Quantum formation of Topological Defects

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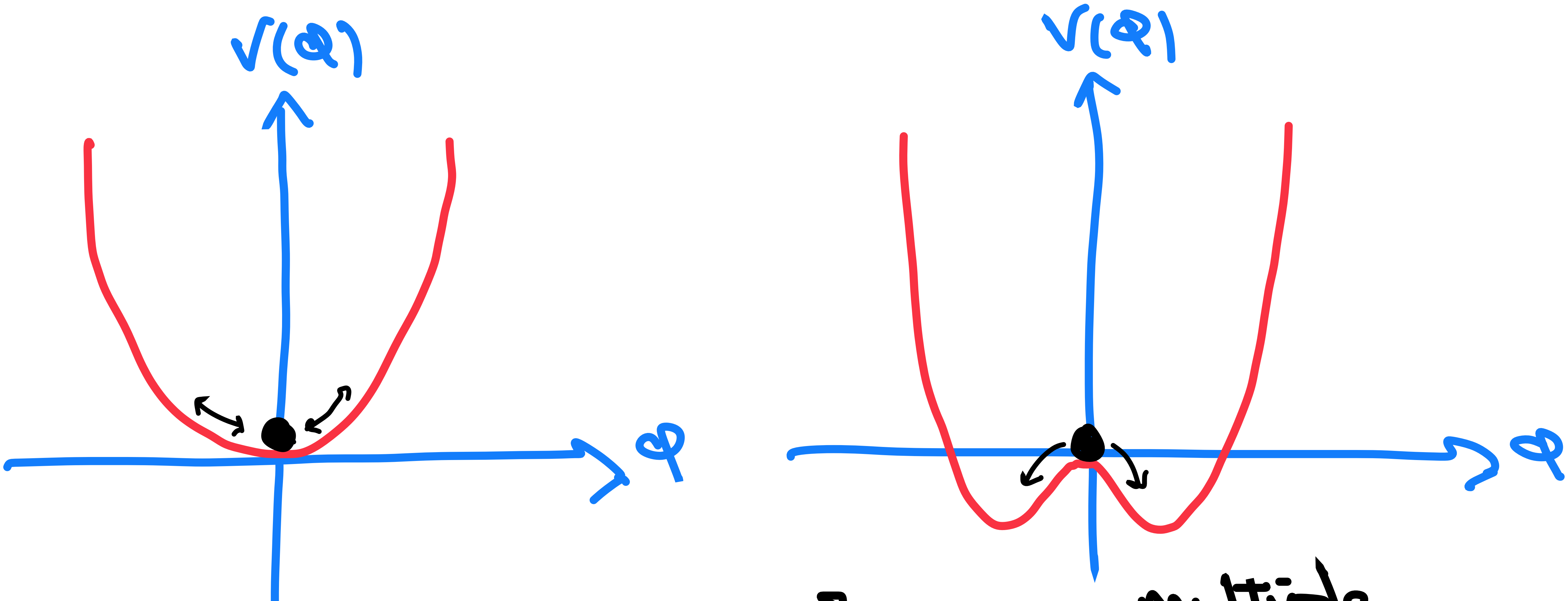
Based on:
- 2004.07249
- 2009.11480
- 2212.11204
- 2404.xxxxx

with M. Mukhopadhyay, T. Vachaspati and O. Pujolas

Setting:

Quantum phase transition in $1+1$ D



one vacuum

time evolution

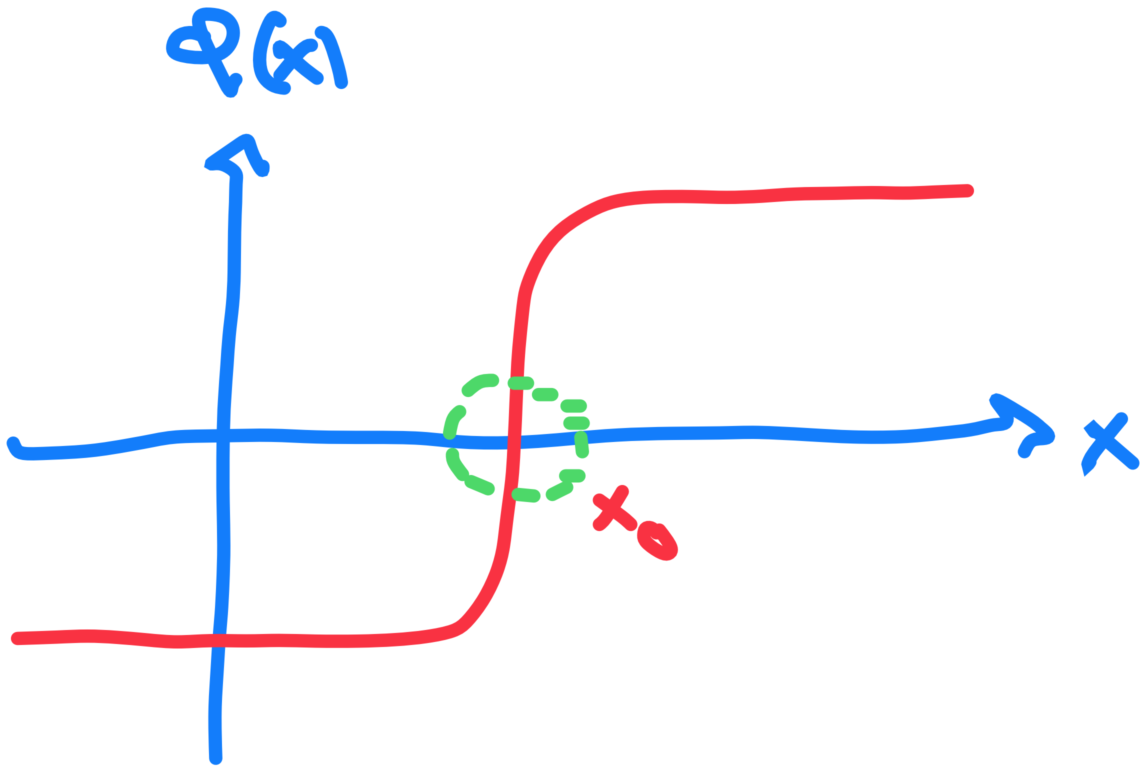
multiple vacua

$\Rightarrow$ Production of kinks and antikinks

CLASSICAL SOLUTIONS

Setting:

Quantum phase transition in 1 + 1 D



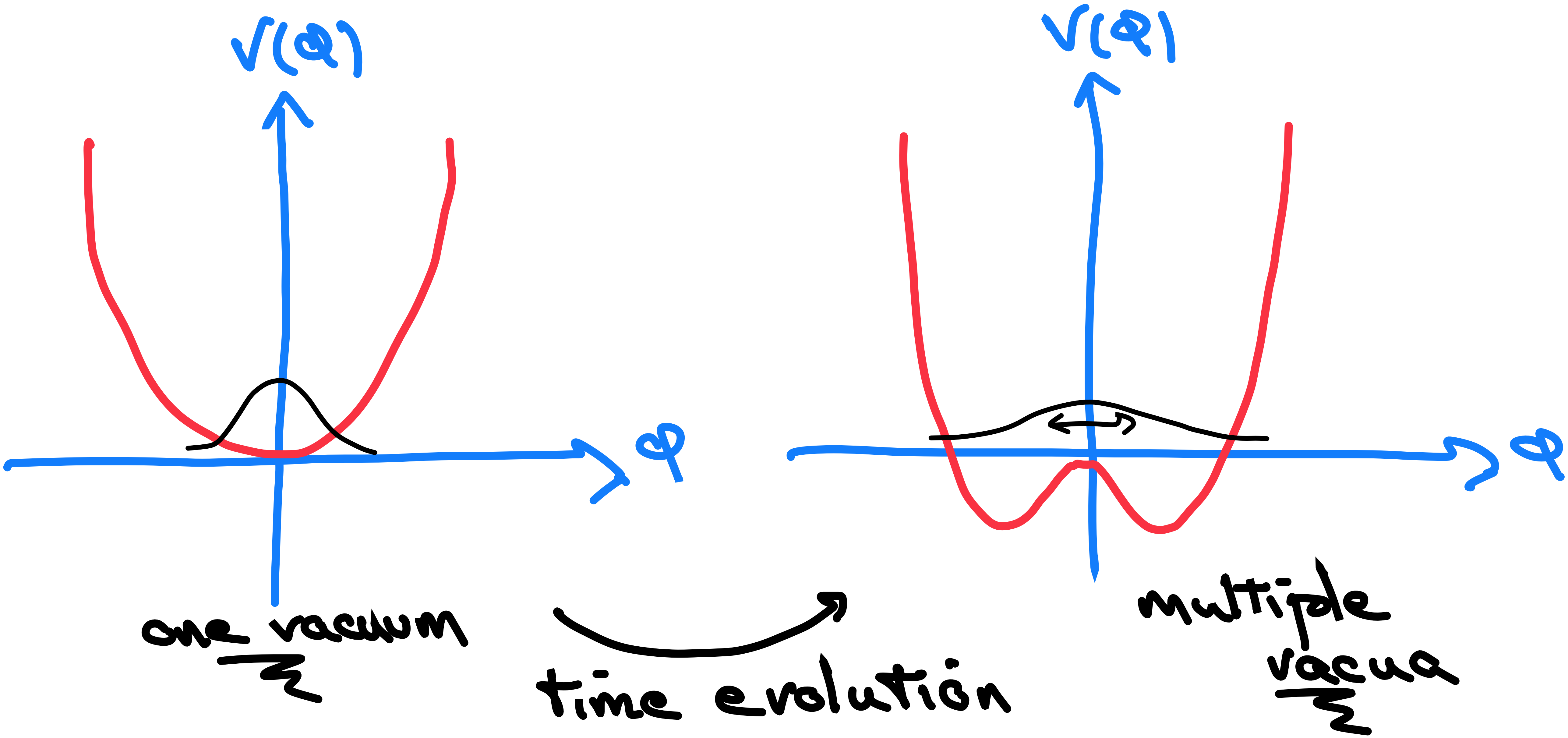
KINK
classical solution interpolating between the vacua (crossing 0 at x_0)

ex: $V(\varphi) = -\frac{m^2}{2}\varphi^2 + \frac{\lambda}{4}\varphi^4$

$$\boxed{\varphi_\pm(x) = \pm\frac{m}{\sqrt{\lambda}}\tanh\left(\frac{mx}{2}\right)}$$

Setting:

Quantum phase transition in 1 + 1 D

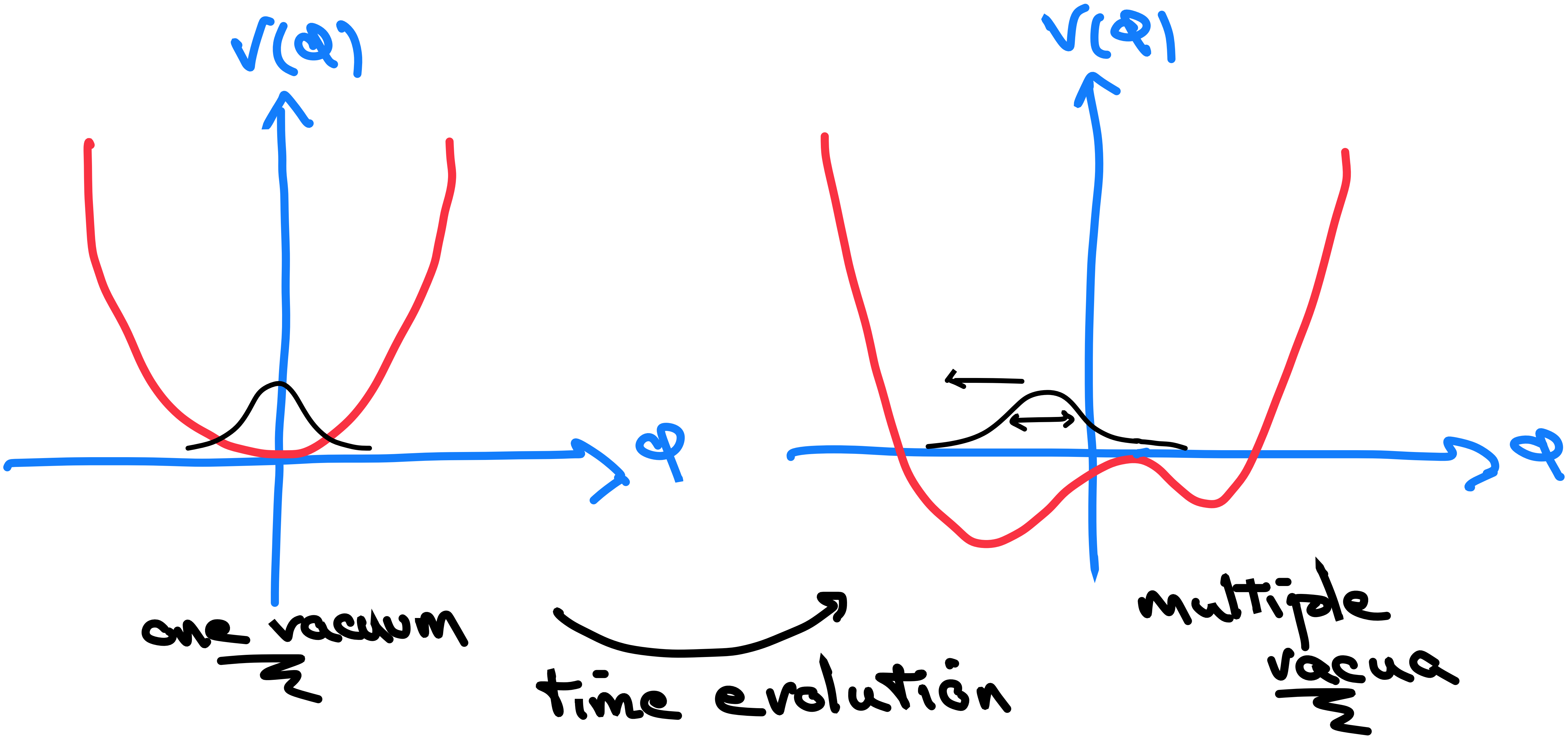


⇒ Production of kinks and antikinks

QUANTUM AVERAGE NUMBER DENSITY

Setting:

Quantum phase transition in 1 + 1 D



$\Rightarrow$ Production of kinks and antikinks

QUANTUM AVERAGE NUMBER DENSITY

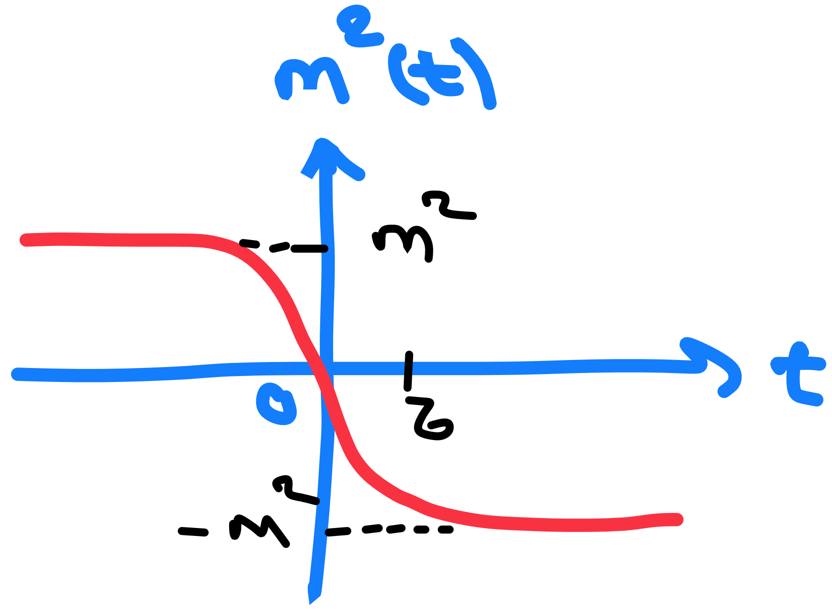
Outline

1. Setup
2. Quantization
3. Kink counting
4. Results + generalizations

Detailed discussion of kink production and dynamics in 1 + 1 D (for concreteness)

① Setup: simplified phase transition

$$\mathcal{L} = \frac{1}{2}\dot{\varphi}^2 - \frac{1}{2}\varphi'^2 - \frac{1}{2}m^2(t)\,\varphi^2 + \text{higher oder}$$



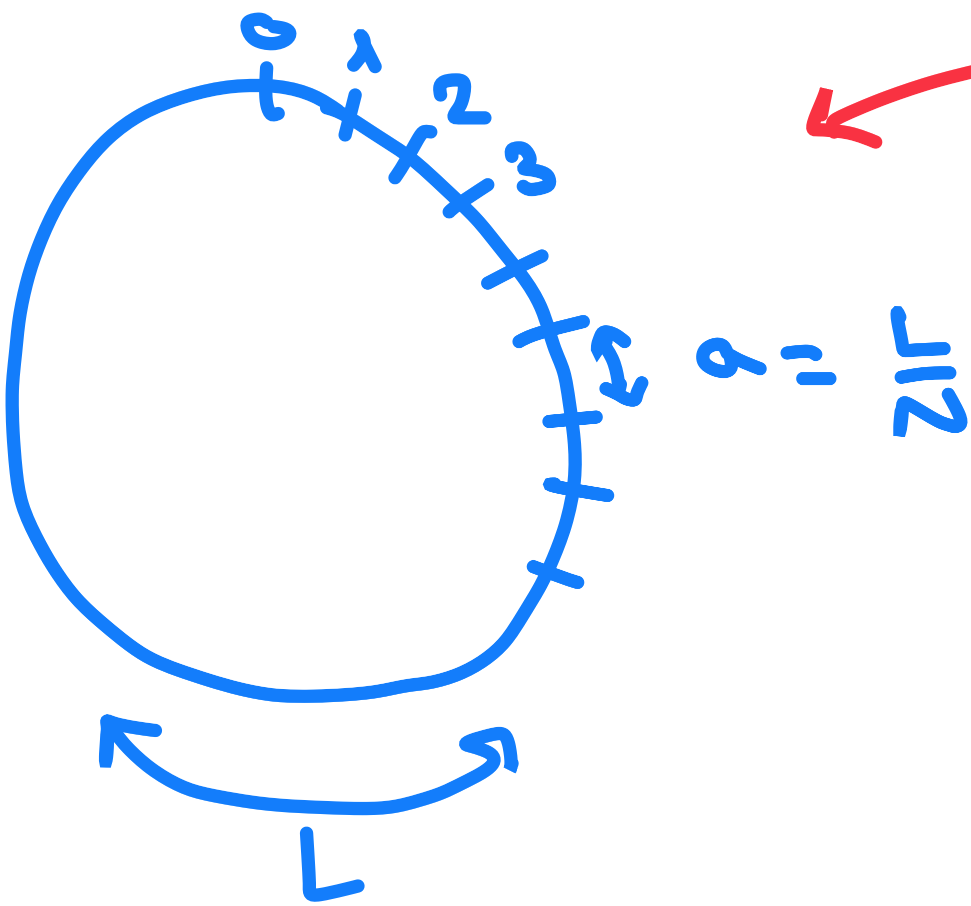
Mass Term becomes Tachyonic in Time τ

IDEA: zooming in on the neighborhood of $\varphi = 0$ in field space so that higher order terms are neglected

Validity: weak coupling, fast phase Transition short time scales ...
($\frac{\lambda\tau}{m} \ll 1$ and spinodal decomposition phase)

① Setup: simplified phase transition

$$\mathcal{L} = \frac{1}{2}\dot{\phi}^2 - \frac{1}{2}\phi'^2 - \frac{1}{2}m^2(t)\,\phi^2 + \cdots$$



periodic lattice with N evenly spaced points

Discretized Lagrangian

$$L = \sum_{i=1}^{N} a\left[\frac{1}{2}\dot{\phi}_i^2 + \frac{1}{2a^2}\phi_i\left(\phi_{i-1} - 2\phi_i + 2\phi_{i+1}\right) - \frac{1}{2}m^2(t)\,\phi_i^2\right]$$

① Setup: simplified phase transition

More compact notation

$$L = \frac{a}{2}\,\dot{\varphi}^T \dot{\varphi} - \frac{a}{2}\,\varphi^T \Omega^2(t)\,\varphi$$

N bilinearly coupled harmonic oscillators

$$\varphi^T = (\varphi_1\ \varphi_2\ \cdots\ \varphi_N)$$

$$\Omega^2(t) = \begin{bmatrix} \frac{2}{a^2} + m^2(t) & -1/a^2 & 0 & \cdots & 0 & -1/a^2 \\ -1/a^2 & \frac{2}{a^2} + m^2(t) & -1/a^2 & & & 0 \\ 0 & -1/a^2 & \ddots & \ddots & & \vdots \\ \vdots & & \ddots & \ddots & \ddots & \vdots \\ 0 & & & \ddots & \ddots & \\ -1/a^2 & & & & & \end{bmatrix}$$

$\Longrightarrow$ QUANTIZE

② Quantization:

Solve functional Schrödinger equation

- Gaussian ansatz:

$$\Psi(\varphi,t) = \mathcal{N}(t)\exp\left[\frac{ia}{2}\varphi^T M(t)\varphi\right]$$

$N\times N$ symmetric matrix

- Schrödinger equation:

$$i\frac{\partial\Psi}{\partial t} = -\frac{1}{2a}\sum_{i=1}^{N}\frac{\partial^2\Psi}{\partial\varphi_i^2} + \frac{a}{2}\varphi^T\Omega^2(t)\varphi\ \Psi$$

$$\Rightarrow \dot{M} + M^2 + \Omega^2(t) = 0\ ,\quad M(t_0) = i\Omega^2(t_0)^{1/2}$$

initial condition corresponding to "vacuum"

② Quantization:

Solve for $M(t)$ in terms of mode functions

$$\ddot{c}_n + \left[\frac{4}{a^2}\sin^2\left(\frac{\pi n}{N}\right) + m^2(t)\right]c_n = 0$$

$$\begin{cases} c_n(t_0) = \dfrac{1}{\sqrt{2a}}\left[\dfrac{4}{a^2}\sin^2\left(\dfrac{\pi n}{N}\right) + m^2(t_0)\right]^{-1/4} \\ \dot{c}_n(t_0) = \dfrac{i}{\sqrt{2a}}\left[\dfrac{4}{a^2}\sin^2\left(\dfrac{\pi n}{N}\right) + m^2(t_0)\right]^{1/4} \end{cases} \qquad 0 \le n < N$$

$$\Rightarrow [M(t)]_{ij} = \frac{1}{N}\sum_{n=0}^{N-1} c_n(t)^{-1}\dot{c}_n(t)\cos\left(\frac{2\pi(i-j)n}{N}\right)$$

BUT we will need the probability density functional $P(\varphi,t) = |\Psi(\varphi,t)|^2$

② Quantization:

$$P(\varphi, t) = \frac{1}{\sqrt{\det(2\pi K)}} \exp\left[-\frac{1}{2}\varphi^T K(t)^{-1} \varphi\right]$$

covariance matrix
$N \times N$

$$\Rightarrow [K(t)]_{ij} = \frac{1}{N} \sum_{n=0}^{N-1} |c_n(t)|^2 \cos\left(\frac{2\pi(i-j)n}{N}\right)$$

NOW LET'S COUNT KINKS!!!

③ Kink counting:

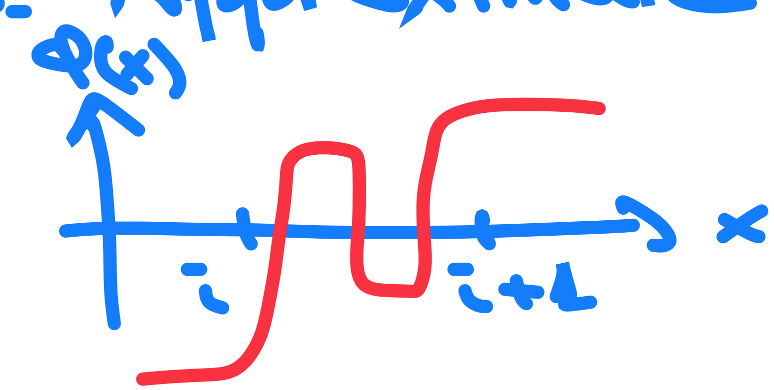
Basic idea: look for kinks among points where $\phi(x,t) = 0$ (among zeros of the field)

• Number density of zeros operator:

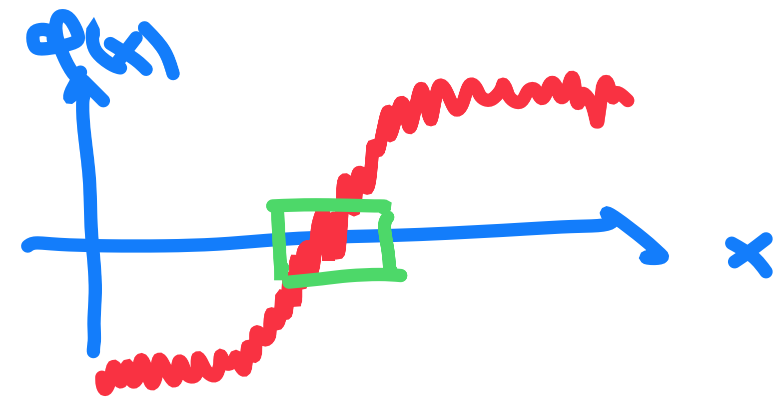
$$\hat{n}_z = \frac{1}{L} \sum_{i=1}^{N} \frac{1}{4} \left[\mathrm{sgn}(\hat{\phi}_i) - \mathrm{sgn}(\hat{\phi}_{i+1}) \right]^2$$

sgn → signum function

N.B. Approximate operator:



undercounting if a is too large



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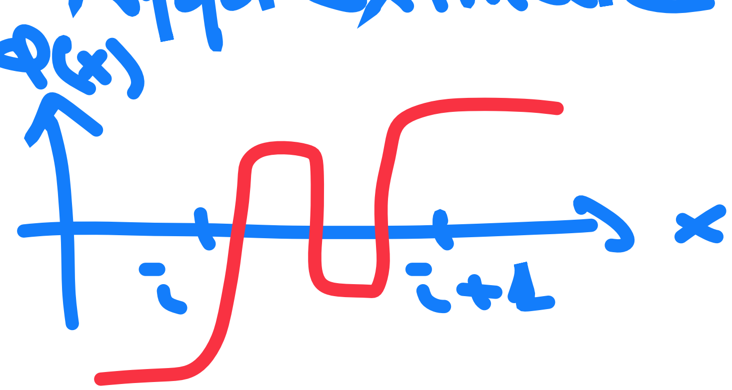

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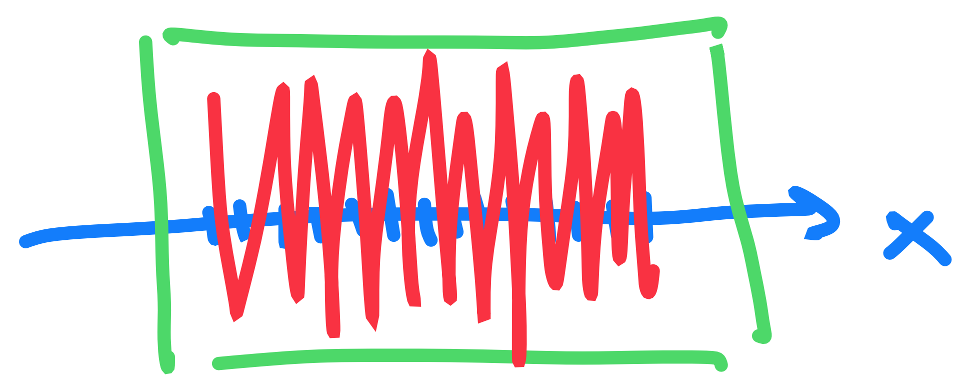


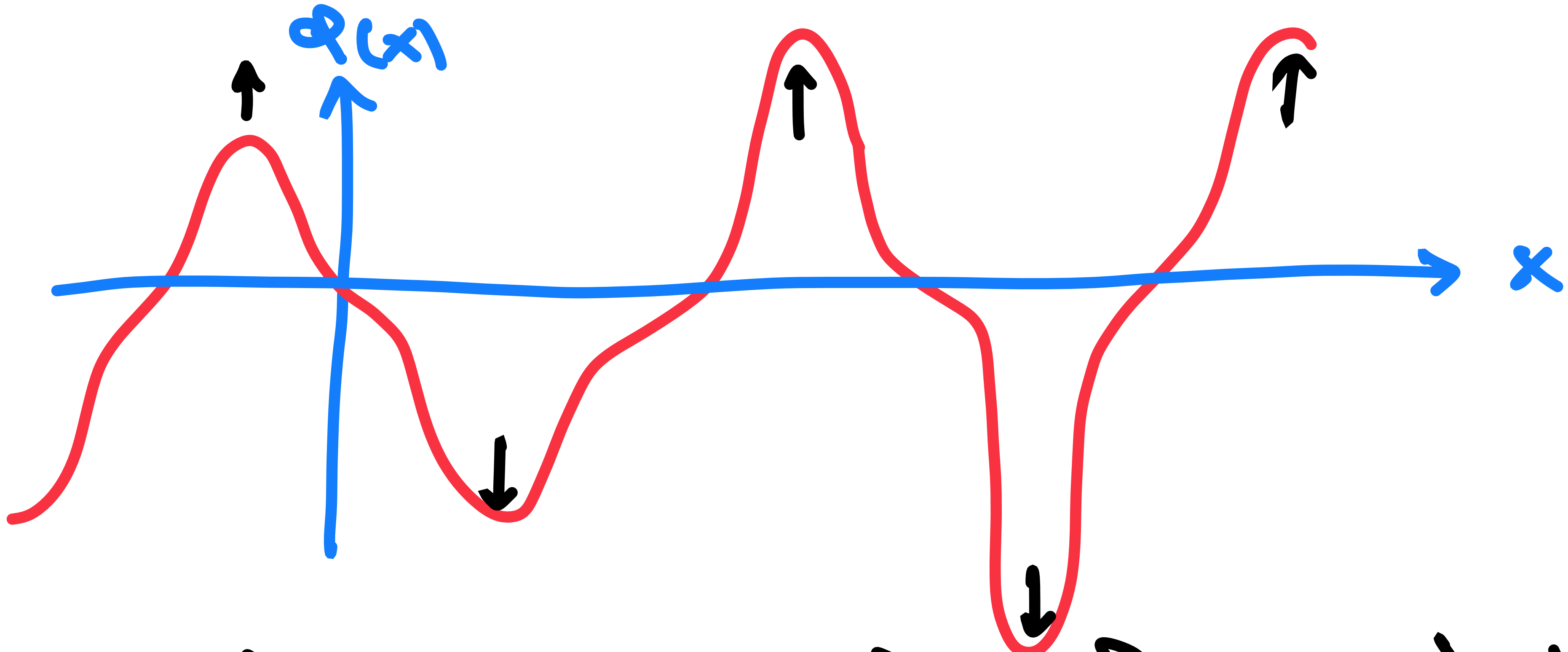
overcounting if a is too small

VACUUM FLUCTUATIONS

③ Kink counting:

We are counting unstabilized (precursor) kinks

⇒ SPINODAL INSTABILITY



sinh profiles (rather than the usual tanh)

Expectation:

- production of kinks and antikinks (random positions and velocities)
- mutual annihilation (ballistic?)
- ? stabilization

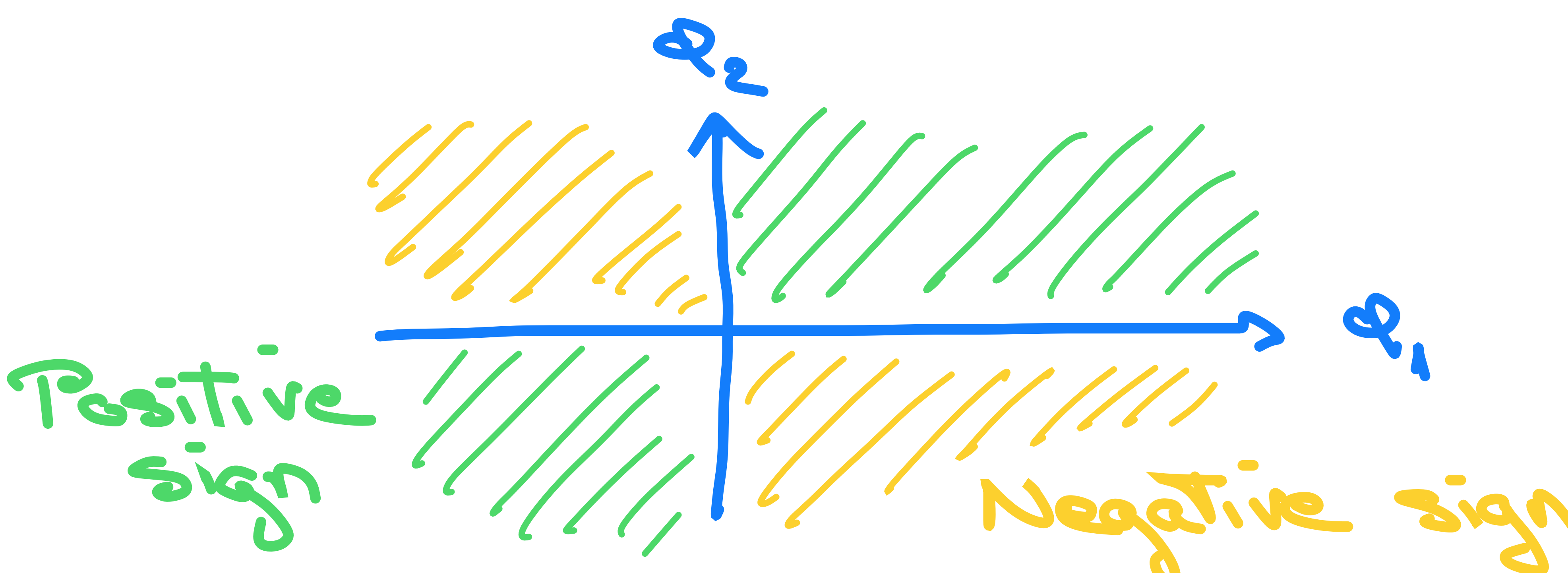
③ Kink counting:

$$\langle \hat{n}_k \rangle = \frac{N}{2L}\left[1 + \langle \mathrm{sgn}(\hat{\varphi}_1 \hat{\varphi}_2) \rangle\right]$$

$$\langle \mathrm{sgn}(\hat{\varphi}_1 \hat{\varphi}_2) \rangle = \langle \mathrm{sgn}(\hat{\varphi}_i \hat{\varphi}_{i+1}) \rangle$$

translational invariance

$\Rightarrow$ Gaussian integral in N-dimensions with sign changes in the (φ_1, φ_2) plane



③ Kink counting:

Result of integration:

$$\boxed{\langle \hat{n}_z \rangle = \frac{N}{\pi L} \arccos(\beta/\alpha)}$$

Where
$$\begin{cases} \alpha(t) = \frac{1}{N} \sum_{n=0}^{N-1} |c_n(t)|^2 \\ \beta(t) = \frac{1}{N} \sum_{n=0}^{N-1} |c_n(t)|^2 \cos\left(\frac{2\pi n}{N}\right) \end{cases}$$

③ Kink counting:

Result of integration:

$$\boxed{n_K = \frac{N}{\pi L} \arccos\left(\bar{\beta}/\bar{\alpha}\right)}$$

Where
$$\begin{cases} \bar{\alpha}(t) = \dfrac{1}{N} \displaystyle\sum_{n=-n_c}^{n_c} |c_n(t)|^2 \\ \bar{\beta}(t) = \dfrac{1}{N} \displaystyle\sum_{n=-n_c}^{n_c} |c_n(t)|^2 \cos\left(\dfrac{2\pi n}{N}\right) \end{cases}$$

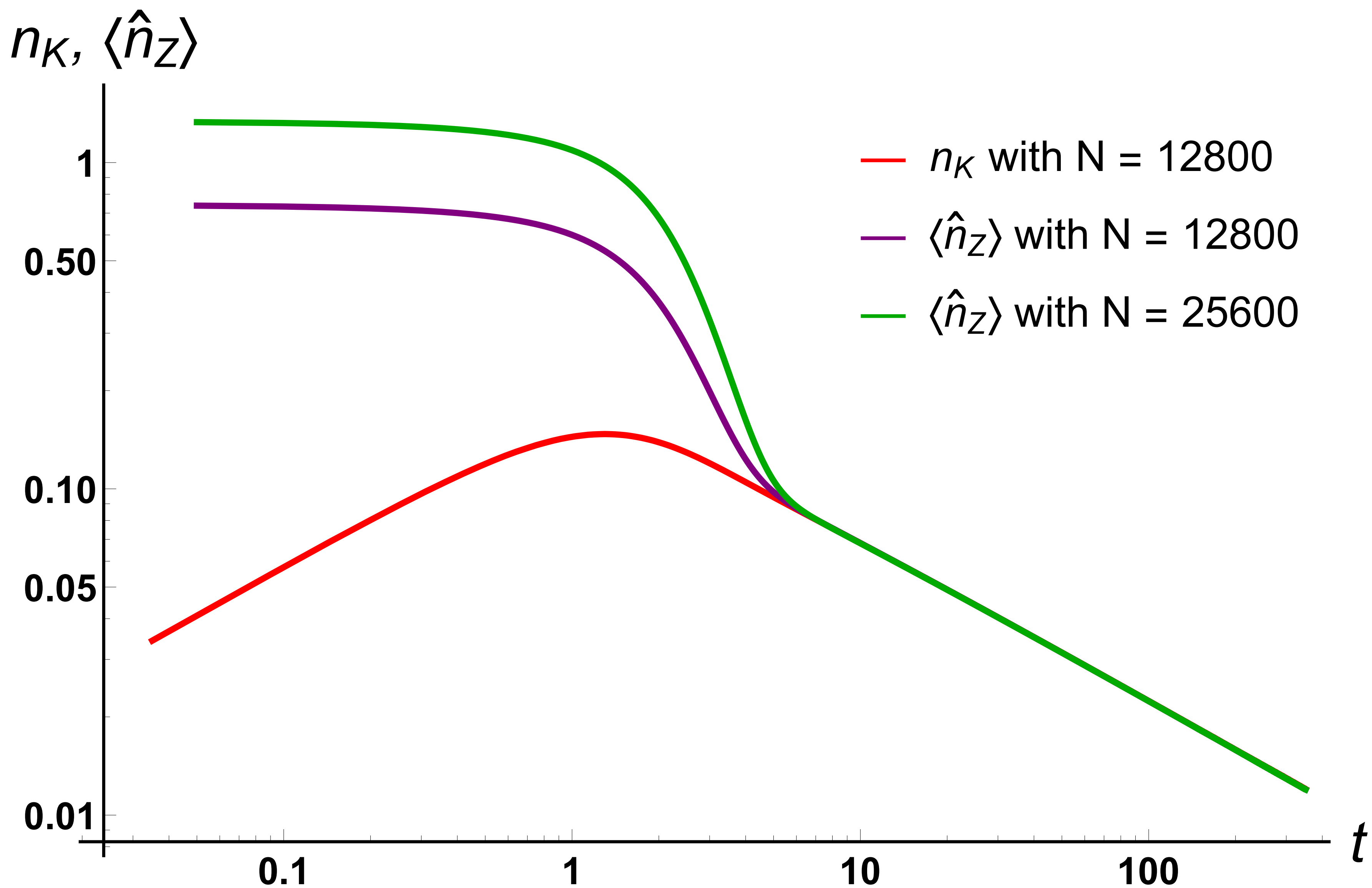
Last ingredient: in $\alpha(t)$, $\beta(t)$ only keep the unstable modes to get rid of UV sensitive component of $\langle \hat{n} \rangle$

$$n \leq n_c = \frac{N}{\pi} \arcsin\left(\frac{a}{2}\sqrt{1-m^2(t)}\right)$$

Z

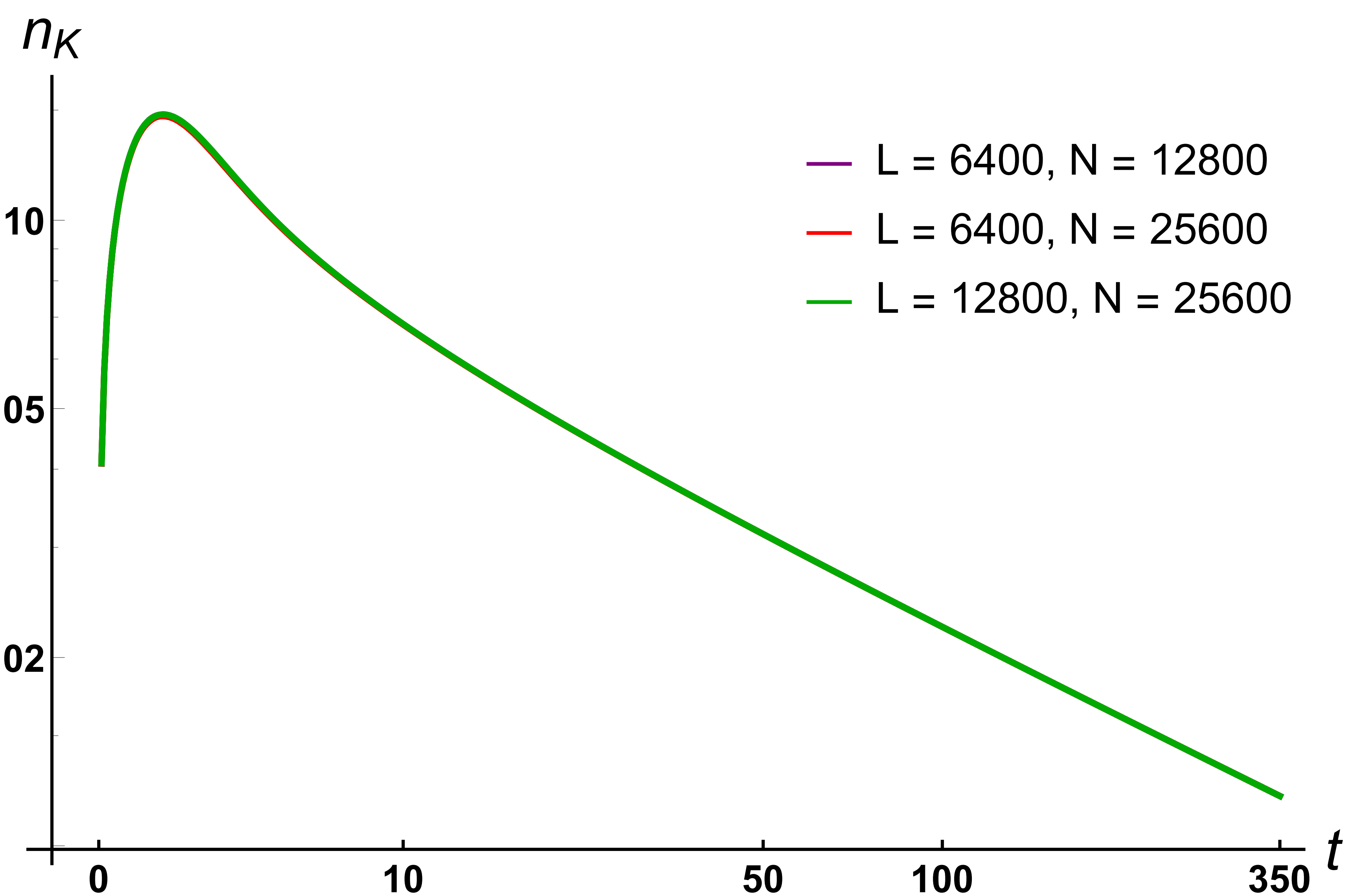
④ Results:

Difference between $\langle\hat{n}\rangle$ and n_K: sensitivity to N (log divergence)



④ Results:

UV and IR stability of n_K:



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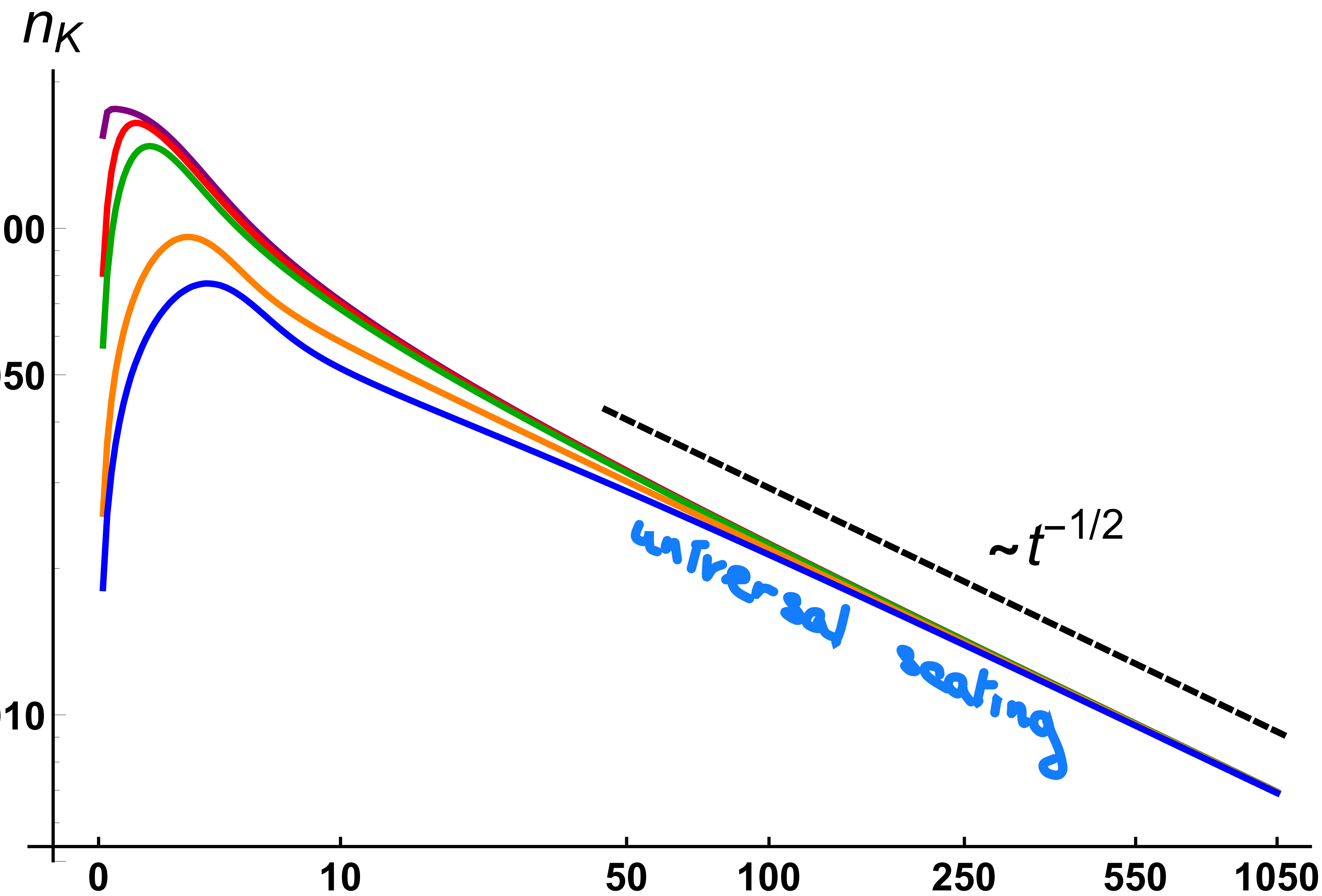

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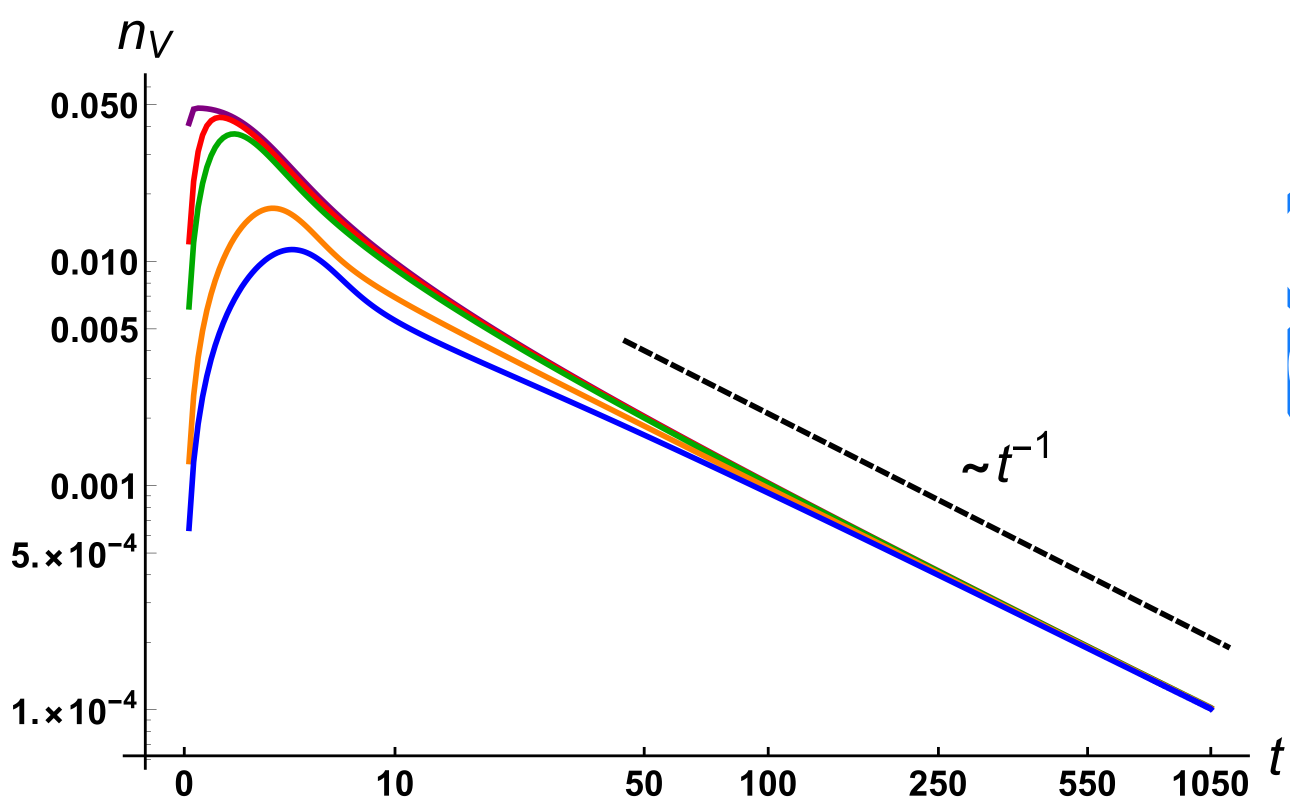
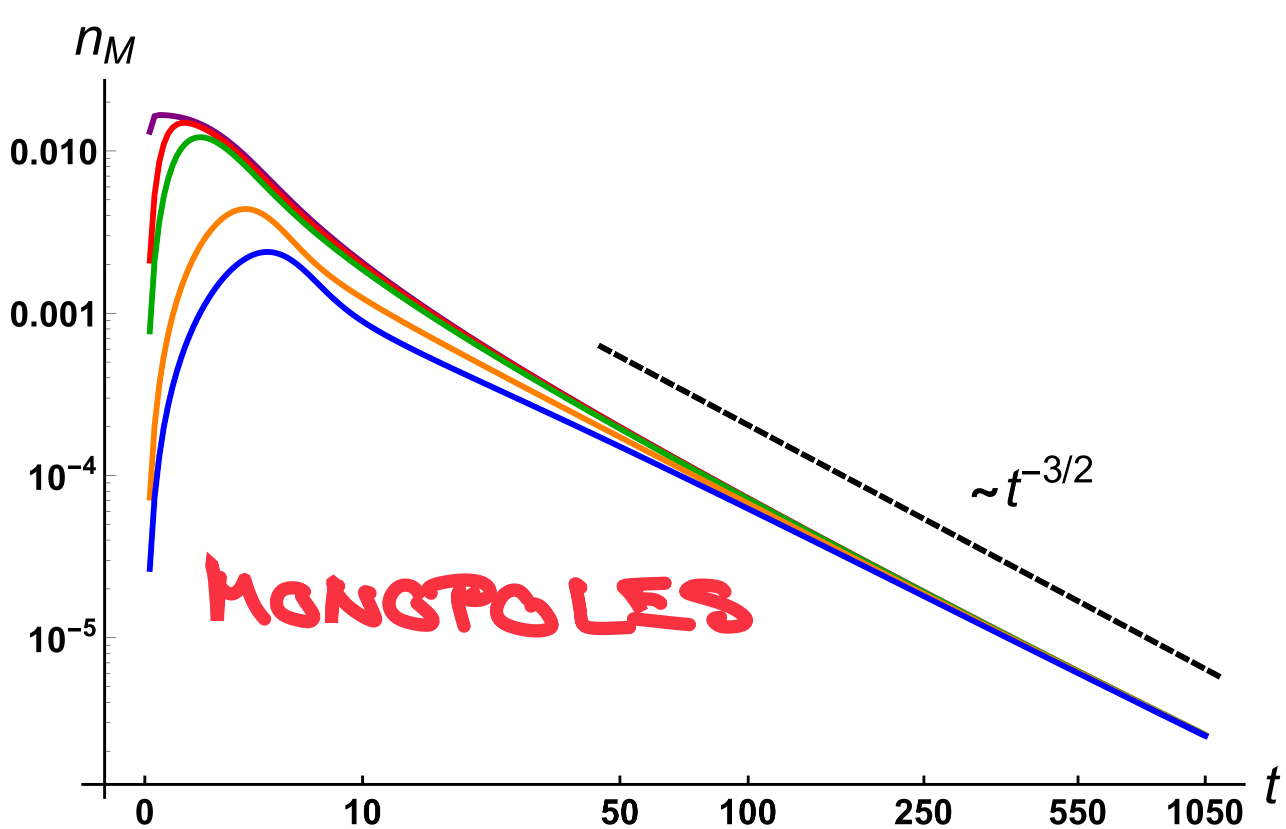
$\cdot\, G(t^{-3/2})$

(similar dynamics for domain wall area density in higher dimensions)

④ Results: higher dimensions





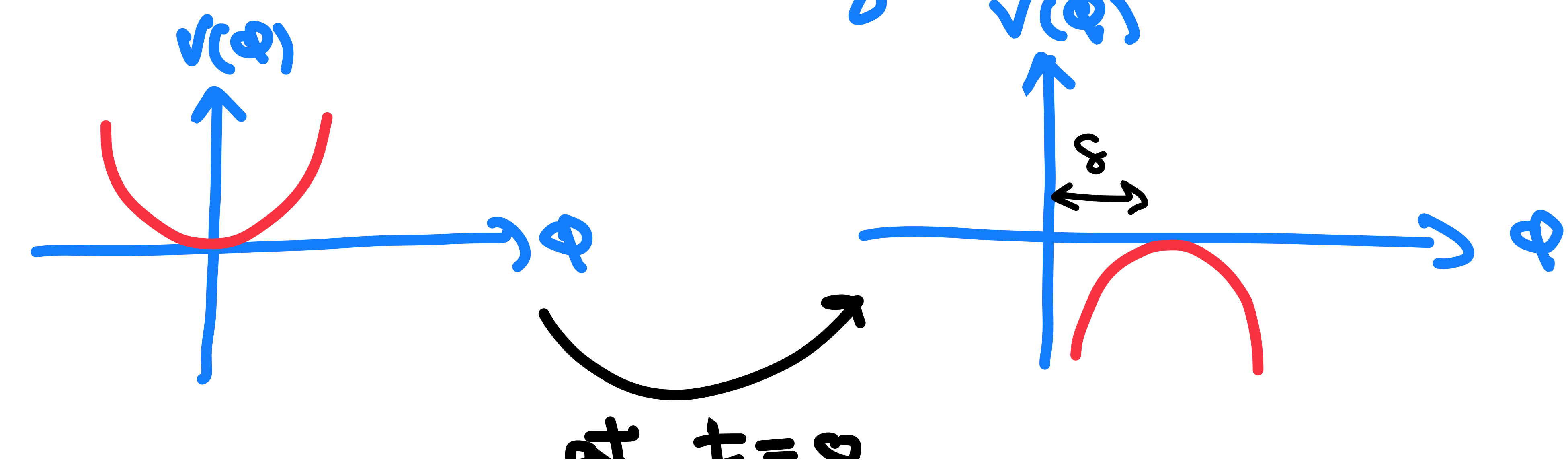
POSSIBLE GENERALIZATIONS IN HIGHER D

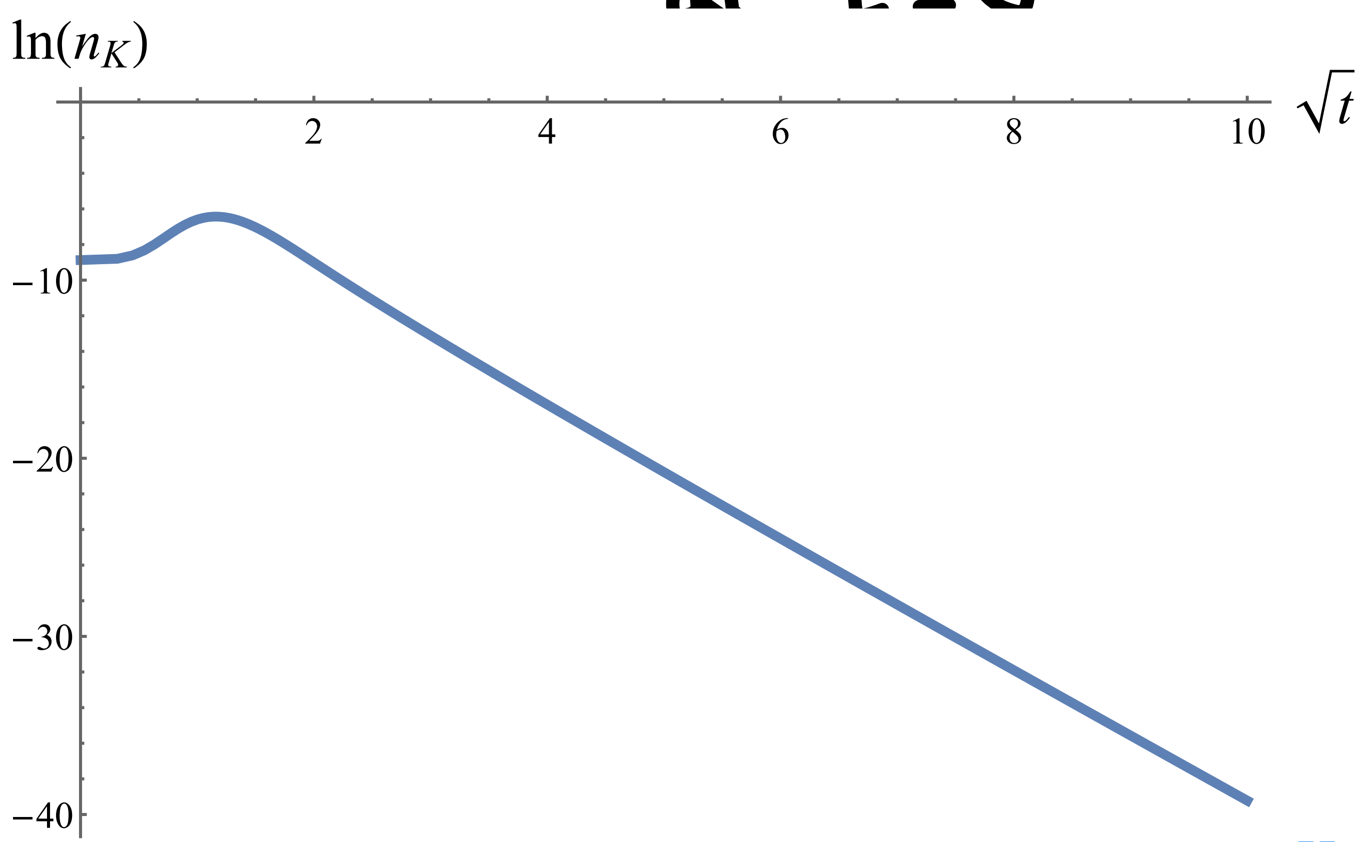
in d spatial dimensions

$$n_\Delta \sim \frac{d!}{2^{d/2}\pi^d}\left(\frac{m}{t}\right)^{d/2} + O\left(t^{-(d+2)/2}\right)$$

④ Results: biased case

Case of the instantaneous phase transition + instantaneous shift

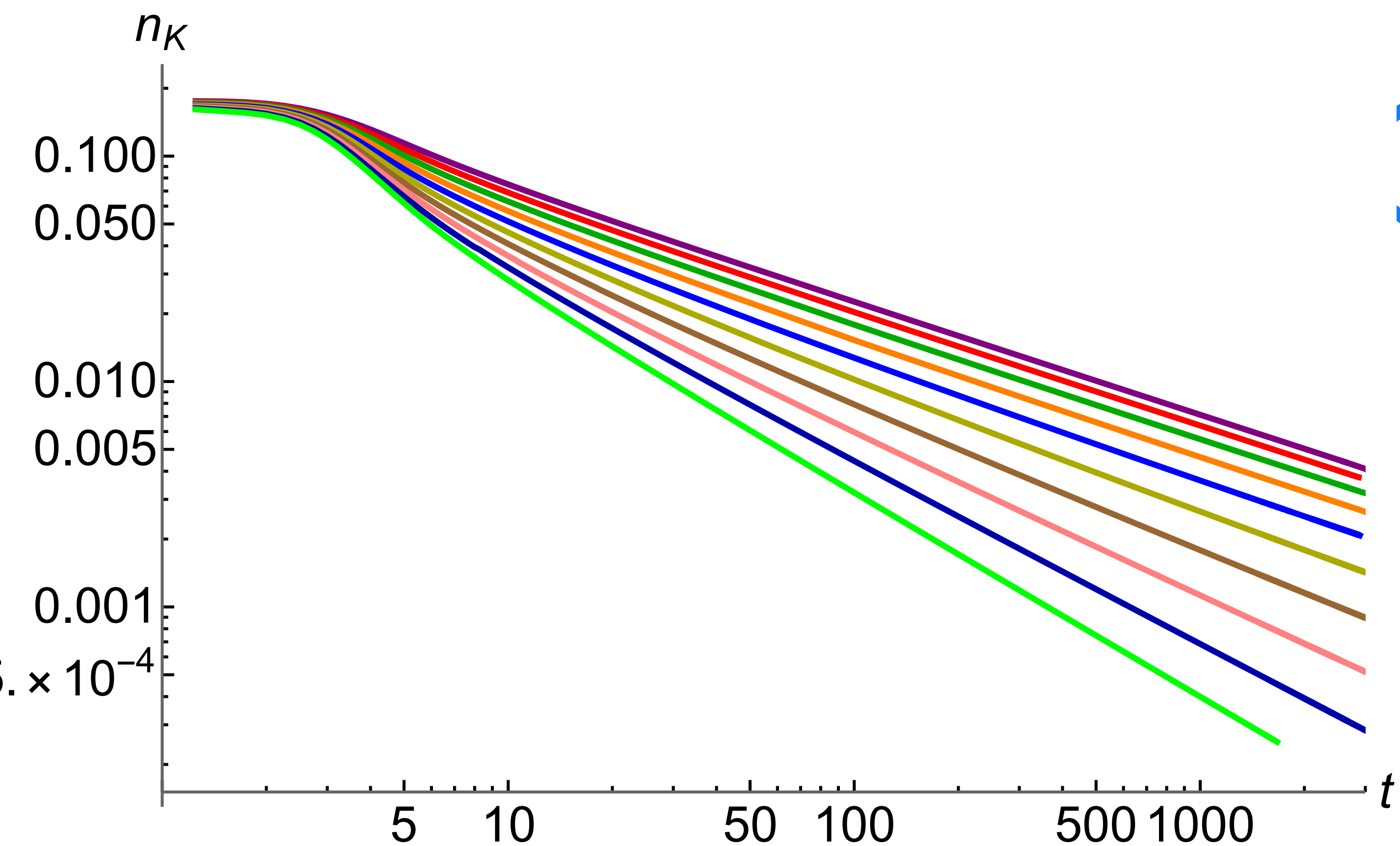




$$n_K \sim \frac{1}{\pi}\sqrt{\frac{m}{2t}}\; e^{-2\sqrt{\pi m t}}$$

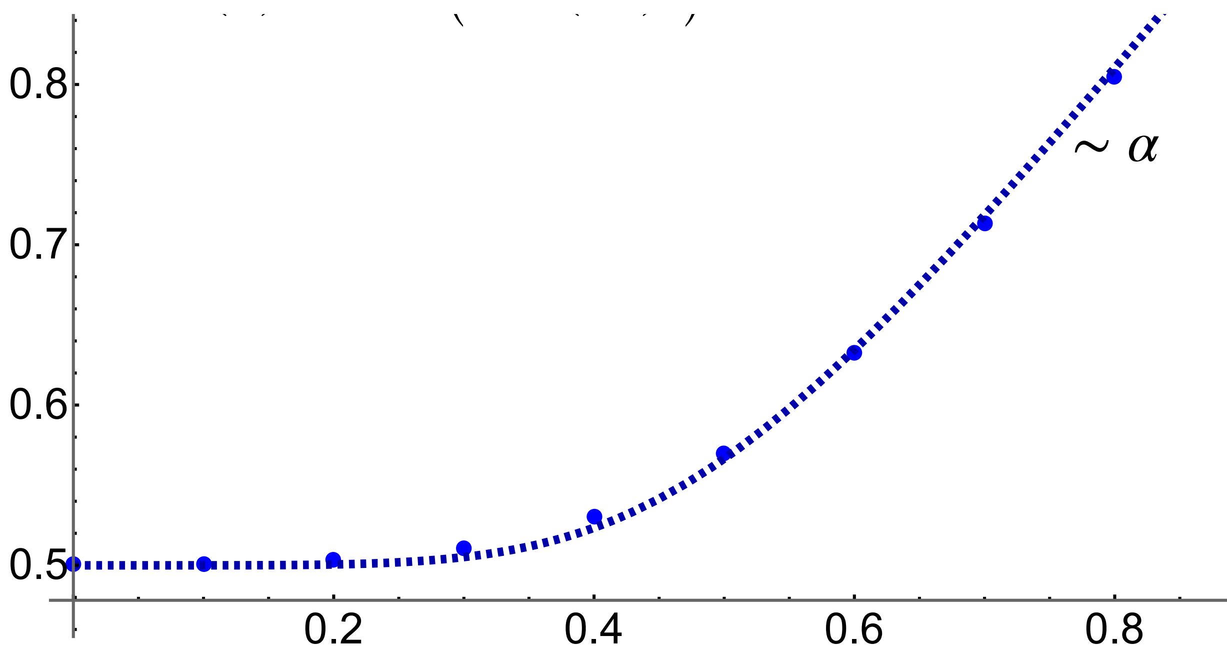
(analytical estimate for $\delta = 1$)

④ Results: What about cosmology?



Flat FLRW in 1+1D: scale factor $\sim t^{\alpha}$

↓ increasing α



← Domain wall gas $n_K \sim 1/a(t)$

Broken power law of VOS modelling

Takeaways

- Full quantum dynamics of topological defect production and annihilation

$n_K \sim \frac{1}{\pi}\sqrt{\frac{m}{2t}}$ (for kinks)

- Limitation : computation valid while Gaussian approximation holds

$\Rightarrow$ spinodal decomposition phase

- generalization
  - vortices , monopoles ---
  - Domain walls in higher dimensions
  - cosmology

THANK YOU