Large-invariant-mass photon pairs production in nucleon-photon scattering at next-to-leading order

Oskar Grocholski

Deutsches Elektronen-Synchrotron DESY

in collaboration with B. Pire, P. Sznajder, L. Szymanowski and J. Wagner.

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Background – Parton Distribution Functions

Deep Inelastic Scattering

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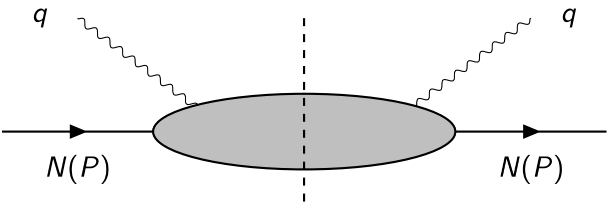
Optical theorem – relate inclusive cross-section to imaginary part of forward scattering amplitude:



Collinear Factorization

Large energy scale: $Q^2 = -q^2 \gg \Lambda^2$ – separate the large- and short- distance physics.

Infinite momentum frame:

- Introduce $\pm$ four-vector components $v^{\pm} = \frac{1}{\sqrt{2}}\left(1, 0, 0, \pm 1\right)^T$.
- Momentum of the nucleon: $\rightarrow p \approx p^+$
- Nucleon-photon interaction $\rightarrow$ sum of scattering amplitudes on a single parton (quark, gluon) carrying the fraction x of the total momentum.

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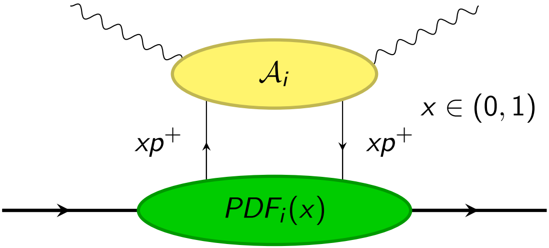

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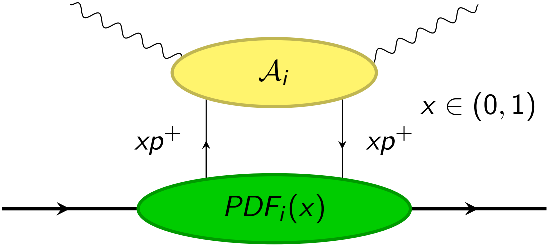

Full amplitude:

$$\sum_i PDF_i(x) \times \mathcal{A}_i(x, Q^2)$$

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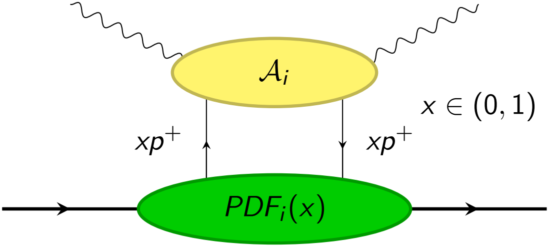

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PDFs defined using matrix elements:

$$\int \frac{dz^-}{2\pi} e^{ixp^+z^-} \langle N(\mathbf{p})| \bar{q}_f(-z/2)\Gamma q_f(z/2) |N(\mathbf{p})\rangle \Big|_{z=z^-} \quad (1)$$

Generalised Parton Distributions

Exclusive processes

$A + N \longrightarrow B + N$, all outgoing states measured.
The problem reduces to $\gamma^* + N \longrightarrow B + N$.

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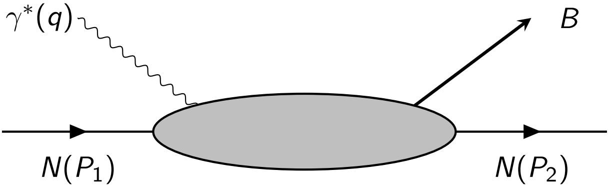

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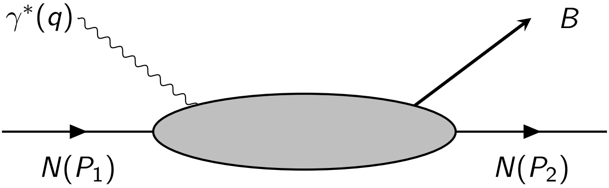

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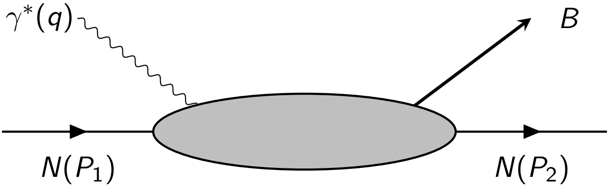

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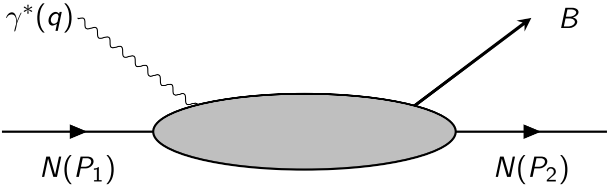

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- DVMP: $\gamma^* + N \longrightarrow N + M$

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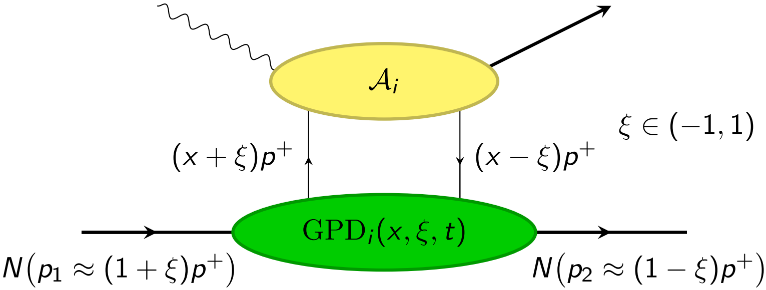

$$t = (p_1 - p_2)^2$$

Amplitude:

$$\mathcal{A} = \sum_i \int_{-1}^{1} dx\, \mathcal{A}_i(x,\xi,Q^2,\ldots) \times \mathrm{GPD}_i(x,\xi,t).$$

Study of exclusive processes

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A new process, which we consider: photoproduction of photon pairs with large invariant mass:

$$\gamma N \to \gamma\gamma N$$

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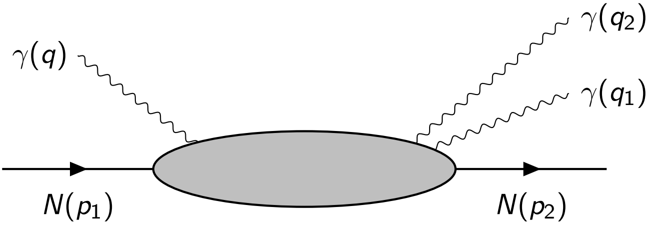
- $p\gamma \to p\gamma\gamma$ – the "theoretical laboratory" to study factorization in $2 \to 3$ reactions.
- Also phenomenologically interesting: the amplitude depends only on charge-odd combinations of GPDs (only valence quarks contribute).

Kinematics



$$S_{\gamma N} = (p_1 + q)^2, \qquad u' = (q_2 - q)^2,$$

$$M^2_{\gamma\gamma} = (q_1 + q_2)^2, \qquad t = (p_1 - p_2)^2.$$

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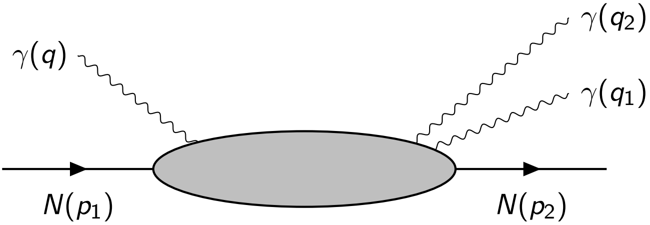

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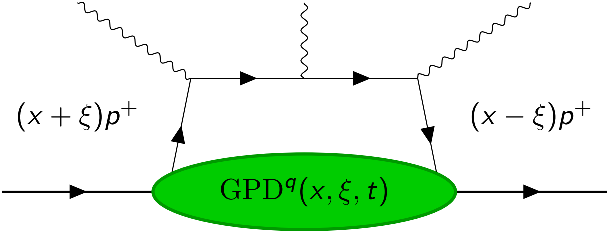
$$M^2_{\gamma\gamma} = (q_1 + q_2)^2, \qquad t = (p_1 - p_2)^2.$$

$$\xi \approx \frac{M^2_{\gamma\gamma}}{2S_{\gamma N}}.$$

The leading order analysis

Pedrak et al. Phys. Rev. D 96 (2017) [arXiv:1708.01043]



LO results: the process can be studied at intense quasi-real photon beam facilities in JLab or EIC.

Next-to-leading order diagrams

NLO factorization and the amplitude

Phys. Rev. D 104 (2021) [2110.00048]

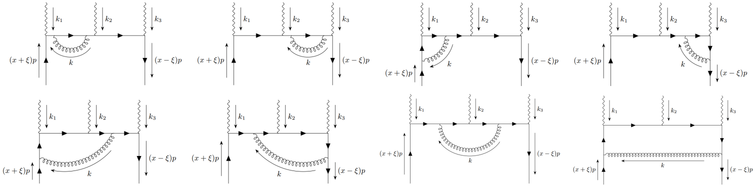
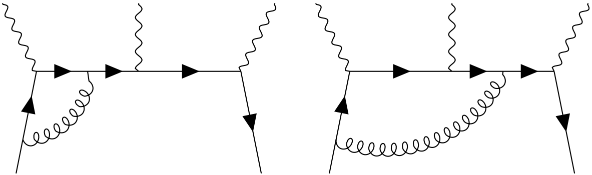


Figure: Considered 1-loop diagrams

Collinear divergences



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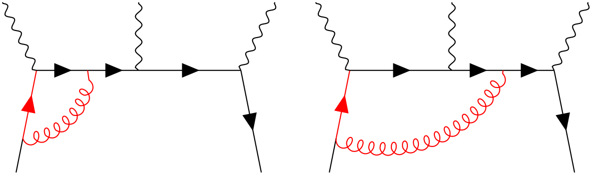

Momentum of the gluon k collinear with the one of the parton $(x \pm \xi)p$

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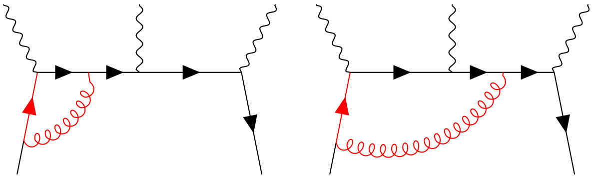

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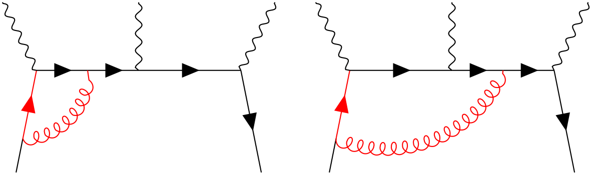

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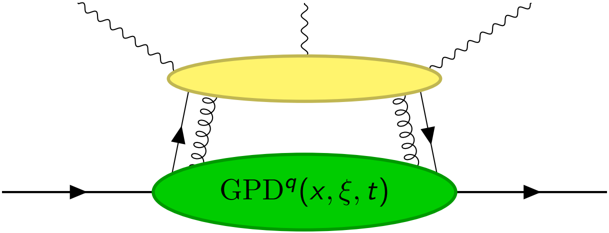
$\implies k^2 = 0$ and $\left((x \pm \xi)p + k\right)^2 = 0$

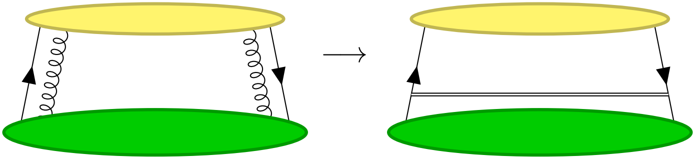
$\implies$ Singularities in loop momenta integrals

General form of a leading-power graph



Higher-order leading power graphs – can attach an arbitrary numbers of collinear, longitudinally polarized gluon to the hard part.

Collinear gluons



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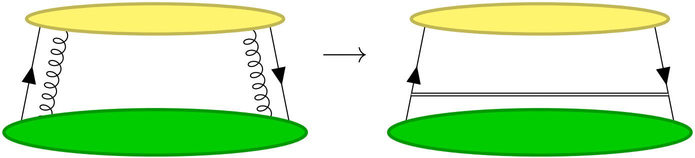
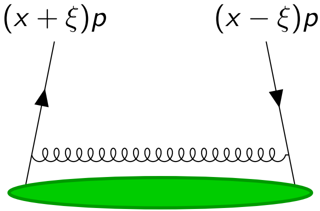


At leading-order in Q^2/Λ^2, the collinear gluons can be taken into accout by inserting the Wilson line operator between the parton field operators in the definition of GPDs $\rightarrow$ preserved form of the factorization formula, but GPDs acquire corrections.

Interactions between spectator partons



$\longrightarrow$

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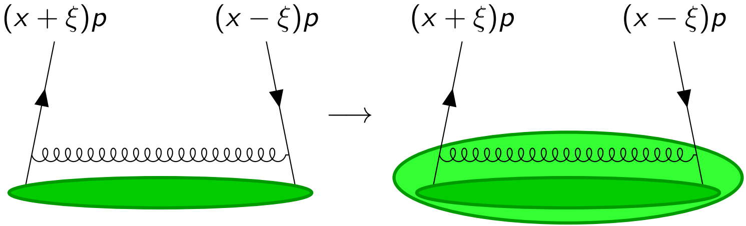

Factorization at NLO

After including the QCD corrections to GPD, we obtain:

$$\mathrm{GPD}^q(x,\xi,t) = \mathrm{GPD}^q_R(x,\xi,t;\mu_F) + \frac{\alpha_S}{2\pi}\left(-\frac{1}{\varepsilon} + \ln\frac{\mu_F^2}{\mu_R^2}\right)\int dx' K^{qq}(x,x',\xi)\mathrm{GPD}^q_R(x',\xi,t;\mu_F). \quad (2)$$

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Hard-scattering amplitude:

$$\mathcal{A}^q(x) = \mathcal{A}^q_0(x) + \frac{\alpha_S}{2\pi}\Big(\frac{M^2_{\gamma\gamma}}{\mu_R^2}\Big)^{-\varepsilon}\Big(\frac{1}{\varepsilon}\mathcal{A}^q_{coll.}(x) + \mathcal{A}^q_1(x)\Big). \quad (3)$$

Cancelation of divergences

IR divergences present in GPDs and hard-scattering amplitudes cancel, if

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If that is true, then the full amplitude is:

$$\mathcal{A} = \sum_q \int_{-1}^{1} dx \, \mathrm{GPD}^q_R(x, \xi, t; \mu_F) \times \left(\mathcal{A}^q_0 + \frac{\alpha_S}{2\pi} \left[\mathcal{A}^q_1 + \ln\left(\frac{\mu_F^2}{M^2_{\gamma\gamma}} \right) \mathcal{A}^q_{coll.} \right] \right). \tag{5}$$

Next-to-leading order results

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- Finite part of a 4-point diagrams: expressible in terms of

$$\mathcal{F}_{nab} := \int_0^1 dy \int_0^1 dz\, y^a z^b \Big(\alpha_1 y + \alpha_2 z + \alpha_3 yz + i\epsilon\Big)^{-n},$$

$$\mathcal{G} := \int_0^1 dy \int_0^1 dz\, z^2 \Big(\alpha_1 y + \alpha_2 z + \alpha_3 yz + i\epsilon\Big)^{-2} \times \log\Big(\alpha_1 y + \alpha_2 z + \alpha_3 yz + i\epsilon\Big).$$

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Large computational power is needed to get stable results.

PARTONS

PARtonic Tomography Of Nucleon Software
B. Berthou et al., Eur. Phys. J. C 78, 478 (2018),
hep-ph/1512.06174

http://partons.cea.fr

Considered GPD models

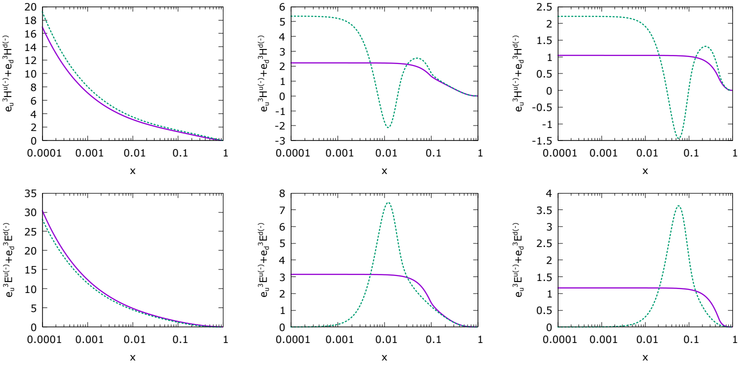

Figure: Comparison between GK [hep-ph/0708.3569] (solid magenta) and MMS [hep-ph/1304.7645] (dotted green) GPD models for $t = -0.1\ \mathrm{GeV}^2$ and the scale $\mu_F^2 = 4\ \mathrm{GeV}^2$.

Differential cross section: u'-dependence

$$\frac{d\sigma}{dt\, du'\, dM^2_{\gamma\gamma}}$$

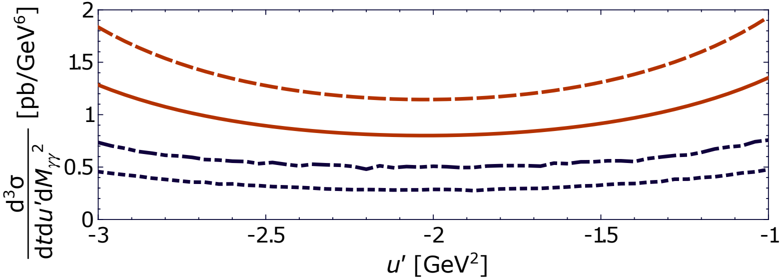

Figure: Differential cross-section as a function of u' for $S_{\gamma N} = 20\ \mathrm{GeV}^2$, $M^2_{\gamma\gamma} = 4\ \mathrm{GeV}^2$ ($\xi \approx 0.12$) and $t = t_0 \approx -0.05\ \mathrm{GeV}^2$ for proton target. LO: solid (dashed) red line, NLO: dotted (dash-dotted) blue line for GK (MMS) GPD model.

Differential cross section: $S_{\gamma N}$-dependence

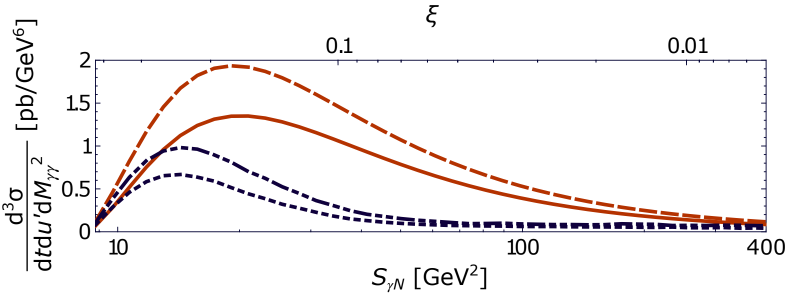

Figure: Differential cross-section as a function of $S_{\gamma N}$ (bottom axis) and the corresponding ξ (top axis) for $M^2_{\gamma\gamma} = 4\ \mathrm{GeV}^2$, $t = t_0$ and $u' = -1\ \mathrm{GeV}^2$.

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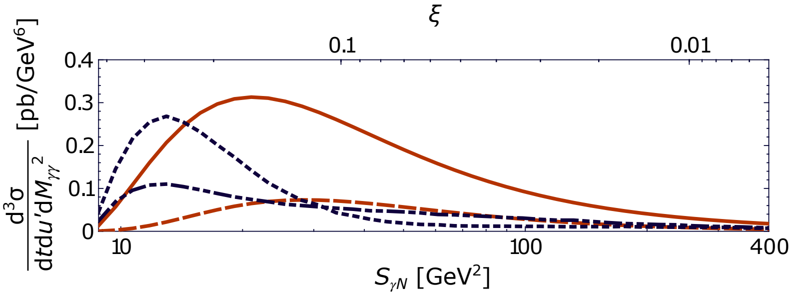

Figure: The same, but for neutron target.

Differential cross section: $M^2_{\gamma\gamma}$-dependence

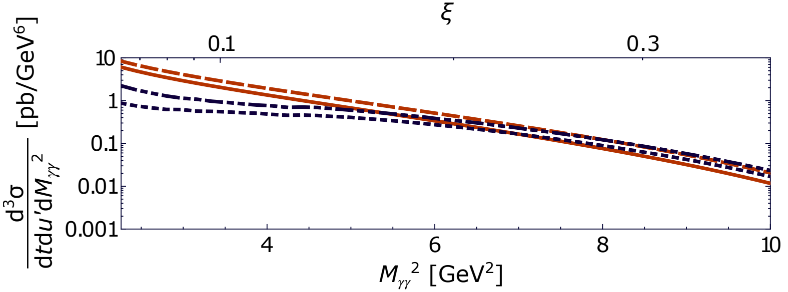

Figure: Differential cross-section as a function of $M^2_{\gamma\gamma}$ (bottom axis) and the corresponding ξ (top axis) for $S_{\gamma N} = 20\ \mathrm{GeV}^2$, $t = t_0$ and $u' = -1\ \mathrm{GeV}^2$.

Differential cross section: ϕ-dependence

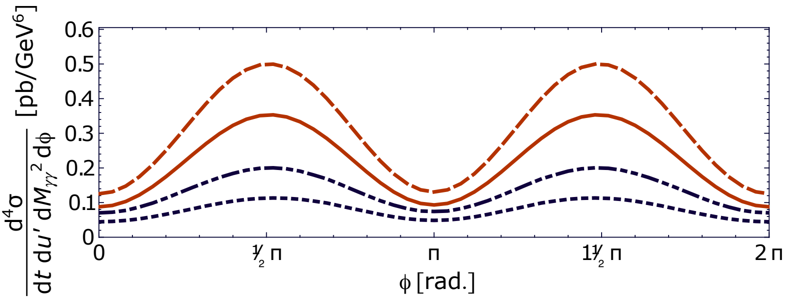

Figure: Differential cross-section as a function of ϕ – the angle between the initial photon polarization and one of the final photon momentum in the transverse plane for $S_{\gamma N} = 20\ \mathrm{GeV}^2$, $M_{\gamma\gamma}^2 = 4\ \mathrm{GeV}^2$ (which corresponds to $\xi \approx 0.12$), $u' = -1\ \mathrm{GeV}^2$ and $t = t_0 \approx -0.05\ \mathrm{GeV}^2$.

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- NLO factorization has been verified. It opens a new class of processes in which the collinear factorization can be studied.
- $\gamma N \to \gamma\gamma N$ can provide valuable information about charge-odd combinations of GPDs.
- We obtained a next-to-leading order scattering amplitude and perfomed phenomenological analysis of the diphoton photoproduction.