Nuclear ab initio studies for neutrino oscillations (and beyond)

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Theory of Particle Physics and Cosmology, 21 March 2024

Cluster of Excellence PRISMA+ Precision Physics, Fundamental Interactions and Structure of Matter
JG|U
Alexander von Humboldt Stiftung/Foundation
MARIE CURIE ACTIONS
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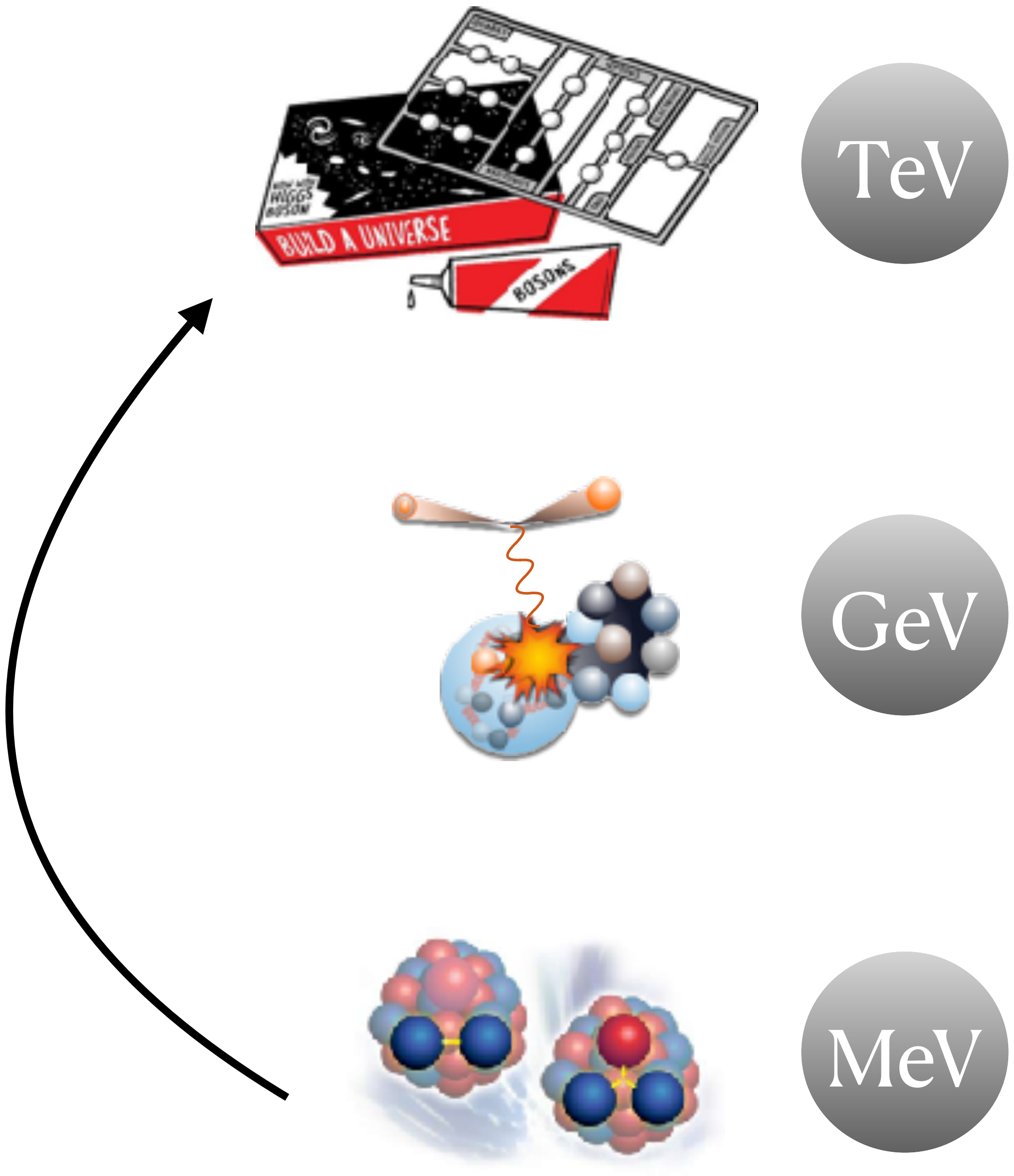
Outline

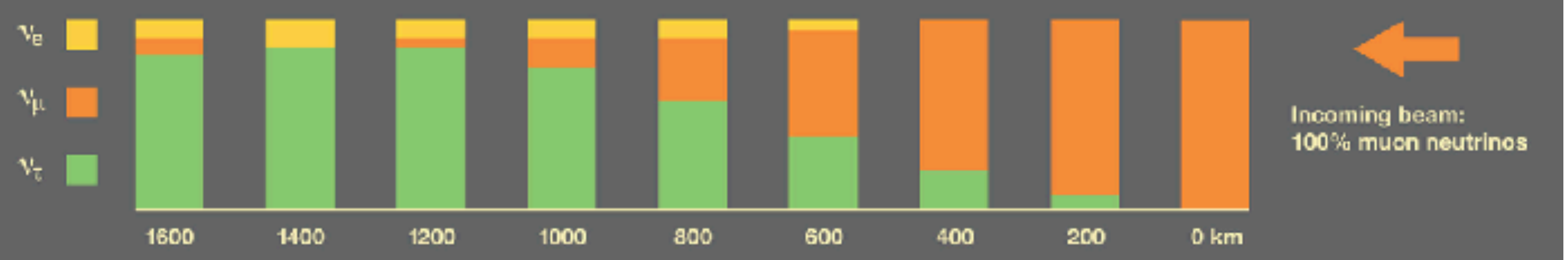


- Neutrino oscillation programs @ Precision Frontier
- Ab initio nuclear methods & uncertainty quantification
- Electroweak physics with nuclear probes
- From matrix elements to continuous nuclear responses
- ...and beyond

Source:
https://www.iop.org/explore-physics/big-ideas-physics/standard-model
https://phys.org/news/2011-07-tango-nuclear-analysis-three-body.html

Neutrino oscillations

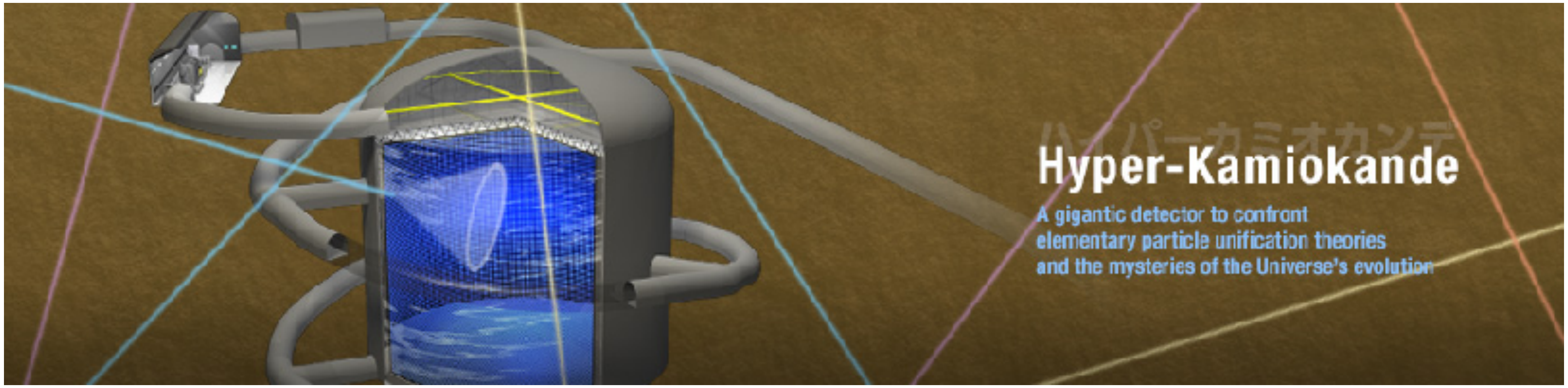
source: www.symmetrymagazine.org

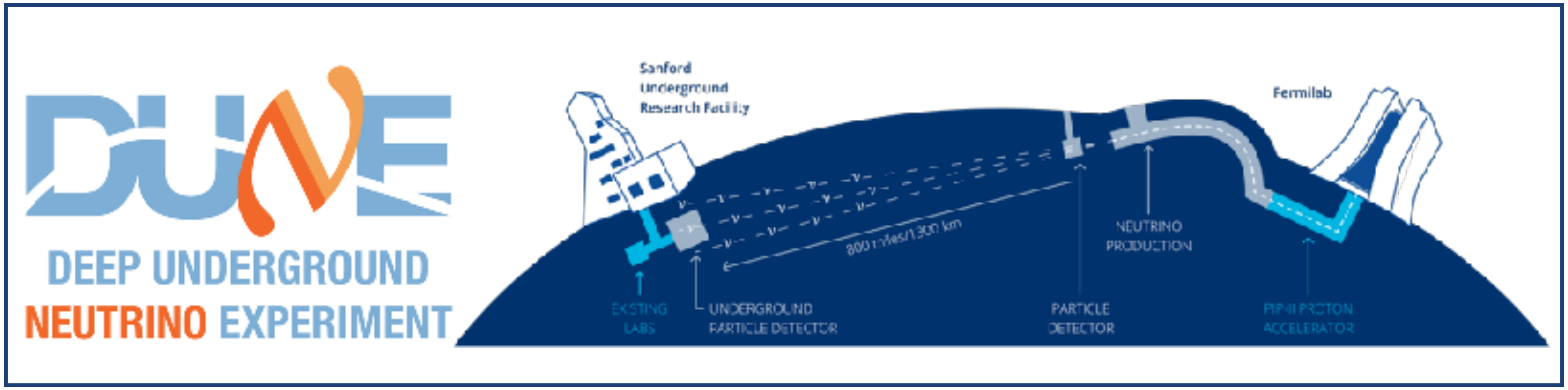


Probability of detecting electron, muon and tau neutrinos

$$\mathscr{P}_{\alpha\to\beta} = \sin^2(2\theta)\sin^2\left(\frac{\Delta m_{12}^2 L}{4E}\right)$$

Next generation experiments



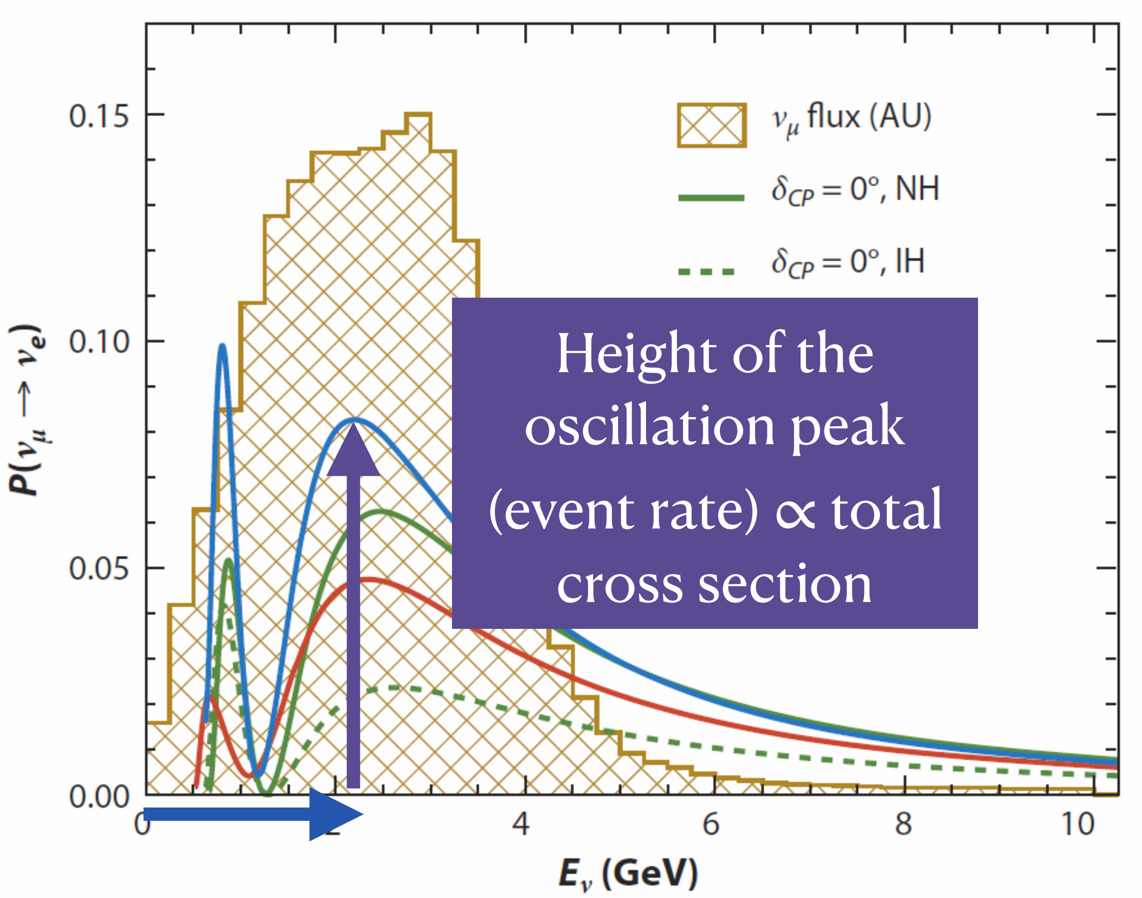


- ✓ **CP-violation measurement**
- ✓ **Determining ν mass ordering**
- ✓ Proton decay searches
- ✓ Cosmic neutrino observation

Aims & challenges

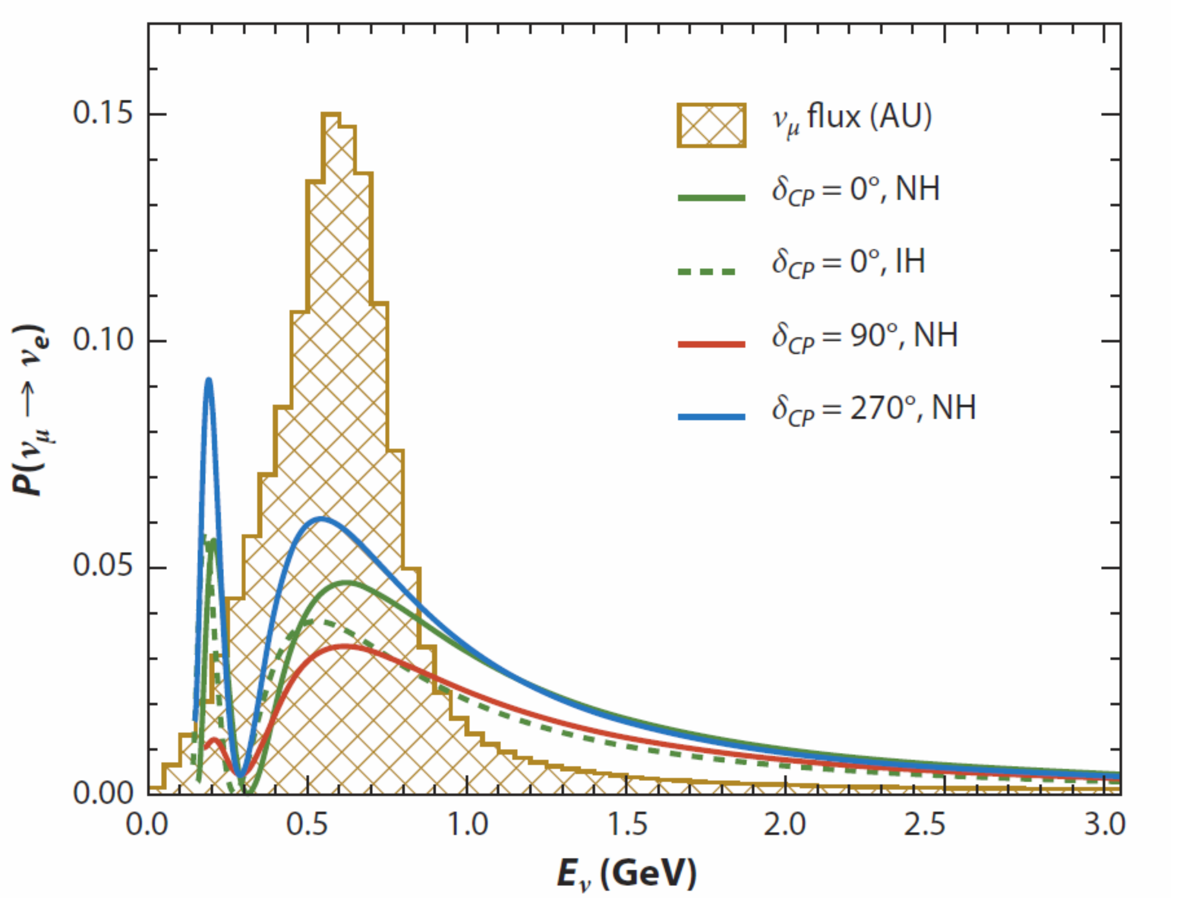
From: Diwan et al, Ann. Rev.Nucl. Part. Sci 66 (2016)

DUNE



Height of the oscillation peak (event rate) $\propto$ total cross section

T2HK

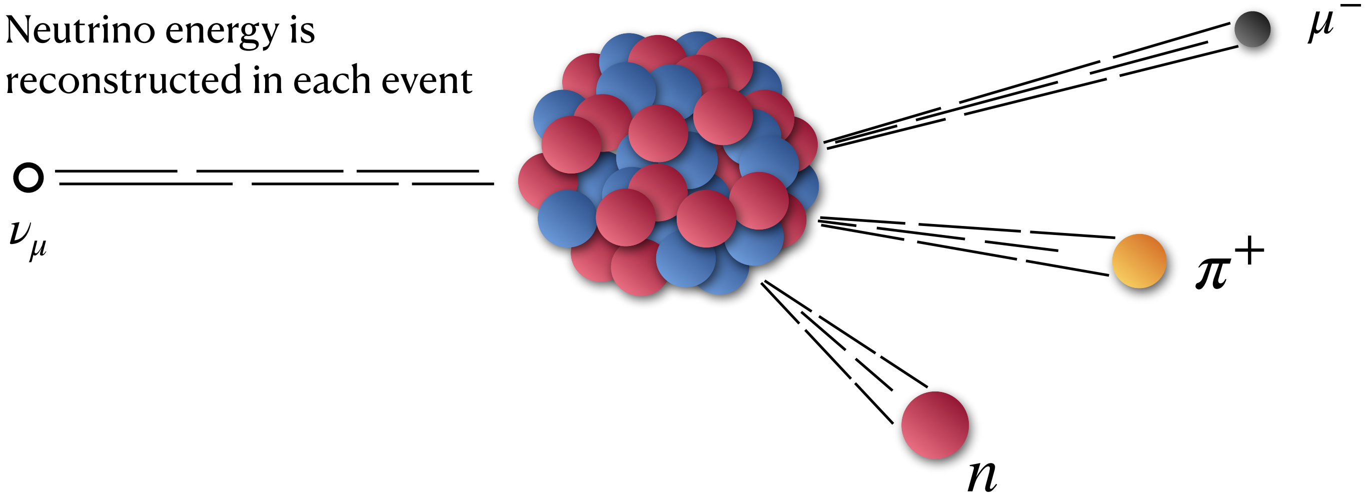


Position of the oscillation peak depends on energy reconstruction

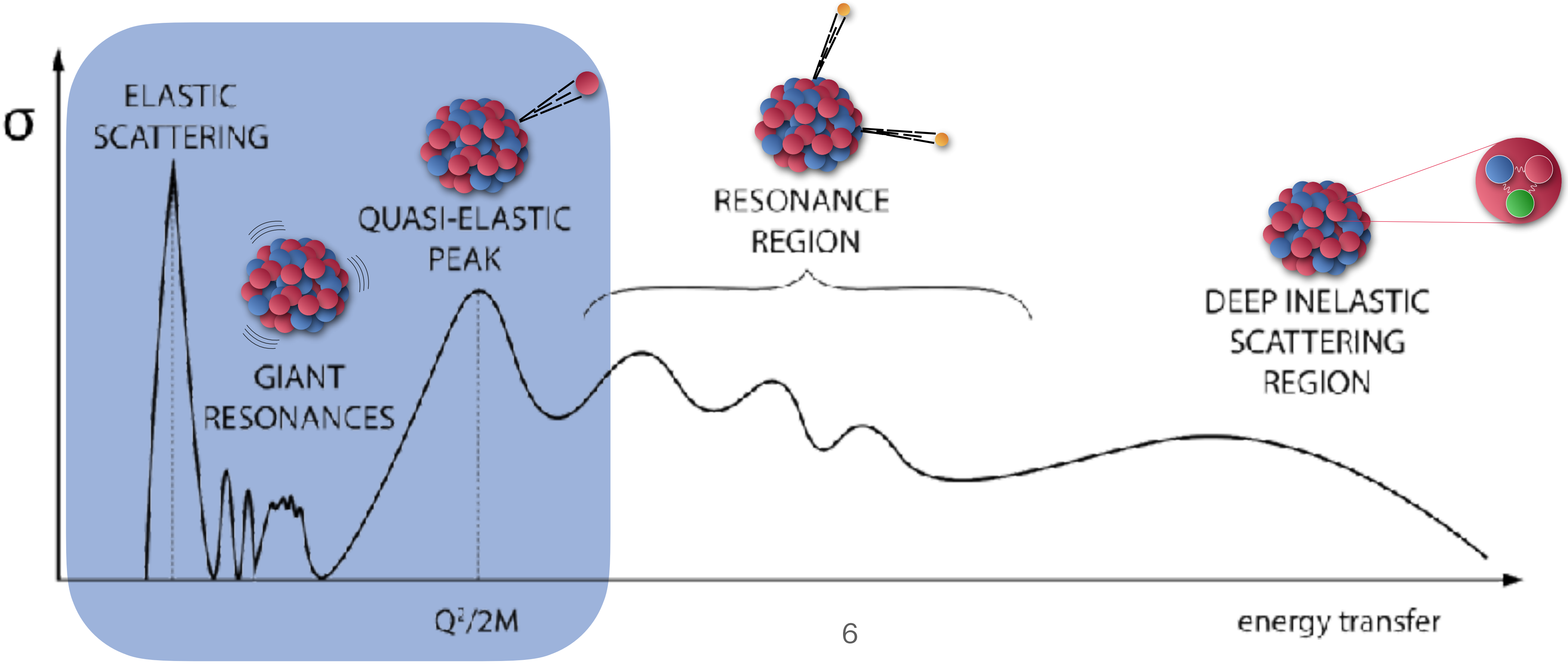
DUNE aims at uncertainties $< 1\,\%$ meaning $\mathcal{O}(25)$ MeV precision of energy reconstruction

Systematic errors should be small since statistics will be high

Motivation

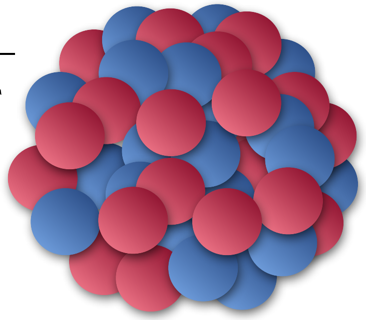


Neutrino energy is reconstructed in each event



“Ab initio” nuclear theory

nucleons — degrees of freedom



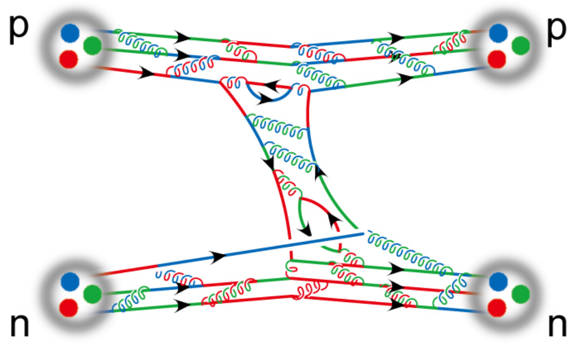
$$\mathscr{H}\,|\Psi\rangle = E\,|\Psi\rangle$$

What is the dynamics of our system?

$$\mathscr{H} = \sum_{i=1}^{A} t_{kin} + \sum_{i>j=1}^{A} v_{ij} + \sum_{i>j>k=1}^{A} v_{ijk} + \dots$$

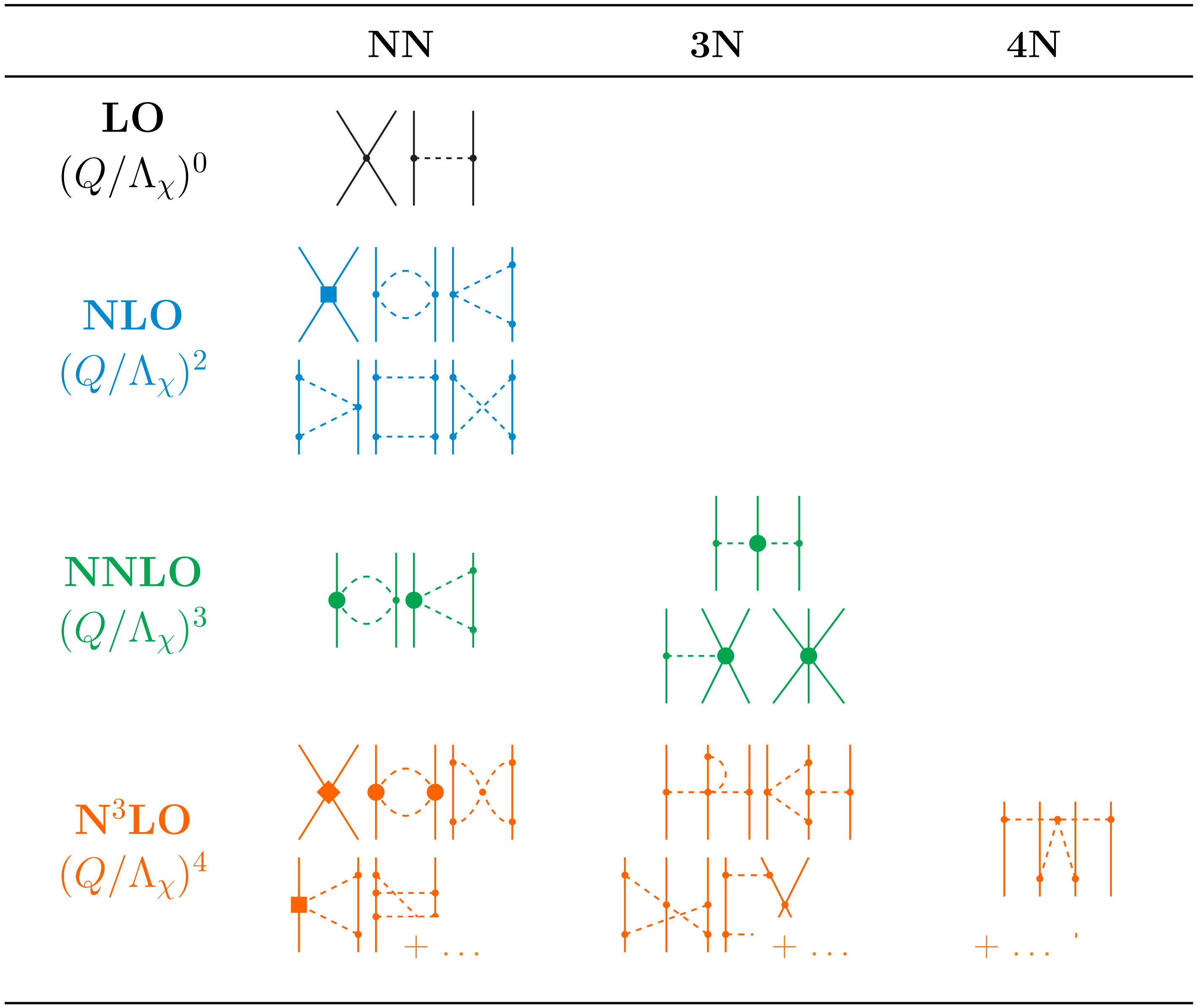
How the nuclear force is rooted in the fundamental theory of QCD?

Quantum Chromodynamics



Nuclei & nuclear matter

Nuclear Hamiltonian



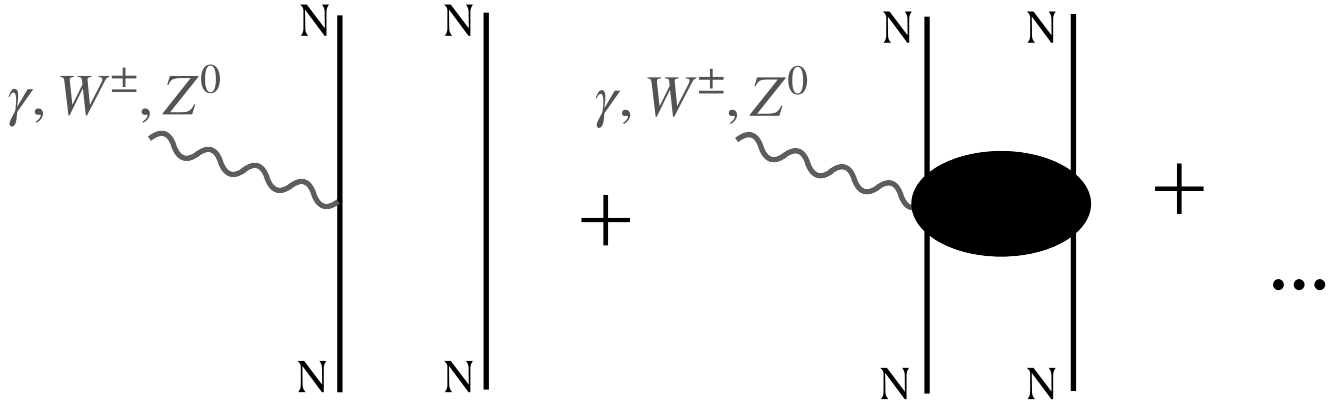
- S. Weinberg, Phys. Lett. **B251**, 288 (1990); Nucl. Phys. **B363**, 3 (1991); Phys. Lett **B295**, 114 (1992)
- Effective chiral Lagrangian $\mathscr{L}_{eff}(\pi, N, \Delta) \rightarrow$ obtain nuclear potential
- Power counting scheme $\left(\frac{Q}{\Lambda_\chi}\right)^n$
- LEC fitted to data
- Uncertainty quantification possible

Epelbaum, Hammer, Meissner, Rev. Mod. Phys. 81, 1773–1825 (2009)

Electroweak interactions

- Chiral EFT allows to construct electroweak currents consistently with the chiral potential

$$j = \sum_{i=1}^{A} j_i + \sum_{i>j=1}^{A} j_{ij} + \sum_{i>j>k=1}^{A} j_{ijk} + \ldots$$



To describe:

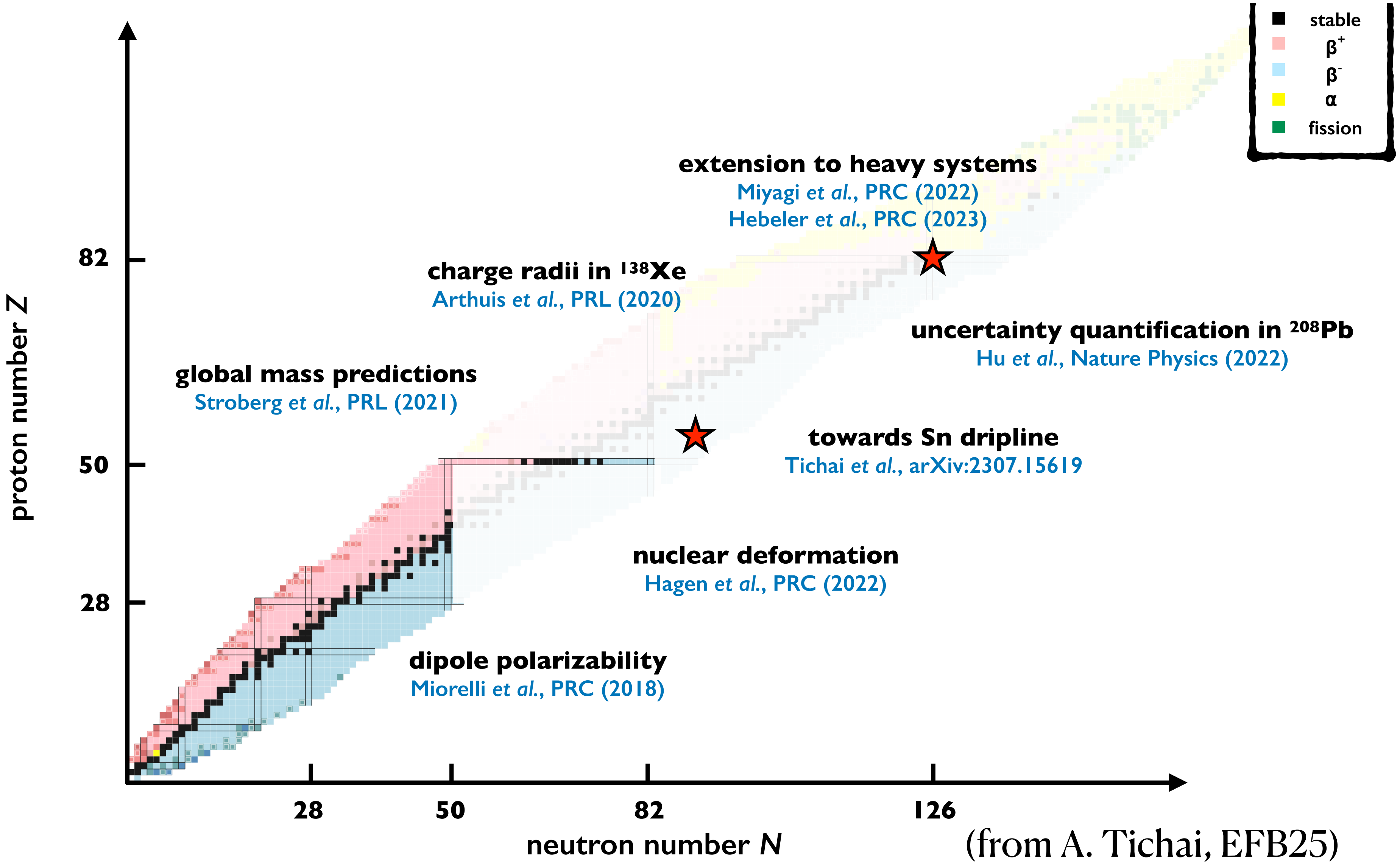
- ➡ Electroweak form-factors
- ➡ Gamow-Teller ME (β decays)
- ➡ Magnetic moments
- ➡ Radiative/weak captures
- ➡ **Electroweak response functions**

Ab initio nuclear theory

$$\mathscr{H}\,|\Psi\rangle = E\,|\Psi\rangle$$

> *"we interpret the ab initio method to be a systematically improvable approach for quantitatively describing nuclei using the finest resolution scale possible while maximizing its predictive capabilities."*

A. Ekström et al, *Front. Phys.*11 (2023) 29094



extension to heavy systems
Miyagi *et al.*, PRC (2022)
Hebeler *et al.*, PRC (2023)

charge radii in ^{138}Xe
Arthuis *et al.*, PRL (2020)

uncertainty quantification in ^{208}Pb
Hu *et al.*, Nature Physics (2022)

global mass predictions
Stroberg *et al.*, PRL (2021)

towards Sn dripline
Tichai *et al.*, arXiv:2307.15619

nuclear deformation
Hagen *et al.*, PRC (2022)

dipole polarizability
Miorelli *et al.*, PRC (2018)

(from A. Tichai, EFB25)

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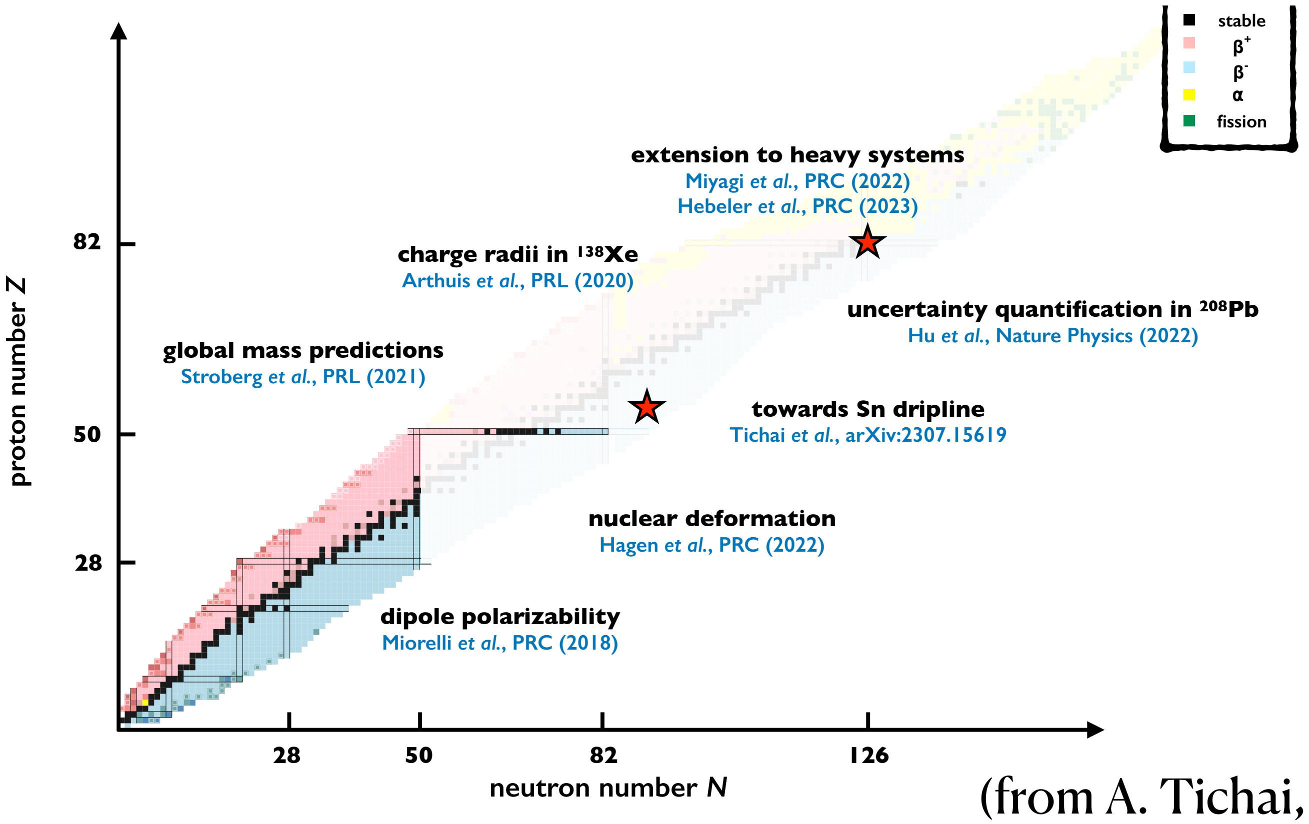

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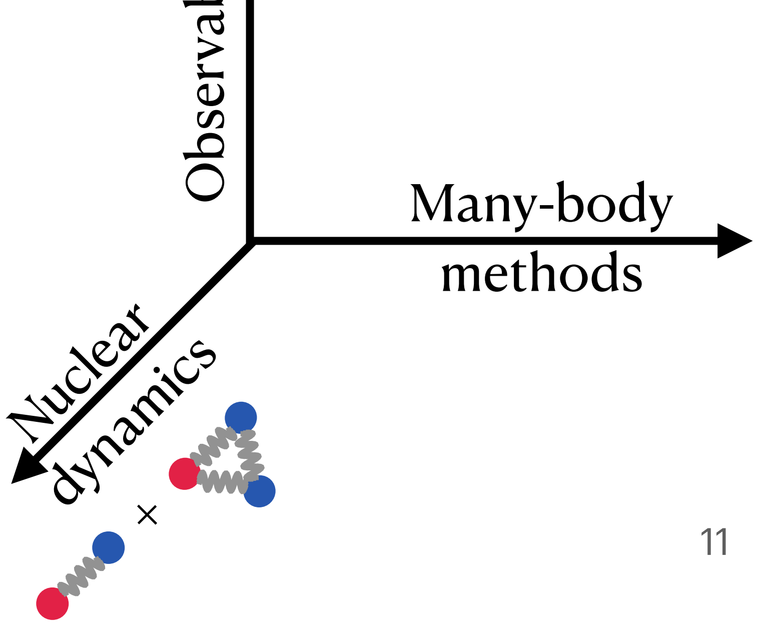


Observables

Many-body methods

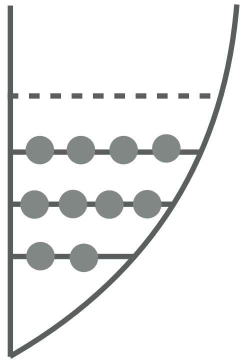
Nuclear dynamics

×

- ✓ Computational power
- ✓ Polynomial scaling with A
- ✓ “Softer” Hamiltonians (better convergence)

Coupled cluster theory

Reference state (Hartree-Fock): $|\Psi\rangle = a_i^\dagger a_j^\dagger \dots a_k^\dagger |0\rangle$



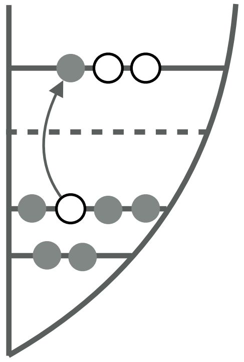
Include **correlations** through e^T operator

$$\mathscr{H}_N e^T|\Psi\rangle = E e^T|\Psi\rangle$$

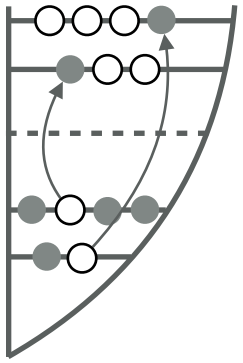
- ✓ Controlled approximation through truncation in T
- ✓ Polynomial scaling with A (predictions for ^{132}Sn and ^{208}Pb)

Expansion: $T = \sum \overset{\text{1p1h}}{t_a^i} a_a^\dagger a_i + \frac{1}{4} \sum \overset{\text{2p2h}}{t_{ab}^{ij}} a_a^\dagger a_b^\dagger a_i a_j + \dots$

singles



doubles



←coefficients obtained through coupled cluster equations

$$\langle\Psi|\overline{\mathscr{H}}|\Psi\rangle = E$$
$$\langle\Psi_i^a|\overline{\mathscr{H}}|\Psi\rangle = 0$$
$$\langle\Psi_{ij}^{ab}|\overline{\mathscr{H}}|\Psi\rangle = 0$$

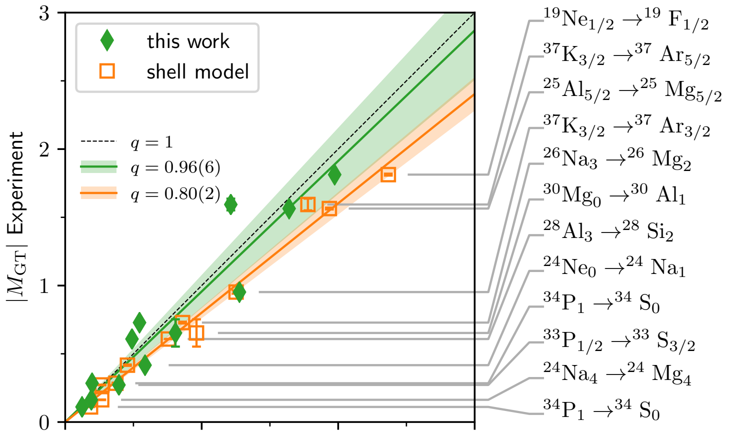
G. Hagen, T. Papenbrock, M. Hjorth-Jensen, D. J. Dean,
Rep. Prog. Phys. 77, 096302 (2014).

Some results

β decays

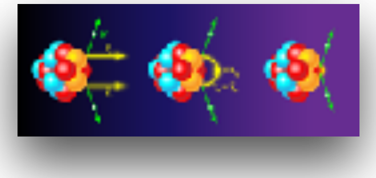
Long standing puzzle in β decays: quenching of a fundamental constant $g_A \approx 1.27$ in nuclei

Nuclear correlations + 2-body el-mag currents

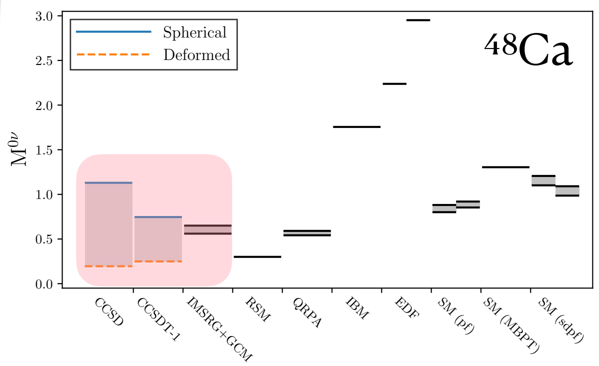


Nature Phys. 15 (2019) 5, 428-431

$0\nu\beta\beta$ decays

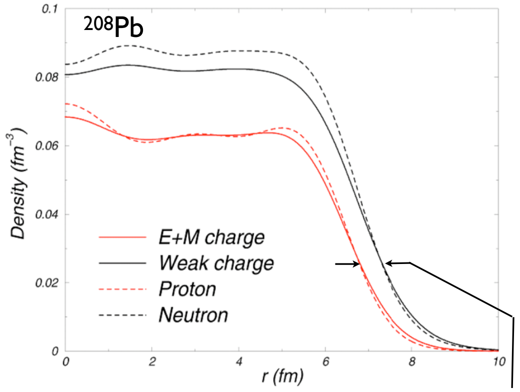


First ab initio results predict relatively low NME

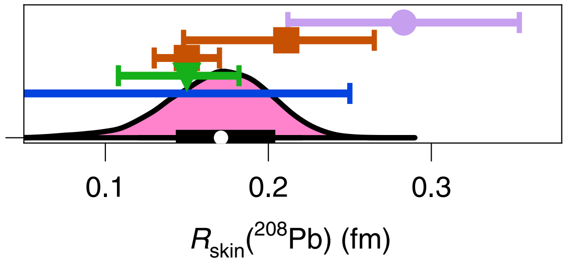


Phys.Rev.Lett. 126 (2021) 18, 182502

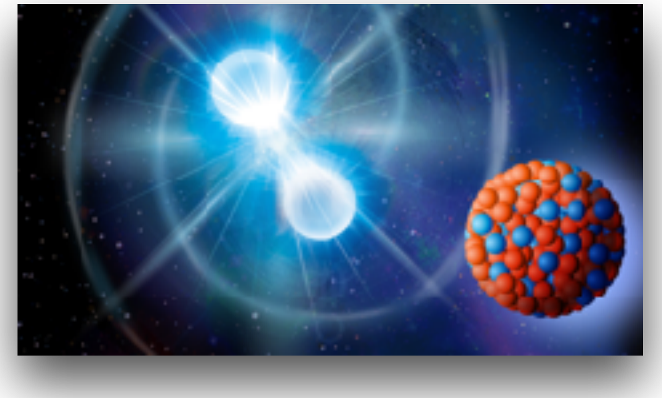
Neutron skin thickness linked to the structure and size of neutron stars



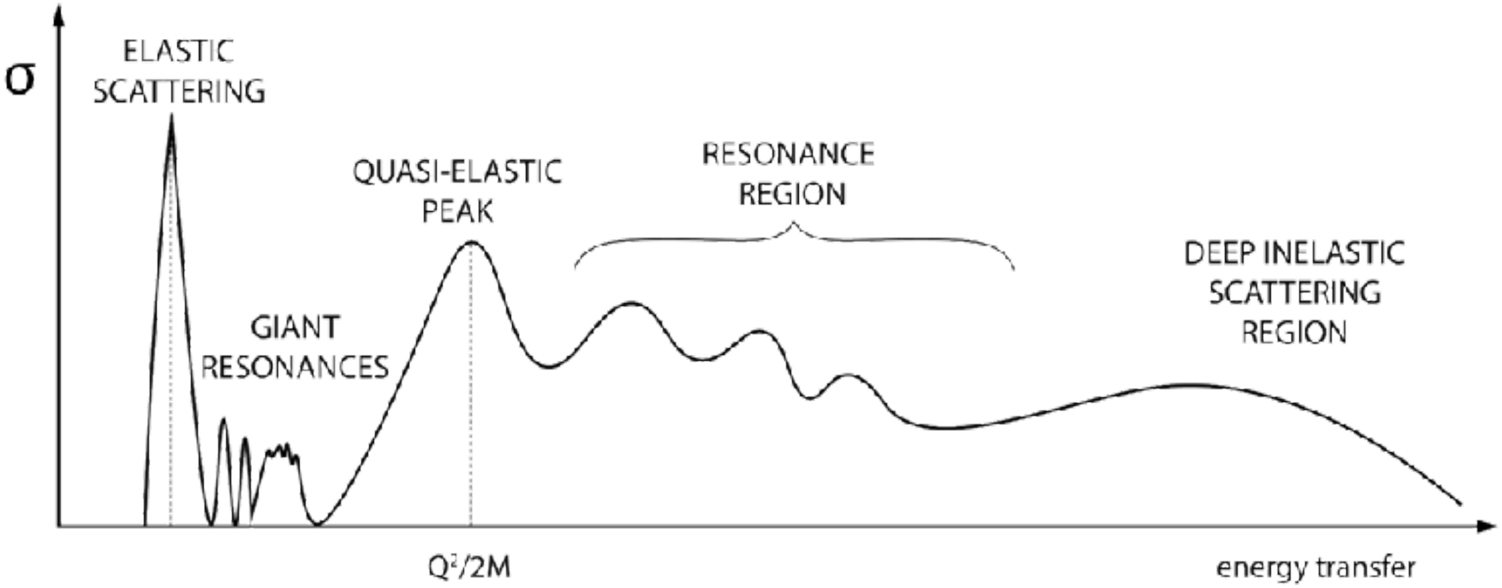
Neutron skin



Nature Phys. 18 (2022) 10, 1196-1200

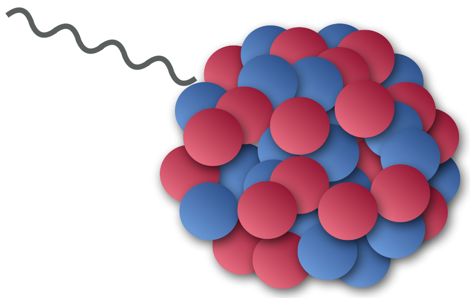


Beyond groundstate: nuclear responses



$$J_\mu = (\rho, \vec{j}) \,|\, \Psi \rangle$$

$\gamma, W^\pm, Z^0$



$$\sigma \propto L^{\mu\nu} R_{\mu\nu}$$

lepton tensor

nuclear responses

$$R_{\mu\nu}(\omega, q) = \sum_f \langle \Psi \,|\, J_\mu^\dagger(q) \,|\, \Psi_f \rangle \langle \Psi_f \,|\, J_\nu(q) \,|\, \Psi \rangle \delta(E_0 + \omega - E_f)$$

Electrons for neutrinos

$$\left.\frac{d\sigma}{dE'd\Omega}\right|_{\nu/\bar{\nu}} = \sigma_0\Big(v_{CC}R_{CC} + v_{CL}R_{CL} + v_{LL}R_{LL} + v_T R_T \pm v_{T'}R_{T'}\Big)$$

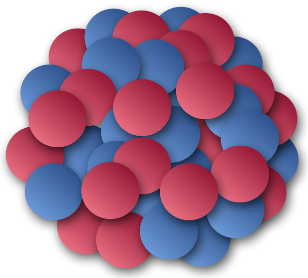
$$\left.\frac{d\sigma}{dE'd\Omega}\right|_{e} = \sigma_M\Big(v_L R_L(\omega, \bar{q}) + v_T R_T(\omega, \bar{q})\Big)$$

✓ much more precise data

✓ we can get access to R_L and R_T separately (Rosenbluth separation)

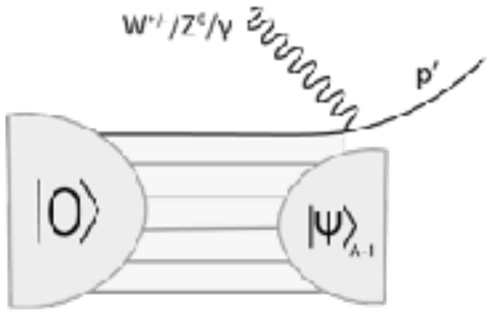
✓ experimental programs of electron scattering in JLab, MAMI, MESA

Low/high energies

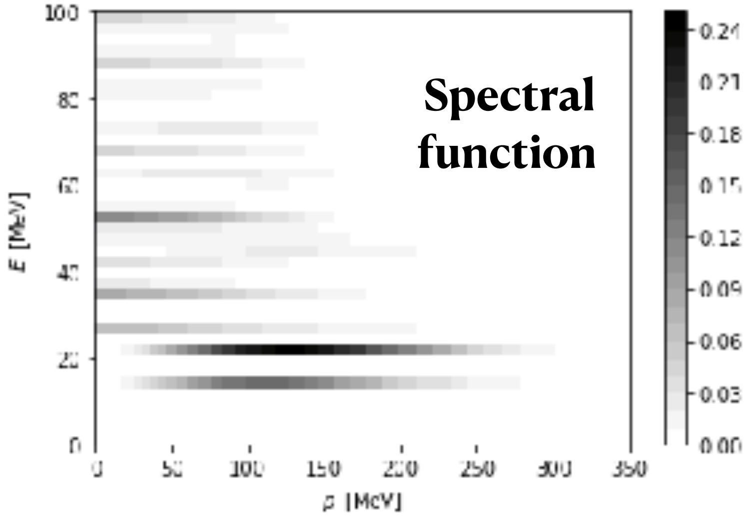


$$\hat{H}|\psi_A\rangle = E|\psi_A\rangle$$

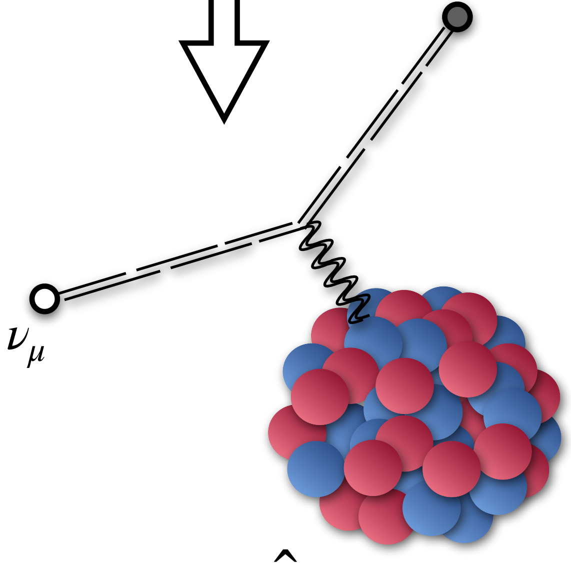
Many-body problem



Impulse Approximation

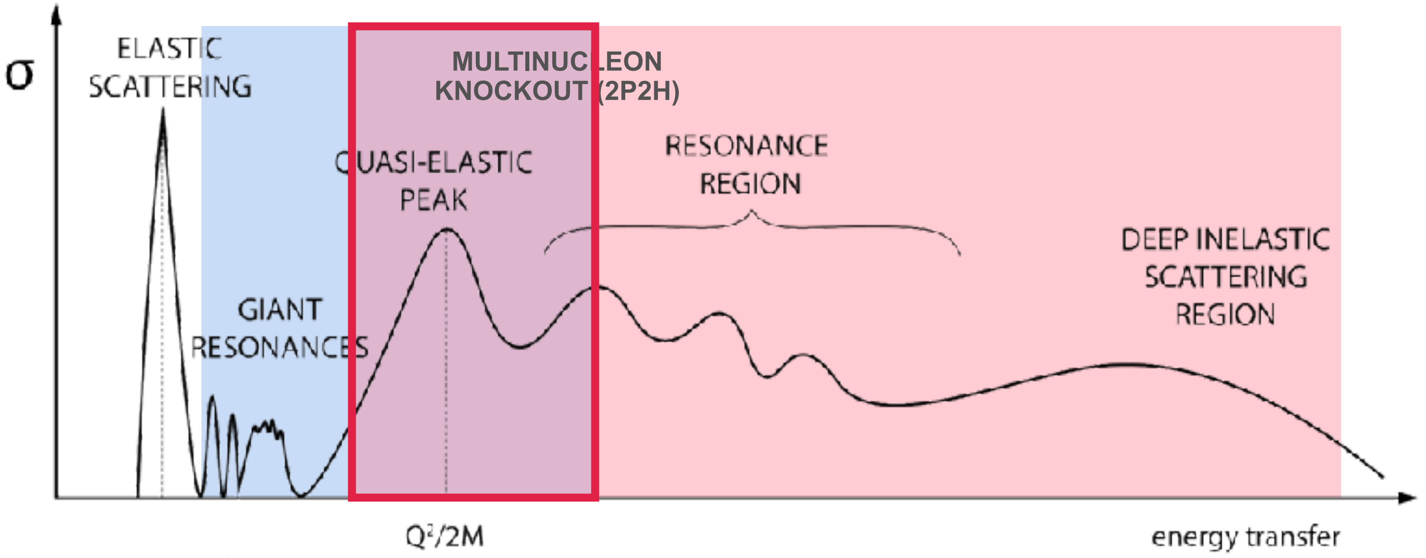


Probability density of finding nucleon $(E, \mathbf{p})$ in ground state nucleus



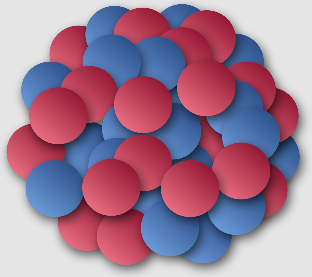
$$\langle\psi_f|\hat{j}|\psi_A\rangle$$

Electroweak responses
consistent treatment of final states



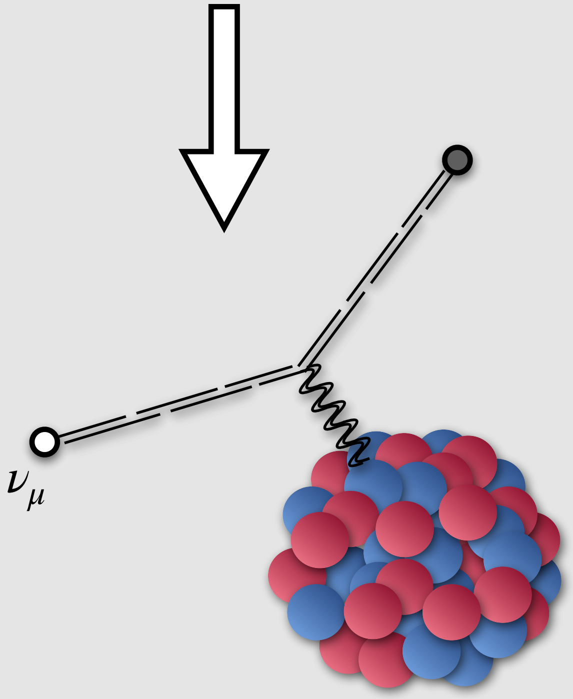
Possible
comparison
within the same
framework

Low/high energies



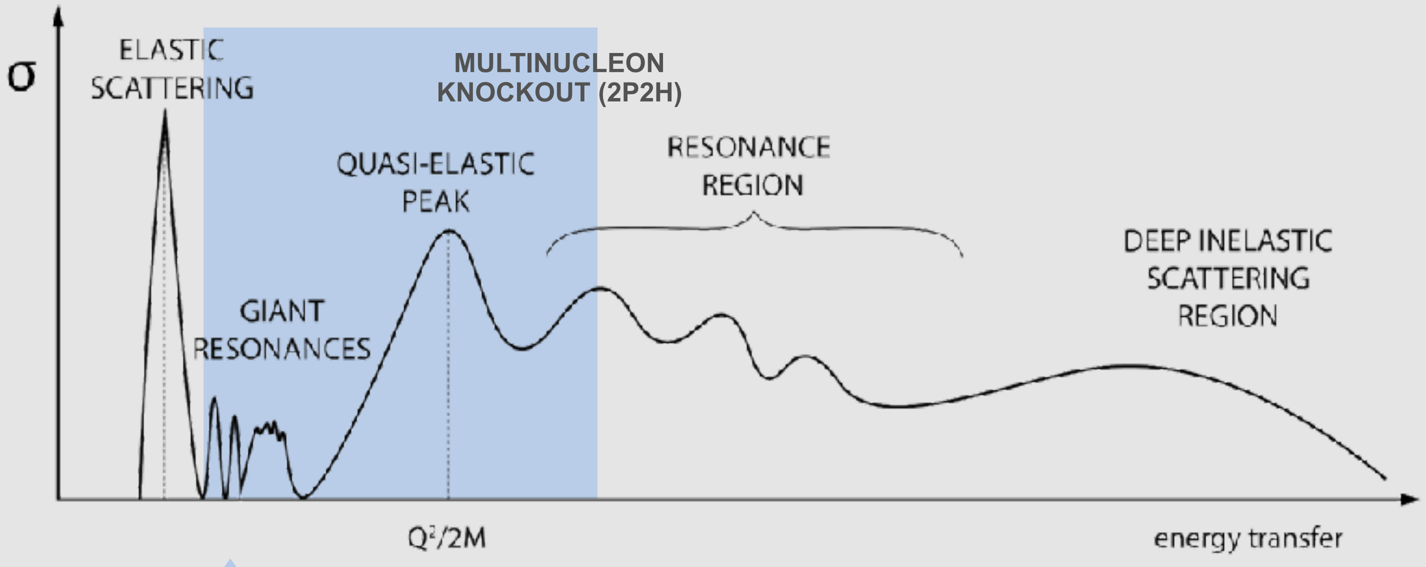
$$\hat{H}\,|\psi_A\rangle = E\,|\psi_A\rangle$$

Many-body problem



$$\langle\psi_f|\,\hat{j}\,|\psi_A\rangle$$

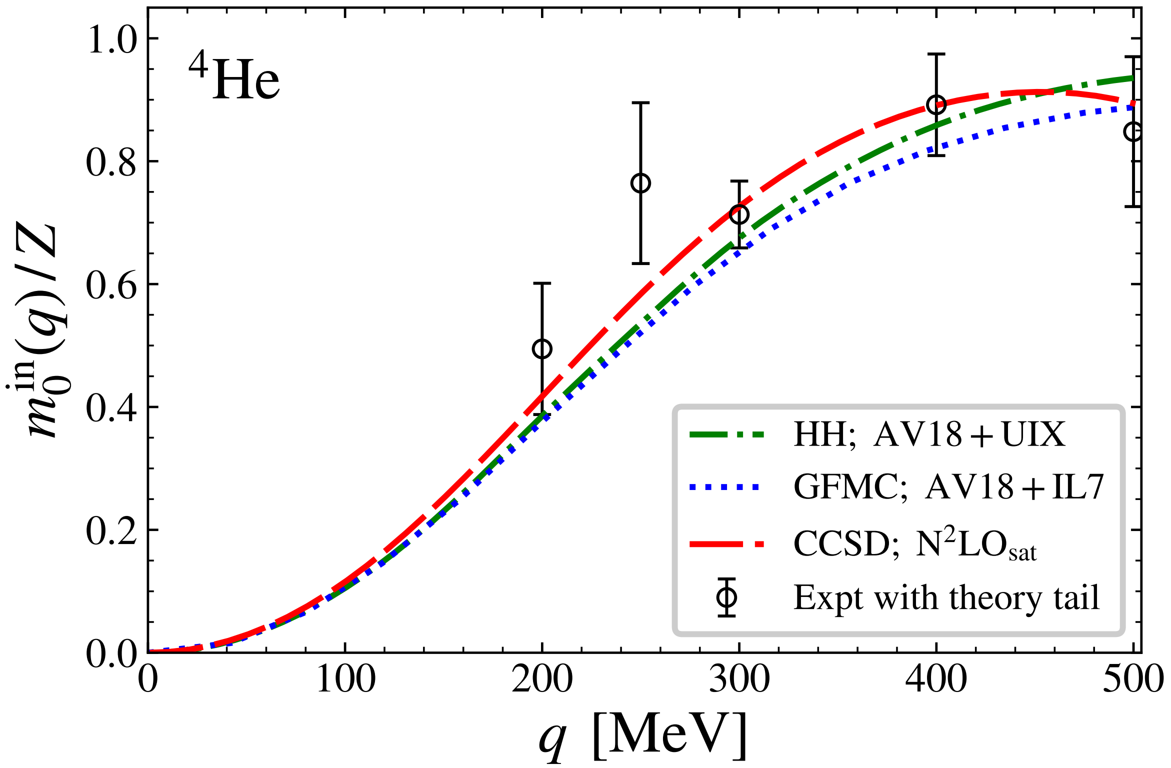
Electroweak responses



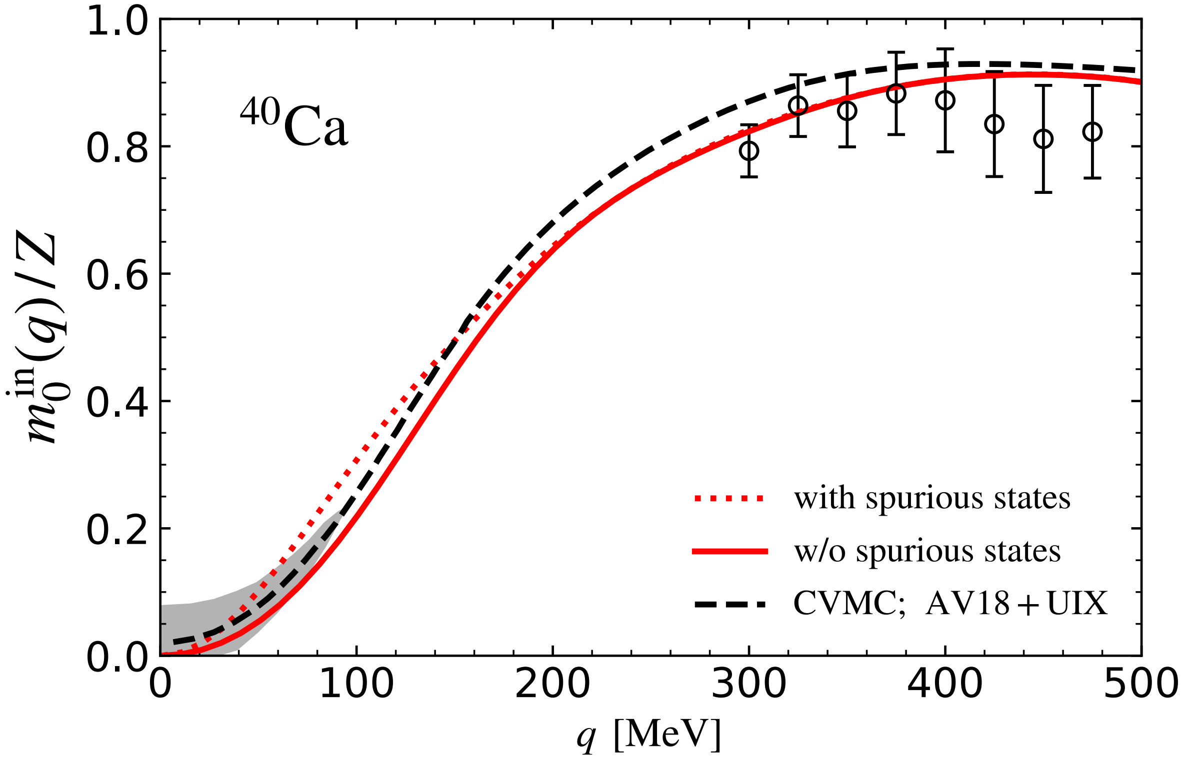
Coulomb sum rule

charge operator
$$\hat{\rho}(q) = \sum_{j=1}^{Z} e^{iqz'_j}$$

$$m_0(q) = \int d\omega R_L(\omega, q) = \sum_{f\neq 0} |\langle \Psi_f | \hat{\rho} | \Psi \rangle|^2 = \langle \Psi | \hat{\rho}^\dagger \hat{\rho} | \Psi \rangle - |F_{el}(q)|^2$$



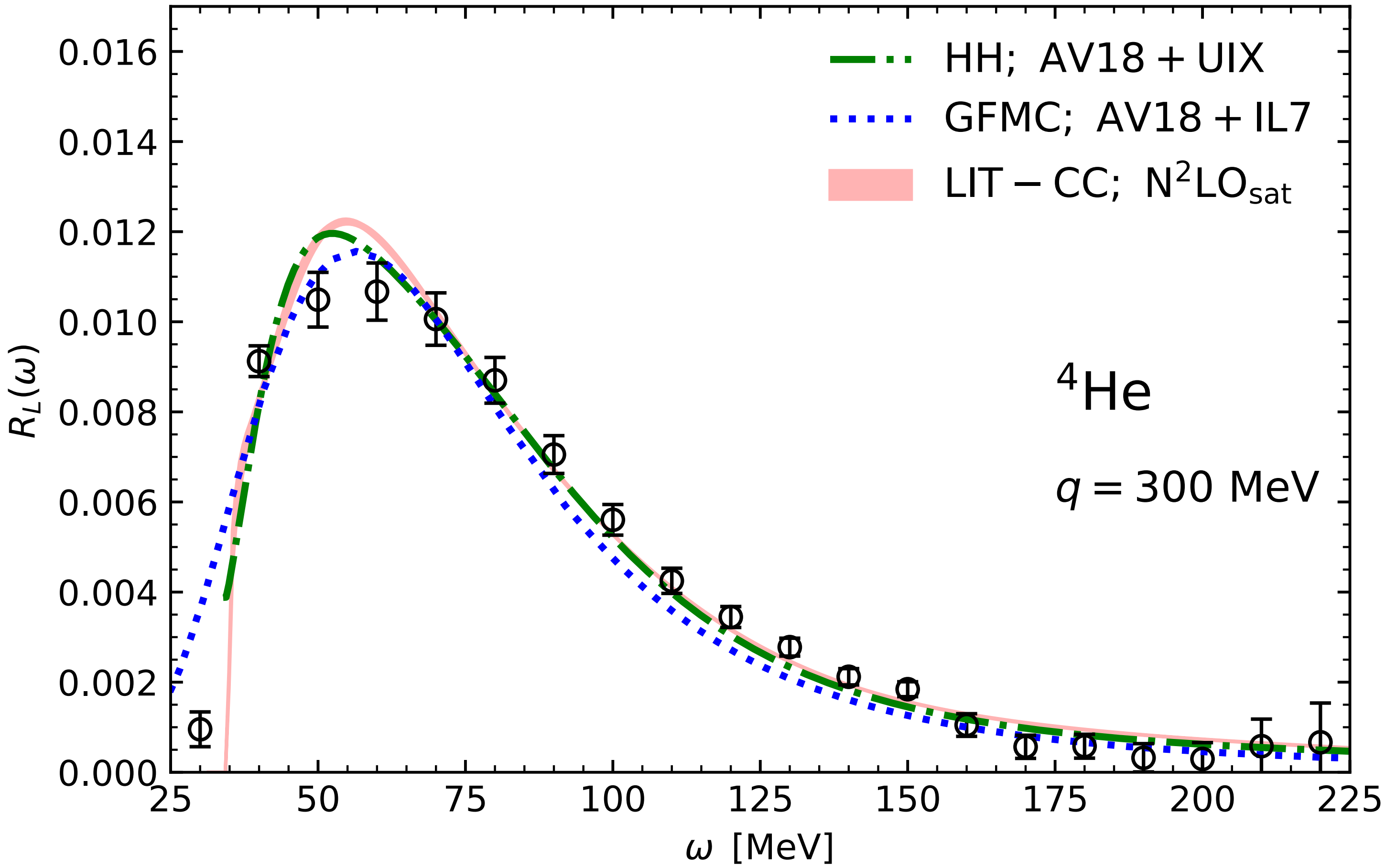
JES, B. Acharya, S.Bacca, G. Hagen Phys.Rev.C 102 (2020) 064312



JES, B. Acharya, S. Bacca, G. Hagen PRL 127 (2021) 7, 072501

Longitudinal response

Lorentz Integral Transform + Coupled Cluster **(LIT-CC)**



JES, B. Acharya, S. Bacca, G. Hagen; *PRL* 127 (2021) 7, 072501

charge operator

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$$R_{\mu\nu}(\omega, q) = \sum\!\!\!\!\!\!\int_f \langle\Psi|J_\mu^\dagger|\Psi_f\rangle\langle\Psi_f|J_\nu|\Psi\rangle\delta(E_0 + \omega - E_f)$$

Consistent treatment of final state interactions.

Lorentz Integral Transform (LIT)

$$R_{\mu\nu}(\omega, q) = \sum\!\!\!\!\!\!\!\int_f \langle \Psi | J_\mu^\dagger | \Psi_f \rangle \langle \Psi_f | J_\nu | \Psi \rangle \delta(E_0 + \omega - E_f)$$

continuum spectrum

Integral transform

$$S_{\mu\nu}(\sigma, q) = \int d\omega K(\omega, \sigma) R_{\mu\nu}(\omega, q) = \langle \Psi | J_\mu^\dagger \, K(\mathscr{H} - E_0, \sigma) \, J_\nu | \Psi \rangle$$

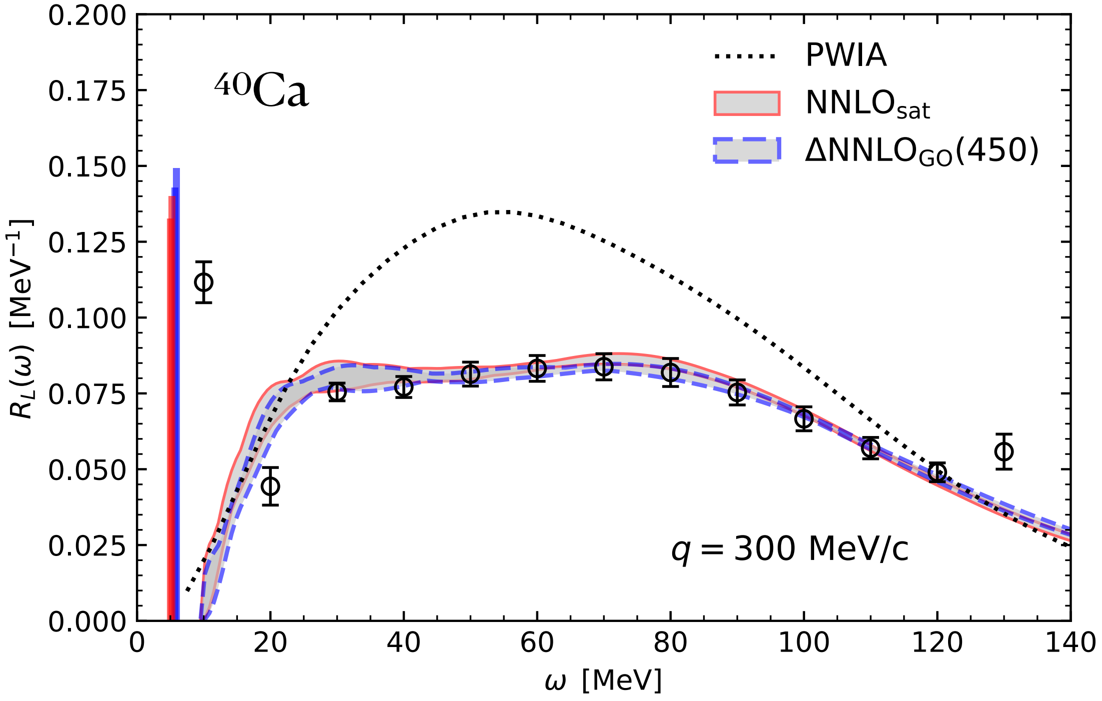
Lorentzian kernel:

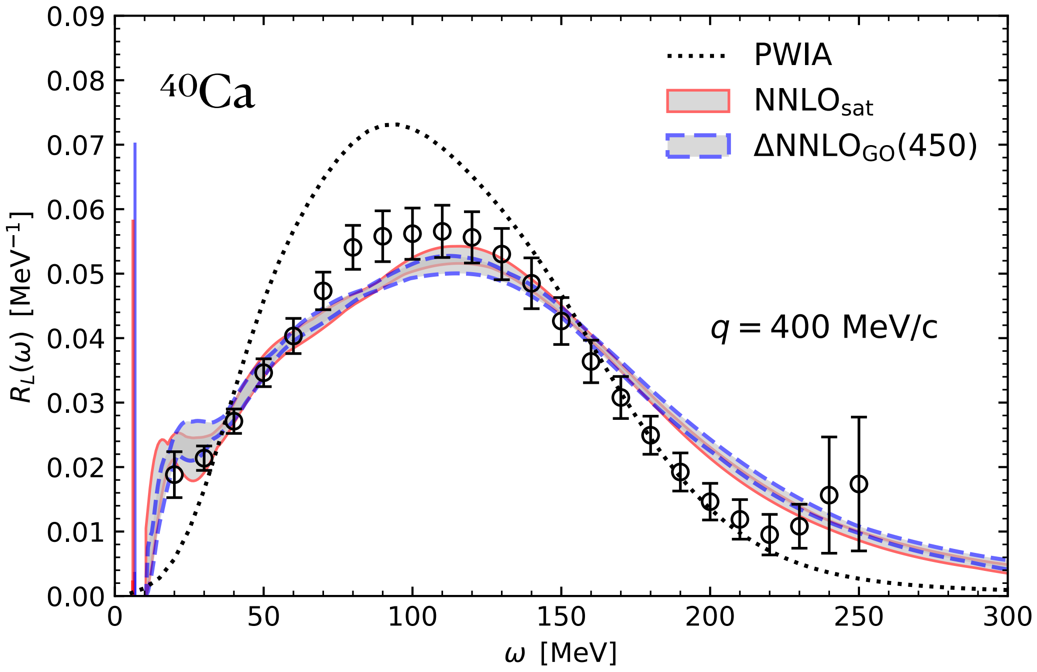
$$K_\Gamma(\omega, \sigma) = \frac{1}{\pi} \frac{\Gamma}{\Gamma^2 + (\omega - \sigma)^2}$$

$S_{\mu\nu}$ has to be inverted to get access to $R_{\mu\nu}$

Longitudinal response ^{40}Ca

Lorentz Integral Transform + Coupled Cluster **(LIT-CC)**





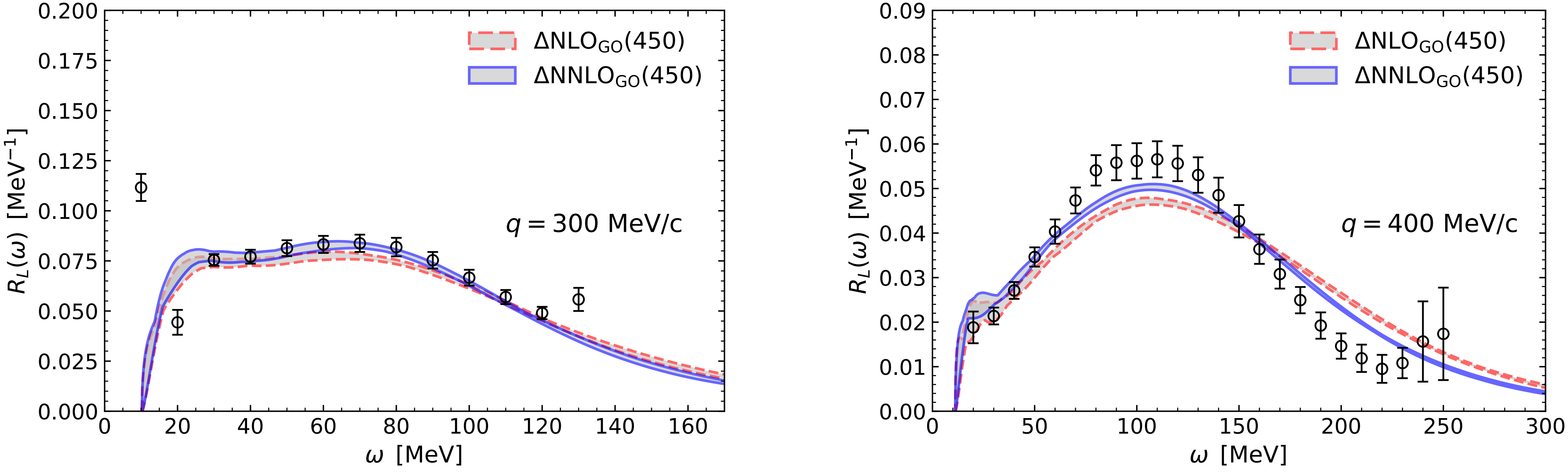
JES, B. Acharya, S. Bacca, G. Hagen; *PRL* 127 (2021) 7, 072501

- ✓ Coupled cluster singles & doubles
- ✓ Two different chiral Hamiltonians
- ✓ Uncertainty from LIT inversion

First ab-initio results for many-body system of 40 nucleons

Chiral expansion for ^{40}Ca

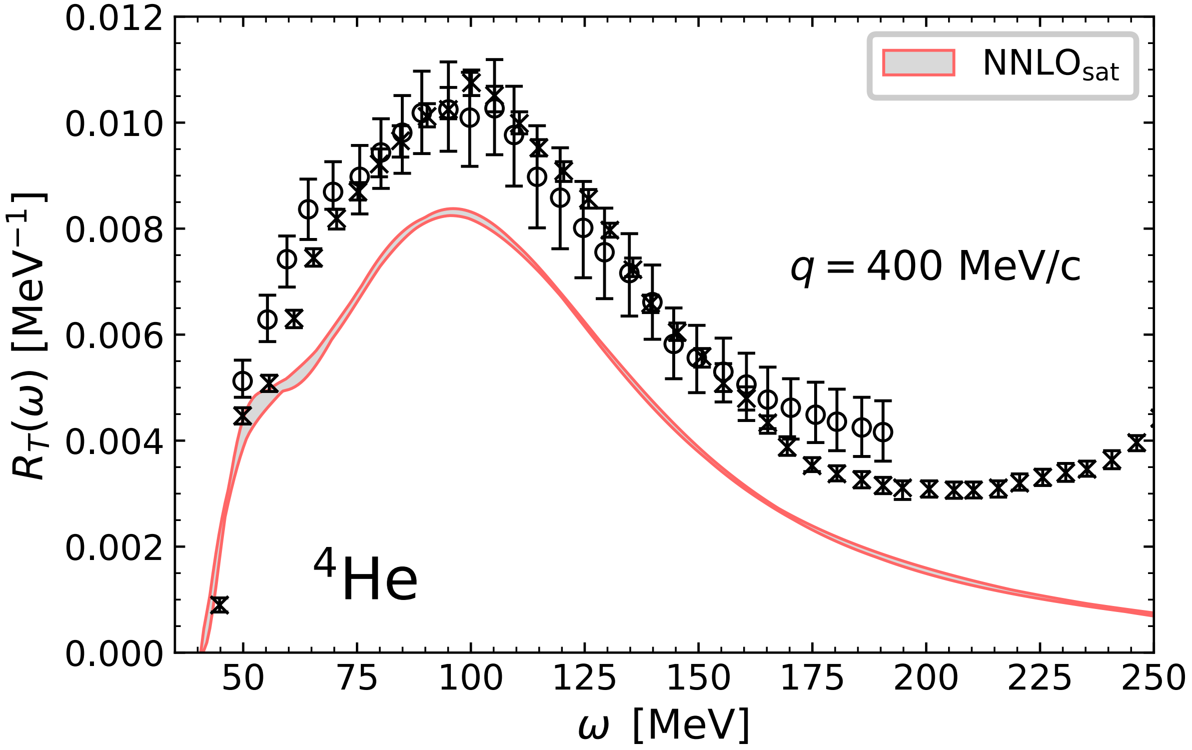
(Longitudinal response)

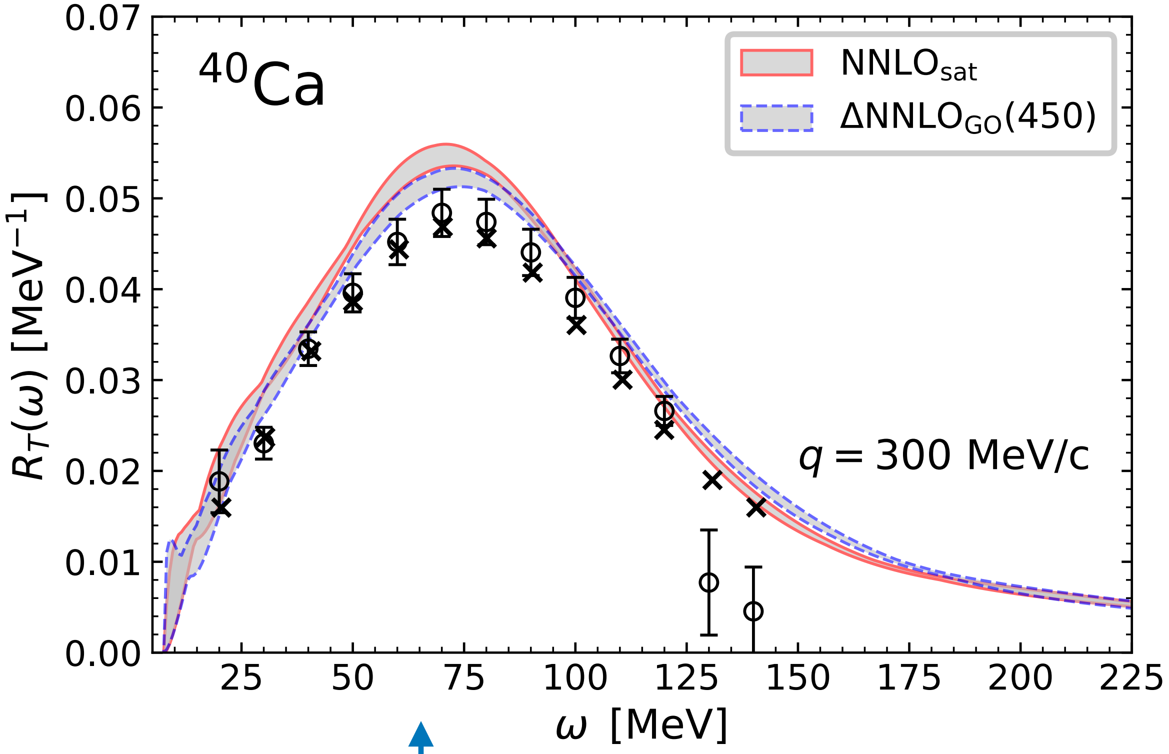


B. Acharya, S. Bacca, JES et al. Front. Phys. 1066035(2022)

- ✓ Two orders of chiral expansion
- ✓ Convergence better for lower q (as expected)
- ✓ Higher order brings results closer to the data

Transverse response





JES, B. Acharya, S. Bacca, G. Hagen;
PRC 109 (2024) 2, 025502

$$\left.\frac{d\sigma}{d\omega dq}\right|_e = \sigma_M\left(v_L R_L + v_T R_T\right)$$

➡ This allows to predict electron-nucleus cross-section

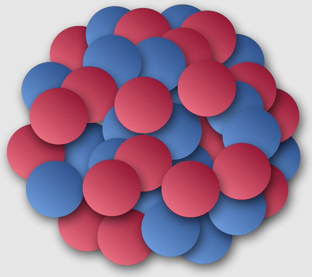
➡ Currently only 1-body current

2-body currents important for ^{4}He

→ more correlations needed?

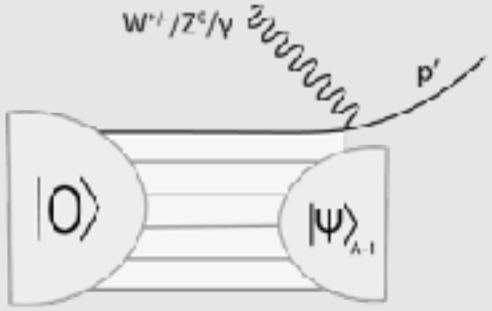
→ 2-body currents strength depends on nucleus?

Low/high energies

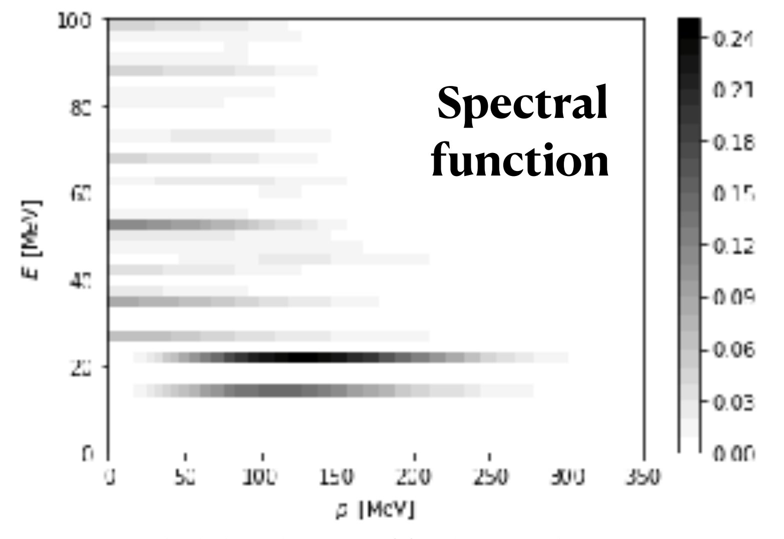


$$\hat{H}|\psi_A\rangle = E|\psi_A\rangle$$

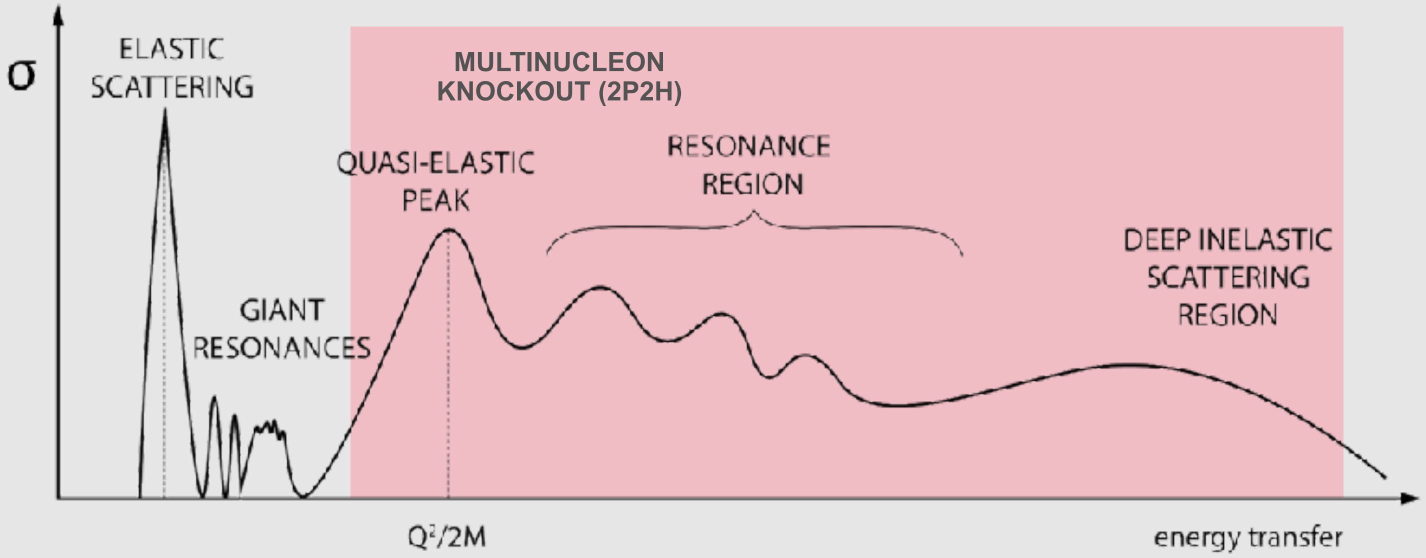
Many-body problem



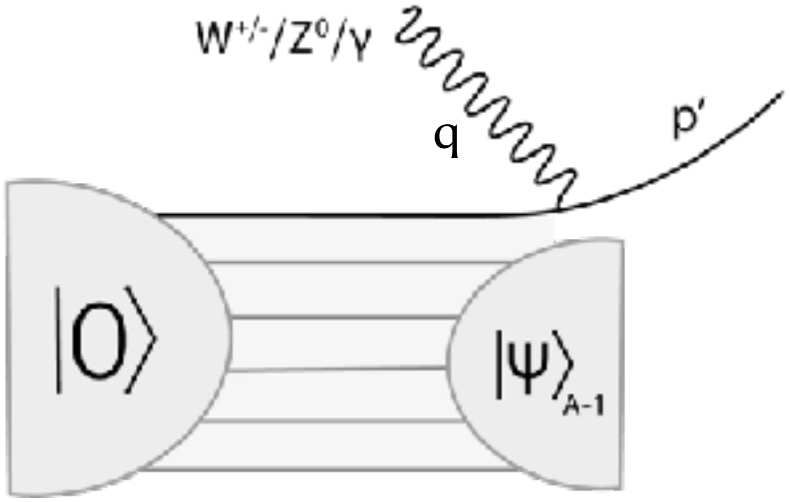
Impulse Approximation



Probability density of finding nucleon $(E, \mathbf{p})$ *in ground state nucleus*



^{4}He spectral function



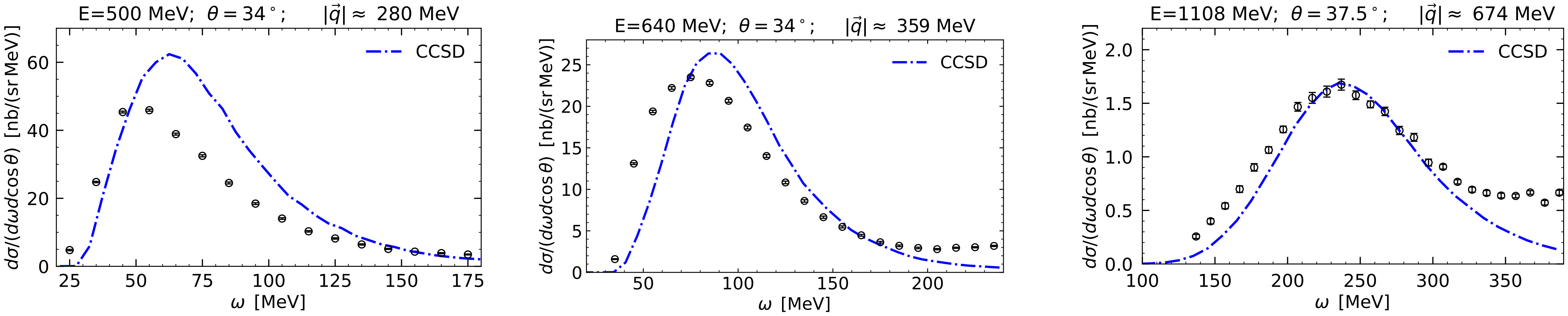
$$\sigma \propto |\mathscr{M}|^2 S(E,p)$$

Factorized interaction vertex (relativistic, pion production...)

Spectral function - nuclear information

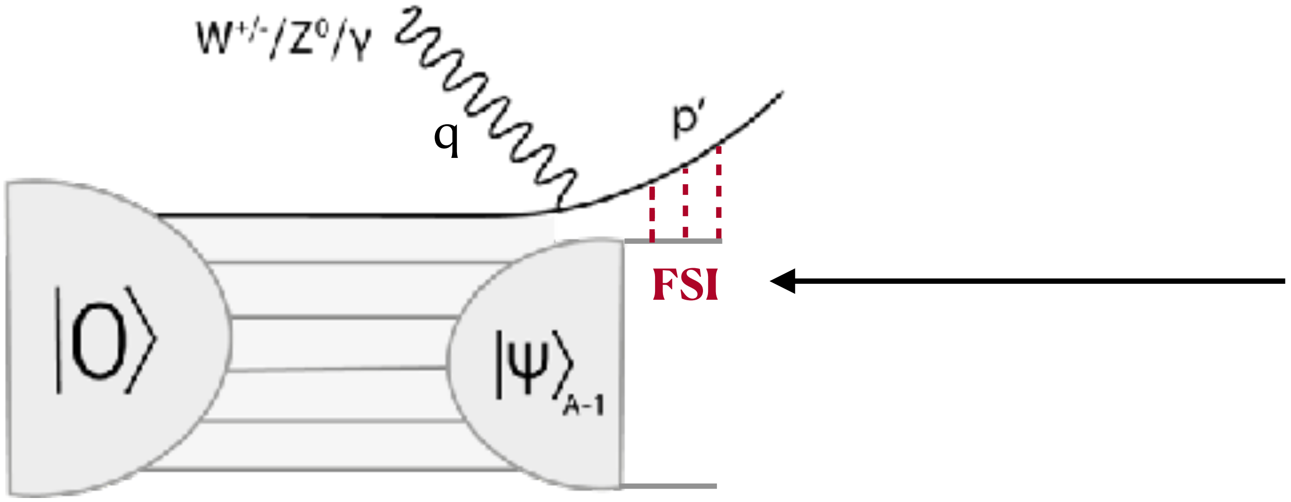
growing **q** momentum transfer → final state interactions play minor role

Scattering off **^{4}He**



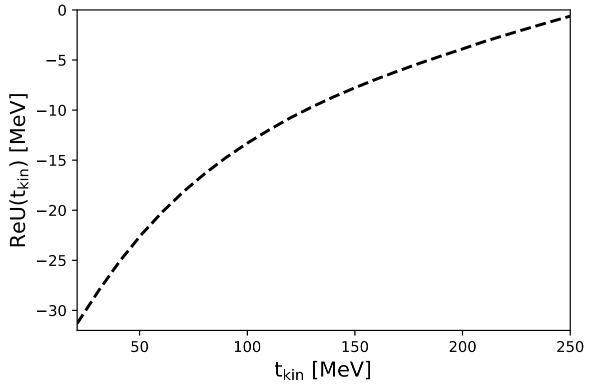
^{16}O spectral function

Error propagation to cross sections



Phenomenological optical potential

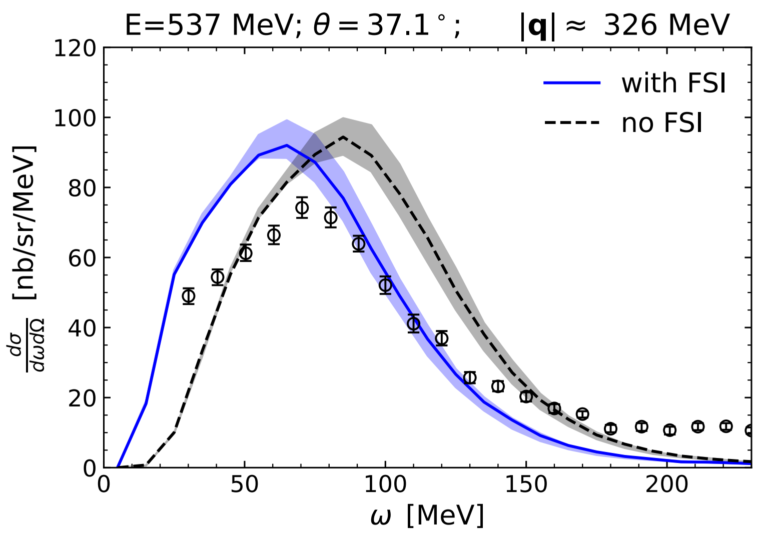
$$E_{p+q} \rightarrow E_{p+q} + \mathrm{Re}U(t_{kin})$$

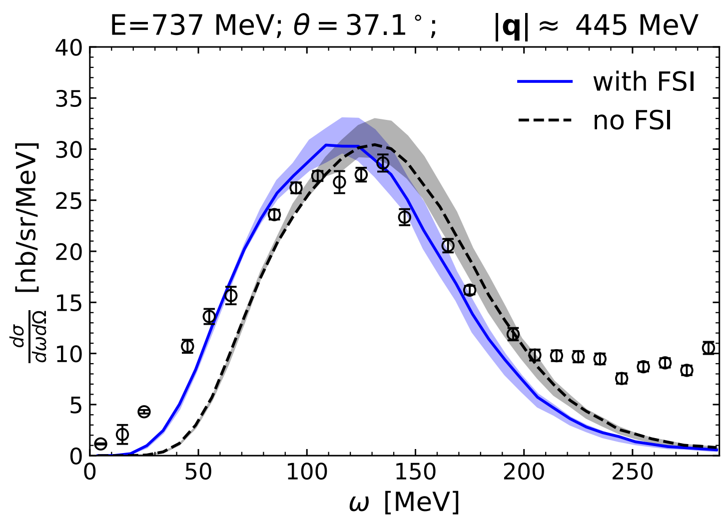


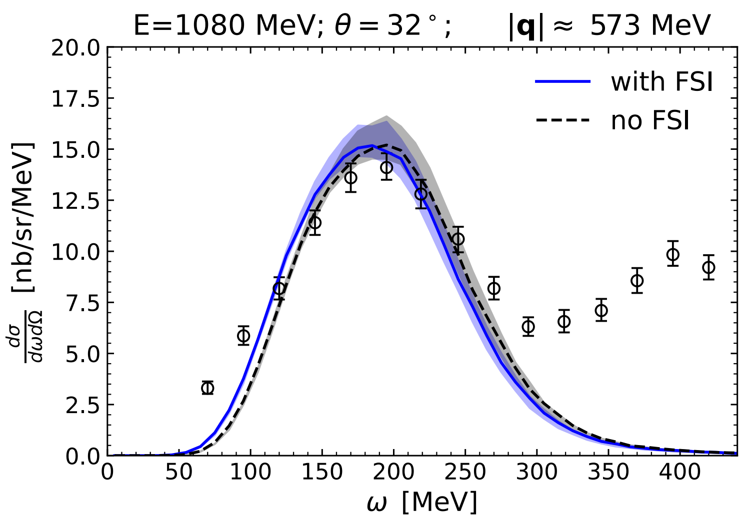
E. D. Cooper et al. *Phys.Rev.C 47, 297–311*

growing **q** momentum transfer → final state interactions play minor role

Scattering off ^{16}O







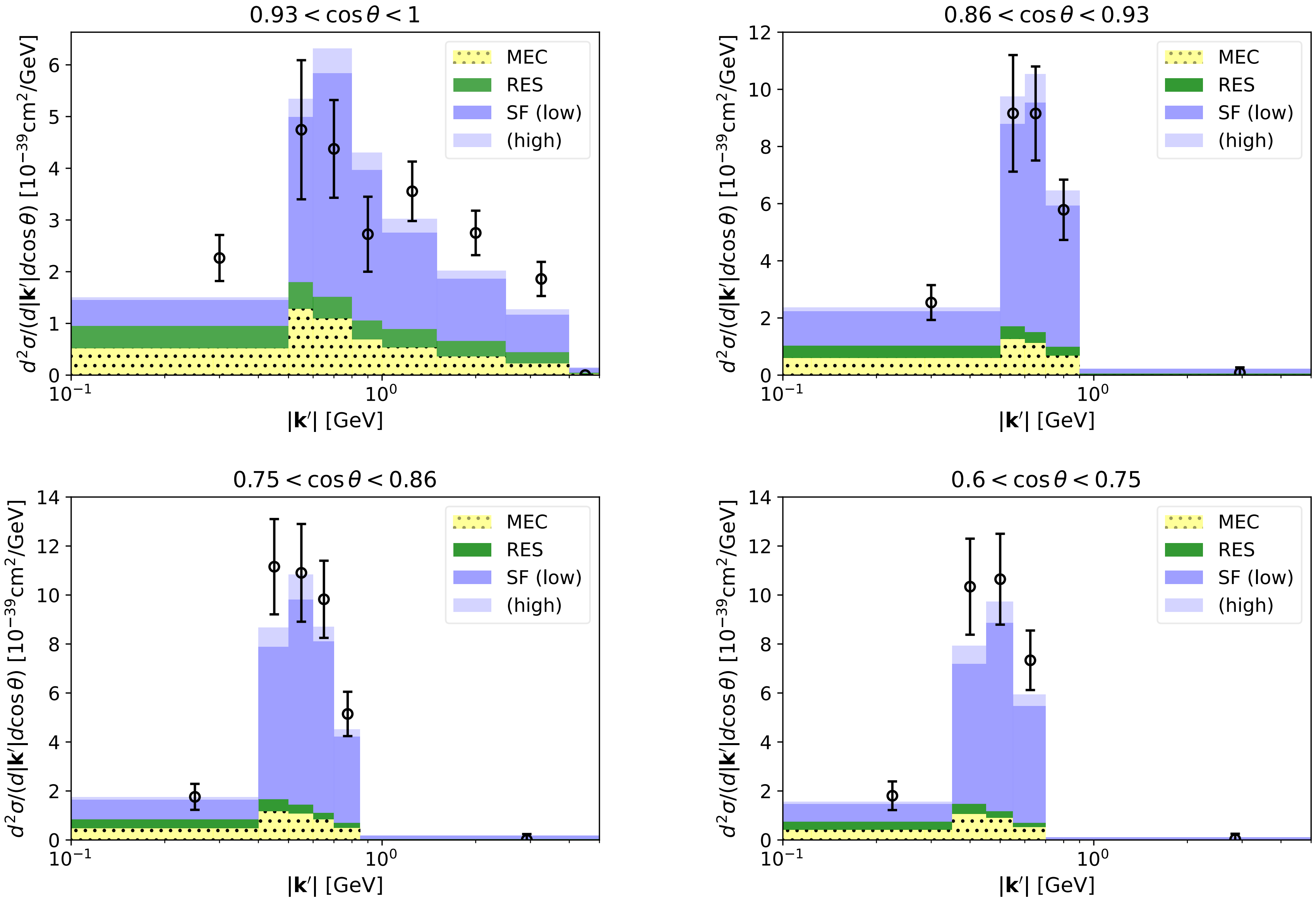
JES , S. Bacca arXiv:2309.00355 *(accepted in PRC)*

^{16}O spectral function

Error propagation to cross sections

- Comparison with T2K long baseline ν oscillation experiment
- CC0π events
- Spectral function implemented into NuWro Monte Carlo generator

$$\nu_\mu + ^{16}\mathrm{O} \rightarrow \mu^- + X$$



Data: Phys. Rev. D 101, 112004 (2020)

JES , S. Bacca arXiv:2309.00355 *(accepted in PRC)*

Spectral function calculation

$$S(E,\mathbf{p}) = \sum_{\alpha,\alpha'} \int_{\Psi_{A-1}} |\langle\Psi|a^\dagger_\alpha|\Psi_{A-1}\rangle\langle\Psi_{A-1}|a_{\alpha'}|\Psi\rangle\langle\mathbf{p}|\alpha\rangle^\dagger\langle\mathbf{p}|\alpha'\rangle\delta(E+E_f^{A-1}-E_0)$$

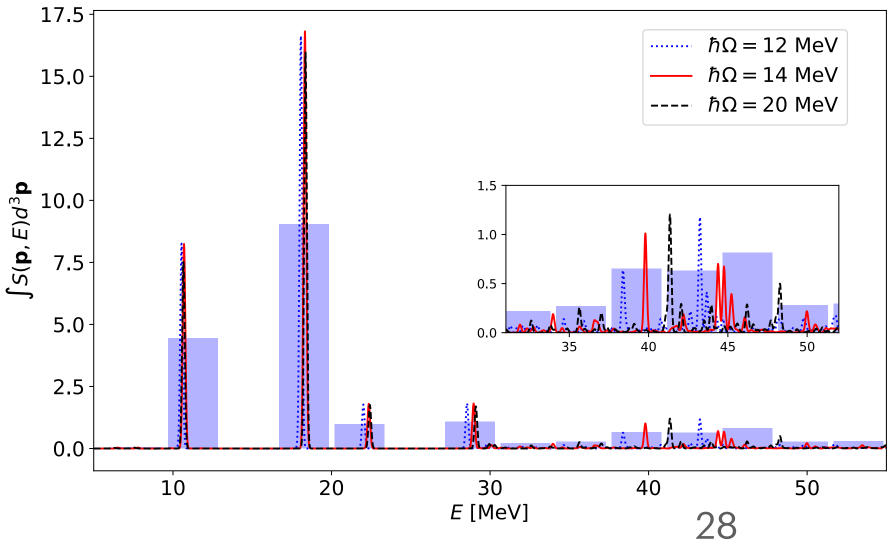
Spectral reconstruction using expansion in Chebyshev polynomials + building histograms

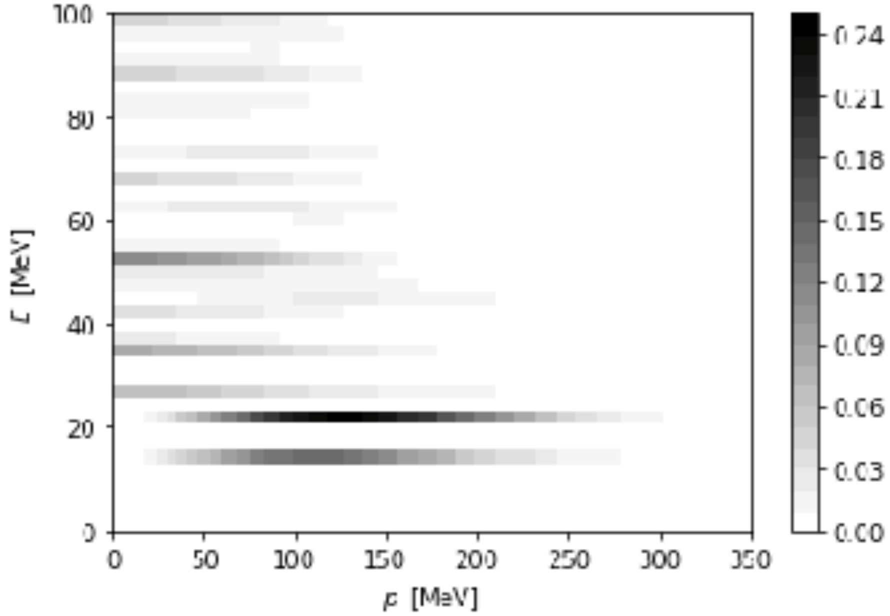
Integral transform

$$S_\Lambda(E,\mathbf{p}) = \int K_\Lambda(E,E')S(E',\mathbf{p})dE'$$

expansion in Chebyshev polynomials

$$K_\Lambda(\omega,\sigma) = \sum_k c_k(\sigma)T_k(\omega)$$





A. Roggero Phys.Rev.A 102 (2020) 2, 022409
JES, A. Roggero Phys.Rev.E 105 (2022) 055310

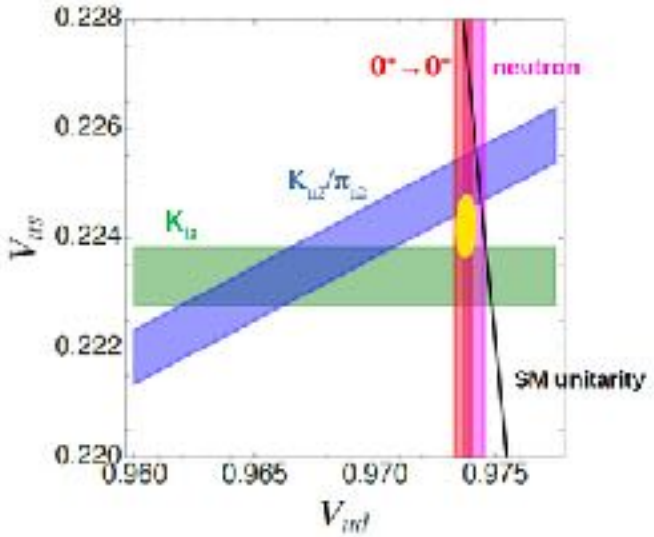
Nuclear ab initio studies for neutrino oscillations (and beyond)

Tests of CKM matrix

The “Cabbibo angle anomaly”

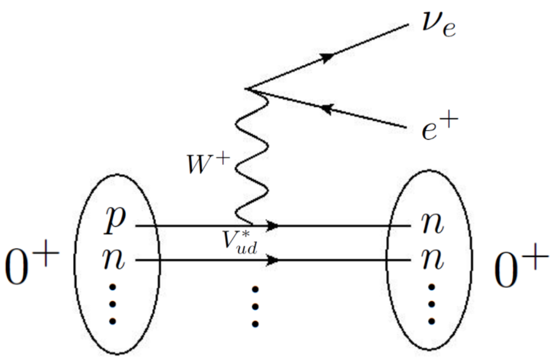
In the SM $\Delta_{\text{CKM}} \equiv |V_{ud}|^2 + |V_{us}|^2 + |\cancel{V_{ub}}|^2 - 1 = 0$

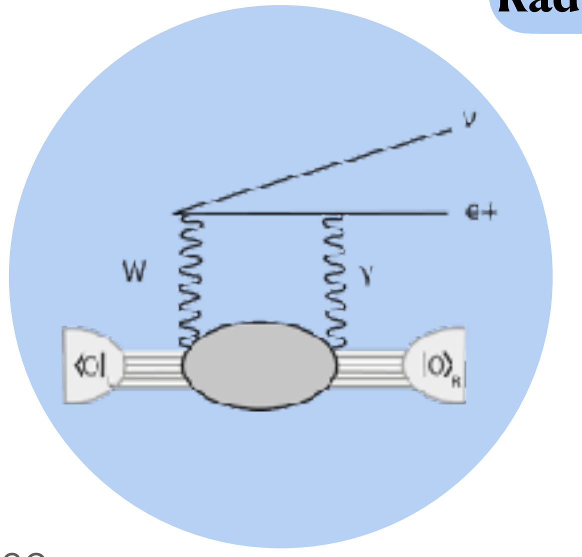
V_{ud}	$0^+ \to 0^+$ $(\pi^\pm \to \pi^0 e\nu)$	$n \to pe\bar{\nu}$ (Mirror transitions)	$\pi \to \mu\nu$
V_{us}	$K \to \pi l \nu$	$(\Lambda \to pe\bar{\nu}, \ldots)$	$K \to \mu\nu$



Superallowed beta decays provide the **best measurement of** V_{ud}

$$|V_{ud}|^2 = \frac{2984.432(3)s}{ft(1 + \underbrace{\Delta_R^V + \delta_R' + \delta_{NS} - \delta_C}_{\textbf{Radiative corrections}})}$$



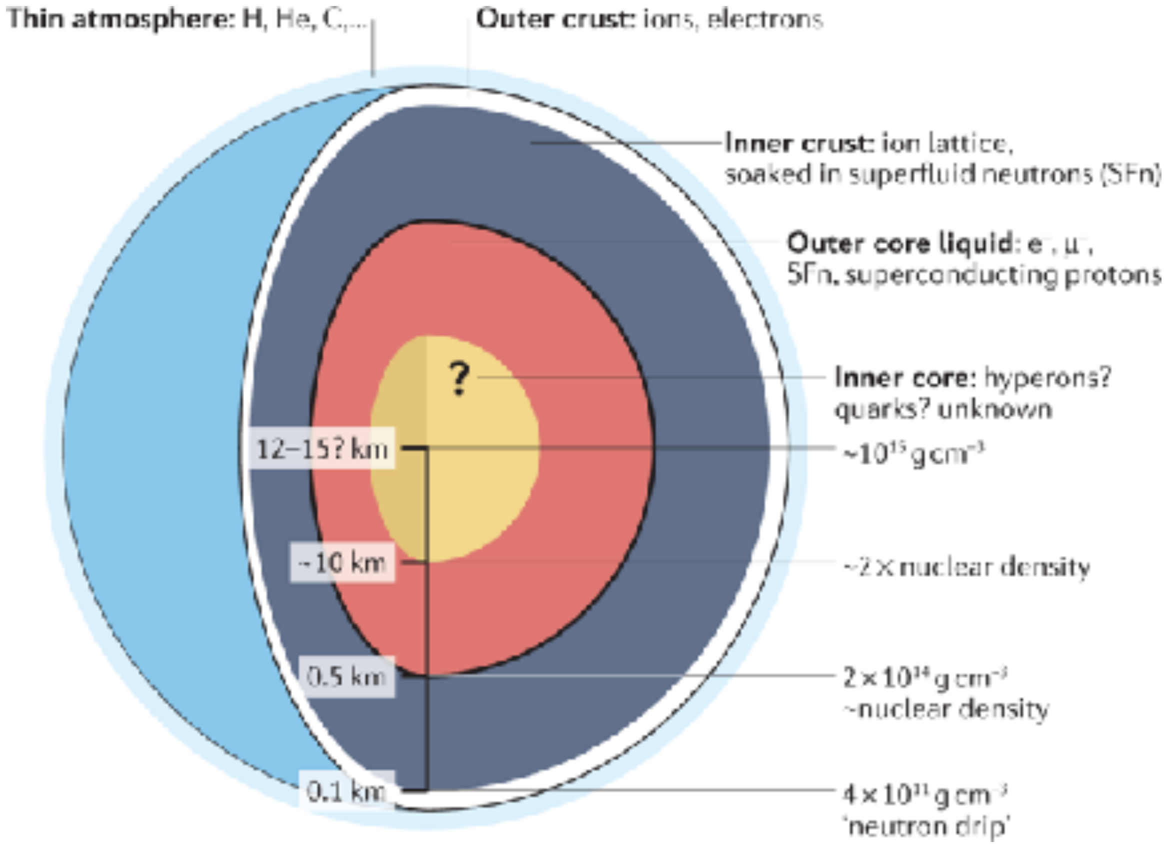


One-loop radiative corrections probe QCD at all scales. **Nuclear scale $\mathcal{O}(100)$ MeV is most uncertain!**

$$V_{ud}^{0^+\to 0^+} = 0.97367(11)_{\text{exp}}(13)_{\Delta_R^V}(27)_{\text{NS}}[32]_{\text{total}}$$

C. Y. Seng, et. al., PRD 100(2019), 013001
M. Gorchtein, PRL 123(2019), 042503

Neutrino propagation in neutron stars



Source: https://www.nature.com/articles/s42254-022-00420-y

- Neutrino emission — mechanism of cooling in neutron stars
- Neutrino energies are low ($\omega \approx 30$ MeV) → the long-wavelength limit is a good approximation. Then: **spin response** becomes important.
- Spin fluctuations strongly depend on many-body effects + the coupling of spin and space in the nuclear force

➡ Coupled-cluster theory for nuclear matter (possible UQ)

Outlook

- Next step: from electromagnetic to electroweak processes
- Extension of the formalism to ^{40}Ar
- Role played by 2-body currents in LIT-CC predictions
- Development in spectral functions (accounting for final state interactions, adding 2-body currents)
- Bayesian analysis of uncertainties in nuclear responses
- …

Thank you!