CP violation in Higgs sector

(A phenomenological & first principle overview)

Norway grants

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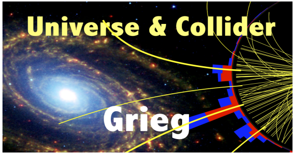

Understanding the Early Universe:
interplay of theory and collider experiments

Joint research project between the
University of Warsaw & University of Bergen

Dibyakrupa Sahoo
University of Warsaw, Warsaw, Poland

Based on **arXiv:2311.16211 [hep-ph]**
in collaboration with

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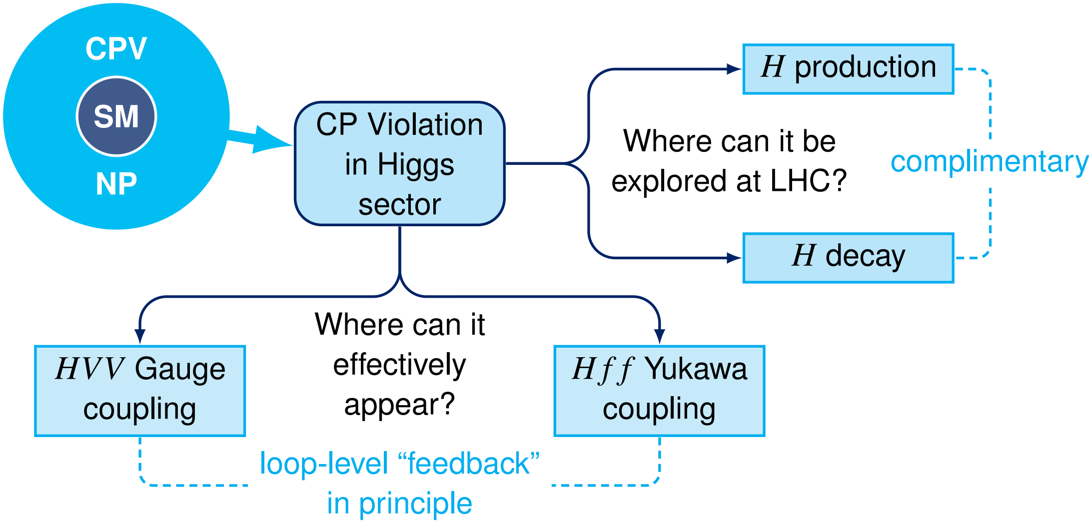
(University of Warsaw, Poland)
E. Aakvaag, N. Fomin & Anna Lipniacka
(University of Bergen, Norway)

Particle Physics and Cosmology Seminar (Faculty of Physics, University of Warsaw)
29 February 2024

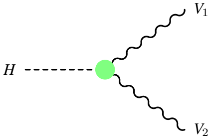
Outline of CP violation studies in Higgs sector

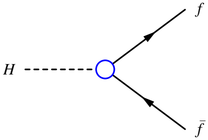


Part 1: Introduction

Sources of CP violation in Higgs sector

- 125 GeV **Higgs boson** (H) is a **scalar** (CP-even) particle
  $\Longrightarrow$ H has *dominant* CP-even couplings
- Any discovery of ***non-zero*** **CP-odd couplings** of Higgs
  $\Longrightarrow$ Clear sign of **New Physics** (NP)
- **Effective field theory formalism:**
  - ✪ **CP-odd Gauge couplings through higher dimension operators** (Λ = scale of NP)





$$\mathscr{L}_{\text{eff}} = \mathscr{L}_{\text{SM}} + \frac{1}{\Lambda^2}\sum_i c_i^{(6)} O_i^{(6)} + \frac{1}{\Lambda^4}\sum_i c_i^{(8)} O_i^{(8)} + \cdots$$

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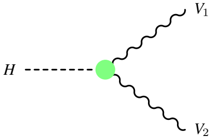

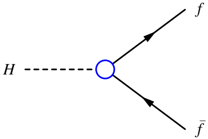

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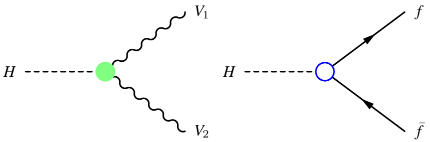
Some gauge invariant dimension-6 operators (CP-even as well as CP-odd) are:

$$\begin{aligned}
O_{HB} &\propto H^\dagger\, H\, B_{\mu\nu}\, B^{\mu\nu}, & O_{H\widetilde{B}} &\propto H^\dagger\, H\, B_{\mu\nu}\, \widetilde{B}^{\mu\nu}, \\
O_{HW} &\propto H^\dagger\, H\, W_{k\mu\nu}\, W_k^{\mu\nu}, & O_{H\widetilde{W}} &\propto H^\dagger\, H\, W_{k\mu\nu}\, \widetilde{W}_k^{\mu\nu}, \\
O_{HWB} &\propto H^\dagger\, \tau_k\, H\, W_{k\mu\nu}\, B^{\mu\nu}, & O_{HW\widetilde{B}} &\propto H^\dagger\, \tau_k\, H\, W_{k\mu\nu}\, \widetilde{B}^{\mu\nu}, \quad (k = 1, 2, 3)
\end{aligned}$$

where $\widetilde{V}^{\mu\nu} = \frac{1}{2}\epsilon^{\mu\nu\rho\sigma}\, V_{\rho\sigma}$ is the dual field strength tensor.

Sources of CP violation in Higgs sector



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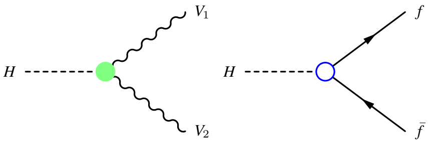
The effective NP Lagrangian can also be recast in the following form for doing phenomenological studies,

$$\mathcal{L}_{\text{NP}} \supset \frac{H}{4v}\left(2\,A_2^{Z\gamma}\,F^{\mu\nu}\,Z_{\mu\nu} + 2\,A_3^{Z\gamma}\,F^{\mu\nu}\,\widetilde{Z}_{\mu\nu} + A_2^{\gamma\gamma}\,F^{\mu\nu}\,F_{\mu\nu} + A_3^{\gamma\gamma}\,F^{\mu\nu}\,\widetilde{F}_{\mu\nu}\right)$$

Sources of CP violation in Higgs sector



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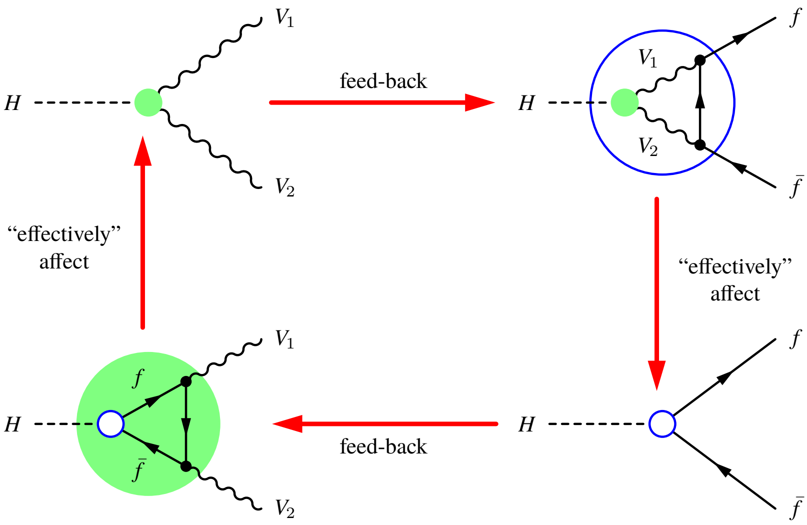
  - ✪ **CP-odd pseudoscalar Hff couplings at tree level** (α = CP mixing angle, κ_f = Yukawa coupling)

$$\mathscr{L}_{\text{Yuk}} = -\sum_f \frac{m_f}{v} H\left[\bar{f}\,\kappa_f\left(\cos\alpha + i\,\sin\alpha\,\gamma^5\right) f\right],$$

CP-even: $\alpha = 0$, CP-odd: $\alpha = \frac{\pi}{2}$, maximal CP violation: $\alpha = \frac{\pi}{4}$.
SM: $\kappa_f = 1, \alpha = 0$. NP: $\kappa_f \neq 1$, $\alpha \neq 0$.
Analogous parametrization: $a_f = \kappa_f \cos\alpha$, $b_f = \kappa_f \sin\alpha$.

In principle "feedback" of CP violation at loop level



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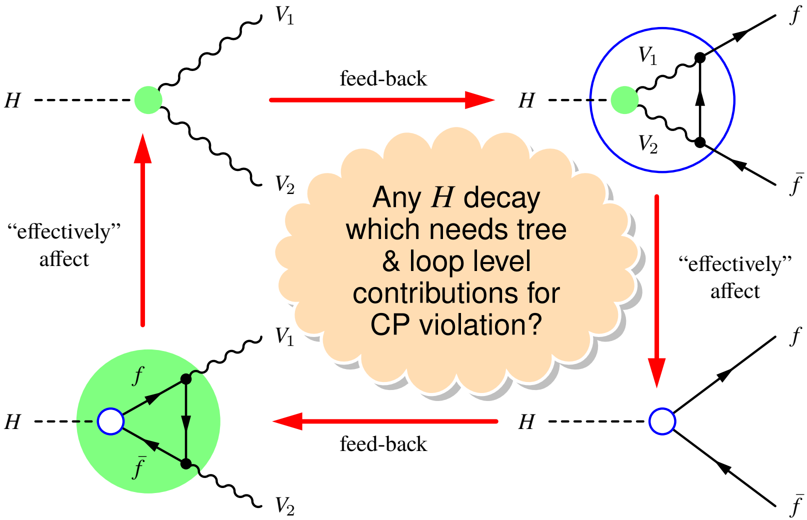


Signature of CP violation in Higgs sector

Various process independent perspectives from first principle analysis

- **Amplitude decomposition in terms of contributions of operators of 4, 6 and higher dimensions:**

$$\mathscr{M} = \mathscr{M}^{(4)} + \mathscr{M}^{(6)} + \mathscr{M}^{(8)} + \cdots$$
$$\Longrightarrow |\mathscr{M}|^2 = \left|\mathscr{M}^{(4)}\right|^2 + \underbrace{2\,\mathrm{Re}\left(\mathscr{M}^{(4)}\,\mathscr{M}^{(6)*}\right)}_{\text{suppressed by } \Lambda^{-2}} + \cdots$$

Optimal Observable: Search for how large this 4- and 6-dimension interference can be.

- **Amplitude decomposition in terms of CP property:**

$$\mathscr{M} = \underbrace{\mathscr{M}_e}_{\text{CP-even}} + \underbrace{\mathscr{M}_o}_{\text{CP-odd}} \qquad \left(\text{where, for example, } \mathscr{M}_{e/o} = \mathscr{M}_{e/o}^{\text{Tree}} + \mathscr{M}_{e/o}^{\text{Loop}}\right)$$
$$\Longrightarrow |\mathscr{M}|^2 = \underbrace{|\mathscr{M}_e|^2 + |\mathscr{M}_o|^2}_{\text{CP-even}} + \underbrace{2\,\mathrm{Re}\left(\mathscr{M}_e\,\mathscr{M}_o^*\right)}_{\text{CP-odd}}$$

Optimal Observable: Search for non-zero CP-odd interference term effects.

Signature of CP violation in Higgs sector

Various process independent perspectives from first principle analysis

- **Formalism analogous to CP violation in meson decays:**
  Amplitude in terms of SM and NP contributions with different CP-even and CP-odd phases (CP-even phase ≡ 'strong' phase = δ, CP-odd phase ≡ 'weak' phase = ϕ):

$$\mathcal{M} = \mathcal{M}_{\text{SM}} + \mathcal{M}_{\text{NP}} = |\mathcal{M}_{\text{SM}}|\, e^{i\,(\delta_{\text{SM}}+\phi_{\text{SM}})} + |\mathcal{M}_{\text{NP}}|\, e^{i\,(\delta_{\text{NP}}+\phi_{\text{NP}})},$$

$$\overline{\mathcal{M}} = \overline{\mathcal{M}_{\text{SM}}} + \overline{\mathcal{M}_{\text{NP}}} = |\mathcal{M}_{\text{SM}}|\, e^{i\,(\delta_{\text{SM}}-\phi_{\text{SM}})} + |\mathcal{M}_{\text{NP}}|\, e^{i\,(\delta_{\text{NP}}-\phi_{\text{NP}})},$$

  such that

$$\mathcal{A} = \frac{\left|\mathcal{M}\right|^2 - \left|\overline{\mathcal{M}}\right|^2}{\left|\mathcal{M}\right|^2 + \left|\overline{\mathcal{M}}\right|^2} \propto |\mathcal{M}_{\text{SM}}|\,|\mathcal{M}_{\text{NP}}|\,\sin\left(\delta_{\text{SM}} - \delta_{\text{NP}}\right)\,\sin\left(\phi_{\text{SM}} - \phi_{\text{NP}}\right),$$

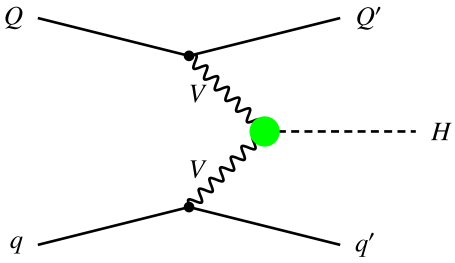
  which is non-zero only when $|\mathcal{M}_{\text{NP}}| \neq 0$, $\delta_{\text{SM}} \neq \delta_{\text{NP}}$ and $\phi_{\text{SM}} \neq \phi_{\text{NP}}$.

  Optimal Observable: Search for asymmetry $\mathcal{A}$.

Two complimentary ways to probe CP violation

1. Exploring correlations in Higgs production process

Probe does not depend on details of the subsequent H decay

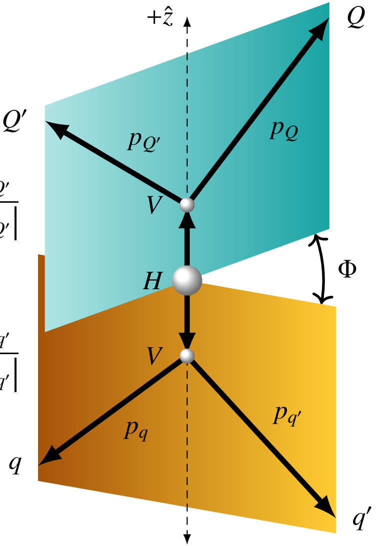


Study CP-odd triple product asymmetry with respect to angle Φ between the two planes:

$$\cos\Phi = \hat{n}_{QQ'} \cdot \hat{n}_{qq'}, \quad \sin\Phi = \left(\hat{n}_{QQ'} \times \hat{n}_{qq'}\right) \cdot \hat{z}$$

$$\hat{n}_{QQ'} = \frac{\vec{p}_Q \times \vec{p}_{Q'}}{\left|\vec{p}_Q \times \vec{p}_{Q'}\right|}$$

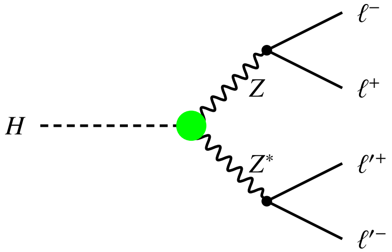
$$\hat{n}_{qq'} = \frac{\vec{p}_q \times \vec{p}_{q'}}{\left|\vec{p}_q \times \vec{p}_{q'}\right|}$$



Two complimentary ways to probe CP violation

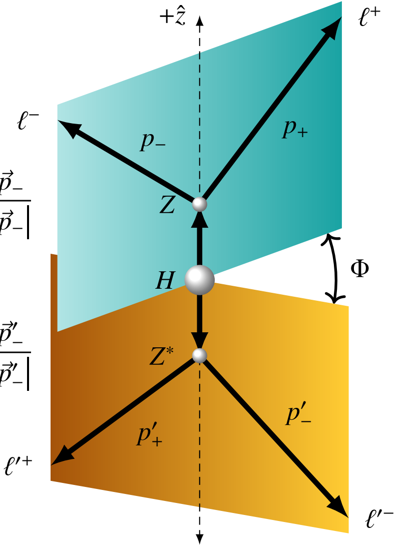
2. Exploring correlations in Higgs decay

Probe does not depend on details of how the H boson was produced



$$\hat{n}_{+-} = \frac{\vec{p}_+ \times \vec{p}_-}{|\vec{p}_+ \times \vec{p}_-|}$$

$$\hat{n}'_{+-} = \frac{\vec{p}'_+ \times \vec{p}'_-}{|\vec{p}'_+ \times \vec{p}'_-|}$$



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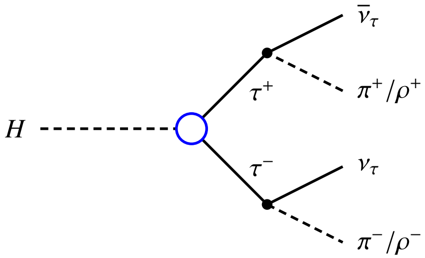
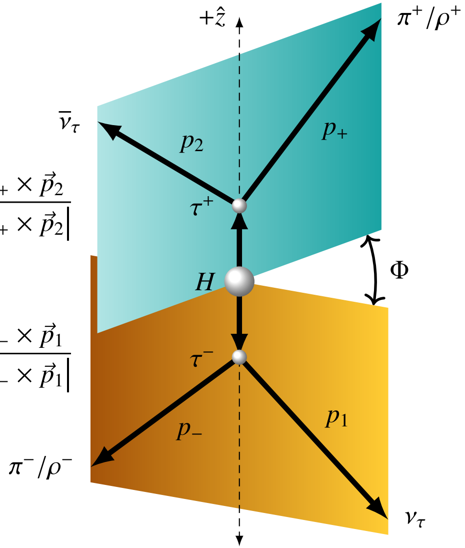


Study CP-odd triple product asymmetry with respect to angle Φ between the two planes:

$$\cos\Phi = \hat{n}_{+2}\cdot\hat{n}_{-1}, \qquad \sin\Phi = (\hat{n}_{+2}\times\hat{n}_{-1})\cdot\hat{z}$$

$$\hat{n}_{+2} = \frac{\vec{p}_+\times\vec{p}_2}{|\vec{p}_+\times\vec{p}_2|}$$

$$\hat{n}_{-1} = \frac{\vec{p}_-\times\vec{p}_1}{|\vec{p}_-\times\vec{p}_1|}$$



Part 2: $H\tau\tau$ Yukawa interaction

Goal: Probe the CP violating $H\tau\tau$ Lagrangian

CP-even CP-odd

$$\mathscr{L}_{H\tau\tau} = -\frac{m_\tau}{v}\,\bar{\tau}\left(a_\tau + i\gamma^5\, b_\tau\right)\tau\, H$$

Standard Model $\Longrightarrow$ $a_\tau = 1$ $b_\tau = 0$

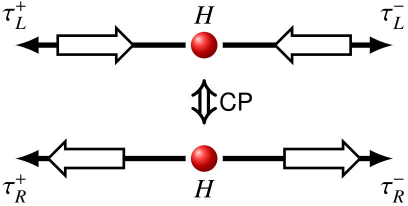
New Physics $\Longrightarrow$ $a_\tau \neq 1$ $b_\tau \neq 0$

where $v = \left(\sqrt{2}\,G_F\right)^{-1/2} \simeq 246$ GeV. Constraint from e^- EDM measurement: $|b_\tau| \lesssim 0.29$ at 90% C.L.

[J. Alonso-Gonzalez, A. de Giorgi, L. Merlo and S. Pokorski, JHEP **05**, 041 (2022).]

The 2-body decay $H \to \tau^+ \tau^-$ is *not* suitable to probe $b_\tau \neq 0$.

- $\mathrm{Br}(H \to \tau^+ \tau^-) = \left(6.0^{+0.8}_{-0.7}\right)\,\%$
  [PDG 2023]



- Only 2 allowed helicity configurations

- Partial decay rate

$$\Gamma_{\tau\tau} = \frac{m_H}{8\,\pi} \left(\frac{m_\tau}{v}\right)^2 \left(a_\tau^2 \left(1 - \frac{4\,m_\tau^2}{m_H^2}\right) + b_\tau^2\right) \times \sqrt{1 - \frac{4\,m_\tau^2}{m_H^2}}\,.$$

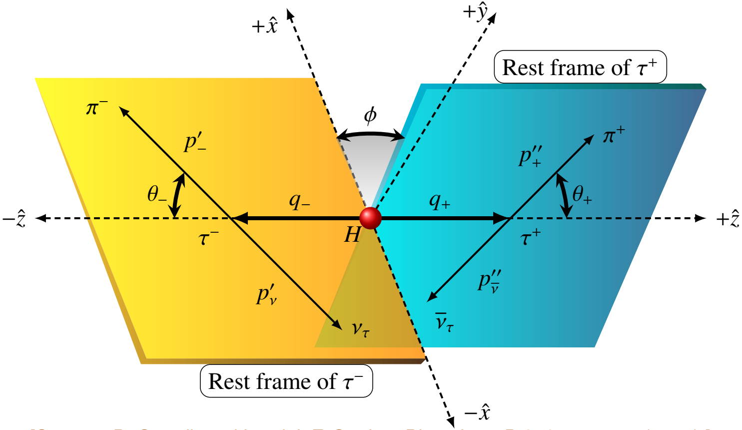
- Experimental constraint:

$$a_\tau^2 + b_\tau^2 \approx 0.93^{+0.14}_{-0.12}$$

[inferred from G. Aad *et al.* [ATLAS], JHEP **08**, 175 (2022), neglecting m_τ]

The 4-body decay $H \to \tau^+ \tau^- \to \pi^+ \pi^- \nu_\tau \bar{\nu}_\tau$ can probe $b_\tau \neq 0$.

- Much richer kinematics: 3 uni-angular distributions possible.



[See e.g. B. Grzadkowski and J. F. Gunion, Phys. Lett. B **350**, 218-224 (1995).]

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- Much richer kinematics: 3 uni-angular distributions possible.

- Full angular distribution:

$$\frac{d^3\Gamma_{\pi\pi\nu\bar{\nu}}}{d\cos\theta_+ \, d\cos\theta_- \, d\varphi} = \frac{\left\langle \left| \mathcal{M}_{\pi\pi\nu\bar{\nu}} \right|^2 \right\rangle}{2^{15}\, \pi^6\, m_H} \left(1 - \frac{4\, m_\tau^2}{m_H^2}\right)^{\frac{1}{2}} \left(1 - \frac{m_\pi^2}{m_\tau^2}\right)^2 ,$$

$$\left\langle \left| \mathcal{M}_{\pi\pi\nu\bar{\nu}} \right|^2 \right\rangle = \left(\frac{G_F}{\sqrt{2}} f_\pi V_{ud}\right)^4 \left(\frac{m_\tau}{v}\right)^2 \left(\frac{\pi}{m_\tau \Gamma_\tau}\right)^2$$

$$\times \Big(8\, a_\tau^2\, m_\tau^4 \left(m_H^2 - 4\, m_\tau^2\right) \left(m_\tau^2 - m_\pi^2\right)^2 \left(1 - \cos\theta_+ \cos\theta_- - \sin\theta_+ \sin\theta_- \cos\varphi\right)$$

$$+ 8\, b_\tau^2\, m_H^2\, m_\tau^4 \left(m_\tau^2 - m_\pi^2\right)^2 \left(1 - \cos\theta_+ \cos\theta_- + \sin\theta_+ \sin\theta_- \cos\varphi\right)$$

$$- 16\, a_\tau\, b_\tau\, m_H\, m_\tau^4 \sqrt{m_H^2 - 4\, m_\tau^2} \left(m_\tau^2 - m_\pi^2\right)^2 \sin\theta_+ \sin\theta_- \sin\varphi \Big).$$

The 4-body decay $H \to \tau^+ \tau^- \to \pi^+ \pi^- \nu_\tau \bar{\nu}_\tau$ can probe $b_\tau \neq 0$.

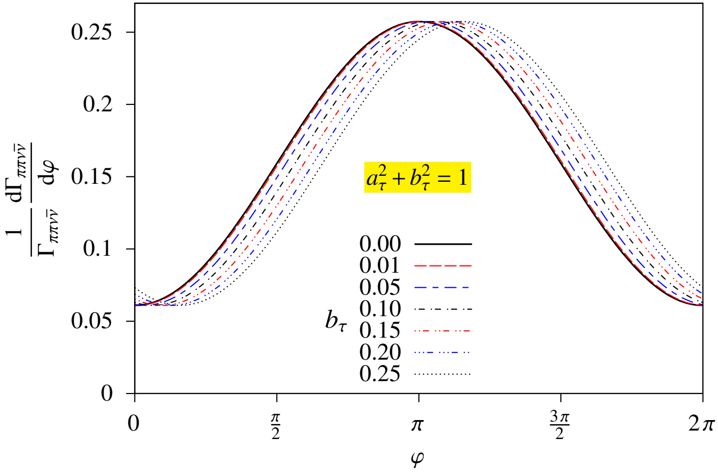
- Much richer kinematics: 3 uni-angular distributions possible.

- Only the uni-angular distribution $\dfrac{\mathrm{d}\Gamma_{\pi\pi\nu\bar{\nu}}}{\mathrm{d}\varphi}$ gets contribution from $a_\tau\, b_\tau$.

$$\frac{1}{\Gamma_{\pi\pi\nu\bar{\nu}}} \frac{\mathrm{d}\Gamma_{\pi\pi\nu\bar{\nu}}}{\mathrm{d}\varphi} = \frac{\begin{pmatrix} a_\tau^2 \left(m_H^2 - 4m_\tau^2\right)\left(16 - \pi^2 \cos\varphi\right) \\ + \; b_\tau^2 \; m_H^2 \left(16 + \pi^2 \cos\varphi\right) \\ - \, 2\,\pi^2 \; a_\tau\, b_\tau \; m_H \sqrt{m_H^2 - 4m_\tau^2}\; \sin\varphi \end{pmatrix}}{32\,\pi\left(a_\tau^2 \left(m_H^2 - 4m_\tau^2\right) + b_\tau^2 m_H^2\right)}.$$

$\therefore$ It is sensitive to **CP violation**.

The 4-body decay $H \to \tau^+ \tau^- \to \pi^+ \pi^- \nu_\tau \bar{\nu}_\tau$ can probe $b_\tau \neq 0$.



The 4-body decay $H \to \tau^+ \tau^- \to \pi^+ \pi^- \nu_\tau \overline{\nu}_\tau$ can probe $b_\tau \neq 0$.

- In H rest frame, τ's are highly boosted $\implies$ final π's and $\nu/\overline{\nu}$ are almost collinear to the parent τs $\implies$ constructing τ decay planes and measuring φ not straightforward.

- Experimentalists prefer $\rho^\pm$ instead of $\pi^\pm$ as $\rho^\pm \to \pi^\pm \pi^0$ make the plane reconstruction easier.
  $\therefore$ Only $H \to \tau^+ \tau^- \to \underbrace{\pi^+ \pi^- \pi^0 \pi^0 \nu_\tau \overline{\nu}_\tau}_{\text{6-body final state}}$ events useful.

- Constraint on b_τ from such studies:

$$|b_\tau| \lesssim 0.34$$

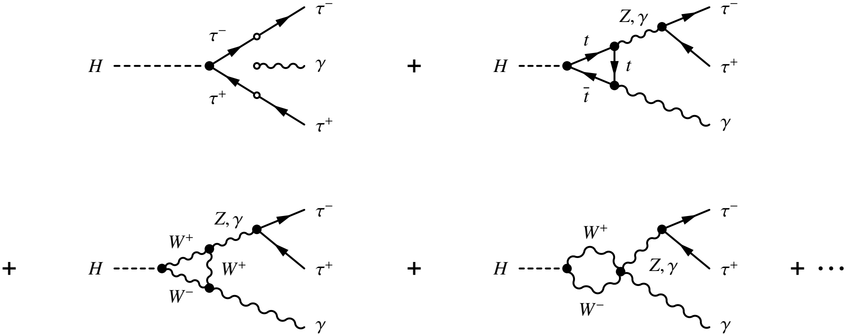
  [A. Tumasyan *et al.* [CMS], JHEP **06**, 012 (2022)]

- Way forward: More data + improved decay plane reconstruction + better angular resolutions.

- **Is there an alternative method, of probing CP violation in $H\tau\tau$ Yukawa interaction, which does not require reconstruction of τ decay planes?**

The 3-body decay $H \to \tau^+ \tau^- \gamma$ offers an alternative methodology.

Decay proceeds via both tree and loop diagrams



$\mathrm{Br}(H \to \tau^+ \tau^- \gamma)_{\mathrm{SM}} \sim 3.24 \times 10^{-3}$ with $E_\gamma > 5$ GeV and angular separation $> 5^\circ$ in rest frame of H

[See for example Phys. Rev. D **55**, 5647-5656 (1997); Phys. Rev. D **90**, no.11, 113006 (2014); Eur. Phys. J. C **74**, no.11, 3141 (2014); JHEP **12**, 111 (2016).]

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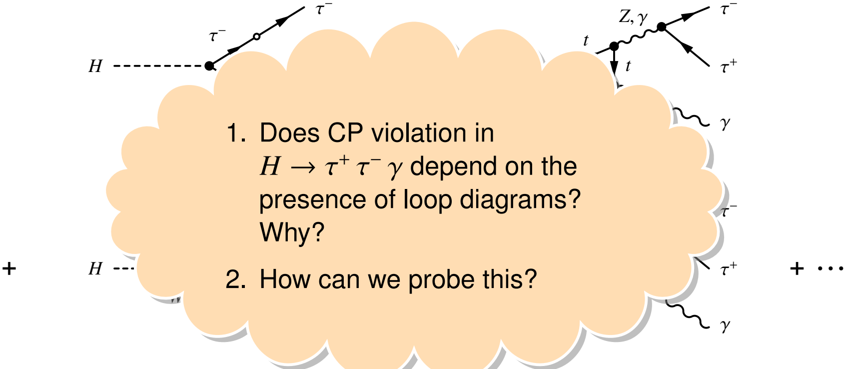

1. Does CP violation in $H \to \tau^+ \tau^- \gamma$ depend on the presence of loop diagrams? Why?
2. How can we probe this?

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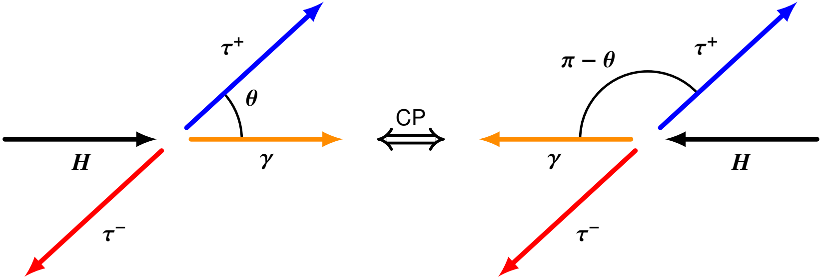
Idea: The 3-body decay $H \to \tau^+ \tau^- \gamma$ must exhibit forward-backward asymmetry if CP is violated.

A first-principle analysis

Gottfried-Jackson (GJ) frame
Center-of-momentum frame of $\tau^+ \tau^-$



CP violation $\Leftrightarrow$ **Asymmetry** under $\theta \leftrightarrow \pi - \theta \equiv \cos\theta \leftrightarrow -\cos\theta$

$\equiv$ Forward-Backward asymmetry

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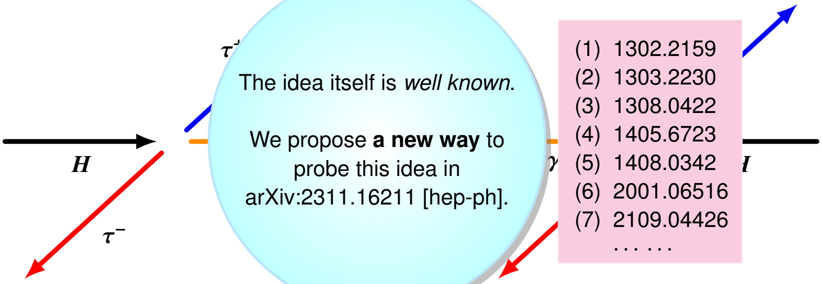

The idea itself is *well known*.

We propose **a new way** to probe this idea in arXiv:2311.16211 [hep-ph].

(1) 1302.2159
(2) 1303.2230
(3) 1308.0422
(4) 1405.6723
(5) 1408.0342
(6) 2001.06516
(7) 2109.04426
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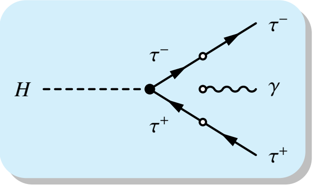
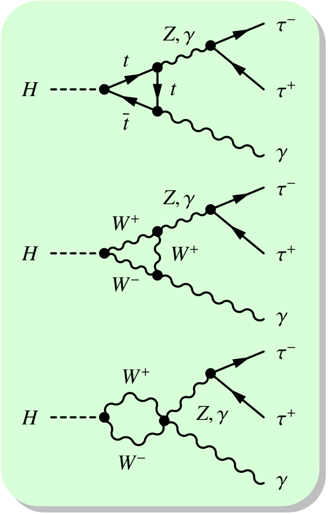
Feynman Diagrams and Amplitude for $H \to \tau^+ \tau^- \gamma$

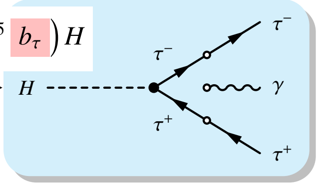


$$\mathcal{M}_{\tau\tau\gamma} = \mathcal{M}^{(\mathrm{Yuk})}_{\tau\tau\gamma} + \mathcal{M}^{(Z\gamma)}_{\tau\tau\gamma} + \mathcal{M}^{(\gamma\gamma)}_{\tau\tau\gamma}$$

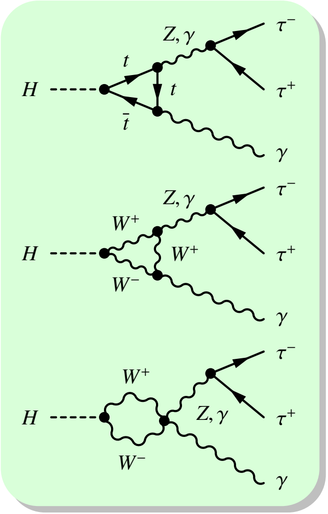


Feynman Diagrams and Amplitude for $H \to \tau^+ \tau^- \gamma$

$$\mathscr{L}_{H\tau\tau} = -\frac{m_\tau}{v}\,\bar{\tau}\left(a_\tau + i\gamma^5 b_\tau\right) H$$



$$\mathscr{M}_{\tau\tau\gamma} = \mathscr{M}^{(\mathrm{Yuk})}_{\tau\tau\gamma} + \mathscr{M}^{(Z\gamma)}_{\tau\tau\gamma} + \mathscr{M}^{(\gamma\gamma)}_{\tau\tau\gamma}$$



$$\mathscr{L}_{H\mathcal{V}\gamma} = \frac{H}{4\,v}\Big(2\,A_2^{Z\gamma} F^{\mu\nu} Z_{\mu\nu} + 2\,A_3^{Z\gamma} F^{\mu\nu} \widetilde{Z}_{\mu\nu} + A_2^{\gamma\gamma} F^{\mu\nu} F_{\mu\nu} + A_3^{\gamma\gamma} F^{\mu\nu} \widetilde{F}_{\mu\nu}\Big),$$

where $\mathcal{V}_{\mu\nu} = \partial_\mu \mathcal{V}_\nu - \partial_\nu \mathcal{V}_\mu$, $\quad \widetilde{\mathcal{V}}_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\rho\sigma}\mathcal{V}^{\rho\sigma}$,

for $\mathcal{V} = Z, \gamma$.

Feynman Diagrams and Amplitude for $H \to \tau^+ \tau^- \gamma$

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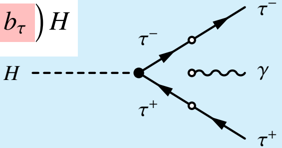
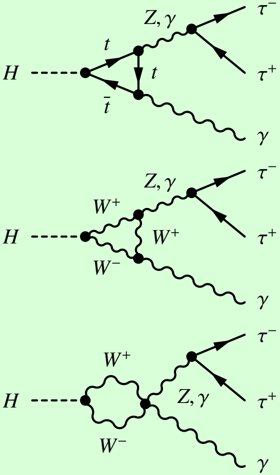


$$\mathscr{M}_{\tau\tau\gamma} = \mathscr{M}^{(\mathrm{Yuk})}_{\tau\tau\gamma} + \mathscr{M}^{(Z\gamma)}_{\tau\tau\gamma} + \mathscr{M}^{(\gamma\gamma)}_{\tau\tau\gamma}$$

SM loop effects only



$$\mathscr{L}_{H\mathcal{V}\gamma} = \frac{H}{4v}\Big(2\, A_2^{Z\gamma} F^{\mu\nu} Z_{\mu\nu} + 2\, A_3^{Z\gamma} F^{\mu\nu} \widetilde{Z}_{\mu\nu} + A_2^{\gamma\gamma} F^{\mu\nu} F_{\mu\nu} + A_3^{\gamma\gamma} F^{\mu\nu} \widetilde{F}_{\mu\nu} \Big),$$

where $\mathcal{V}_{\mu\nu} = \partial_\mu \mathcal{V}_\nu - \partial_\nu \mathcal{V}_\mu$, $\quad \widetilde{\mathcal{V}}_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\rho\sigma}\mathcal{V}^{\rho\sigma}$,

for $\mathcal{V} = Z, \gamma$. *We shall consider* $A_3^{\gamma\gamma} = 0 = A_3^{Z\gamma}$.

The interference of tree-level and loop-level amplitudes of $H \to \tau^+ \tau^- \gamma$ is sensitive to $b_\tau \neq 0$.

$$|\mathcal{M}|^2 = \underbrace{\left|\mathcal{M}^{(\mathrm{Yuk})}\right|^2 + \left|\mathcal{M}^{(Z\gamma)}\right|^2 + \left|\mathcal{M}^{(\gamma\gamma)}\right|^2 + 2\,\mathrm{Re}\left(\mathcal{M}^{(\mathrm{Yuk})}\,\mathcal{M}^{(\gamma\gamma)*}\right)}_{\text{even under } \cos\theta \leftrightarrow -\cos\theta}$$

$$+ \underbrace{2\,\mathrm{Re}\left(\mathcal{M}^{(\gamma\gamma)}\,\mathcal{M}^{(Z\gamma)*}\right)}_{\text{has a term linear in } \cos\theta \text{ which vanishes when } A_3^{\gamma\gamma} = 0 = A_3^{Z\gamma}} + \underbrace{2\,\mathrm{Re}\left(\mathcal{M}^{(\mathrm{Yuk})}\,\mathcal{M}^{(Z\gamma)*}\right)}_{\text{has a term} \propto b_\tau \text{ \& linear in } \cos\theta\text{, which survives even when } A_3^{\gamma\gamma} = 0 = A_3^{Z\gamma}},$$

- non-zero CP-odd (“weak”) phase difference $\Longleftarrow$ $b_\tau \neq 0,\, A_3^{\gamma\gamma} \neq 0,\, A_3^{Z\gamma} \neq 0,$
- non-zero CP-even (“strong”) phase difference $\Longleftarrow$ $\mathrm{Im}\left[\left((p_+ + p_-)^2 - m_Z^2 + i\, m_Z\,\Gamma_Z\right)^{-1}\right].$

The amplitude square can be expressed using Lorentz invariant mass-squares.

- Only 3 Lorentz invariant mass-squares possible,

$$m_{+-}^2 \equiv (p_H - p_0)^2 = (p_+ + p_-)^2\,, \qquad \Longrightarrow 4\,m_\tau^2 \leqslant m_{+-}^2 \leqslant m_H^2$$
$$m_{+0}^2 \equiv (p_H - p_-)^2 = (p_+ + p_0)^2\,, \qquad \Longrightarrow m_\tau^2 \leqslant m_{+0}^2 \leqslant (m_H - m_\tau)^2$$
$$m_{-0}^2 \equiv (p_H - p_+)^2 = (p_- + p_0)^2\,. \qquad \Longrightarrow m_\tau^2 \leqslant m_{-0}^2 \leqslant (m_H - m_\tau)^2$$

Note: $m_{+-}^2 + m_{+0}^2 + m_{-0}^2 = m_H^2 + 2\,m_\tau^2.$ $\Longrightarrow$ Only 2 *independent* mass-squares.

- In the GJ frame,

$$\left.\begin{aligned} m_{+0}^2 &= M^2 - M'^2 \cos\theta, \\ m_{-0}^2 &= M^2 + M'^2 \cos\theta, \end{aligned}\right\} \Longrightarrow \begin{cases} & \theta \leftrightarrow \pi - \theta \\ \equiv & \cos\theta \leftrightarrow -\cos\theta \\ \equiv & m_{+0}^2 \leftrightarrow m_{-0}^2 \end{cases}$$

where $M^2 = \frac{1}{2}\left(m_H^2 + 2\,m_\tau^2 - m_{+-}^2\right), \quad M'^2 = \frac{1}{2}\left(m_H^2 - m_{+-}^2\right)\sqrt{1 - 4\,m_\tau^2/m_{+-}^2}.$

Other choices of variables are frame dependent.

	$\left(m_{+0}^2, m_{-0}^2\right)$	$\left(m_{+-}^2, \cos\theta\right)$	$\left(E_+, E_-\right)$
Differential Decay rate	$\frac{\mathrm{d}^2\Gamma_{\tau\tau\gamma}}{\mathrm{d}m_{+0}^2\,\mathrm{d}m_{-0}^2}$	$\frac{\mathrm{d}^2\Gamma_{\tau\tau\gamma}}{\mathrm{d}m_{+-}^2\,\mathrm{d}\cos\theta}$	$\frac{\mathrm{d}^2\Gamma_{\tau\tau\gamma}}{\mathrm{d}E_+\,\mathrm{d}E_-}$
Frame of reference	Any frame	GJ frame	H rest frame
Need to boost?	No	Yes	Yes

- $E_\pm$ = energy of $\tau^\pm$ in H rest frame. $m_{\pm 0}^2 = m_H^2 - 2\,m_H\,E_\pm$ & $m_{+0}^2 \leftrightarrow m_{-0}^2 \equiv E_+ \leftrightarrow E_-$
- Differential decay rate is frame dependent:

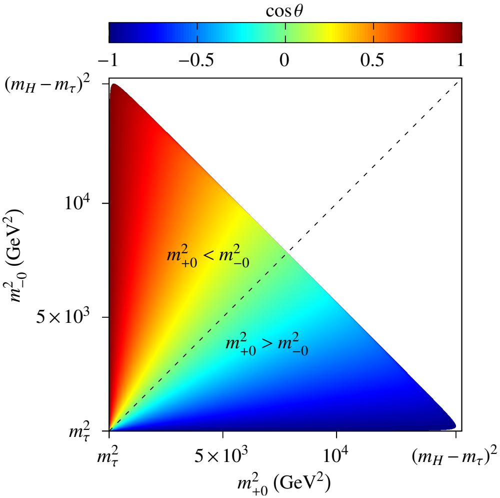
$$\left(\frac{\mathrm{d}^2\Gamma_{\tau\tau\gamma}}{\mathrm{d}m_{+0}^2\,\mathrm{d}m_{-0}^2}\right)_{\text{H rest}} = \frac{\left|\mathcal{M}_{\tau\tau\gamma}\right|^2}{256\,\pi^3\,m_H^3},$$

$$\left(\frac{\mathrm{d}^2\Gamma_{\tau\tau\gamma}}{\mathrm{d}m_{+-}^2\,\mathrm{d}\cos\theta}\right)_{\text{H rest}} = \frac{m_H^2 - m_{+-}^2}{512\,\pi^3\,m_H^3}\sqrt{1 - \frac{4\,m_\tau^2}{m_{+-}^2}}\,\left|\mathcal{M}_{\tau\tau\gamma}\right|^2,$$

$$\left(\frac{\mathrm{d}^2\Gamma_{\tau\tau\gamma}}{\mathrm{d}m_{+-}^2\,\mathrm{d}\cos\theta}\right)_{\text{GJ}} = \frac{m_{+-}\left(m_H^2 - m_{+-}^2\right)}{256\,\pi^3\,m_H^2\left(m_H^2 + m_{+-}^2\right)}\sqrt{1 - \frac{4\,m_\tau^2}{m_{+-}^2}}\,\left|\mathcal{M}_{\tau\tau\gamma}\right|^2.$$

The forward-backward asymmetry can be easily observed in Lorentz invariant Dalitz plot distribution.

Notations, Regions & Expectations



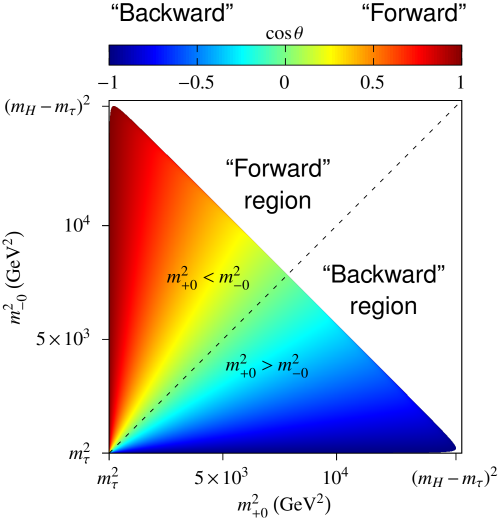
- Let $\mathcal{D}\left(m_{+0}^2, m_{-0}^2\right) \equiv \dfrac{\mathrm{d}^2\Gamma_{\tau\tau\gamma}}{\mathrm{d}m_{+0}^2\,\mathrm{d}m_{-0}^2}$ denote **distribution of events in the m_{+0}^2** vs. **m_{-0}^2 Dalitz plot**.

- Area of the Dalitz plot $\propto$ Available phase space

- One can also choose to work with
$$\mathcal{D}\left(m_{+0}, m_{-0}\right) \equiv \frac{\mathrm{d}^2\Gamma_{\tau\tau\gamma}}{\mathrm{d}m_{+0}\,\mathrm{d}m_{-0}}$$

The forward-backward asymmetry can be easily observed in Lorentz invariant Dalitz plot distribution.

Notations, Regions & Expectations



Notation:

Region	"Forward"	"Backward"
$\cos\theta$	$[0, 1]$	$[-1, 0]$
$m_{\pm 0}^2$	$m_{+0}^2 < m_{-0}^2$	$m_{+0}^2 > m_{-0}^2$
Distribution	$\mathcal{D}\left(m_{+0}^2 < m_{-0}^2\right)$	$\mathcal{D}\left(m_{+0}^2 > m_{-0}^2\right)$
No. of events	N_F	N_B

Expectation: CP violation $(b_\tau \neq 0) \implies$

- $\mathcal{D}\left(m_{+0}^2 < m_{-0}^2\right) \neq \mathcal{D}\left(m_{+0}^2 > m_{-0}^2\right)$
- $N_F \neq N_B$

The forward-backward asymmetry can be easily observed in Lorentz invariant Dalitz plot distribution.

How to quantify Dalitz Plot Asymmetries to probe CP violation?

- **Non-integrated or distribution asymmetry:** Compare the distribution of events *across* the Dalitz plot in the "forward" and "backward" regions.

$$\mathcal{A}\left(m_{+0}^2, m_{-0}^2\right) = \frac{\left|\mathcal{D}\left(m_{+0}^2 < m_{-0}^2\right) - \mathcal{D}\left(m_{+0}^2 > m_{-0}^2\right)\right|}{\mathcal{D}\left(m_{+0}^2 < m_{-0}^2\right) + \mathcal{D}\left(m_{+0}^2 > m_{-0}^2\right)}.$$

Or equivalently,

$$\mathcal{A}\left(m_{+0}, m_{-0}\right) = \frac{\left|\mathcal{D}\left(m_{+0} < m_{-0}\right) - \mathcal{D}\left(m_{+0} > m_{-0}\right)\right|}{\mathcal{D}\left(m_{+0} < m_{-0}\right) + \mathcal{D}\left(m_{+0} > m_{-0}\right)}.$$

The forward-backward asymmetry can be easily observed in Lorentz invariant Dalitz plot distribution.

How to quantify Dalitz Plot Asymmetries to probe CP violation?

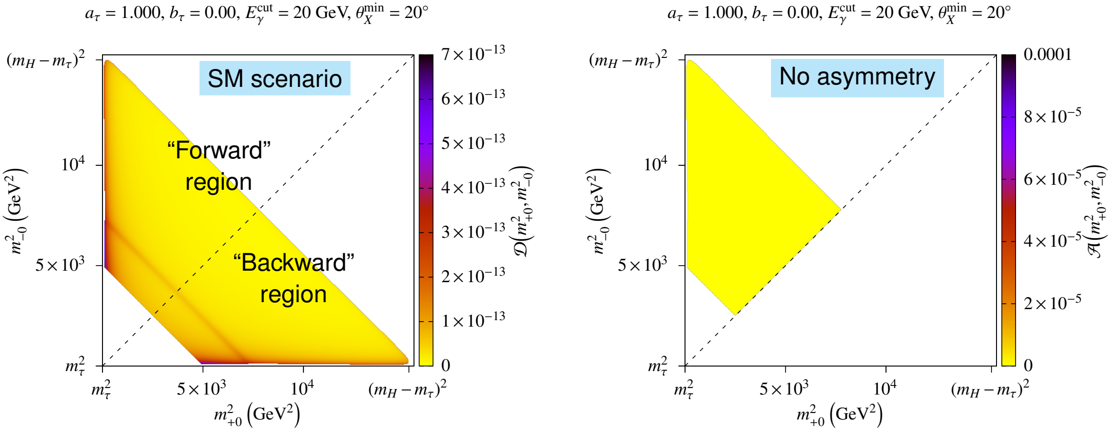
- **Regional integrated asymmetries:** Count and compare the number of events in 'islands' sitting in opposite regions of the Dalitz plot, e.g. in the forward and backward regions surrounding the Z-pole,

$$A(n) = \frac{\left| \iint \left[\mathcal{D}\left(m_{+0}^2 < m_{-0}^2\right) - \mathcal{D}\left(m_{+0}^2 > m_{-0}^2\right) \right] \Pi\left(m_{+-}^2, n\right) \, \mathrm{d}m_{+0}^2 \, \mathrm{d}m_{-0}^2 \right|}{\iint \mathcal{D}\left(m_{+0}^2, m_{-0}^2\right) \Pi\left(m_{+-}^2, n\right) \, \mathrm{d}m_{+0}^2 \, \mathrm{d}m_{-0}^2}.$$

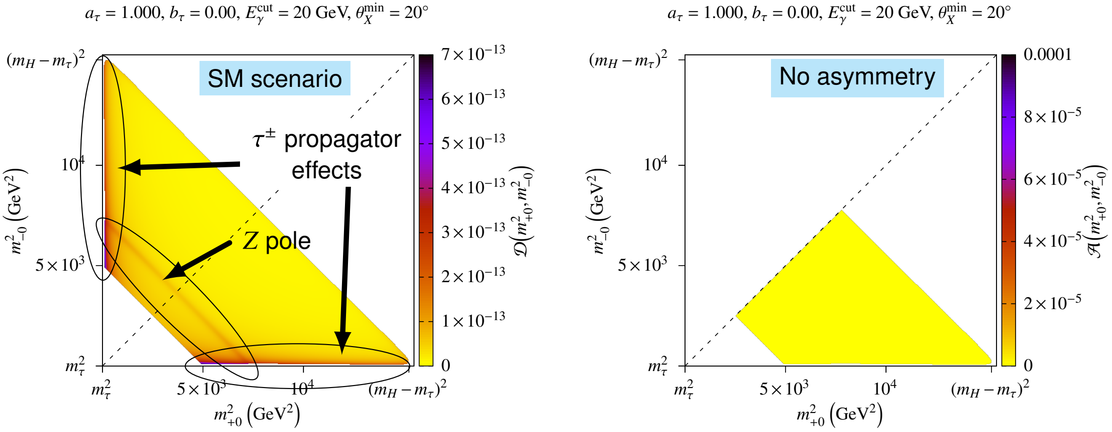
where the function $\Pi\left(m_{+-}^2, n\right)$ is defined as,

$$\Pi\left(m_{+-}^2, n\right) = \begin{cases} 1, & \text{for } |m_{+-} - m_Z| \leqslant n\,\Gamma_Z, \\ 0, & \text{otherwise.} \end{cases}$$

The forward-backward asymmetry can be easily observed in Lorentz invariant Dalitz plot distribution.



The forward-backward asymmetry can be easily observed in Lorentz invariant Dalitz plot distribution.



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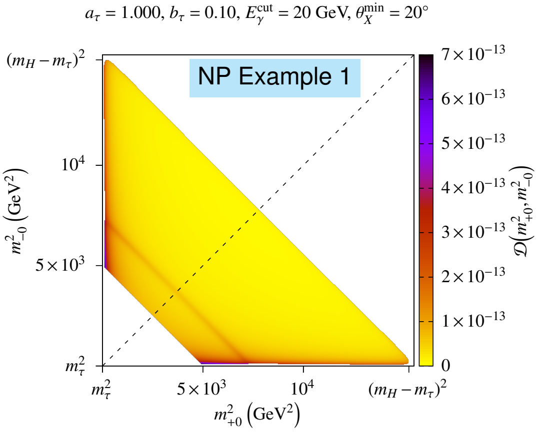

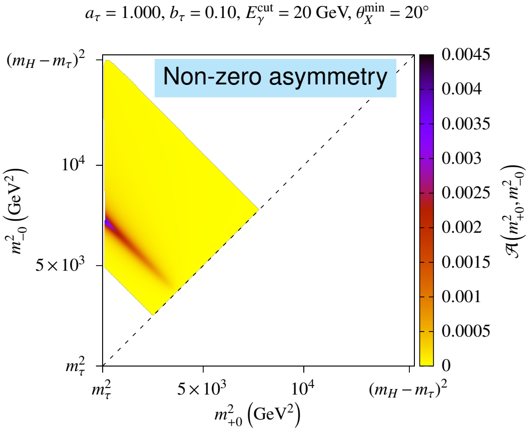

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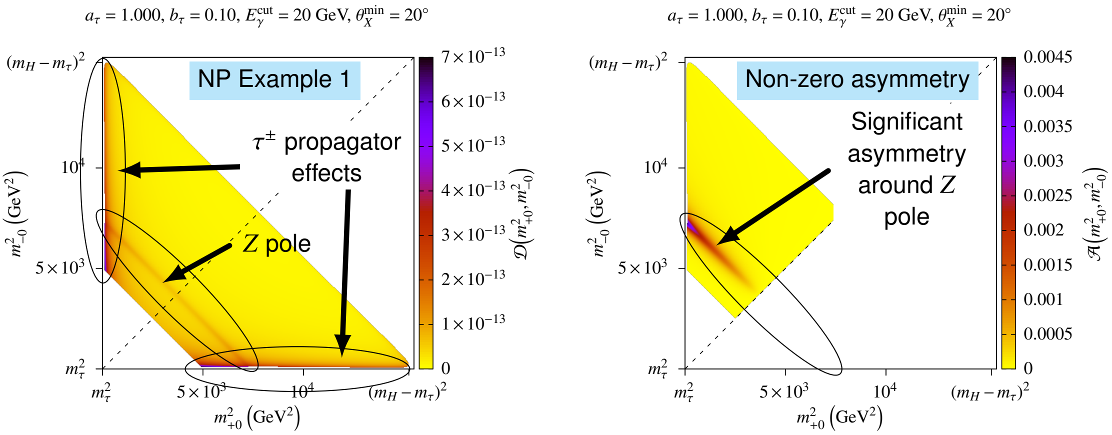

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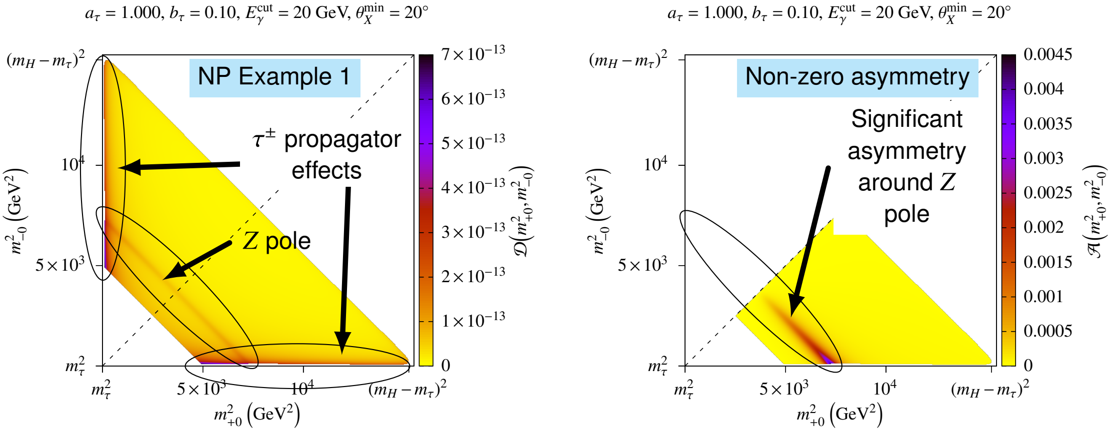

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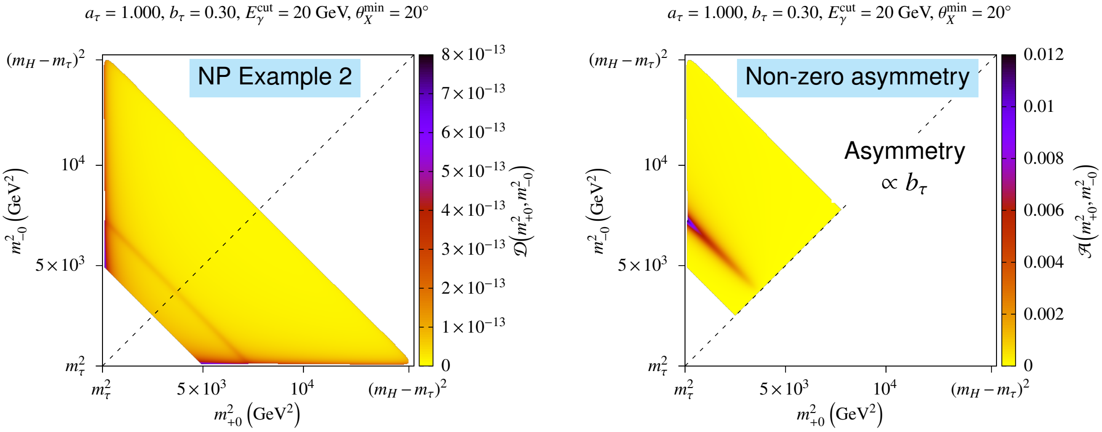

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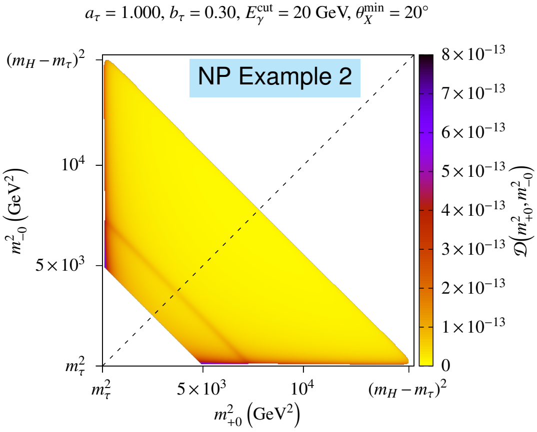
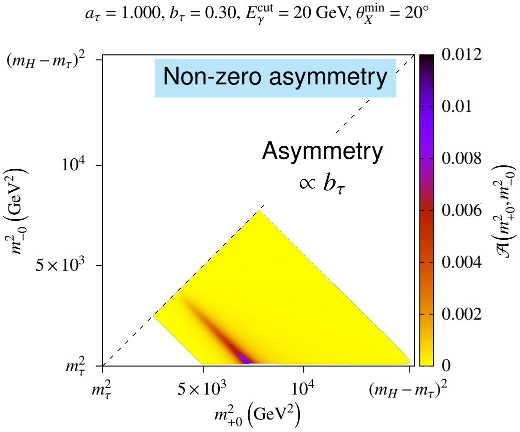
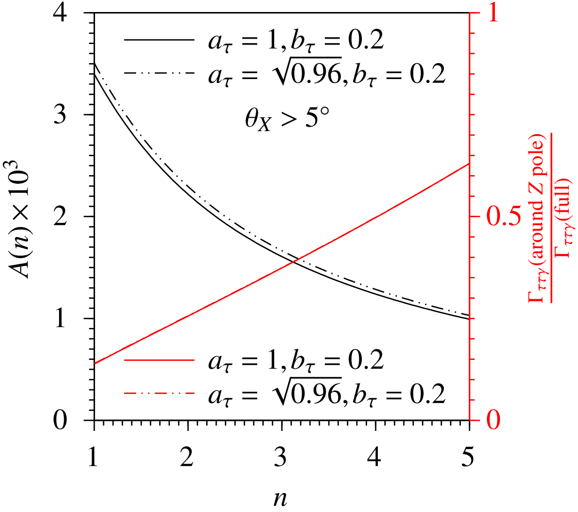




Probing the forward-backward asymmetry in the neighbourhood of Z pole could be tricky.

$|m_{+-} - m_Z| \leqslant n\,\Gamma_Z$



Summary of theoretical expectation

We have noticed that

(1) CP violation $(b_\tau \neq 0) \implies$ Forward-Backward asymmetry in Gottfried-Jackson frame

(2) Forward-Backward asymmetry ≡ Asymmetry in m^2_{+0} vs. m^2_{-0} (or equivalently m_{+0} vs. m_{-0}) Dalitz plot under $m_{+0} \leftrightarrow m_{-0}$:

$$\underbrace{\mathcal{A}\left(m^2_{+0}, m^2_{-0}\right) \neq 0,}_{\text{full distribution asymmetry}} \quad \left(\text{maximum is } \sim 4 \times 10^{-3} \text{ for } b_\tau = 0.1\right), \qquad \underbrace{A(n) \neq 0.}_{\text{asymmetry around } Z \text{ pole}}$$

(3) m^2_{+0} vs. m^2_{-0} (or m_{+0} vs. m_{-0}) Dalitz plot can be obtained in *any frame of reference* including the laboratory frame, and

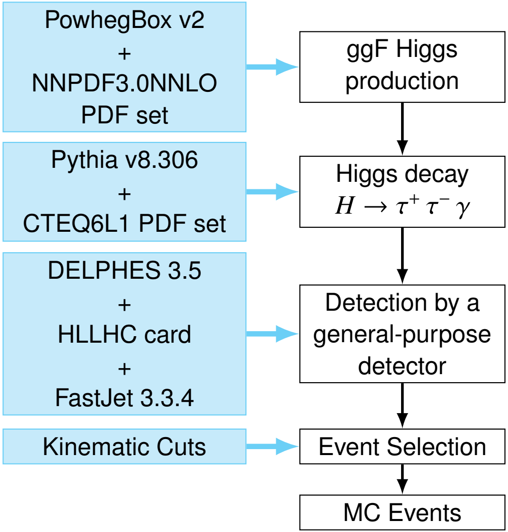
(4) the asymmetry is most prominent in region surrounding the Z pole.

Studying Lorentz invariant Dalitz plot distribution to probe CP violation would be new for HL-LHC.

- We expect that at HL-LHC,
  1. about 1.6×10^8 Higgs will be produced via gluon-gluon fusion,
  2. about 2.24×10^5 Higgs would decay via $H \to \tau^+_{\text{had}} \tau^-_{\text{had}} \gamma$,

  with appropriate kinematic cuts ($p_T^\gamma > 10$ GeV, $p_T^\tau > 15$ GeV, photon isolation cone with $\Delta R \leqslant 0.3$ and radius parameter R $= 0.4$ for seed jets for hadronically decaying τs).

- In a simple Monte Carlo study we reweight the MC signal sample to emulate the effect of interference term, for many b_τ values, akin to the "interpolation" approach of G. Aad *et al.* [ATLAS], JHEP **10** (2021), 013.



Our simple MC study suggests some way forward.

- The b_τ values extracted from our *simple* MC study yields large uncertainties, e.g.

$$b_\tau = 0.32 \pm 2.24 \text{ for the input } b_\tau = 0.1.$$

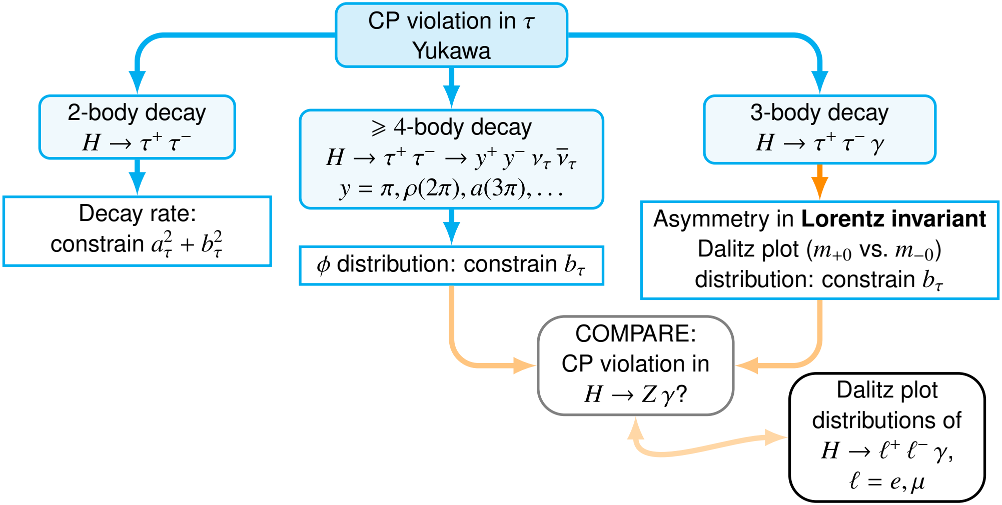
- Our simple MC study suggests three directions to take in future.
  1. **Take interference effects at the generator level to avoid reweighting.**
     $\Longrightarrow$ Better modeled MC event sample to study forward-backward asymmetry.
  2. **Include events with one of the τs decaying leptonically as well.**
     $\Longrightarrow$ Increase number of signal events $\Longrightarrow$ Bigger dataset $\Longrightarrow$ Smaller statistical uncertainty.
  3. **Employ 2D unbinned analysis techniques for Dalitz plot distribution.**
     $\Longrightarrow$ We focused on *binned* 1D event distribution w.r.t. m_{+-} in forward and backward regions.
     $\Longrightarrow$ Study of full 2D distribution of events inside the Dalitz plot would be useful.

- Already existing methods to do 2D distribution study of Dalitz plot:
  1. 'Miranda' Procedure a.k.a. Dalitz plot significance anisotropy, [e.g. PRD **80** (2009) 096006]
  2. Method of energy test statistic, [e.g. PRD **84** (2011), 054015]
  3. Method of Wasserstein (earth mover's) distance. [e.g. JHEP06 (2023) 098]

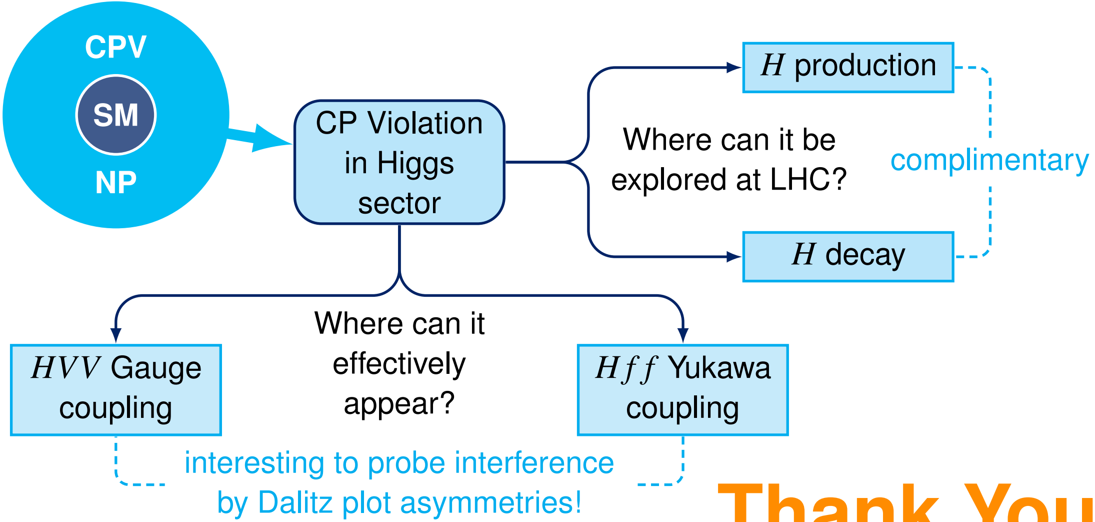
Part 3: Conclusion

The 3-body decay $H \to \tau^+ \tau^- \gamma$ is an interesting and complementary avenue to probe CP violation.

Dalitz plot analysis has a bigger role to play in future.



CP Violation in Higgs Sector



Thank You