Renormalization Group Equations in generic Effective Field Theories

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In collaboration with J. Aebischer, P. Mieszkalski, M. Misiak and N. Selimović

1. **Motivation and assumptions**
2. **Results for dimension-four operators**
3. **Classification of dimension-six operators**
4. **One-loop calculations and sample results**
5. **Identities stemming from gauge invariance**
6. **Automatic computations**
7. **Passing to the on-shell basis**
8. **Verification of the preliminary results**
9. **Current status of the one-loop RGE computation**
10. **Outlook: methods for proceeding to two loops and beyond**

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[3] I. Schienbein, F. Staub, T. Steudtner and K. Svirina, "Revisiting RGEs for general gauge theories," Nucl. Phys. B 939 (2019) 1, Nucl. Phys. B 966 (2021) 115339 (E), [arXiv:1809.06797].

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$$- \frac{1}{2}\left(Y^a_{jk}\phi_a\psi^T_j C\psi_k + \text{h.c.}\right) + \mathcal{L}_{\text{g.f.}} + \mathcal{L}_{\text{FP}} + \frac{1}{\Lambda^2}\sum Q_N + \mathcal{O}\left(\frac{1}{\Lambda^4}\right).$$

Let's absorb the gauge couplings into the structure constants and generators. Then $F^A_{\mu\nu} = \partial_\mu V^A_\nu - \partial_\nu V^A_\mu - f^{ABC}V^B_\mu V^C_\nu$,

$(D_\mu\phi)_a = \left(\delta_{ab}\partial_\mu + i\theta^A_{ab}V^A_\mu\right)\phi_b$, $\quad (D_\mu\psi)_j = \left(\delta_{jk}\partial_\mu + it^A_{jk}V^A_\mu\right)\psi_k$, $\quad (D_\rho F_{\mu\nu})^A = \partial_\rho F^A_{\mu\nu} - f^{ABC}V^B_\rho F^C_{\mu\nu}$.

The quantities Q_N stand for linear combinations of dimension-six operators multiplied by their Wilson coefficients.

Renormalization of the dimension-four part:

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"2. Yukawa Couplings," Nucl. Phys. B 236 (1984) 221,
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Classification of dimension-six operators (off shell):

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$$Q_1 = \frac{1}{6!} W^{(1)}_{abcdef}\, \phi_a \phi_b \phi_c \phi_d \phi_e \phi_f, \qquad Q_2 = \frac{1}{4} W^{(2)}_{abcd}\, (D_\mu \phi)_a (D^\mu \phi)_b \phi_c \phi_d,$$

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$$Q_{15} = \tfrac{1}{2} W^{(15)}_{a,jk}\, \phi_a (D_\mu\psi)^T_j C\sigma^{\mu\nu} (D_\nu\psi)_k + \text{h.c.},$$

$$Q_{16} = \tfrac{i}{2} W^{(16)}_{ab,jk}\, \phi_a\phi_b \left[(\bar\psi \overleftarrow{\not{D}})_j\psi_k - \bar\psi_j(\not{D}\psi)_k\right],$$

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$$Q_{19} = \frac{1}{2} W^{(19)A}_{a,jk}\, \phi_a F^A_{\mu\nu}\, \psi^T_j C\sigma^{\mu\nu}\psi_k + \text{h.c.},$$

$$Q_{20} = iW^{(20)A}_{jk}\, F^A_{\mu\nu} \left[(\bar\psi\overleftarrow{D}^\nu)_j\gamma^\mu\psi_k - \bar\psi_j\gamma^\mu(D^\nu\psi)_k\right],$$

$$Q_{21} = iW^{(21)A}_{jk}\, \widetilde{F}^A_{\mu\nu}\, \bar\psi_j\gamma^\mu(D^\nu\psi)_k,$$

$$Q_{22} = W^{(22)A}_{jk}\, (D^\mu F_{\mu\nu})^A\, \bar\psi_j\gamma^\nu\psi_k.$$

Classification of dimension-six operators (off shell):

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Here, $W^{(N)}$ contain both the Wilson coefficients and the necessary Clebsch-Gordan coefficients that select singlets from various tensor products of the gauge group representations.

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In general, each $W^{(N)}$ may contain many independent Wilson coefficients.

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$$Q_{13} = \tfrac{1}{2}W^{(13)}_{a,jk}\,\phi_a(D_\mu\psi)^T_j C(D^\mu\psi)_k + \text{h.c.}, \qquad Q_{14} = W^{(14)}_{a,jk}\,\phi_a\psi^T_j C(D_\mu D^\mu\psi)_k + \text{h.c.},$$

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One-loop calculations and sample results

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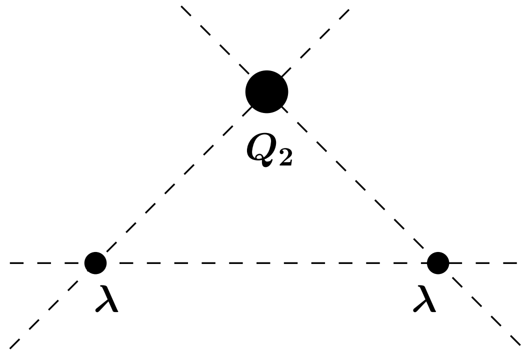
A sample diagram:



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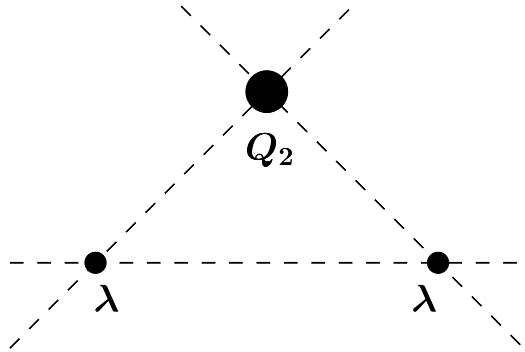

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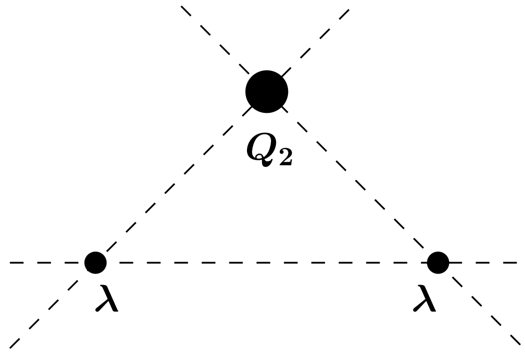

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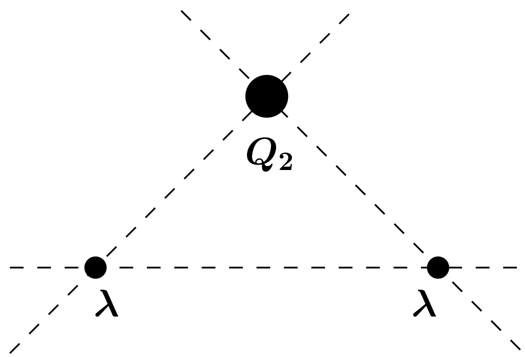

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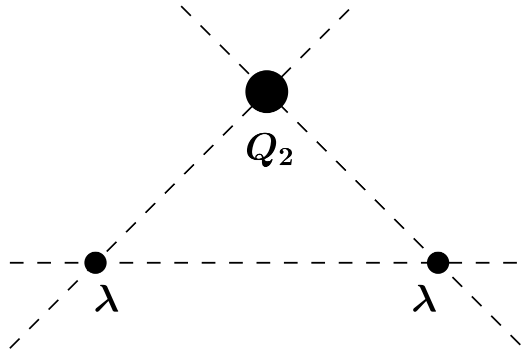

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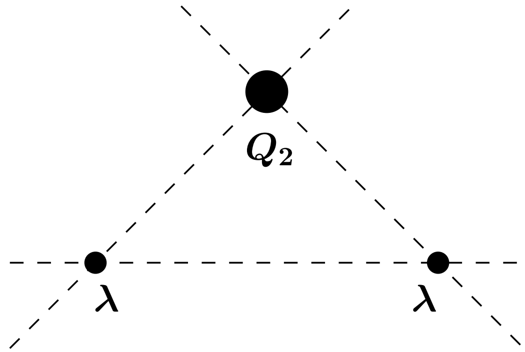

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The scalar field anomalous dimensions in $X^{(2)}$ are given by $(\gamma_\phi)_{ab} = \frac{1}{32\pi^2}\left[Y^a_{ij}Y^{b*}_{ij} + Y^b_{ij}Y^{a*}_{ij} - 4\theta^A_{ac}\theta^A_{cb}\right]$.

Identities stemming from gauge invariance

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Example – an identity for the Yukawa couplings

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$$\boldsymbol{Y^a_{jk}\phi_a(\psi_j)^T C P_L \psi_k \to (Y^a_{jk}(\delta_{ab} - i\epsilon^A\theta^A_{ab})\phi_b[(\delta_{jl} - i\epsilon^B t^B_{jl})\psi_l]^T C P_L(\delta_{kn} - i\epsilon^C t^C_{kn})\psi_n.}$$

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Identities for the Wilson coefficients with bosonic indices only are obtained by contracting each index with an appropriate generator, and adding the product to the r.h.s. of the equation

$$\begin{aligned} 0 = {} & if^{BEA_1}W^{(N)BA_2\ldots A_k}_{a_1\ldots a_m} + \ldots + if^{BEA_k}W^{(N)A_1\ldots B}_{a_1\ldots a_m} \\ & + \theta^E_{ba_1}W^{(N)A_1\ldots A_k}_{ba_2\ldots a_m} + \ldots + \theta^E_{ba_m}W^{(N)A_1\ldots A_k}_{a_1\ldots b}. \end{aligned}$$

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Both types of terms arise on the r.h.s. for operators that involve both the fermionic and bosonic fields.

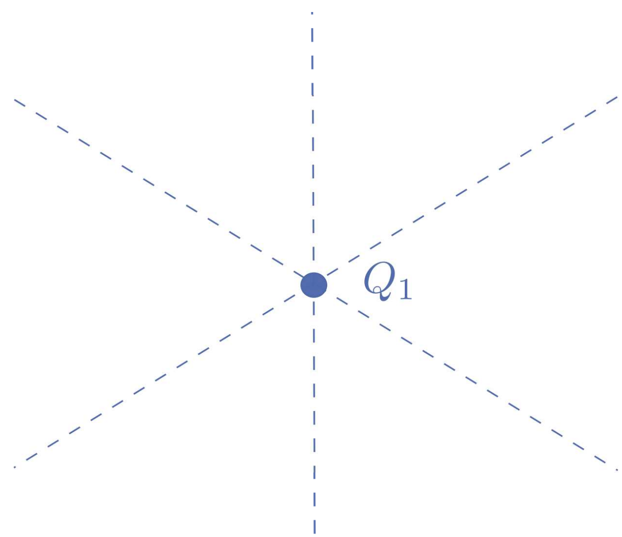
Automatic computations

Automatic computations

FeynRules

Automatic computations

FeynRules



$$-i\,W^{(1)}_{abcdef}$$

Automatic computations

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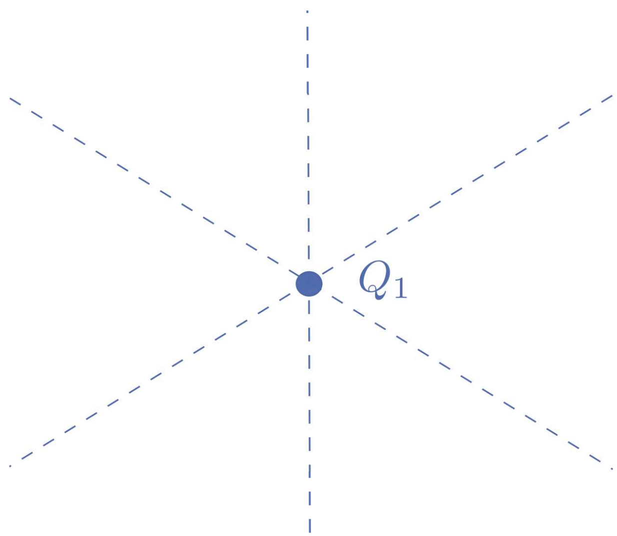

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FeynRules $\Longrightarrow$ FeynArts

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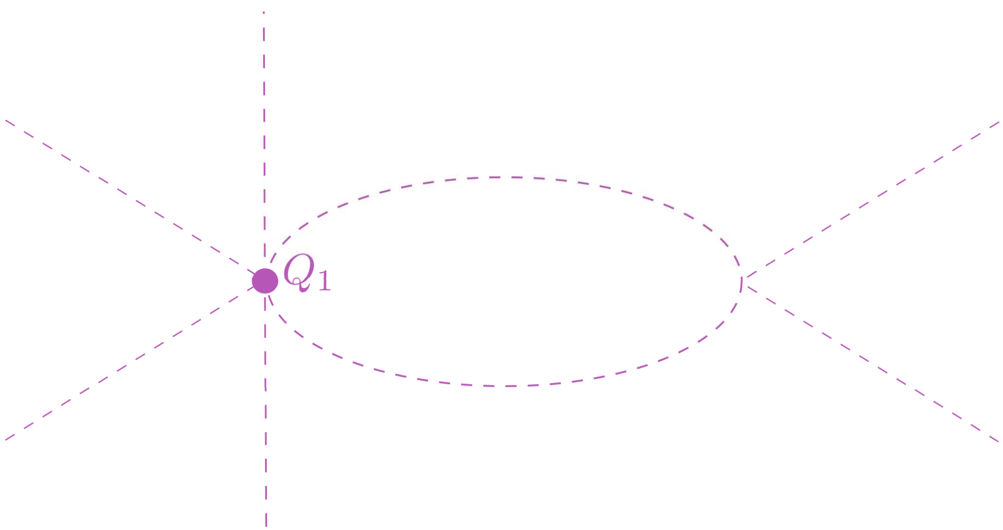

$$-W^{(1)}_{abcdgh}\lambda_{ghef}\,\mu^{2\epsilon}\int\frac{d^Dq}{(2\pi)^D}\frac{1}{(q^2-m^2+i\varepsilon)^2}$$

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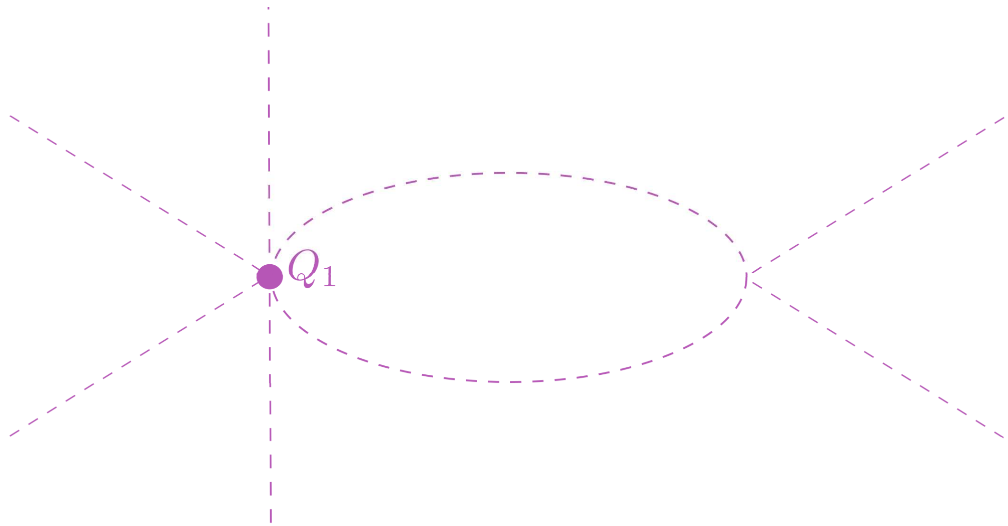

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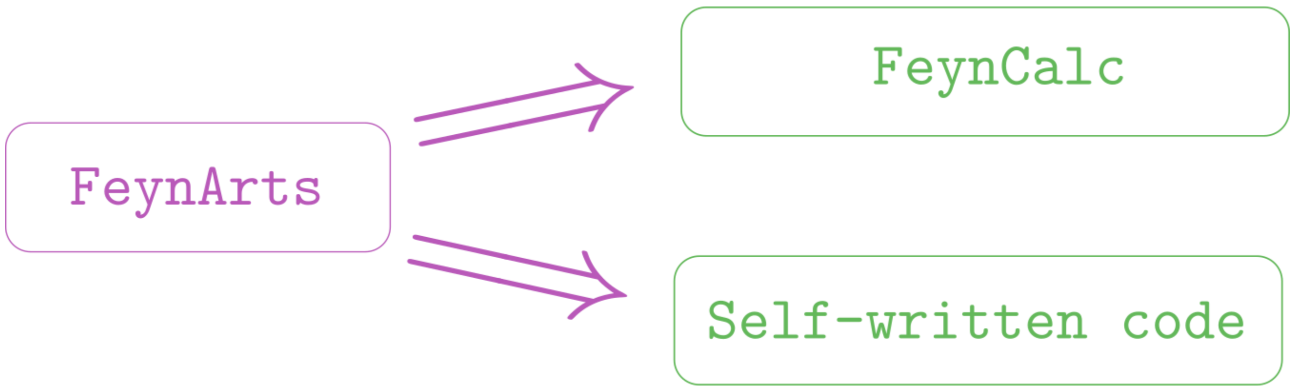

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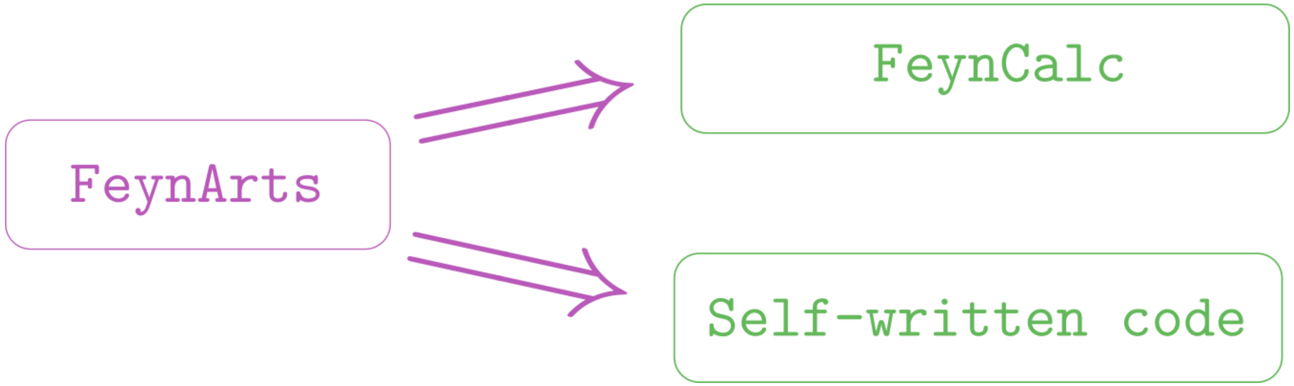

$$\mathrm{Div}\left(\mu^{2\epsilon}\int\frac{d^Dq}{(2\pi)^D}\frac{1}{(q^2-m^2+i\varepsilon)^2}\right)=\frac{i}{(4\pi)^2\epsilon}$$

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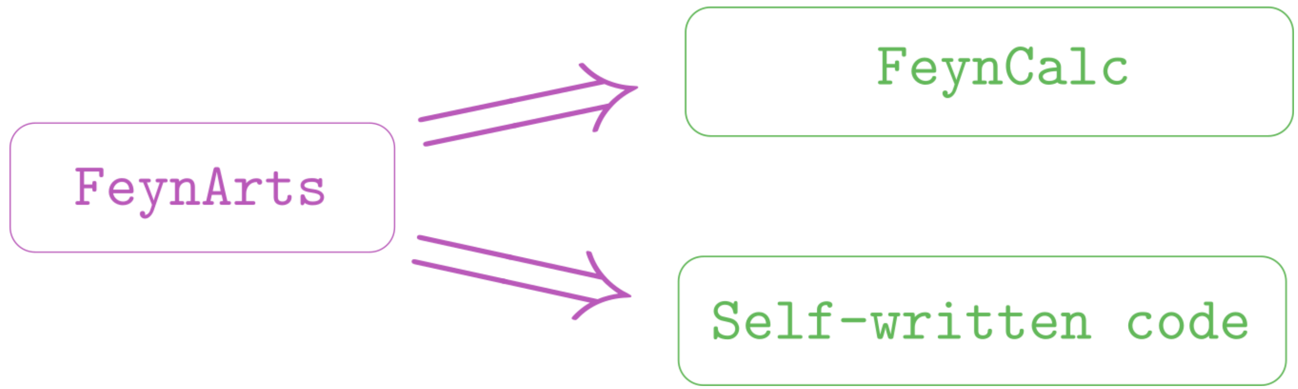

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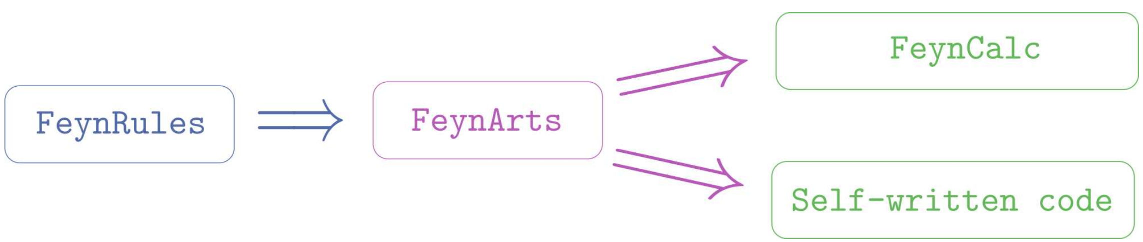

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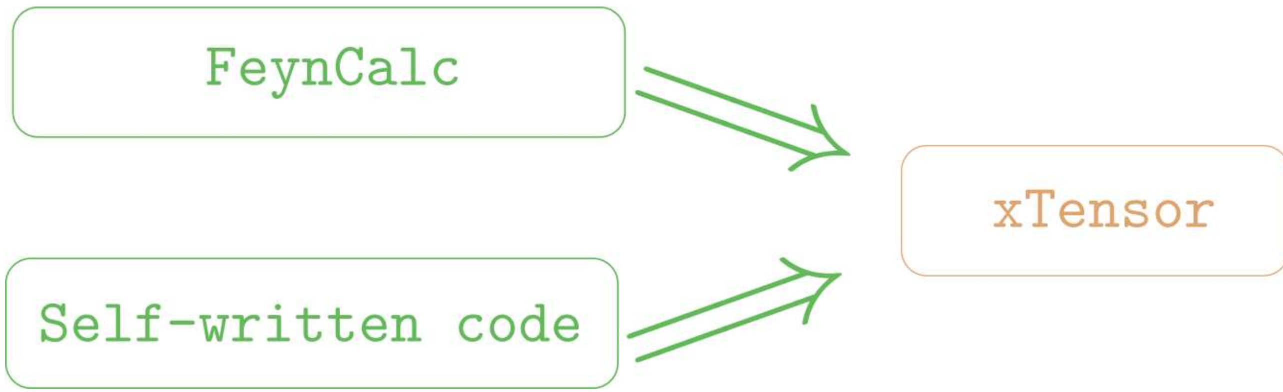

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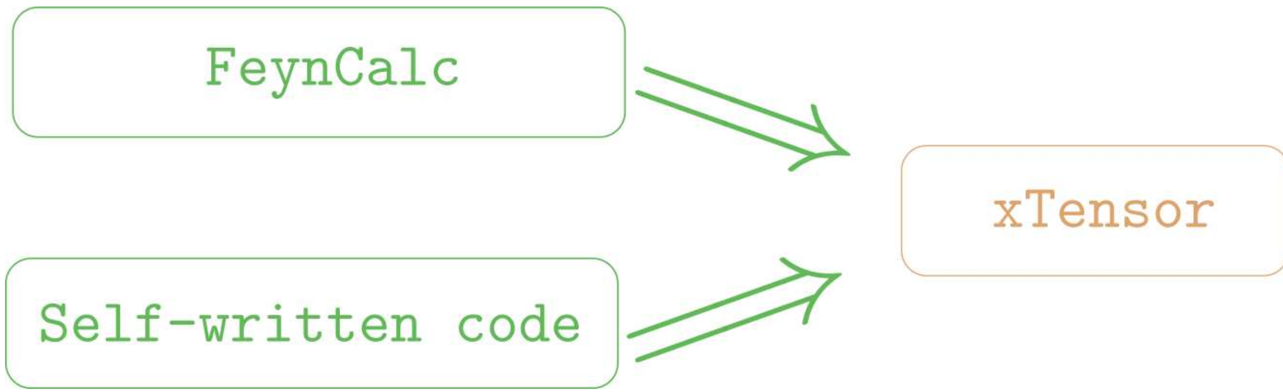

$$\mu \frac{dW^{(1)}_{abcdef}}{d\mu} = \frac{1}{(4\pi)^2}\left[(\ldots) - X^{(3)} + (\ldots)\right]_{abcdef}$$

$$X^{(3)}_{abcdef} := \frac{1}{48}\sum W^{(1)}{}_{cdefgh}\lambda_{abgh}$$

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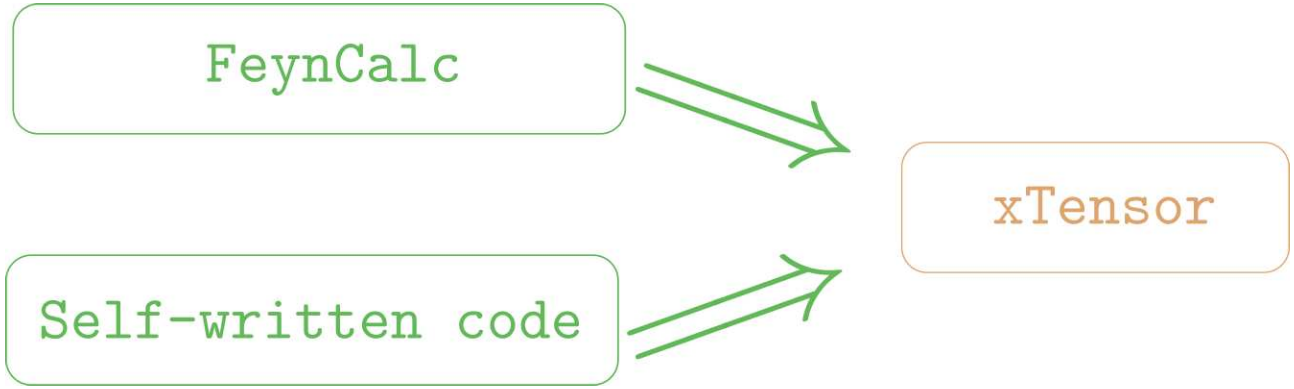

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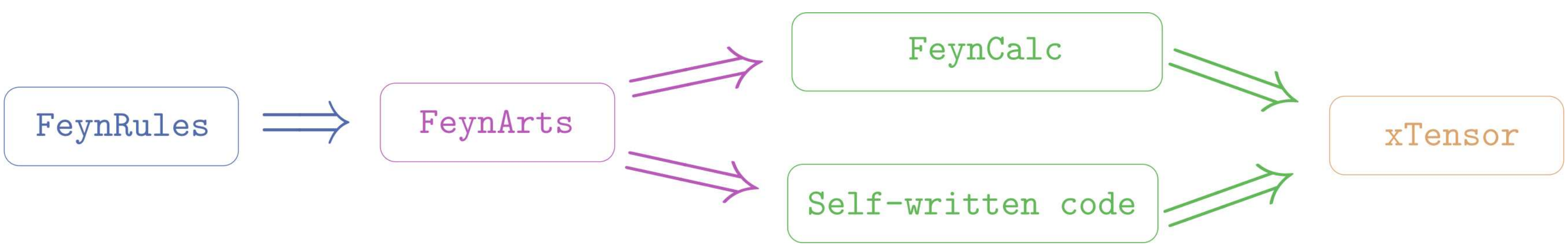

Passing to the on-shell basis

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Example – Deriving the on-shell RGEs for the Wilson coefficient of $Q_5 = \frac{1}{4} W^{(5)AB}_{ab}\, \phi_a \phi_b F^A_{\mu\nu} F^{B\,\mu\nu}$ **.**

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The operator $Q_7 = \frac{1}{2} W^{(7)AB} \left(D^\mu F_{\mu\nu}\right)^A \left(D_\rho F^{\rho\nu}\right)^B$ is reducible by the EOM

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After a simple redefinition

$$\widetilde{Q}_7 := Q_7 + \frac{1}{2} Q'_4 + \frac{1}{4} Q'_5 + (\ldots)_\psi.$$

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we obtain an operator $\widetilde{Q_7}$ that vanishes on-shell.

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$$\overline{W}^{(4)\;A}_{\;\;ab} := W^{(4)\;A}_{\;\;ab} - i W^{(7)\;AC} \theta^C{}_{ab}, \qquad \overline{\mathrm{W}}^{(5)\;\mathrm{AB}}_{\;\;\mathrm{ab}} := \mathrm{W}^{(5)\;\mathrm{AB}}_{\;\;\mathrm{ab}} - \frac{1}{4}\mathrm{W}^{(7)\;\mathrm{AC}}\theta^{\mathrm{C}}_{\mathrm{ac}}\theta^{\mathrm{B}}_{\mathrm{bc}}.$$

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$$\mu\frac{\mathrm{d}\widetilde{\mathrm{W}}^{(5)\;\mathrm{AB}}_{\;\;\mathrm{ab}}}{\mathrm{d}\mu} := \mu\frac{\mathrm{d}\mathrm{W}^{(5)\;\mathrm{AB}}_{\;\;\mathrm{ab}}}{\mathrm{d}\mu} + \frac{\mathrm{i}}{4}\sum\left(\mu\frac{\mathrm{d}\mathrm{W}^{(4)\;\mathrm{A}}_{\;\;\mathrm{ac}}}{\mathrm{d}\mu}\theta^{\mathrm{B}}_{\mathrm{bc}} - \mathrm{W}^{(4)\;\mathrm{A}}_{\;\;\mathrm{ac}}\theta^{\underline{\mathrm{B}}}_{\mathrm{bc}}\gamma_{\underline{\mathrm{B}}}\right).$$

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Here, $\gamma_B = \frac{1}{48\pi^2}[11C_2(G_B) - \frac{1}{2}\mathrm{tr}(\theta^A_{\underline{B}}\theta^A_{\underline{B}}) - 2\mathrm{tr}(t^A_{\underline{B}}t^A_{\underline{B}})]$ and $C_2(G_{\underline{B}})\delta^{\underline{B}C} = f^{BDE}f^{CDE}$.

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[1] E. E. Jenkins, A. V. Manohar, and M. Trott. "Renormalization group evolution of the standard model dimension six operators. I: formalism and λ dependence.", Journal of High Energy Physics 10 (2013) 087 [hep-ph/1308.2627].

[2] E. E. Jenkins, A. V. Manohar, and M. Trott. "Renormalization group evolution of the standard model dimension six operators. II: Yukawa dependence." Journal of High Energy Physics 01 (2014) 035 [hep-ph/1310.4838].

[3] E. E. Jenkins, A. V. Manohar, and M. Trott. "Renormalization group evolution of the standard model dimension six operators. III: gauge coupling dependence and phenomenology" Journal of High Energy Physics 04 (2014) 159 [hep-ph/1312.2014].

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[1] E. Braaten, C. S. Li, and T. C. Yuan "The evolution of Weinberg's gluonic CP-violation operator," Phys. Rev. Lett. 64 (1990) 1709.

[2] E. Braaten, C. S. Li, and T. C. Yuan "The gluon color-electric dipole moment and its anomalous dimension", Phys. Rev. D. 42 (1990) 276.

Current status of the one-loop RGEs computation

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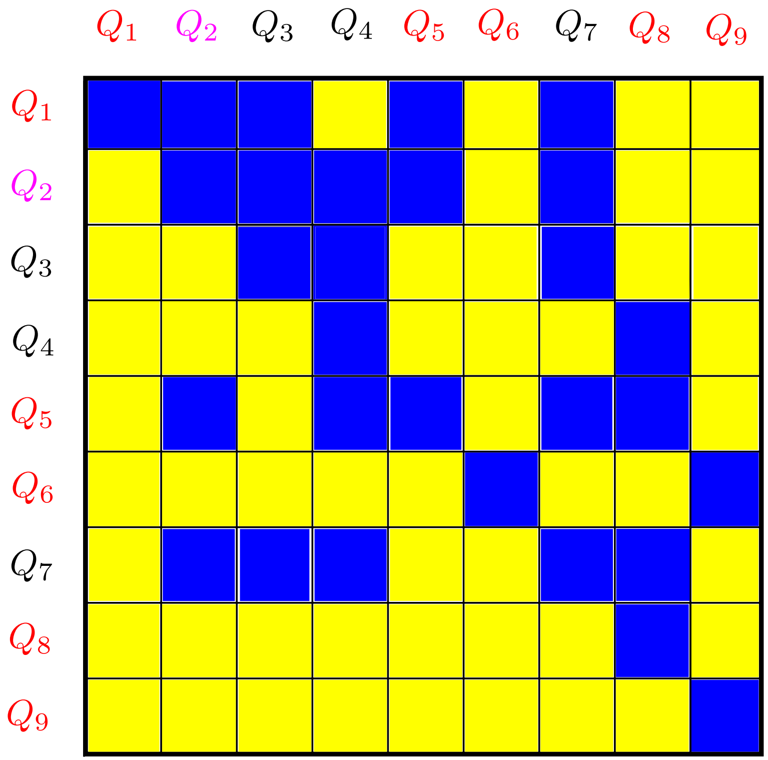
Bosonic operators



Left plot:

blue (yellow) – the operator contributes (does not contribute) to the off-shell RGE.

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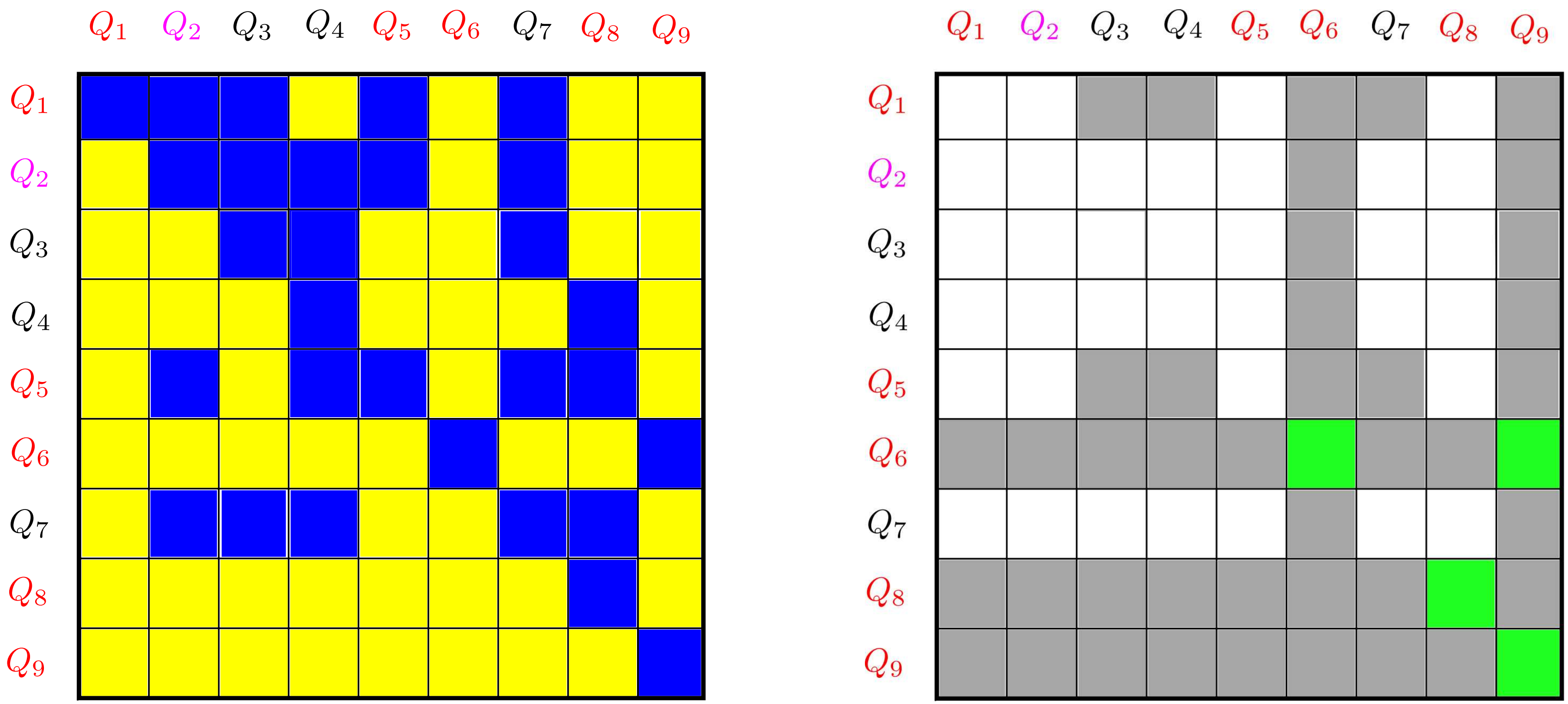

Left plot:

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Right plot:

green (gray) – the operator contributes (does not contribute) to the on-shell RGE

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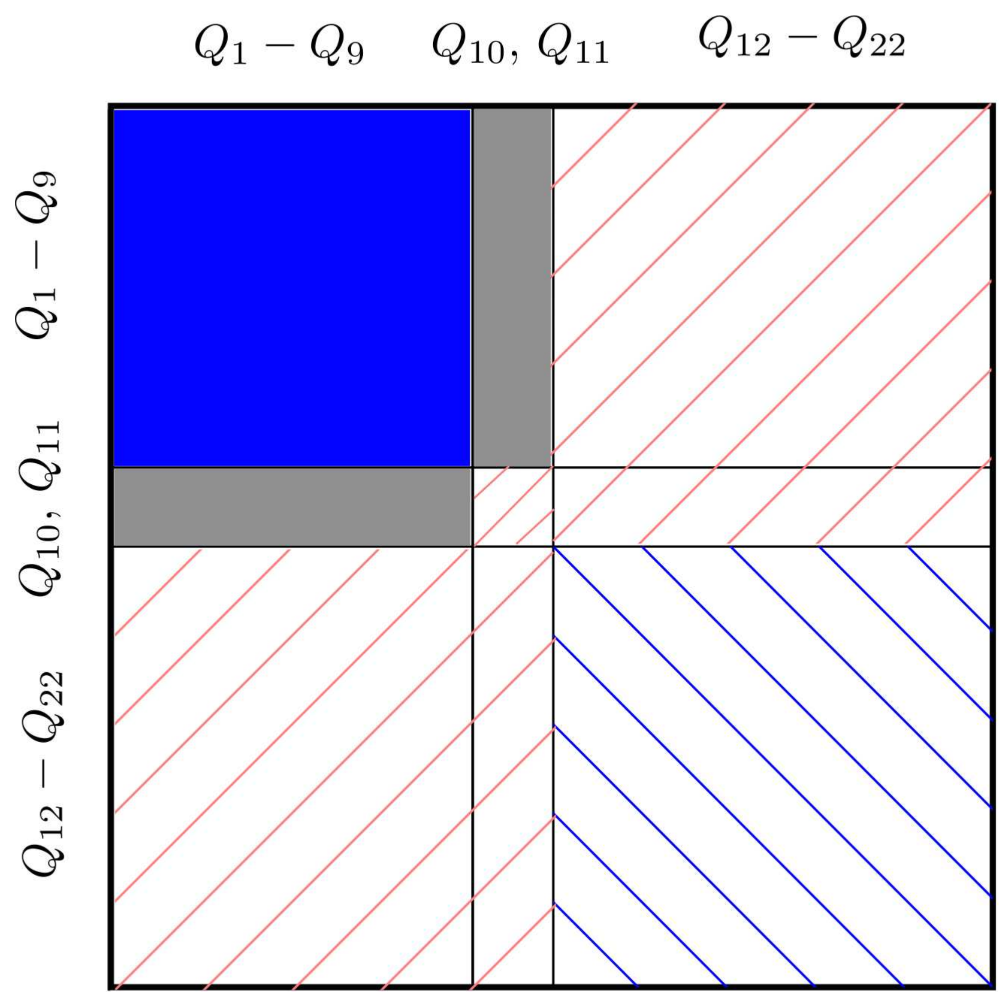
General view



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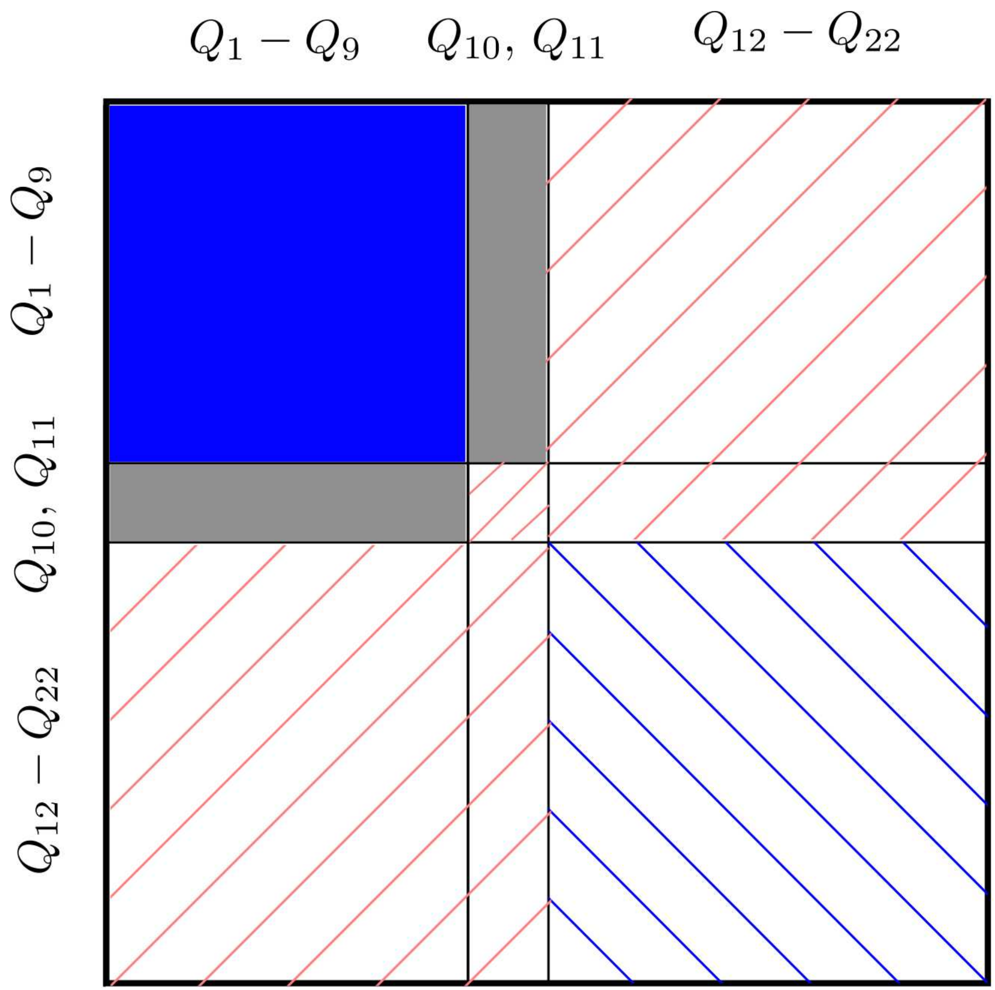

Legend:

blue – The RGEs computed in the off-shell basis (hatching denotes preliminary results).
hatched red – The contribution to RGEs that were not computed yet.
gray – No contribution to the RGEs at one loop.

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