Bubble-Wall Velocity from Hydrodynamical Simulations

T. Krajewski, M. Lewicki, and M. Zych arXiv:2303.18216 and 2402.15408

Mateusz Zych

University of Warsaw

Theory of Fundamental Interactions and Cosmology Seminar

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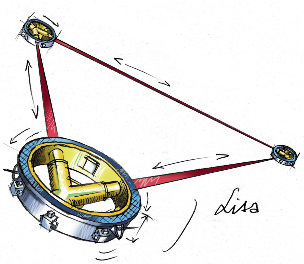
Outline

1. Introduction: basics of cosmological phase transitions
2. Bubble-wall expansion: analytical approach
3. Benchmark model: scalar singlet extension
4. Bubble-wall expansion: real-time simulations
5. Conclusions

Motovation

- Cosmological phase transitions are present in a variety of particle-physics models beyond SM.
- If they are first order, they could create an environment for the generation of baryon asymmetry and production of a stochastic gravitational wave background, which could be potentially observed with the next generation of detectors.
- Evaluation of the bubble-wall velocity in the stationary state, which has a crucial impact both on the amplitude of GW signal and baryon-asymmetry production, remains one of the most problematic issues.



Cosmological first order phase transitions

Let us consider theory of scalar field given by Lagrangian density:

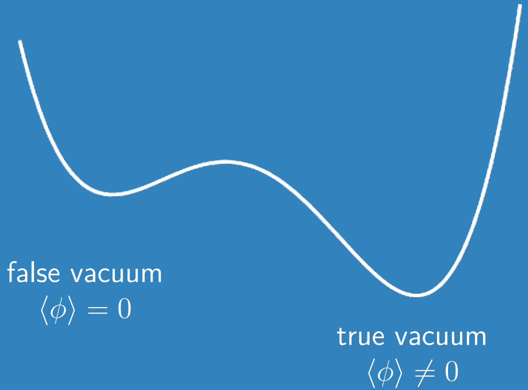
$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)(\partial^\mu \phi) - V(\phi, T),$$

leading to the equation of motion in the form:

$$\frac{\partial^2 \phi}{\partial r^2} + \frac{2}{r}\frac{\partial \phi}{\partial r} = \frac{\partial V(\phi, T)}{\partial \phi},$$

where T is temperature.

Scalar effective potential $V(\phi, T)$



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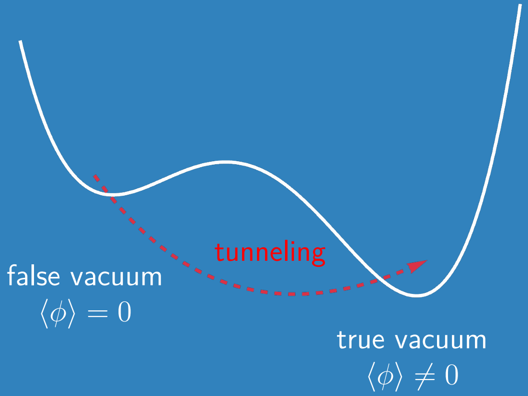

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Tunneling bubbles

Nucleation rate:

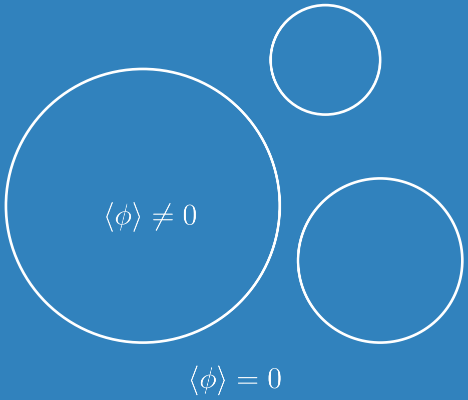
$$\Gamma(T) = A(T) \cdot \exp(-S)$$

For tunneling in finite temperatures:

$$S = \frac{S_3}{T} \qquad A(T) = T^4 \left(\frac{S_3}{2\pi T} \right)^{\frac{3}{2}}$$

where S_3 is an action of $O(3)$-symmetric solution of the eom.
Nucleation condition:

$$\frac{\Gamma(T_n)}{H^4} \approx 1$$

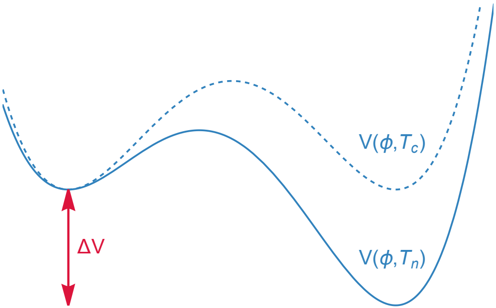


Phase transition parameters

- Critical and nucleation temperatures: T_c, T_n
- Level of supercooling: T_n/T_c
- Transition strength: $\alpha \sim \Delta V/\rho_r$
  In this work:

$$\alpha_{\bar{\theta}} = \frac{\Delta\bar{\theta}}{3w_s}, \qquad \text{with} \qquad \bar{\theta} = e - \frac{p}{c_b^2}$$

  with the speed of sound in the broken phase c_b and model-dependent energy ϵ, pressure p and enthalpy w.
- Bubble-wall velocity: v_w



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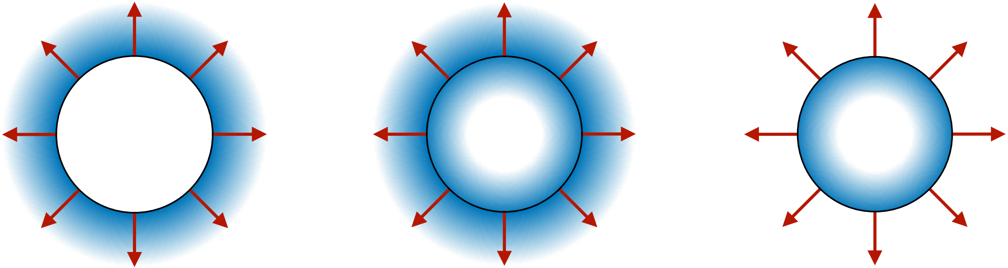
Expansion of bubbles

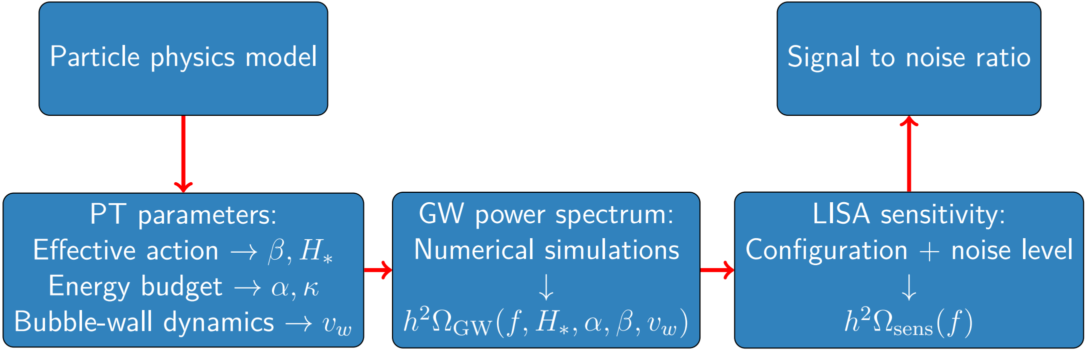
Different modes depending on bubble-wall velocity v_w and transition strength α_θ:

deflagration	hybrid	detonation
$v_w < c_s$	$c_s < v_w < c_J$	$c_J < v_w$



Where Jouget velocity is $c_J = \frac{1}{\sqrt{3}} \frac{1+\sqrt{3\alpha^2+2\alpha}}{1+\alpha}$.

Overview for analyzing cosmological phase transitions



This talk: Real-time hydrodynamical simulations of the bubble-wall and plasma dynamics to determine v_w, comparison with analytical methods.

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Dynamics of the steady state expansion

Integrated EoM of the growing bubble:

$$\int \mathrm{d}z \frac{\mathrm{d}\phi}{\mathrm{d}z}\left(\Box\phi + \frac{\partial V_{\mathrm{eff}}}{\partial\phi} + \sum_i \frac{\mathrm{d}m_i^2(\phi)}{\mathrm{d}\phi}\int\frac{\mathrm{d}^3p}{(2\pi)^3 2E_i}\delta f_i(p,x)\right) = 0$$

$$\downarrow \quad \frac{\mathrm{d}\phi}{\mathrm{d}z}\frac{\partial V_{\mathrm{eff}}}{\partial\phi} = \frac{\mathrm{d}V_{\mathrm{eff}}}{\mathrm{d}z} - \frac{\partial V_{\mathrm{eff}}}{\partial T}\frac{\mathrm{d}T}{\mathrm{d}z}$$

$$\Delta V_{\mathrm{eff}} = \boxed{\int \mathrm{d}z \frac{\partial V_{\mathrm{eff}}}{\partial T}\frac{\mathrm{d}T}{\mathrm{d}z}} - \boxed{\sum_i \int \mathrm{d}\phi \frac{\mathrm{d}m_i^2(\phi)}{\mathrm{d}\phi}\int\frac{\mathrm{d}^3p}{(2\pi)^3 2E_i}\delta f_i(p,x)}$$

driving force $=$ hydrodynamic backreaction $+$ non-equilibrium friction

- Boltzmann eq. $+$ EoM (different approaches: e.g fluid ansatz)
- LTE approximation (only hydrodynamic backreaction)
- Numerical simulations with effective friction η parametrizing δf

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Stationary profiles

Energy-momentum tensor for the plasma is given by

$$T^{\mu\nu} = \omega u^\mu u^\nu - g^{\mu\nu} p$$

Conservation of $T^{\mu\nu}$ along the flow and its projection orthogonal to the flow leads to

$$\partial_\mu (u^\mu \omega) - u_\mu \partial^\mu p = 0 \qquad \bar{u}^\nu u^\mu \omega \partial_\mu u_\nu - \bar{u}^\nu \partial_\mu p = 0.$$

Spherical symmetry + scale invariance:

$$u_\mu \partial^\mu = -\frac{\gamma}{t}(\xi - v)\partial_\xi \qquad \bar{u}_\mu \partial^\mu = \frac{\gamma}{t}(1 - \xi v)\partial_\xi$$

Hydrodynamic equation

$$2\frac{v}{\xi} = \gamma^2 (1 - v\xi) \left[\frac{\mu^2}{c_s^2} - 1\right] \partial_\xi v,$$

with $\mu(\xi, v) = \frac{\xi - v}{1 - \xi v}$ and $\xi = r/t$.

Stationary profiles

Hydrodynamic equation

$$2\frac{v}{\xi} = \gamma^2(1 - v\xi)\left[\frac{\mu^2}{c_s^2} - 1\right]\partial_\xi v,$$

Matching equations

1 $\omega_-\gamma_-^2 v_- = \omega_+\gamma_+^2 v_+$

2 $\omega_-\gamma_-^2 v_-^2 + p_- = \omega_+\gamma_+^2 v_+^2 + p_+$

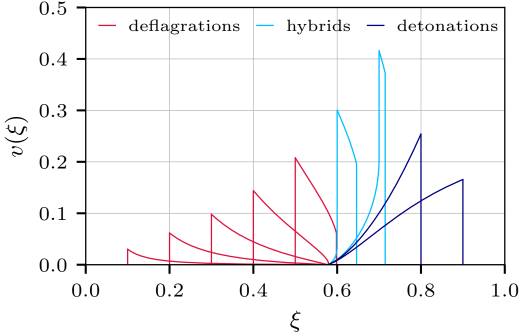
3 $s_-\gamma_- v_- = s_+\gamma_+ v_+$ (if $\delta f = 0$)

Bag equation of state

$\epsilon_s = 3a_s T_s^4 + \theta_s$ $\epsilon_b = 3a_b T_b^4 + \theta_b$

$p_s = a_s T_s^4 - \theta_s$ $p_b = a_b T_b^4 - \theta_b$

Solving hydrodynamic equation assuming bag model and proper boundary conditions (1 and 2), we get analytical profiles $v(\xi)$ depending on ξ_w, α_θ.



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LTE approximation $(\delta f = 0)$

Matching method: conservation of entropy accross the bubble-wall (matching eq. 3)

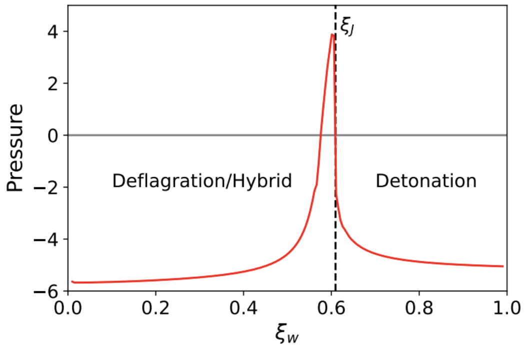

Figure: Total pressure acting on the wall[1]

Evaluation of bubble-wall velocity based on

- transition strength α_θ
- enthalpy ratio $\Psi_N = \frac{\omega_b}{\omega_s}$
- speed of sound in the plasma c_s, c_b

Stationary state (deflagration or hybrid) can be typically found for not too large α_θ
There are no stable detonations in LTE.

[1]Wen-Yuan Ai, Benoit Laurent, Jorinde van de Vis, *Model-independent bubble wall velocities in local thermal equilibrium*, JCAP 07 (2023) 002

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Scalar singlet extension

Model: SM Higgs dublet H and Z_2-symmetric real singlet s.
Tree-level potential (unitary gauge):

$$V_0(h,s) = \frac{1}{2}\mu_h^2 h^2 + \frac{1}{4}\lambda_h h^4 + \frac{1}{4}\lambda_{hs} h^2 s^2 + \frac{1}{2}\mu_s^2 s^2 + \frac{1}{4}\lambda_s s^4$$

$$\lambda_h = \frac{m_h^2}{2v^2} \qquad \text{and} \qquad \mu_h^2 = -\lambda_h v^2,$$

with $m_h = 125.09$ GeV and $v = 246.2$ GeV.

$$\boxed{\text{free parametres: } m_s, \lambda_s, \lambda_{hs}}$$

Effective potential:

$$V_{\text{eff}}(h,s,T) = V_0(h,s) + V_{\text{CW}}(h,s,T) + V_{\text{T}}(h,s,T)$$

- $V_{\text{CW}}(h,s,T)$ - Coleman-Weinberg potential (here neglected)
- $V_{\text{T}}(h,s,T)$ - thermal potential

Thermal potential

Thermal functions

$$J_{b/f}(x) = \pm \int_0^\infty \mathrm{d}y y^2 \log\left(1 \mp \exp\left(-\sqrt{y^2+x^2}\right)\right)$$

High-temperature expansion: $(x \ll 1)$:

$$J_b(x) \approx -\frac{\pi^4}{45} + \frac{\pi^2}{12}x^2 + \mathcal{O}(x^3) \qquad J_f(x) \approx -\frac{7}{8}\frac{\pi^4}{45} + \frac{\pi^2}{24}x^2 + \mathcal{O}(x^4 \log x^2)$$

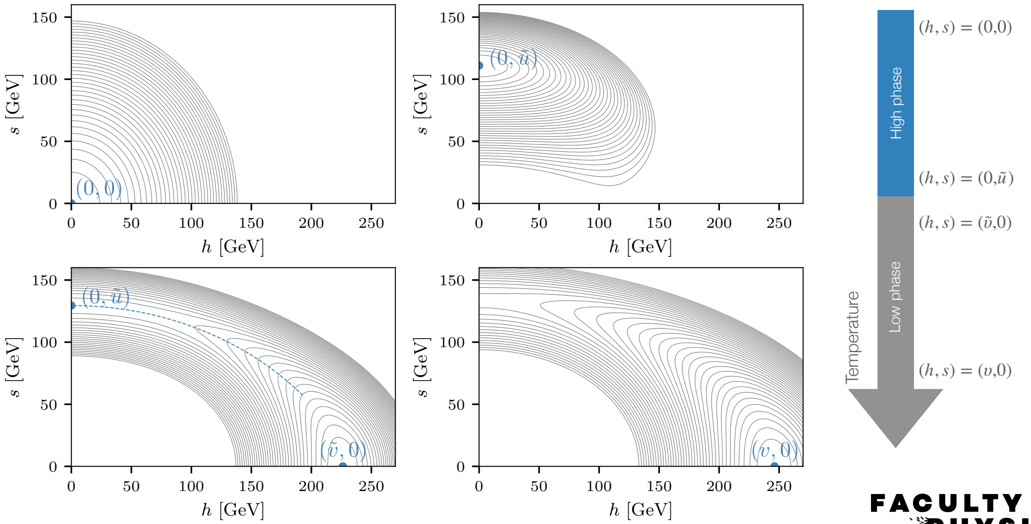
$$V_T = \sum_i \frac{n_i T^4}{2\pi^2} J_{b/f}\left(\frac{m_i(h,s)}{T}\right) \overset{m_i \ll T}{\approx} -\frac{g_* \pi^2}{90} T^4 + \sum_i \frac{c_i n_i}{24} m_i^2(h,s) T^2$$

Effectively tree-level potential with temperature-dependent mass terms

$$\mu_h^2(T) \coloneqq \mu_h^2 + c_h^2 T^2 \quad \text{and} \quad \mu_s^2(T) \coloneqq \mu_s^2 + c_s^2 T^2,$$

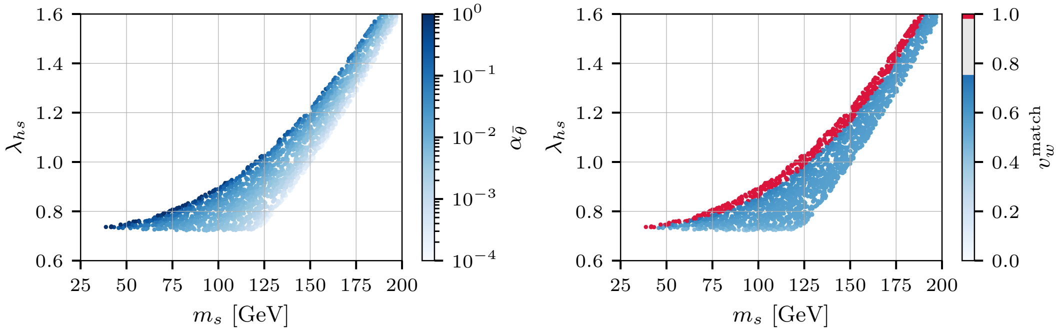
$$c_h^2 = \frac{1}{48}\left(9g^2 + 3g'^2 + 12y_t^2 + 24\lambda_h + 2\lambda_{hs}\right) \quad \text{and} \quad c_s^2 = \frac{1}{12}\left(2\lambda_{hs} + 3\lambda_s\right)$$

Transition pattern



Parameter space

- Scan of the parameter space with $\lambda_s = 1$.
- Wall velocity determined analytically with matching conditions assuming LTE.



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Scalar fields and perfect fluid

The system consists of

- relativistic perfect fluid
- real scalar fields h, s.

The fields acquires a temperature-dependent effective potential V_{eff}.

Equation of state

$$p(h,s,T) = -V_{\text{eff}}(h,s,T),$$

$$e(h,s,T) = V_{\text{eff}}(h,s,T) - T\frac{\mathrm{d}V_{\text{eff}}(h,s,T)}{\mathrm{d}T},$$

$$w(h,s,T) = -T\frac{\mathrm{d}V_{\text{eff}}(h,s,T)}{\mathrm{d}T}.$$

Energy-momentum tensor

$$T^{\mu\nu} = T^{\mu\nu}_{\text{field}} + T^{\mu\nu}_{\text{fluid}}$$

$$T^{\mu\nu}_{\text{field}} = \partial^\mu\phi\partial^\nu\phi - g^{\mu\nu}\left(\frac{1}{2}\partial_\alpha\phi\partial^\alpha\phi\right)$$

$$T^{\mu\nu}_{\text{fluid}} = wu^\mu u^\nu + g^{\mu\nu}p$$

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Equations of motion

Total energy-momentum tensor in conserved, but both contributions are not, due to the extra coupling term parametrized by effective non-equilibrium friction η

$$\nabla_\mu T^{\mu\nu}_{\text{field}} = \frac{\partial V_{\text{eff}}}{\partial \phi}\partial^\nu \phi + \eta u^\mu \partial_\mu \phi \partial^\nu \phi, \quad \nabla_\mu T^{\mu\nu}_{\text{fluid}} = -\frac{\partial V_{\text{eff}}}{\partial \phi}\partial^\nu \phi - \eta u^\mu \partial_\mu \phi \partial^\nu \phi$$

Local thermal equilibrium: $\eta = 0$

EoM - scalar fields

$$-\partial_t^2 h + \frac{1}{r^2}\partial_r(r^2 \partial_r h) - \frac{\partial V_{\text{eff}}}{\partial h} = 0$$

$$-\partial_t^2 s + \frac{1}{r^2}\partial_r(r^2 \partial_r s) - \frac{\partial V_{\text{eff}}}{\partial s} = 0$$

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Local thermal equilibrium: $\eta = 0$

EoM - plasma

$$\partial_t \tau + \frac{1}{r^2}\partial_r (r^2(\tau + p)v) = \frac{\partial V_{\text{eff}}}{\partial h}\partial_t h + \frac{\partial V_{\text{eff}}}{\partial s}\partial_t s$$

$$\partial_t Z + \frac{1}{r^2}\partial_r \left(r^2 Z v\right) + \partial_r p = -\frac{\partial V_{\text{eff}}}{\partial h}\partial_r h - \frac{\partial V_{\text{eff}}}{\partial s}\partial_r s$$

where $Z := w\gamma^2 v$ and $\tau := w\gamma^2 - p$

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Spatial discretization

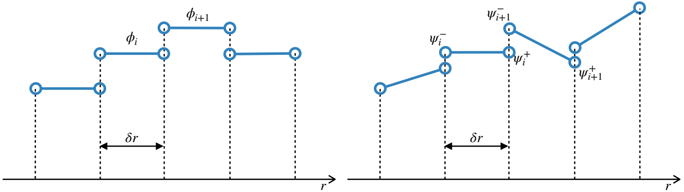
Spatial discretization of the field: discontinous Galerkin method

$$0 = \forall_q \int_0^\infty dr r^2 f(r, \phi(r), \psi(r)) q \quad \rightarrow \quad \forall_q \sum_{i=0}^{N-1} \int_{r_i}^{r_{i+1}} dr r^2 f(r, \phi(r), \psi(r)) q$$

with an auxiliary variable $\psi := \partial_r \phi$. We introduce following interpolations:

$$\phi(t,r)\big|_{I_i} = \phi_i(t)$$

$$\psi(t,r)\big|_{I_i} = \psi_i^+(t) \frac{r_{i+1} - r}{r_{i+1} - r_i} + \psi_{i+1}^-(t) \frac{r - r_i}{r_{i+1} - r_i}.$$



Boundary conditions:

$$\psi\big|_{r=0} = 0$$

$$\phi\big|_{r=\infty} = 0$$

Similar approach for the thermodynamic variables: Z, τ

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Temporal discretization for the fields

Temporal discretization of the fields: Strömer-Verlet method

$$\phi_{i,j+1/2} = \phi_{i,j} + \frac{1}{2}\delta t \dot{\phi}_{i,j}$$

$$\dot{\phi}_{i,j+1} = \dot{\phi}_{i,j} - \delta t \left(\frac{\partial V}{\partial \phi}(\phi_{i,j+1/2}) - \Delta_d \phi_{i,j+1/2} \right)$$

$$\phi_{i,j+1} = \phi_{i,j+1/2} + \frac{1}{2}\delta t \dot{\phi}_{i,j+1}$$

Can be interpreted as discontinuous Galerkin method in time.

Temporal discretization for plasma

For high order spacial discretization we use explicit midpoint method:

$$U_{i,j+1/2} = U_{i,j} + \frac{t_{j+1} - t_j}{2} \left[\mathcal{F}_{i+1/2}(U_{\cdot,j}) - \mathcal{F}_{i-1/2}(U_{\cdot,j}) + \mathcal{G}(U_{i,j}, \phi_{i,j}, \nabla_d \phi_{i,j}, \dot{\phi}_{i,j}, r_i)\right],$$

$$U_{i,j+1} = U_{i,j} + (t_{j+1} - t_j) \left[\mathcal{F}_{i+1/2}(U_{\cdot,j+1/2}) - \mathcal{F}_{i-1/2}(U_{\cdot,j+1/2}) + \mathcal{G}\left(U_{i,j+1/2}, \phi_{i,j+1/2}, \nabla_d \phi_{i,j+1/2}, \dot{\phi}_{i,j+1/2}, r_i\right)\right].$$

Temporal discretization of low order scheme: forward/backward Euler method:

$$U_{i,j+1} = U_{i,j} + \delta t \left[\theta \left(\mathcal{F}_{i+1/2}(U_{\cdot,j+1}) - \mathcal{F}_{i-1/2}(U_{\cdot,j+1}) + \mathcal{G}(U_{i,j+1})\right) + (1-\theta) \left(\mathcal{F}_{i+1/2}(U_{\cdot,j}) - \mathcal{F}_{i-1/2}(U_{\cdot,j}) + \mathcal{G}(U_{i,j})\right)\right]$$

We use implicit ($\theta = 1$) for low order spacial discretization since explicit one ($\theta = 0$) turned out to be unstable.

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Flux corrected transport

1. Compute F^L using low order method guaranteed not to generate unphysical values.
2. Compute F^H using high order method accurate in smooth regions of the solution.
3. Compute the "antidiffusive fluxes":

$$A = F^H - F^L$$

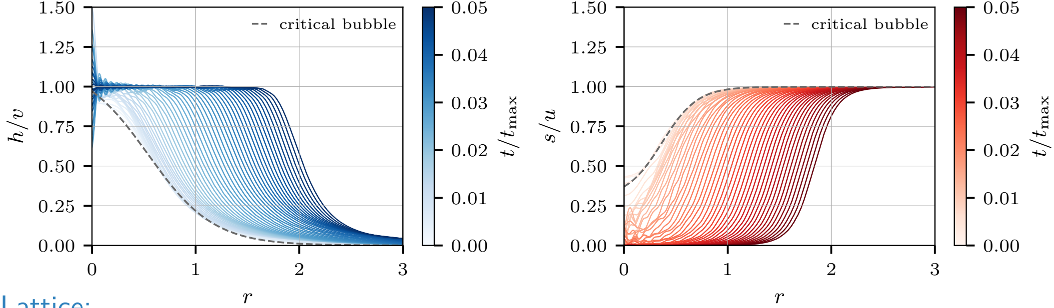
4. Compute numerical solution U^L with low order method.
5. Limit the "antidiffusive fluxes":

$$A_\alpha = \alpha A, \qquad 0 \leq \alpha \leq 1$$

   such that $\alpha \sim 1$ in the smooth regions of the solution and $\alpha \sim 0$ around shocks.
6. Apply the limited "antidiffusive fluxes" to U^L in order to obtain final solution reproducing high order scheme in the smooth regions of the solution.

Evolution: early stages

- Evolution is initialized with $h(r)$ and $s(r)$ profiles correspond to the critical bubble at nucleation temperature T_n.
- Plasma initially remains at rest $v(r) = 0$ with $T(r) = T_n$
- Bubble-wall quickly achieves constant velocity v_w



Lattice:

$t_{\max} = 100\ \text{GeV}^{-1}$ $\quad r_{\max} = ct_{\max}$ $\quad \delta r = 10\delta t = 10^{-2}\ \text{GeV}^{-1}$

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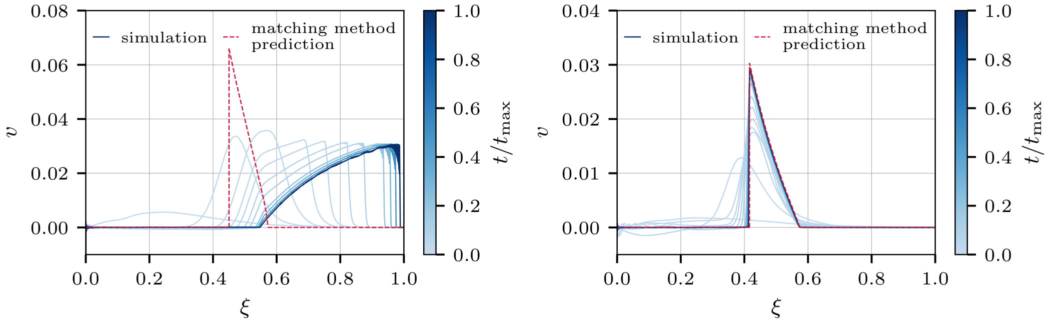
Evolution: late stages

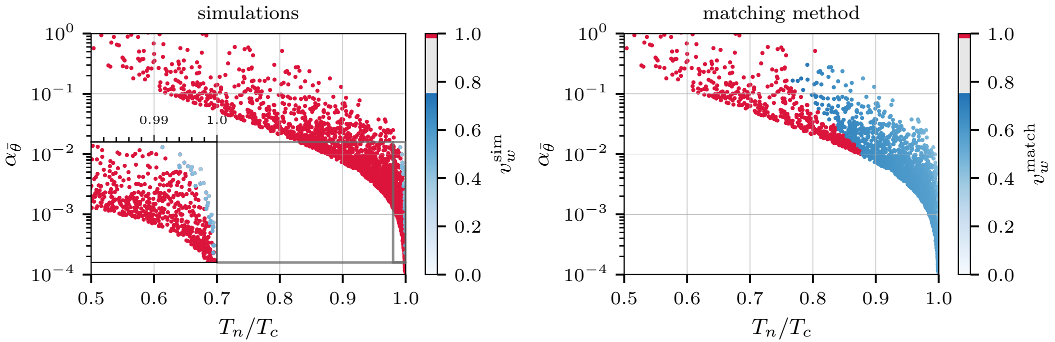
Self-similar profiles: $\xi = r/t$



Two possible scenarios for the growing bubble in LTE:

- rapid expansion beyond Chapman-Jouguet velocity leading to a runaway scenario
- evolution toward a stationary state predicted by matching conditions

Analytical treatment vs real-time simulations



While matching equations predict significant number of stationary deflagrations and hybrids, in real-time simulations only few indeed evolve towards stationary state.

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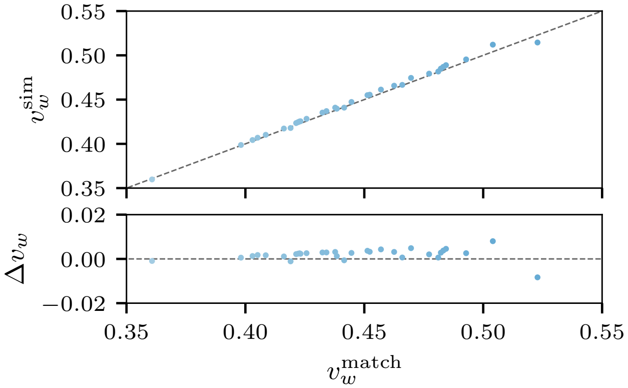
Analytical treatment vs real-time simulations



If the stationary state is achieved for a given model, bubble-wall velocity is very accurately predicted by the matching equations.

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We investigated fluid solutions in the presence of growing bubbles of the scalar field in cosmological FOPT using numerical lattice simulations:

- Without non-equilibrium friction bubbles generically expand as runaways.
- Stationary profiles are dynamically achieved only for tiny supercooling ($T_n/T_c \lesssim 1$).
- If steady state is achieved, it matches to equilibrium prediction with high precision.

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Thank you for your attention!