Determination of V_{cb} from inclusive semileptonic B-meson decays

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- Dirac mass terms would explicitly break the SM gauge symmetry.
- Instead, quark masses are generated through Yukawa interactions:

$$\mathscr{L}_{SM} \supset \mathscr{L}_Y^{quark} = -\sum_{i,j=1}^{3} \left(Y_{ij}^{(d)} \bar{q}_L^i \phi d_R^j + Y_{ij}^{(u)} \bar{q}_L^i (i\sigma_2 \phi^*) u_R^j + h.c. \right).$$

- After symmetry breaking, the part proportional to the Higgs vev takes the form

$$\mathscr{L}_Y^{quark} \supset \mathscr{L}_{mass}^{quark} = -\sum_{i,j=1}^{3} \left(\bar{d}_L^i M_{ij}^{(d)} d_R^j + \bar{u}_L^i M_{ij}^{(u)} u_R^j + h.c. \right), \qquad q_L \equiv (u_L, d_L)^T.$$

- The matrices $M^{(u)}$ and $M^{(d)}$ are diagonalized via Singular Value Decomposition:

$$M^{(u)} = U_L^{(u)} M_D^{(u)} U_R^{(u)\dagger}, \qquad M^{(d)} = U_L^{(d)} M_D^{(d)} U_R^{(d)\dagger},$$

- The unitary matrices $U_{L/R}^{(u)/(d)}$ are absorbed into definitions of quark fields:

$$u_L \to U_L^{(u)} u_L \qquad u_R \to U_R^{(u)} u_R \qquad d_L \to U_L^{(d)} d_L \qquad d_R \to U_R^{(d)} d_R,$$

which transforms the quark mass terms as:

$$\mathscr{L}_{mass}^{quark} \to -\sum_{i=1}^{3} \left(\bar{d}_L^i \left(M_D^{(d)} \right)_{ii} d_R^i + \bar{u}_L^i \left(M_D^{(u)} \right)_{ii} u_R^i + h.c. \right) = -\sum_{i=1}^{3} \left(\bar{d}^i \left(M_D^{(d)} \right)_{ii} d^i + \bar{u}^i \left(M_D^{(u)} \right)_{ii} u^i \right).$$

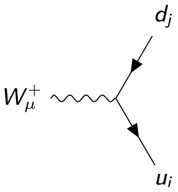
- The quark field redefinition affects their interactions with the charged gauge bosons:

$$\mathscr{L}_{SM} \supset \mathscr{L}_{CC} = \frac{g_{ew}}{\sqrt{2}} \left(W^+_\mu \bar{u}_L \gamma^\mu d_L + W^-_\mu \bar{d}_L \gamma^\mu u_L \right) \to \frac{g_{ew}}{\sqrt{2}} \left(W^+_\mu \bar{u}_L \gamma^\mu V d_L + W^-_\mu \bar{d}_L \gamma^\mu V^\dagger u_L \right),$$

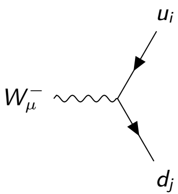
where

$$V \equiv U_L^{(u)\dagger} U_L^{(d)}, \qquad V^\dagger V = \mathbb{1}.$$

- The vertices following from $\mathscr{L}_{CC}$ are



$$= -i \frac{g_{ew}}{\sqrt{2}} \gamma_\mu P_L V_{ij}$$



$$= -i \frac{g_{ew}}{\sqrt{2}} \gamma_\mu P_L V^*_{ij}$$

$$|V| = \begin{pmatrix} |V_{ud}| & |V_{us}| & |V_{ub}| \\ |V_{cd}| & |V_{cs}| & |V_{cb}| \\ |V_{td}| & |V_{ts}| & |V_{tb}| \end{pmatrix},$$

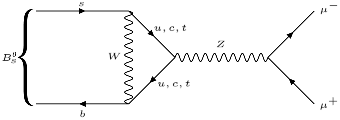
$\mathcal{O}(1)$
$\mathcal{O}(10^{-1})$
$\mathcal{O}(10^{-2})$
$\mathcal{O}(10^{-3})$

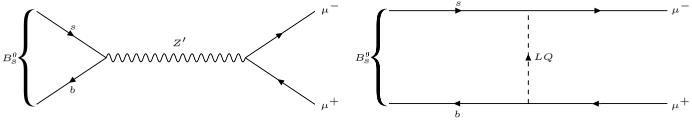
- Unitarity of the CKM matrix V gives constraints on its entries. There are only 4 independent, real parameters.
- One can choose a parametrization that uses $|V_{cb}|$ explicitly:

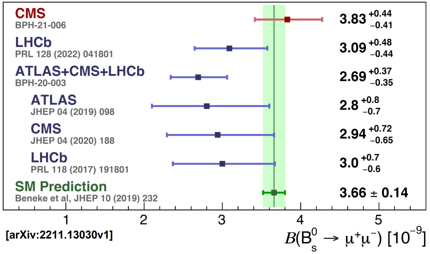
$$V \approx \begin{pmatrix} 1 - \frac{\lambda^2}{2}, & \lambda, & |V_{ub}|e^{-i\gamma}, \\ -\lambda, & 1 - \frac{\lambda^2}{2}, & |V_{cb}|, \\ |V_{cb}|\lambda - |V_{ub}|e^{i\gamma}\left(1 - \frac{\lambda^2}{2}\right), & -|V_{cb}|\left(1 - \frac{\lambda^2}{2}\right) - |V_{ub}|\lambda e^{i\gamma}, & 1 \end{pmatrix},$$

where in each entry, relative corrections $\mathcal{O}(\lambda^3)$ were neglected.

- Parameter λ is the sine of the Cabibbo angle $\theta_C \approx 13^\circ$ and γ is the standard parametrization of the CKM phase, measured to be $\gamma \approx 66^\circ$.

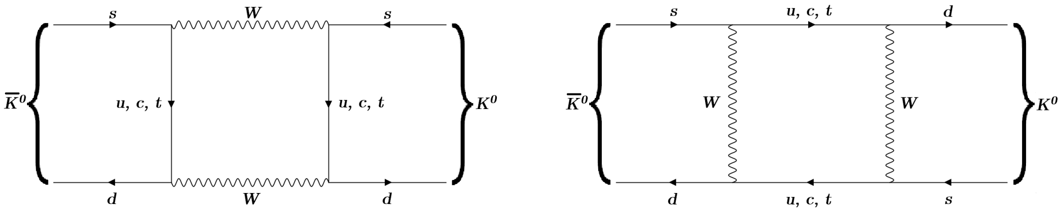






- The rare B meson decay $B_s^0 \to \mu^+\mu^-$ is mediated by the FCNC and thus loop suppressed in the SM.
- It is also sensitive to many New Physics (NP) models.
- Uncertainty due to $|V_{cb}|$ is the main factor limiting the precision of the SM prediction:

$$\delta\mathcal{B}(B_s^0 \to \mu^+\mu^-) = \sqrt{\underbrace{(3.0\%)^2}_{|V_{cb}|} + \underbrace{(2.3\%)^2}_{other}}.$$

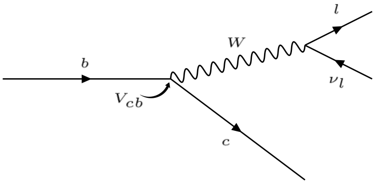


- The value of $|V_{cb}|$ is one of the main sources of theoretical uncertainty in neutral Kaon mixing. The uncertainty budget of the CP-violating parameter $|\epsilon_K|$ is [arXiv:1911.06822]

$$\delta|\epsilon_K| = \sqrt{\underbrace{(5.3\%)^2}_{|V_{cb}|} + \underbrace{(6.4\%)^2}_{other}}.$$

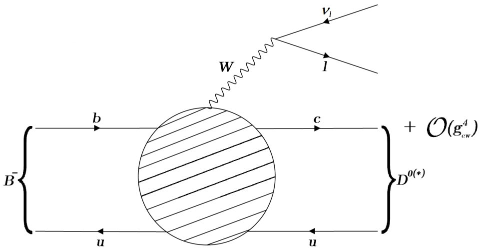
- The experimental error is already below 1% [PDG, Prog. Theor. Exp. Phys. 2022, 083C01]:

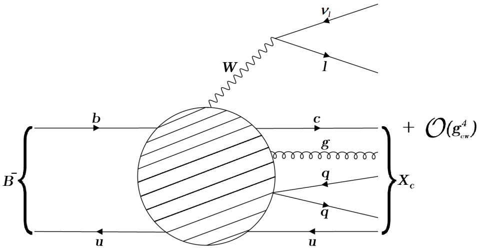
$$|\epsilon_K|_{exp} = (2.228 \pm 0.011) \times 10^{-3}.$$



- The standard method of extracting the value of $|V_{cb}|$ is to study the semileptonic transition $b \to c l \bar{\nu}_l$, $l \in \{e, \mu\}$.

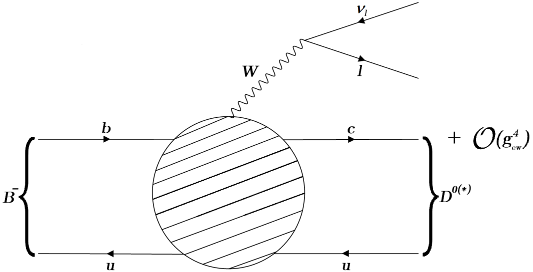
- On the hadronic level it can be realized in an exclusive or inclusive way:





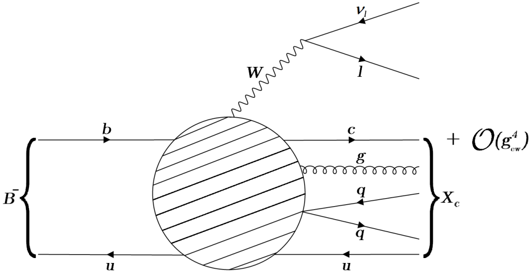
- Exclusive $B^- \to D^{0(*)} l \bar{\nu}_l$ decay.

- Inclusive $B^- \to X_c l \bar{\nu}_l$ decay. All hadronic final states with $C = 1$ are summed over.



- Calculation of the exclusive decay rate requires knowledge of hadronic form factors.
- These are obtained using Lattice QCD for large 4-momentum transfer (q^2) to the leptons, or Light-Cone Sum Rules for small q^2.
- The HFLAV average for the exclusive determination of $|V_{cb}|$ is [arXiv:1909.12524]

$$|V_{cb}|_{excl} = (39.25 \pm 0.56) \times 10^{-3}$$



- After integrating out top quarks and weak gauge bosons, the inclusive differential rate can be written as

$$\frac{d\Gamma}{dq^2 dr^2 dE_l} \propto G_F^2 |V_{cb}|^2 L_{\alpha\beta} W^{\alpha\beta},$$

where G_F is the Fermi constant, $q \equiv k_l + k_{\bar{\nu}}$, $r \equiv p_B - q$, and

$$L^{\alpha\beta} \equiv \frac{1}{2} \sum_{s_l s_{\bar{\nu}}} A_l^\alpha A_l^{\dagger\beta} = k_l^\alpha k_{\bar{\nu}}^\beta + k_l^\beta k_{\bar{\nu}}^\alpha - (k_l k_{\bar{\nu}}) g^{\alpha\beta} - i\epsilon^{\alpha\rho\beta\sigma} k_{l_\rho} k_{\bar{\nu}_\sigma}, \qquad A_l^\alpha \equiv \bar{u}_l^{(s_l)} \gamma^\alpha P_L v_{\bar{\nu}}^{(s_{\bar{\nu}})}.$$

- Assuming massless leptons, one can integrate $L^{\alpha\beta}$ over E_l to obtain

$$\frac{d\Gamma}{dq^2 dr^2} \propto G_F^2 |V_{cb}|^2 \frac{|\vec{q}|}{3} (q_\alpha q_\beta - q^2 g_{\alpha\beta}) W^{\alpha\beta},$$

- The hadronic tensor $W^{\alpha\beta}$ is defined as

$$W^{\alpha\beta} \equiv \sum_{X_c} \langle B|J_H^{\alpha}|X_c\rangle \langle X_c|J_H^{\dagger\beta}|B\rangle\,, \qquad J_H^{\alpha} \equiv \bar{b}\gamma^{\alpha}P_L c,$$

- The optical theorem can be used to relate $W^{\alpha\beta}$ to the correlator of currents J_H^{α}:

$$W^{\alpha\beta} = \frac{1}{\pi}\mathrm{Im}\,\langle B|i\int d^4x e^{iqx}\,T[J_H^{\alpha}(x)J_H^{\dagger\beta}(0)]|B\rangle\,.$$

- Now, one introduces the re-phased b-quark field $b_v(x)$:

$$b_v(x) \equiv \exp(im_b v x)b(x), \qquad v \equiv \frac{p_B}{m_B},$$

  so $W^{\alpha\beta}$ becomes

$$W^{\alpha\beta} = \frac{1}{\pi}\mathrm{Im}\,\langle B|i\int d^4x e^{-im_b\left(v-\frac{q}{m_b}\right)x}\,T[J_{Hv}^{\alpha}(x)J_{Hv}^{\dagger\beta}(0)]|B\rangle\,.$$

- In the above expression the relevant field modes have momenta $\mathcal{O}(\Lambda)$, where $\Lambda \sim m_B - m_b, \Lambda_{QCD}$.

- Away from the $r^2 \approx m_D^2$ endpoint, short distance contributions dominate, and one can perform the Operator Product Expansion (OPE):

$$\frac{i}{m_b^2}\int d^4x e^{-im_b\left(v-\frac{q}{m_b}\right)x} T[J_{Hv}^{\alpha}(x)J_{Hv}^{\dagger\beta}(0)] = \sum_k \frac{\widetilde{C}_k O_k^{(n)\alpha\beta}(x=0)}{m_b^{n(k)}}.$$

- Coefficients $\widetilde{C}_k$ can be computed in perturbative QCD by equating free parton matrix elements of both sides of the above equation.
- After the OPE, the differential decay rate takes form

$$\frac{d\Gamma}{dq^2 dr^2 dE_l} \propto G_F^2 |V_{cb}|^2 \sum_k \frac{C_k \langle B|O_k^{(n)}|B\rangle}{m_b^{n(k)}}.$$

- At the leading order,

$$\langle B|O_k^{(0)}|B\rangle = \langle B|\bar{b}_v b_v|B\rangle \equiv 2m_B\mu_3 = 2m_B\left(1+\mathcal{O}\left(\frac{\Lambda^2}{m_b^2}\right)\right),$$

and so the leading term proportional to $(\Lambda/m_b)^0$ does not depend on any non-perturbative matrix elements.

- The operator appearing at $\mathcal{O}(\Lambda/m_b)$ vanishes via equations of motion up to corrections $\mathcal{O}\left((\Lambda/m_b)^2\right)$ so the first nonperturbative correction is suppressed by $(\Lambda/m_b)^2$.

- One can define spectral moments of the semileptonic decay as phase space integrals:

$$\langle M[w]\rangle \propto G_F^2 |V_{cb}|^2 \int d\Phi w[v, k_l, k_{\bar{\nu}}] W^{\alpha\beta} L_{\alpha\beta},$$

with a certain weight function $w[v, k_l, k_{\bar{\nu}}]$:
  - $\Gamma = \langle M[1]\rangle$,
  - $\langle (q^2)^k \rangle = \langle M[((k_l + k_{\bar{\nu}})^2)^k]\rangle$,
  - $\frac{d\Gamma}{dE_l} = \langle M[\delta(vk_l - E_l)]\rangle$,
  - $\frac{d\Gamma}{dq^2} = \langle M[\delta((k_l + k_{\bar{\nu}})^2 - q^2)]\rangle$,
  - $\langle (E_l)^k \rangle_{E_l > E_{cut}} = \langle M[(vk_l)^k \theta(vk_l - E_{cut})]\rangle$.
- The OPE can be applied to $\langle M[w]\rangle$ the same way as for the differential rate:

$$\langle M[w]\rangle \propto G_F^2 |V_{cb}|^2 \sum_{k=0}^{\infty} \frac{C_k[w] \langle B|O_k^{(n)}|B\rangle}{m_b^{n(k)}}.$$

- The value of $|V_{cb}|$ is extracted together with quark masses and nonperturbative matrix elements $\langle B|O_k^{(n)}|B\rangle$ from a global fit of SM predictions of various $\langle M[w]\rangle$ to experimental data:

$$\{\langle M[w]\rangle_{exp} = \langle M[w]\rangle_{SM}\}_w \Longrightarrow \{|V_{cb}|, \langle B|O_k^{(n)}|B\rangle, m_b, m_c\}.$$

- Currently, the most precise fits use moments of E_l and r^2 up to $\mathcal{O}\left((\Lambda/m_b)^3\right)$. The most accurate determination of $|V_{cb}|$ is [arXiv:2107.00604v2]:

$$|V_{cb}|_{incl} = (42.16 \pm 0.51) \times 10^{-3}.$$

- Reparametrization (RP) invariance can be used to eliminate some non-perturbative operators from the OPE [T. Mannel, K. K. Vos, JHEP 06 (2018) 115].
- The choice of the 4-velocity v in the definition of $b_v(x)$ is arbitrary, as long as $k_b \equiv p_b - m_b v$ is $\mathcal{O}(\Lambda)$.
- The RP transformation $v \to v + \delta v$ does not change the tensors $W^{\alpha\beta}$ and $L_{\alpha\beta}$.
- Moments $\langle M[w] \rangle$ are RP invariant as long as weight function is RP invariant:

$$w[v, k_l, k_{\bar{\nu}}] = w[k_l, k_{\bar{\nu}}].$$

- Examples of RP invariant (RPI) quantities: Γ, $\langle (q^2)^k \rangle$, $d\Gamma/dq^2$, ...
- Moments and spectra in E_l and r^2 are not RPI.
- At the tree level, all terms of the OPE take the form

$$\sum_{n=const} \frac{C_k[w] \, \langle B | O_k^{(n)} | B \rangle}{m_b^{n(k)}} = \frac{C_n^{\mu_1 \dots \mu_n}[w] \, \langle B | \bar{b}_v iD_{\mu_1} \dots iD_{\mu_n} b_v | B \rangle}{m_b^n}.$$

The RP invariance implies relations between coefficients $C_n[w]$:

$$\delta_{RP} w = 0 \Longrightarrow \delta_{RP} C_n^{\mu_1 \dots \mu_n}[w] = m_b \delta v_\alpha \left(C_{n+1}^{\alpha\mu_1 \dots \mu_n}[w] + C_{n+1}^{\mu_1 \alpha \dots \mu_n}[w] + \dots + C_{n+1}^{\mu_1 \dots \mu_n \alpha}[w] \right).$$

- These relations can be solved order by order in n to find redundant matrix elements in the OPE.

$\mathcal{O}\left((\Lambda/m_b)^n\right)$	New (tree level) RPI matrix elements
2	$\langle B\vert\bar{b}_v b_v\vert B\rangle \equiv 2m_B\mu_3 \equiv 2m_B\left(1 - \frac{1}{2m_b^2}(\mu_\pi^2 - \mu_G^2)\right)$ $\langle B\vert\bar{b}_v(iD_\alpha)(iD_\beta)(-i\sigma^{\alpha\beta})b_v\vert B\rangle \equiv 2m_B\mu_G^2$
3	$\langle B\vert\bar{b}_v\left[(iD_\mu),\left[\left(ivD + \frac{1}{2m_b}(iD)^2\right),(iD)^\mu\right]\right]b_v\vert B\rangle \equiv 4m_B\rho_D^3$
4	$\langle B\vert\bar{b}_v[(iD_\mu),(iD_\nu)][(iD^\mu),(iD^\nu)]b_v\vert B\rangle \equiv 2m_B r_G^4$ $\langle B\vert\bar{b}_v[(ivD),(iD_\mu)][(ivD),(iD^\mu)]b_v\vert B\rangle \equiv 2m_B r_E^4$ $\langle B\vert\bar{b}_v[(iD_\mu),(iD_\alpha)][(iD^\mu),(iD_\beta)](-i\sigma^{\alpha\beta})b_v\vert B\rangle \equiv 2m_B s_B^4$ $\langle B\vert\bar{b}_v[(ivD),(iD_\alpha)][(ivD),(iD_\beta)](-i\sigma^{\alpha\beta})b_v\vert B\rangle \equiv 2m_B s_E^4$ $\langle B\vert\bar{b}_v[iD_\mu,[iD^\mu,[iD_\alpha,iD_\beta]]](-i\sigma^{\alpha\beta})b_v\vert B\rangle \equiv 2m_B s_{qB}^4$

- Considering RPI quantities reduces the number of fitting parameters at each order of the OPE.
- This can be used to improve fit precision up to $\mathcal{O}\left((\Lambda/m_b)^3\right)$ or extract the value of $|V_{cb}|$ with corrections up to $\mathcal{O}\left((\Lambda/m_b)^4\right)$.

$\mathcal{O}\left((\Lambda/m_b)^n\right)$	# of operators with RPI	# of operators without RPI
0	0	0
1	0	0
2	2	2
3	3	4
4	8	13

- The first extraction of inclusive $|V_{cb}|$ from q^2 moments was published in May 2022 and reads [arXiv:2205.10274v1]:

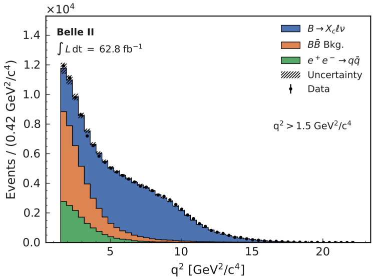
$$|V_{cb}|_{incl} = (41.69 \pm 0.63) \times 10^{-3}.$$

- It includes corrections up to $\mathcal{O}\left((\Lambda/m_b)^4\right)$, as well as most of the available perturbative QCD corrections to coefficients $C_n[w]$:

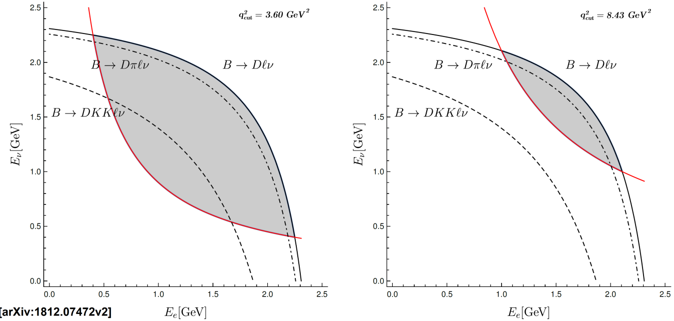
Γ	α_s^0	α_s^1	α_s^2	α_s^3
$n=0$	✓	✓	✓	✓
$n=2$	✓	✓		
$n=3$	✓	✓		
$n=4$	✓			

$\langle (q^2)^{k\leq 4}\rangle$	α_s^0	α_s^1	α_s^2	α_s^3
$n=0$	✓	✓	●	●
$n=2$	✓	●		
$n=3$	✓	●		
$n=4$	✓			

$\langle (q^2)^{k>4}\rangle$	α_s^0	α_s^1	α_s^2	α_s^3
$n=0$	✓	✓		
$n=2$	✓	●		
$n=3$	✓	●		
$n=4$	✓			

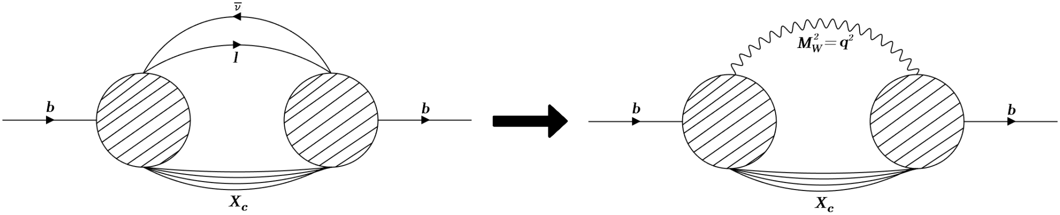


- On the experimental side of the fit, the recent results of q^2 spectrum from Belle [arXiv:2109.01685v2] and Belle II [arXiv:2205.06372v1] were used.
- Measurements of $B \to X_c l \bar{\nu}$ are not sensitive to events with small lepton energies.



[arXiv:1812.07472v2]

- Imposing a lower bound on $q^2 \geq q^2_{cut}$ results in a cut on E_l.
- The lower bound is from the relation $q^2_{cut} \leq 4E_l E_{\bar{\nu}}$.
- The upper from $m^2_{X_c} \leq m^2_B - 2m_b(E_l + E_{\bar{\nu}}) + 4E_l E_{\bar{\nu}}$
  - Choosing $q^2_{cut} \geq q^2_{cr}(E_{cut}) = 2E_{cut}m_B - 2E_{cut}m^2_D/(m_B - 2E_{cut})$ eliminates all events with $E_l < E_{cut}$.
  - One has to extrapolate experimental results to small lepton energies or impose a proper q^2 cut in the theoretical calculation.

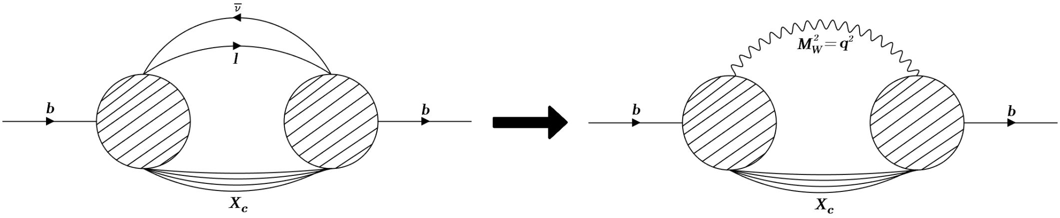


- Knowledge of the whole spectrum $d\Gamma/dq^2$ allows for a straightforward extraction of q^2 moments and inclusion of a q^2 cut.
- Currently, at partonic level, the spectrum is known only up to $\mathcal{O}(\alpha_s^1)$.
- The transverse structure of

$$\frac{d\Gamma}{dq^2 dr^2} \propto G_F^2 |V_{cb}|^2 \frac{|\vec{q}|}{3} (q_\alpha q_\beta - q^2 g_{\alpha\beta}) W^{\alpha\beta}$$

can be reproduced by polarization vectors ε_μ of an auxiliary final state W-boson with $M_W^2 = q^2$:

$$\sum_{polarizations} \varepsilon_\alpha \varepsilon_\beta = \frac{q_\alpha q_\beta - q^2 g_{\alpha\beta}}{q^2} \propto \int dE_l L_{\alpha\beta}.$$

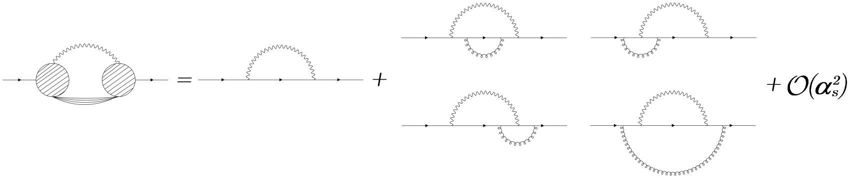


- Comparing the $d\Gamma/(dq^2 dr^2)$ spectrum of the $B \to X_c l \bar{\nu}$ decay with the spectrum $d\Gamma_W/dr^2$ of the $B \to X_c W$ yields a relation:

$$\frac{d\Gamma}{dq^2} = \frac{1}{48\pi^2} \frac{q^2}{M^4_{W(SM)}} \Gamma_W,$$

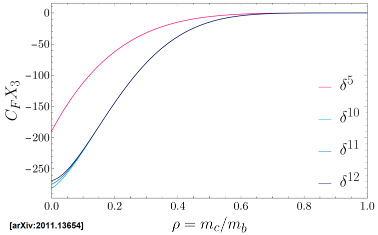
where the numerical pre-factor comes from different phase space volumes of the two processes.

- The replacement $(B \to X_c l \bar{\nu}) \longrightarrow (B \to X_c W)$ allows one to retain the q^2 dependence of the process that would normally be integrated out when using the optical theorem.
- In this way, the spectrum can be computed from standard Feynman diagrams, without inclusion of an additional delta function in the phase space.
- Additionally, the lepton loop is integrated out for the price of an additional scale q^2.

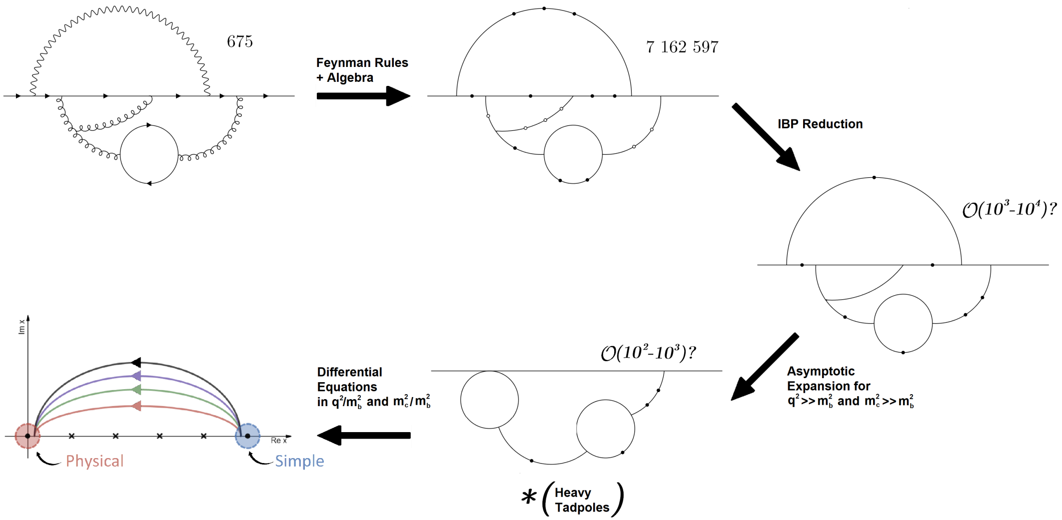


- The rate Γ_W is computed in perturbative QCD in powers of α_s.
- At order α_s^k one has to compute $k+1$-loop integrals with 3 mass scales: m_b, m_c, and q^2 (assuming $m_u = m_d = m_s = 0$).

- It was observed in [arXiv:0810.0543v1] that an asymptotic expansion in $\delta \equiv 1 - m_c/m_b \approx 0.7$ converges rapidly. Employing it at the amplitude level reduces the number of mass scales to 2.
- This can aid the IBP reduction but makes further asymptotic expansion more difficult.



[arXiv:2011.13654]



675
Feynman Rules
+ Algebra
7 162 597
IBP Reduction
$\mathcal{O}(10^3\text{-}10^4)$?
Asymptotic
Expansion for
$q^2 >> m_b^2$ and $m_c^2 >> m_b^2$
$\mathcal{O}(10^2\text{-}10^3)$?
* (Heavy Tadpoles)
Differential
Equations
in q^2/m_b^2 and m_c^2/m_b^2
Im x
Re x
Physical
Simple

Summary

- The CKM matrix describes mixing of different quark flavors. Values of its elements are not predicted by the SM but are extracted from fits to experimental data.
- The value of $|V_{cb}|$ is among the main sources of theoretical uncertainty of observables sensitive to NP.
- $|V_{cb}|$ is extracted from semileptonic decays of the B mesons.
- The inclusive rate of $B \to X_c l\bar{\nu}$ decay is computed using the OPE.
- The number of non-perturbative matrix elements in the OPE is reduced when one considers RPI quantities.
- The q^2 spectrum can be computed using the replacement $l\bar{\nu} \to W$, where $M_W^2 = q^2$.
- Computation of $\mathcal{O}(\alpha_s^2)$ and $\mathcal{O}(\alpha_s^3)$ perturbative corrections to $d\Gamma/dq^2$ is in progress.