Zhang'23]

- $m_t \gg m_H, m_Z, m_W$, check that ξ_Z, ξ_W drop out
- expansion up to $1/m_t^{10}$
- on-shell renormalization (exact in m_t, m_H, m_Z, m_W)

NLO: ratio to m_t^0

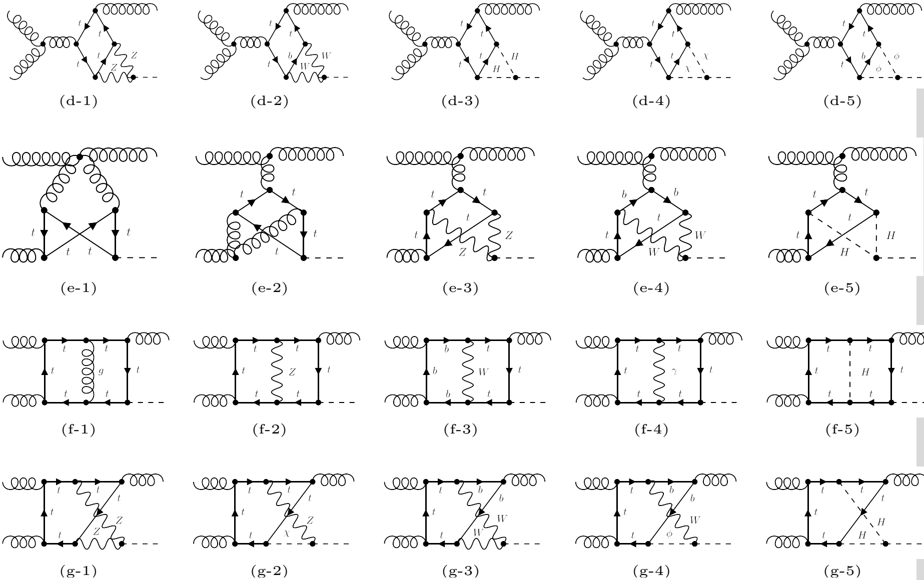


no $\sqrt{s} = m_t + m_W$ cut



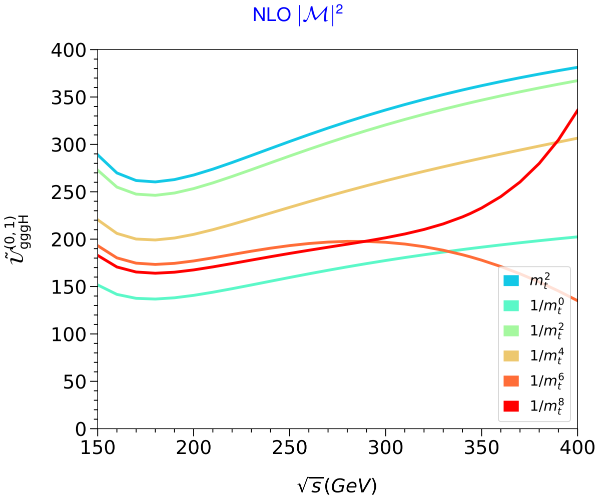
Electroweak corrections for H+jet production

(a-1) (a-2) (b-1) (c-1) (c-2)



Full electroweak corrections to $gg \to Hg$ in large-m_t limit

[Davies,Mishima,Schönwald,Steinhauser,Zhang'23]



Conclusions

- Higgs boson pair production:
  experimentally challenging
  theoretically complicated
- Analytic expansion:
  Combine large-m_t, high-energy, $t \to 0$ expansions
  Analytic and semi-analytic expressions ➪ fast and flexible
- "Expand and match"
- QCD: towards 3-loop $gg \to HH$
- Electroweak corrections:
  expand (in addition) in mass differences, e.g. $(m_t - m_H)/m_t$
- Apply techniques to $gg \to ZH$, $gg \to ZZ$, $gg \to Hg$, ...
  (with massive m_t, m_H, m_Z)