Gravity as Portal to (almost) Everything!

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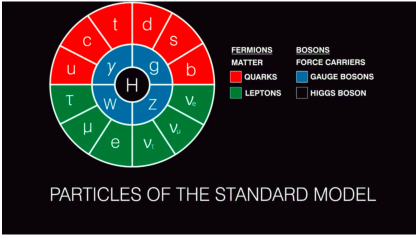
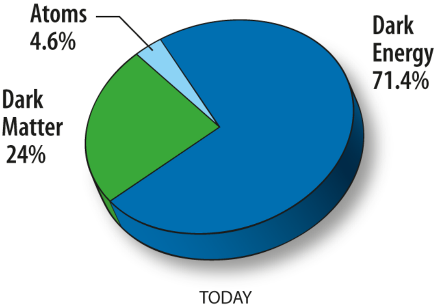
(based on arxiv: 2210.05716, in collaboration with S.Cléry, R.T.Co, Y.Mambrini & K.A. Olive)

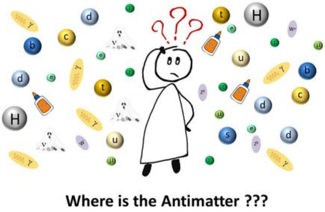
Plan

- A brief introduction
  - Beyond the realms of the SM
- The Framework
  - The underlying model
  - Gravitational production channels
- Detectable Primordial Gravitational Wave (PGW)
- Case of a stable and a decaying Dark Matter (DM)
  - *along with BAU*
  - Explaining IceCube excess
- Light DM via gravity portal
- Conclusion

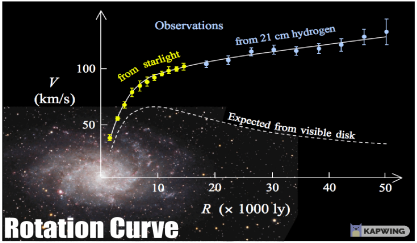
Why go beyond the Standard Model?

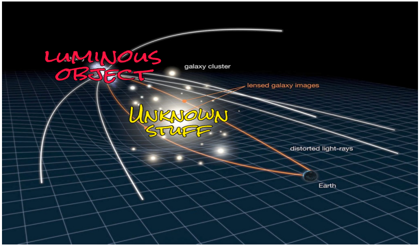


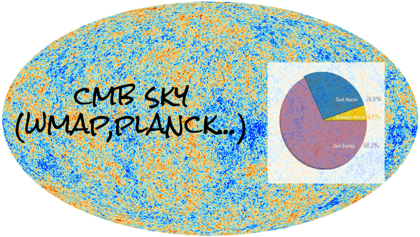




Here and there be Dark Matter!





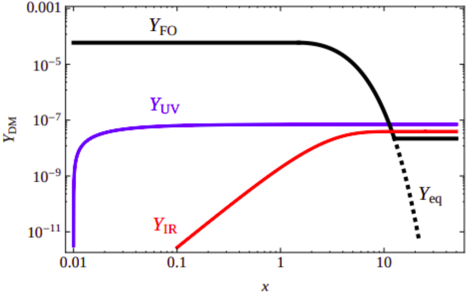


- ✓ Couples very weakly to photons
- ✓ Massive
- ✓ Still around today (stable)

- SM ν?
  - too light to satisfy abundance
  - too hot to form structures

→ Motivation to go beyond the SM

- $T \gg m_X$: DM in thermal equilibrium
- Before $n_X \to 0$, DM is rescued by **freeze-out** $\Gamma_{\text{int}} < H$.
- Weak scale interaction accounts for DM relic

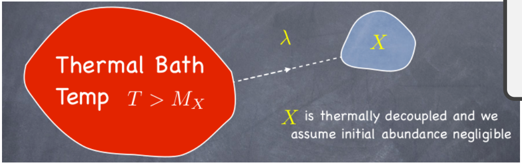


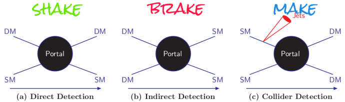
- Canonical WIMP scenario getting cornered by experiments
- A possible alternative:

  Freeze-in (0911.1120 ,1410.6157...).

  IR: $\frac{n_{\text{DM}}}{s} \equiv Y_{\text{DM}} \propto \left(\frac{\lambda}{10^{-10}}\right)^2 \frac{M_P}{T} \implies$ favors low T

  UV: $Y_{\text{DM}} \propto \frac{M_P T^{2n-1}}{\Lambda^{2n}} \implies$ favors high T





Matter over anti-matter

- Independent observations from BBN and CMB

$$Y_{\Delta B} \equiv \frac{n_B - n_{\bar{B}}}{s} \sim 8 \times 10^{-11}$$

- Dynamical generation of BAU: Sakharov conditions Sakharov (JETP Lett.5(1967)24)

  $\rightarrow$ $\not{B}$, ($\not{C}$; $\not{CP}$) , Departure from thermal equilibrium

✓ B-number is an *accidental symmetry* in the SM

⊗ $\not{CP} \implies$ Jarlskog invariant PRL55:1039,1985 $\sim 10^{-20}$ is *too small*

⊗ EWPT is *not strongly* 1^{st} *order* (1206.2942)

- A possible way:

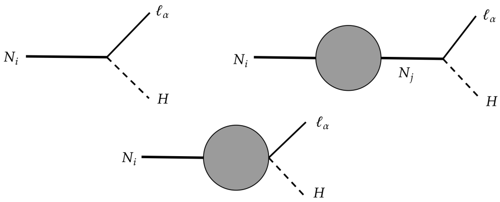
  $\rightarrow$ Baryogenesis via Leptogenesis (Fukugita,Yanagida PLB174:45,1986 ,hep-ph/0401240 ,0802.2962) $\rightarrow$ connects ν-mass with BAU

Matter over anti-matter

- Type-I Seesaw:

$$\mathcal{L} \supset -\frac{1}{2} \overbrace{M_N \, \overline{N^c} N}^{\text{Majorana mass}(\Delta L=2)} - \overbrace{y_N}^{\text{complex Yukawa}} \, \overline{N} \, \widetilde{H}^\dagger \, L + \text{h.c.} \, .$$

- CP-asymmetry: interference between tree- & loop
- Out of equilibrium provided by expansion of the universe
- $Y_L \xrightarrow{\text{"sphaleron"}} Y_B (T \sim \mathcal{O}(10^2 \text{ GeV}))$



- Thermal: produced via scatterings in the thermal bath
  - *Hierarchical:* $M_1 \ll M_{2,3}$
  - *Davidson-Ibarra* (*hep-ph/0202239*)

$$\epsilon \lesssim \frac{M_N \, m_\nu}{\langle H \rangle^2}$$

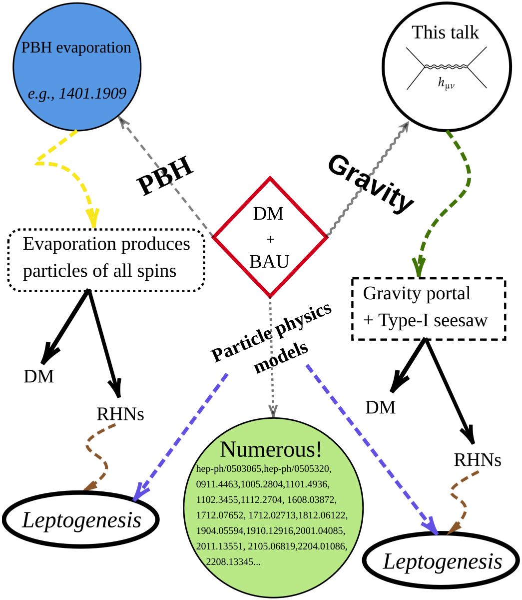
$$T_{rh} > M_N \gtrsim 10^8 - 10^{10} \text{ GeV}$$

$\rightarrow$ *gravitino overproduction*

- Non-thermal: $M_N > T_{\rm rh}$
  - *Produced from perturbative inflaton decay* $\phi \rightarrow N N$ (*PLB258(1991)305*)
  - *From preheating* (*hep-ph/9905242*)

$$\epsilon_i = \frac{\Gamma(N_i \to LH) - \Gamma(N_i \to \bar{L}H^\dagger)}{\Gamma(N_i \to LH) + \Gamma(N_i \to \bar{L}H^\dagger)} \sim \sum_{i \neq j} \frac{Im\left[\left(Y_\nu^\dagger Y_\nu\right)_{ij}^2\right]}{\left(Y_\nu^\dagger Y_\nu\right)_{ii}} \frac{M_i}{M_j} \, ; \, M_i \ll M_j$$

2 birds, 1 (non-standard) stone



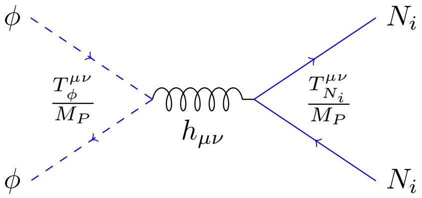
The Set-up

- SM+3 RHNs (Majorana)+1 scalar (inflaton)
- gravity portal arises from perturbation around flat metric

$$\eta_{\mu\nu} + h_{\mu\nu}/M_P$$

$$\to -\frac{1}{M_P} h_{\mu\nu} \left(\overbrace{T^{\mu\nu}_{SM}}^{\text{stress-energy}} + \underbrace{T^{\mu\nu}_{\phi}}_{\text{inflaton}} + \underbrace{T^{\mu\nu}_{X}}_{\text{DM}} \right)$$

- Consider *massless gravitons* and from $T_{\mu\nu}$ we can compute the amplitudes for the processes



- Potential for ϕ (T-model (1306.5220))

$$V(\phi) = \lambda M_P^4 \left| \sqrt{6} \tanh\left(\frac{\phi}{\sqrt{6} M_P} \right) \right|^k$$

$$\approx \lambda \phi^k / M_P^{k-4} \, ; \phi \ll M_P$$

- $w \equiv \frac{p_\phi}{\rho_\phi} = \frac{k-2}{k+2}$ is the general inflaton EOS
- CMB measurement at $k = k_\star$

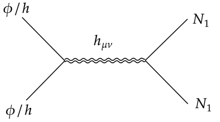
$$n_s \simeq 1 - 6\epsilon_\star + 2\eta_\star \, , r \simeq 16\epsilon_\star$$

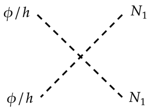
$$A_{S\star} \simeq \frac{V_\star}{24\pi^2 \epsilon_\star M_P^4} \to \lambda \simeq \frac{18\pi^2 A_{S\star}}{6^{k/2} N_\star^2}$$

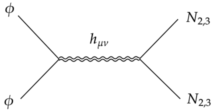
- Only renorm. interaction: $\mathcal{L} \supset -(y_N)_{ij} \, \overline{N}_i \, \widetilde{H}^\dagger \, L_j$
- No perturbative decay of ϕ into RHN
- ϕ interacts with any other matter field *only* gravitationally
- stable DM: $(y_N)_{i1} \to 0$, decaying DM: $(y_N)_{i1} \neq 0$

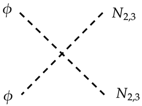
Gravitational Production

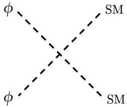
- DM: $\phi\,\phi \to N_1\,N_1$, SM SM $\to N_1\,N_1$
- (Non-thermal) Leptogenesis: $\phi\,\phi \to N_{2,3}\,N_{2,3}$
- Reheating: $\phi\phi \to$ SM SM











- *Graviton exchange processes* sufficient to reheat for $k > 9$ (2201.02348,2205.01689)
- Can be relaxed by adding *non-minimal contribution* to radiation production (but still $k > 4$ (1905.06823))
- $k > 4 \to w > 1/3 \implies$ *a stiff epoch*

$$\mathcal{L}^J_{\text{non-min}} \supset \left[-\frac{M_P^2}{2}\,\Omega^2\,\widetilde{\mathcal{R}} + \widetilde{\mathcal{L}}_\phi + \widetilde{\mathcal{L}}_h + \widetilde{\mathcal{L}}_{N_i} \right]$$

with $\Omega^2 \equiv 1 + \dfrac{\xi_\phi\,\phi^2}{M_P^2} + \dfrac{\xi_h\,|h|^2}{M_P^2}$

$|\xi_i|\chi^2/M_P^2 \ll 1$ (*leading order in small-field limit*)

$$\to \mathcal{L}^E_{\text{non-min}} \supset -\sigma^\xi_{hN}\,|h|^2\,N\,N - \sigma^\xi_{\phi N_i}\,\phi^2\,N\,N$$

$$\sigma^\xi_{\phi N} = \frac{M_{N_i}}{2M_P^2}\,\xi_\phi\,, \quad \sigma^\xi_{hN} = \frac{M_{N_i}}{2M_P^2}\,\xi_h$$

- small field limit: $\xi_\phi \ll 1\,,\ \mathcal{O}(1)(\mathrm{GW}) \lesssim \xi_h \lesssim \mathcal{O}(10^{15})$ (collider [1211.0281])

Primordial Gravitational Wave

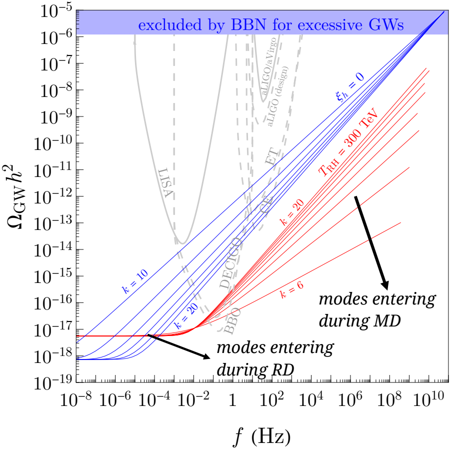
- *PGW is a crucial prediction of inflation*
- For horizon crossing happening during RD, PGW spectrum is *scale-invariant*
- The spectrum is *blue-tilted* for a stiff equation of state (gr-qc/0105121,1412.0743)

$$\Omega_{\rm GW} = \frac{1}{\rho_c}\frac{d\rho_{\rm GW}}{d\log k} \propto a^{3\,w-1}$$

- Excessive blue-tilting is BBN forbidden: *rules out* $\xi_h = 0$ (1811.04093,1905.06823)
- This is relaxed for non-minimal coupling
- PGW spectrum today

$$\Omega^{(0)}_{\rm GW} \propto \Omega^{(0)}_{\gamma} \begin{cases} 1\,, & f < f_{\rm MD} \\ f^{\frac{2\,(3w-1)}{(1+3w)}}\,, & f_{\rm MD} \le f \le f_e \\ 0\,, & f > f_e \end{cases}$$

- Slope of the spectrum can probe shape of the inflaton potential



Case of a stable DM: $(y_N)_{i1} \to 0$

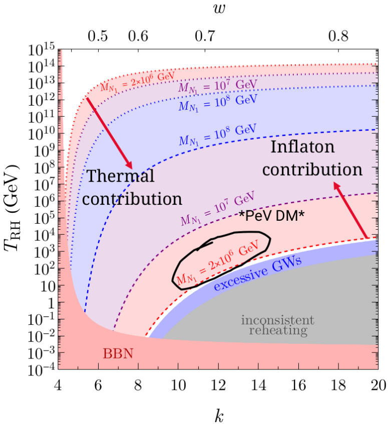
Fields	Z_2
N_1	$-$
$N_{2,3}$	$+$
SM	$+$

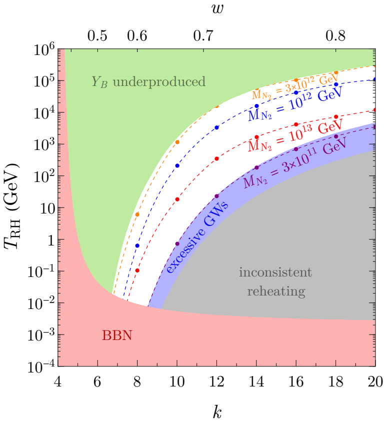
$$\Omega_{N_1} h^2 = \left(\Omega^T_{N_1} + \Omega^\phi_{N_1} \right) h^2 \sim 0.1$$

- $(y_N)_{i1} \to 0$: *e.g.*, Z_2-odd
- Relic is satisfied twice:
  - Low $T_{\rm rh}$: ϕ scattering dominates
  - High $T_{\rm rh}$: SM scattering dominates

$$Y_B \propto \epsilon_{\Delta L}\, n^\phi_{N_2}(T_{\rm rh})/s(T_{\rm rh}) \sim 10^{-11}$$

- Non-thermal leptogensis: N_2 produced *only* during reheating
- kinematical suppression: $M_{N_2} \to m_\phi$





Case of a decaying DM: $(y_N)_{i1} \neq 0$

- "PeV excess" @ IceCube: 1.04, 1.14, 2.2 PeV (1304.5356)
- Maximum energy observed @ IceCube $E_\nu^{\mathrm{max}} \approx m_{\mathrm{DM}}/2$ can be explained by decaying DM (1308.1105,1311.5864,1410.5979,1607.05283...)
- DM lifetime with simplest Yukawa $y_{N_1} \bar{L}\, H\, N_1$

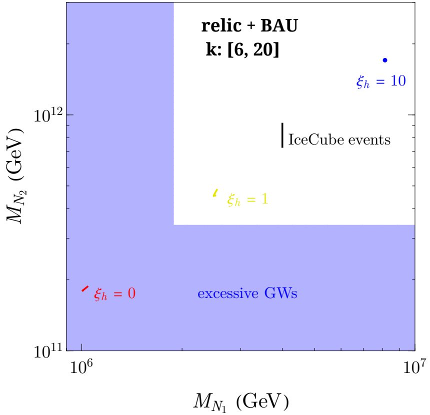
$$\tau \simeq 10^{28}\,\mathrm{s} \left(\frac{4\times 10^{-29}}{y_{N_1}}\right)^2 \left(\frac{1\,\mathrm{PeV}}{M_{N_1}}\right)$$

- Freeze-in production via same needs

$$\Omega_{N_1} h^2 \simeq 0.12 \left(\frac{y_{N_1}}{1.2\times 10^{-12}}\right)^2 \left(\frac{M_{N_1}}{1\,\mathrm{PeV}}\right)$$

- minimal $d = 4$ operator is insufficient (alternatives:1606.04517,1607.05283,2206.12910...)
- Gravity portal naturally provides PeV-scale DM

$k = 8$			
ξ_h	T_{rh}	$\frac{M_{N_1}}{\mathrm{PeV}}$	$\frac{M_{N_2}}{\mathrm{GeV}} \times 10^{11}$
1	0.0084 (excluded)	–	–
2.5	0.11	4.0⋆	7.3
10	8.1	8.1	17



A second avatar

- Strong dependence on RHN mass can be overcome considering an intermediate state

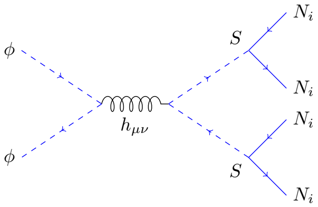
$$\mathcal{L} \supset -y_R^i \, S \, \overline{N_i^c} \, N_i + \text{h.c.}$$

- Each S decaying into 2 RHNs with branching

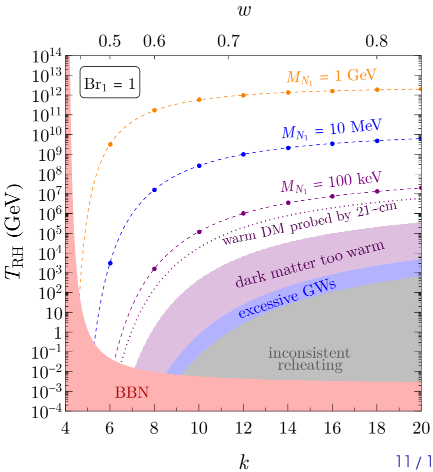
$$\text{Br}_i = \frac{M_{N_i}^2}{M_{N_1}^2 + M_{N_2}^2 + M_{N_3}^2}$$

$\rightarrow R_{N_1}^{\phi S} / R_{N_1}^{\phi} \sim \text{Br}_1 \, m_\phi^2 / M_{N_1}^2$

$\rightarrow$ DM mass is much lower if Br is large



- DM could be $\sim \mathcal{O}$ (keV) such that *warm* DM limit applies
  - $\rightarrow$ *e.g.*, $m_{\text{DM}} \gtrsim 4$ keV for thermal relic (1512.01981 ,2209.14220)



A second avatar

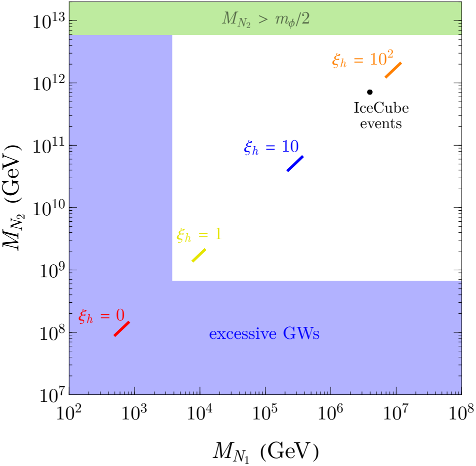
$k = 8$			
ξ_h	T_{rh}	M_{N_1}	$M_{N_2} \times 10^{11}$
1	0.0084 (excluded)	–	–
10	8.1	220 TeV	0.4
68	2.6×10^3	$4.0\star$	7.1
100	8.1×10^3	7.1 PeV	13

- Inflaton contribution dominates over *thermal* for spin-0 production (2112.14668, 2112.15214)

$$R_0^{\phi^k}/R_0^T\Big|_{a=a_{\text{max}}} \sim (\rho_e/\rho_{\text{RH}})^{2/k} \gg 1$$

$$\left[\text{for spin } 1/2 \sim \left(M_N/m_\phi\right)^2 (\rho_e/\rho_{\text{RH}})^{2/k}\right]$$

- Kinematically: $2m_\phi > 4M_N$
- Opens up lower mass and larger ξ_h



Remarks

- A pure gravitational origin of DM+BAU+(gravitational) reheating
- GW overproduction rules out minimal coupling demanding non-minimality
- Possible explanation to IceCube high energy neutrino events via decaying DM
- Testable in future GW detectors because of blue-tilted PGW spectrum

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Thank you for your attention!

Backup Slides

Production Rates

Generic solution to inflaton EoM

$$\ddot{\phi} + 3H\dot{\phi} + V_{,\phi}(\phi) = 0$$
$$\rightarrow \phi(t) = \varphi(t)\,.\mathcal{P}(t)$$

- Rapid oscillations of $\mathcal{P}(t)$ are damped by decreasing envelope $\varphi(t)$ due to redshift
- Period of oscillation

$$\mathcal{T} = 2\pi/\omega\,, \omega = m_\phi \sqrt{\frac{\pi\, k}{2\,(k-1)}}\,\Gamma\left(\frac{1}{2}+\frac{1}{k}\right) \Big/ \Gamma\left(1/k\right)$$

- Each Fourier mode contributes to scattering amplitude with energy $E_n = n\,.\omega$
- Thermal rate

$$R_j^T = \beta_j \frac{T^8}{M_P^4}\text{ , for spin 'j' final states}$$

- Inflaton rate

$$R_\phi^0 = \frac{\rho_\phi^2}{256\pi\, M_P^4} \sum_n \left(1 + 2\,\frac{m_X^2}{E_n^2}\right)^2 |(\mathcal{P}^k)_n|^2 \left[1 - \frac{4m_X^2}{E_n^2}\right]^{1/2}$$

$$R_\phi^{1/2} = \frac{\rho_\phi^2}{256\pi\, M_P^4} \sum_n \frac{m_X^2}{E_n^2} |(\mathcal{P}^k)_n|^2 \left[1 - \frac{4m_X^2}{E_n^2}\right]^{3/2}$$

BEQs

EoM of ϕ

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0$$

Averaging over oscillation leads to

$$\begin{aligned}
&\langle \dot{\phi}^2 \rangle \simeq \langle \phi V'(\phi) \rangle \\
&\implies \rho_\phi \simeq \frac{1}{2}\langle \dot{\phi}^2 \rangle + \langle V(\phi) \rangle \simeq \frac{k+2}{2}\langle V(\phi) \rangle = V(\varphi) \\
&\& \quad p_\phi \simeq \frac{1}{2}\langle \dot{\phi}^2 \rangle - \langle V(\phi) \rangle \simeq \frac{k-2}{2}\langle V(\phi) \rangle = \frac{k-2}{k+2} V(\varphi)
\end{aligned}$$

EOS: $w = p/\rho = (k-2)/(k+2)$

$$\begin{aligned}
&\dot{\rho}_\phi + 3H(1+w_\phi)\rho_\phi \simeq -\Gamma_\phi (1+w_\phi)\rho_\phi \\
&\dot{\rho}_R + 4H\rho_R \simeq +(1+w_\phi)\Gamma_\phi \rho_\phi
\end{aligned}$$

Energy-momentum tensors

$$T_0^{\mu\nu} = \partial^\mu S \partial_\mu S - g^{\mu\nu}\left[\frac{1}{2}\,\partial^\alpha S \partial_\alpha S - V(S)\right]$$

$$T_{1/2}^{\mu\nu} = \frac{i}{8}\left[\bar{\chi}\gamma^\mu \overset{\leftrightarrow}{\partial^\nu}\chi + \bar{\chi}\gamma^\nu \overset{\leftrightarrow}{\partial^\mu}\chi\right] - g^{\mu\nu}\left[\frac{i}{4}\bar{\chi}\gamma^\alpha \overset{\leftrightarrow}{\partial_\alpha}\chi - \frac{m_\chi}{2}\,\overline{\chi^c}\chi\right]$$

Graviton propagator

$$\Pi^{\mu\nu\rho\sigma}(p) = \frac{\eta^{\rho\nu}\eta^{\sigma\mu} + \eta^{\rho\mu}\eta^{\sigma\nu} - \eta^{\rho\sigma}\eta^{\mu\nu}}{2p^2}\,,$$

in harmonic (de Donder) gauge where $\partial^\mu\, h_{\mu\nu} = \frac{1}{2}\,\partial_\nu\,\mathrm{Tr}\,(h_{\mu\nu})$

Number densities

- Minimal gravitational production of RHN

Thermal: $$n_{N_i}^{T}(a_{\rm RH}) \simeq \frac{\beta_{1/2}\,(k+2)\,\rho_{\sf RH}^{\frac{3}{2}}}{12\,\sqrt{3}M_P^3\,c_*^2}\,\frac{2(7-4k)^2}{(k+5)(k-1)(5k-2)}\,\left(a_{\sf RH}/a_e\right)^{\frac{10+2k}{k+2}}$$

Inflaton: $$n_{N_i}^{\phi^k}(a_{\rm RH}) \simeq \frac{M_{N_1}^2\,\sqrt{3}\,(k+2)\,\rho_{\rm RH}^{\frac{1}{2}+\frac{2}{k}}}{24\,\pi\,k(k-1)\lambda^{\frac{2}{k}}\,M_P^{1+\frac{8}{k}}}\left(\frac{\rho_{\rm end}}{\rho_{\rm RH}}\right)^{\frac{1}{k}}\,\Sigma_{1/2}^k$$

- Non-minimal gravitational production of RHN

Thermal: $$n_{N_i}^{T(\xi_h\neq 0)} = \left(\frac{\sqrt{3}\,N_h\,\zeta(3)^2\,\xi_h^2}{32\,\pi^5\,c_*^{3/2}}\,\frac{M_{N_i}^2\,\rho_{\sf RH}}{M_P^3}\right)\frac{(k+2)\,\left(1-\left(\frac{\rho_e}{\rho_{\sf RH}}\right)^{\frac{7}{3\,k}-\frac{4}{3}}\right)^{-3/2}}{72\,(5-4k)\,\Gamma\left(\frac{29-20k}{14-8k}\right)}$$

$$\times\left[9\sqrt{\pi}(5-4k)\left(\frac{\rho_e}{\rho_{\sf RH}}\right)^{1/k}\Gamma\left(\frac{4k-4}{4k-7}\right)+4\left(\frac{\rho_e}{\rho_{\sf RH}}\right)^{\frac{16k^2+4k+169}{21k-12k^2}}\Gamma\left(\frac{29-20k}{14-8k}\right)\,\mathcal{G}\right]$$

- Gravitational production of scalar

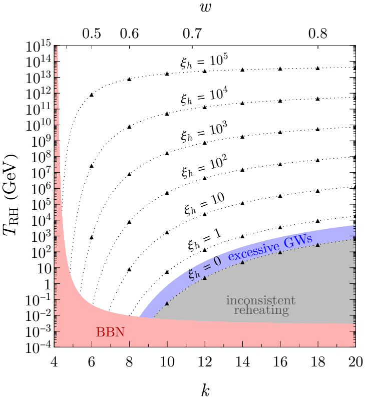
Inflaton: $$n_{N_i}^{S\phi^k}(a_{\rm RH}) \simeq {\sf Br}_i \times \frac{\sqrt{3}\rho_{\rm RH}^{3/2}}{4\pi M_P^3}\,\frac{k+2}{6k-6}\left(\frac{\rho_{\rm end}}{\rho_{\rm RH}}\right)^{1-\frac{1}{k}}\,\Sigma_0^k$$

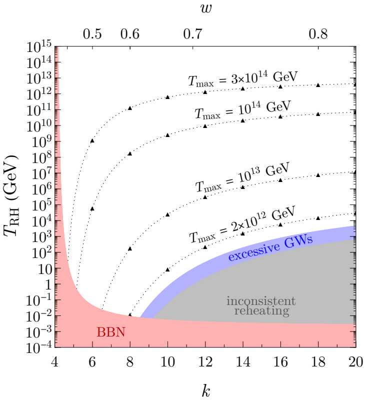
BBN bound on reheating temperature

$\rho_\phi = \rho_R$ at $T = T_{\rm rh}$

$$\implies 1 = \frac{\rho_\phi}{\rho_R}\bigg|_{T_{\rm rh}} = \frac{\rho_\phi}{\rho_R}\bigg|_{T_{\rm BBN}} \frac{(a_{\rm BBN}/a_{\rm rh})^{\frac{6k}{k+2}}}{(T_{\rm rh}/T_{\rm BBN})^4}$$

$$\rho_\phi(T_{\rm BBN}) = \frac{7}{8}\left(\frac{4}{11}\right)^{\frac{4}{3}} \underbrace{\Delta N_\nu}_{<0.226} \frac{\pi^2}{30} T_{\rm BBN}^4$$





Interactions

$$\mathcal{S}_J = \int d^4x\sqrt{-\tilde{g}}\left[-\frac{M_P^2}{2}\,\Omega^2\,\widetilde{\mathcal{R}} + \widetilde{\mathcal{L}}_\phi + \widetilde{\mathcal{L}}_h + \widetilde{\mathcal{L}}_{N_i}\right]$$

where

$$\widetilde{\mathcal{L}}_\phi = \frac{1}{2}\,\partial_\mu\phi\,\partial^\mu\phi - V(\phi)$$

$$\widetilde{\mathcal{L}}_h = \partial_\mu h\,\partial^\mu h^\dagger - V(hh^\dagger)$$

$$\widetilde{\mathcal{L}}_{N_i} = \frac{i}{2}\,\overline{\mathcal{N}_i}\,\overleftrightarrow{\tilde{\nabla}}\,\mathcal{N}_i - \frac{1}{2}\,M_{N_i}\,\overline{(\mathcal{N})^c}_i\,\mathcal{N}_i + \widetilde{\mathcal{L}}_{\text{yuk}}$$

$$\widetilde{\mathcal{L}}_{\text{yuk}} = -y_{N_i}\,\overline{\mathcal{N}_i}\,\widetilde{h^\dagger}\,\mathbb{L} + \text{h.c.}$$

- Conformal transformation: $\Omega^2 \equiv 1 + \frac{\xi_\phi\,\phi^2}{M_P^2} + \frac{\xi_h\,|h|^2}{M_P^2} \implies g_{\mu\nu} = \Omega^2\,\tilde{g}_{\mu\nu}$

$$\mathcal{S}_E = \int d^4x\sqrt{-g}\Bigg[-\frac{M_P^2\,\mathcal{R}}{2} + \frac{K^{ab}}{2}\,g^{\mu\nu}\partial_\mu S_a\,\partial_\nu S_b + \frac{i}{2\,\Omega^3}\,\overline{N_i}\,\overleftrightarrow{\nabla}\,N_i - \frac{1}{\Omega^4}\left(\frac{M_{N_i}}{2}\,\overline{N_i^c}\,N_i + \mathcal{L}_{\text{yuk}}\right) - \frac{3i}{4\Omega^4}\,\overline{N_i}\left(\overleftrightarrow{\not{\partial}}\,\Omega\right)N_i - \frac{1}{\Omega^4}\,(V_\phi + V_h)\Bigg],$$

- Field re-definitons: $L \to \Omega^{3/2}\,L\,,\,N \to \Omega^{3/2}\,N$

$$\mathcal{S}_E = \int d^4x\sqrt{-g}\Bigg[-\frac{M_P^2\,\mathcal{R}}{2} + \underbrace{\frac{K^{ab}}{2}\,g^{\mu\nu}\partial_\mu S_a\,\partial_\nu S_b}_{\text{non-canonical kinetic term}} - \frac{1}{\Omega^4}\,(V_\phi + V_h) + \frac{i}{2}\,\overline{N_i}\,\overleftrightarrow{\nabla}\,N_i + \frac{1}{\Omega}\,\mathcal{L}_{\text{yuk}}\Bigg]$$

Interactions

- Inflaton mass

$$m_\phi^2(k) \equiv \frac{\partial^2 V}{\partial \phi^2} = \lambda^{2/k}\, k\,(k-1)\, M_P^2 \left(\frac{\rho_\phi}{M_P^4}\right)^{1-\frac{2}{k}}$$

$$m_\phi^2\Big|_{a_e} \propto \left(\frac{\rho_e^\phi}{M_P^4}\right)^{1-\frac{2}{k}}, \rho_e^\phi \simeq V(\phi_e) \text{ with } \phi_{\text{end}} \simeq \sqrt{\frac{3}{8}} M_P \ln\left[\frac{1}{2} + \frac{k}{3}\left(k + \sqrt{k^2+3}\right)\right]$$

- Reheating temperature

$$T_{\mathsf{RH}}^4 = \frac{30}{\pi^2\, g_{\rm RH}}\, M_P^4 \left(\frac{\rho_e}{M_P^4}\right)^{\frac{4k-7}{k-4}} \left(\frac{\alpha_k(\xi)\,\sqrt{3}\,(k+2)}{8k-14}\right)^{\frac{3k}{k-4}}$$

- Maximum temperature

$$\rho_{\mathsf{max}} \simeq \sqrt{3}\,\alpha_k(\xi)\, M_P^4 \left(\frac{\rho_{\rm end}}{M_P^4}\right)^{\frac{2k-1}{k}} \frac{k+2}{12k-16} \left(\frac{2k+4}{6k-3}\right)^{\frac{2k+4}{4k-7}} \equiv c_*\, T_{\rm max}^4$$

Derivation of $\Omega_{\rm GW}$

- Assuming "hc" occurs during inflaton-domination

$$\Omega_{\sf GW}(\tau,k) = \frac{1}{12 a(\tau)^2\, H(\tau)^2}\, \mathcal{P}_T(k)\, \left[\chi(\tau,k)'\right]^2$$

$$= \frac{1}{12 a(\tau)^2\, H(\tau)^2}\, \mathcal{P}_T(k)\, \frac{k_{gw}^2}{2} \left(\frac{a_{\rm hc}}{a_0}\right)^2$$

$$= \frac{\Omega_\gamma\, h^2}{3}\, \frac{\rho_\phi}{\rho_{\rm RH}} \left[\frac{g_{\star\rho,rh}}{2} \left(\frac{g_{\star\rho,rh}}{g_{\star s,dec}}\right)^{-\frac{4}{3}} \left(\frac{a_{\rm hc}}{a_{\rm rh}}\right)^4\right] \left(\frac{H_{\rm end}}{2\pi\, M_P}\right)^2$$

$$f = \frac{k_{hc}}{2\pi a_0} = \frac{a_{\rm hc}\, H_{hc}}{2\pi a_0} = \frac{a_{\rm hc}}{a_0}\, \frac{1}{2\pi}\, H_{rh} \sqrt{\frac{\rho_{\rm hc}}{\rho_{\rm RH}}} = \frac{\sqrt{\rho_{\rm RH}}}{2\pi\sqrt{3}\, M_P}\, \frac{a_{\rm hc}}{a_0} \left(\frac{a_{\rm rh}}{a_{\rm hc}}\right)^{3k/(k+2)}$$

Then

$$\Omega_{\rm GW}\, h^2 = \frac{\Omega_\gamma\, h^2}{3}\, \frac{g_{\star\rho,rh}}{2} \left(\frac{g_{\star\rho,rh}}{g_{\star s,dec}}\right)^{-\frac{4}{3}} \left(\frac{H_{\rm end}}{2\pi\, M_P}\right)^2 \left(\frac{a_0}{a_{\rm rh}}\right)^{\frac{k-4}{k-1}} \left(\frac{2\pi\sqrt{3}\, M_P}{\sqrt{\rho_{\rm RH}}}\, f\right)^{\frac{k-4}{k-1}}$$

- Assuming "hc" occurs during RD

$$\Omega_{\rm GW}\, h^2 = \frac{\Omega_\gamma\, h^2}{3}\, \frac{g_{\star\rho,rh}}{2} \left(\frac{g_{\star\rho,rh}}{g_{\star s,dec}}\right)^{-\frac{4}{3}} \left(\frac{H_{\rm end}}{2\pi\, M_P}\right)^2$$

$$\Longrightarrow\ \Omega_{\rm GW}\, h^2 \simeq \Omega_{\rm GW}\, h^2\Big|_{\rm RD} \times \mathcal{W}(f\,, w)$$