Cosmological implications of the Higgs vacuum metastability during inflation

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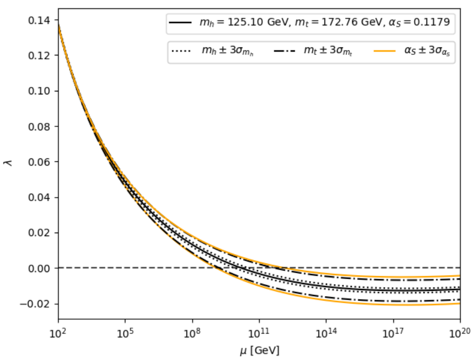
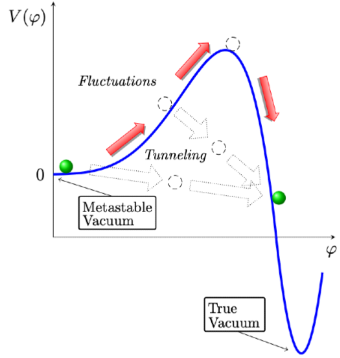
Introduction: vacuum metastability

Experimental values of SM particle masses m_h, m_t indicate that:

- currently in metastable EW vacuum $\rightarrow$ constrain fundamental physics.

$$V_{\mathrm{H}}(h,\mu,R) = \frac{\xi(\mu)}{2} R h^2 + \frac{\lambda(\mu)}{4} h^4$$





Markkanen *et al*, "Cosmological Aspects of Higgs Vacuum Metastability", 2018.

Introduction: vacuum decay formalism

- Decay expands at c with singularity within $\rightarrow$ true vacuum bubbles:

$$d\langle\mathcal{N}\rangle = \Gamma d\mathcal{V} \Rightarrow \langle\mathcal{N}\rangle = \int_{\text{past}} d^4x\sqrt{-g}\Gamma(x)$$

- Universe still in metastable vacuum $\rightarrow$ no bubbles in past light-cone:

$$\langle\mathcal{N}\rangle \lesssim 1$$

- Low decay rate Γ today, but higher rates in the early Universe.

Vacuum bubbles expectation value (during inflation)

$$\langle\mathcal{N}\rangle = \frac{4\pi}{3}\int_0^{N_{\text{start}}} dN \left(\frac{a_{\text{inf}}\left(\eta_0 - \eta\left(N\right)\right)}{e^N}\right)^3 \frac{\Gamma(N)}{H(N)} \leq 1$$

Overview of calculation

1. Calculate $\Delta V_{\rm H}$ and plug it in $\Gamma \approx \left(\frac{R}{12}\right)^2 e^{-\frac{384\pi^2 \Delta V_{\rm H}}{R^2}}$.
2. Cosmological quantities according to the inflationary model $V_{\rm I}(\phi)$.
3. Complete calculation of $\langle \mathcal{N} \rangle$ imposing the condition $\langle \mathcal{N} \rangle \leq 1$.

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4. Result: constraints on $\xi \geq \xi_{\langle \mathcal{N} \rangle = 1}$ and cosmological implications from the time of predominant bubble nucleation.

Overview of calculation

Aims

- study the electroweak (EW) vacuum decay during inflation.
- constrain the Higgs-curvature coupling ξ in a “reallistic” scenario.

Previous approaches

- dS spacetime where H is a constant free parameter.
- Tree-level effective Higgs potential (usually).
- Scale choices: $\mu = h$, $\mu^2 = ah^2 + bR^2$.

Improvements/differences

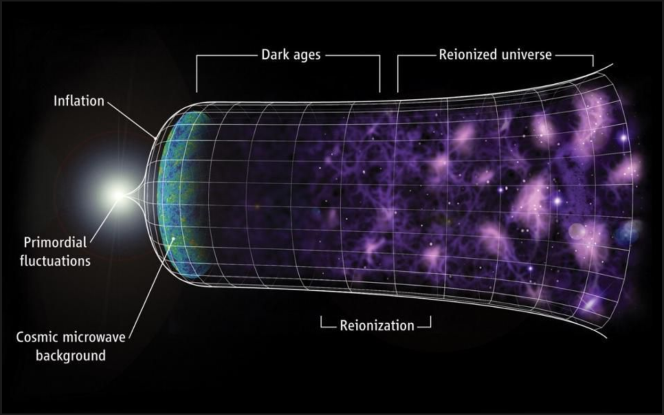
- Realistic inflationary model with $H(t)$ beyond slow-roll.
- RGI Higgs potential with 3-loop running in a curved background to 1-loop with additional terms from the conformal transformation.

Ema ’17, Gorbunov ’11, Fumagalli *et al* ’19, Markkanen *et al* ’18, Markkanen - Rajantie - Stopyra ’18, Espinosa ’18, Rajantie - Stopyra ’17, East *et al* ’17, Czerwińska *et al* ’16, Espinosa *et al* ’15, Hook *et al* ’15, Kamada ’15, Kearney *et al* ’15, ...

Cosmological inflation

Overview of inflation

- Period of exponential expansion of the universe before the HBB.
- Originally proposed to solve the cosmological problems at the time.
- Success: links the origin of LSS to the initial quantum fluctuations.
- Evidence: CMB anisotropies.



C. Faucher-Giguère *et al*, "Numerical Simulations Unravel the Cosmic Web", 2008.

Mathematical formalism of inflation

- Driven by the inflaton ϕ (scalar field) with EoM:

$$\ddot{\phi} + 3H\dot{\phi} + V_{\mathrm{I}}'(\phi) = 0\,.$$

- Energy density and pressure of the universe:

$$\rho = \frac{1}{2}\dot{\phi}^2 + V_{\mathrm{I}}(\phi), \quad p = \frac{1}{2}\dot{\phi}^2 - V_{\mathrm{I}}(\phi)\,.$$

- Friedmann eq. for the evolution of the expansion of the universe:

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{\rho}{3M_P^2}\,.$$

- Measure amount of inflation in e-foldings: $N(t) = \ln\left(\frac{a_{\mathrm{inf}}}{a(t)}\right)$.
- In slow-roll, $V_{\mathrm{I}}(\phi)$ slowly varies with ϕ, i.e. it is approximately flat:

$$H^2 = \frac{V_{\mathrm{I}}(\phi)}{3M_P^2}, \quad 3H\dot{\phi} = -V_{\mathrm{I}}'(\phi)\,.$$

Inflationary models

- *Quadratic inflation*, where $m = 1.4 \times 10^{13}$ GeV, with

$$V_{\rm I}(\phi) = \frac{1}{2} m^2 \phi^2$$

- *Quartic inflation*, where $\lambda = 1.4 \times 10^{-13}$, with

$$V_{\rm I}(\phi) = \frac{1}{4} \lambda \phi^4$$

- *Starobinsky inflation* (Starobinsky-like power-law model), where $M = 1.1 \times 10^{-5}$, with

$$V_{\rm I}(\phi) = \frac{3}{4} M^2 M_P^4 \left(1 - e^{-\sqrt{\frac{2}{3}} \frac{\phi}{M_P}}\right)^2$$

Quadratic and quartic models are simple but not realistic; Starobinsky inflation complies with data and can link different inflationary models.

Numerical solution for vacuum decay

Solve the system of coupled differential equations beyond slow-roll:

$$
\frac{d^2\phi}{dN^2} = \frac{V_{\mathrm{I}}(\phi)}{M_P^2 H^2}\left(\frac{d\phi}{dN} - M_P^2 \frac{V_{\mathrm{I}}'\phi)}{V_{\mathrm{I}}(\phi)}\right)
$$

$$
\frac{d\tilde{\eta}}{dN} = -\tilde{\eta}(N) - \frac{1}{a_{\mathrm{inf}} H(N)}
$$

$$
\frac{d\langle \mathcal{N} \rangle}{dN} = \gamma(N) = \frac{4\pi}{3}\left[a_{\mathrm{inf}}\left(\frac{3.21 e^{-N}}{a_0 H_0} - \tilde{\eta}(N)\right)\right]^3 \frac{\Gamma(N)}{H(N)}
$$

where $\tilde{\eta} = e^{-N}\eta$ with η: conformal time and

$$
H^2 = \frac{V_{\mathrm{I}}(\phi)}{3M_P^2}\left[1 - \frac{1}{6M_P^2}\left(\frac{d\phi}{dN}\right)^2\right]^{-1},
$$

$$
R = 6\left(\frac{\dot{a}^2}{a^2} + \frac{\ddot{a}}{a}\right) = 12H^2\left[1 - \frac{1}{4M_P^2}\left(\frac{d\phi}{dN}\right)^2\right].
$$

The effective potential

Decay rate from Hawking-Moss instanton

- Classical solutions to the tunneling process from false to true vacuum.

- High H's during inflation, CdL→HM instanton with action difference

$$B_{\rm HM}(R) \approx \frac{384\pi^2 \Delta V_{\rm H}}{R^2}$$

where $\Delta V_{\rm H} = V_{\rm H}(h_{\rm bar}) - V_{\rm H}(h_{\rm fv})$: barrier height → decay rate

$$\Gamma_{\rm HM}(R) \approx \left(\frac{R}{12}\right)^2 e^{-B_{\rm HM}(R)}$$

- Curvature effects enter at tree level via non-minimal coupling ξ:

$$V_{\rm H}(h, \mu, R) = \frac{\xi(\mu)}{2} R h^2 + \frac{\lambda(\mu)}{4} h^4$$

Coleman (1977), Coleman and De Lucia (1980), Hawking and Moss (1987).

Higgs potential in curved space-time

- Minkowski terms to 3-loops, curvature corrections in dS at 1-loop:

$$V_{\rm H}(h,\mu,R) = \frac{\xi(\mu)}{2} R h^2 + \frac{\lambda(\mu)}{4} h^4 + \frac{\alpha(\mu)}{144} R^2 + \Delta V_{\rm loops}(h,\mu,R)\,,$$

where the loop contribution can be parametrized as

$$\Delta V_{\rm loops} = \frac{1}{64\pi^2} \sum_{i=1}^{31} \left\{ n_i \mathcal{M}_i^4 \left[\log\left(\frac{|\mathcal{M}_i^2|}{\mu^2} \right) - d_i \right] + \frac{n_i' R^2}{144} \log\left(\frac{|\mathcal{M}_i^2|}{\mu^2} \right) \right\}$$

- RGI: choose $\mu = \mu_*(h,R)$ such that $\Delta V_{\rm loops}(h,\mu_*,R) = 0 \rightarrow$

RGI effective Higgs potential

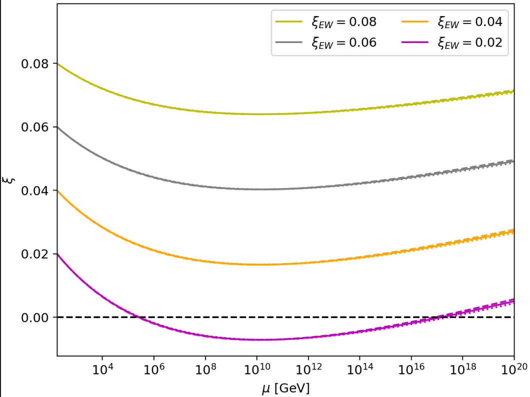
$$V_{\rm H}^{\rm RGI}(h,R) = \frac{\xi(\mu_*(h,R))}{2} R h^2 + \frac{\lambda(\mu_*(h,R))}{4} h^4 + \frac{\alpha(\mu_*(h,R))}{144} R^2$$

Markkanen *et al*, "The 1-loop effective potential for the Standard Model in curved spacetime", 2018.

Running of the non-minimal coupling

Calculate barrier height for Γ: entire SM particle spectrum, running of couplings $\lambda, y_t, g', g, \xi, \alpha$ (β-functions, pole-matching).

$$16\pi^2\beta_\xi = 16\pi^2\frac{d\xi}{d\ln\mu} = \left(\xi - \frac{1}{6}\right)\left(12\lambda + 6y_t^2 - \frac{3}{2}g'^2 - \frac{9}{2}g^2\right)$$



Overview of calculation

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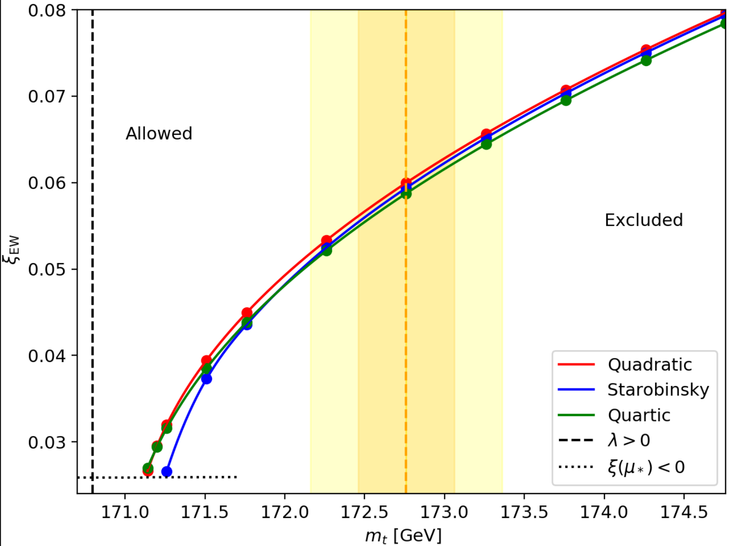
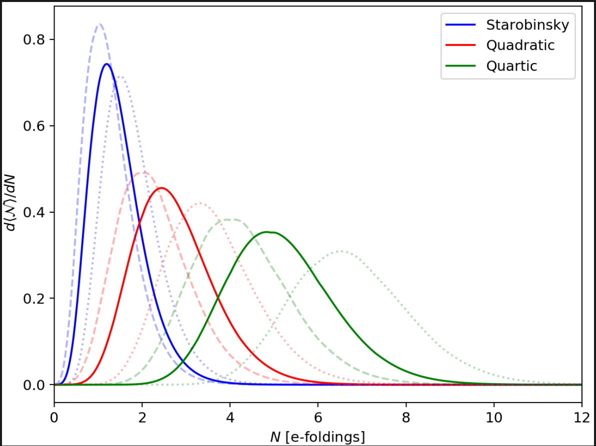
4. Result: constraints on $\xi \geq \xi_{\langle \mathcal{N} \rangle = 1}$ and cosmological implications from the time of predominant bubble nucleation.

Results: Bounds on ξ



Results: Bubble nucleation time

- If bubbles form at $N < 1$ $\rightarrow$ bounds maybe unreliable due to $B_{\mathrm{HM}}^{\mathrm{dS}}$.
- If bubbles form at $N \gg 60$ $\rightarrow$ bounds would depend on early times.



Results: Significance of the total duration of inflation

- Inflation can last for many orders of magnitude longer than 60 e-folds.

- We study early time behavior by splitting the $\langle\mathcal{N}\rangle$-integral

$$\langle\mathcal{N}\rangle(N_{\text{start}}) = \langle\mathcal{N}\rangle(60) + \int_{60}^{N_{\text{start}}} \frac{d\mathcal{V}}{dN}\Gamma(N)\, dN \ ,$$

where we set $\langle\mathcal{N}\rangle(60) = 1$ and slow roll applies to the 2nd term.

- $B_{\text{HM}} \approx$ constant at early times, so that

$$\langle\mathcal{N}\rangle(N_{\text{start}}) \approx 1 + \frac{4\pi e^{-B_{\text{HM}}}}{3} N_{\text{start}} \, .$$

- Contributing if $N_{\text{start}} \gtrsim e^{B_{\text{HM}}} \sim 10^{60} \gg 60\, e$-folds but not infinite.

Vacuum decay in $R + R^2$ gravity

SM Higgs in $R + R^2$ gravity

- Starobinsky '80 adds geometric terms to EH action.
- Scalar field (Higgs) non-minimally coupled to gravity + R^2-term:

$$S = \int d^4x \sqrt{-g_J} \left[\frac{M_P^2}{2} \left(1 - \frac{\xi h^2}{M_P^2} \right) R_J + \frac{R_J^2}{12M^2} + \frac{g_J^{\mu\nu}}{2} \partial_\mu h \partial_\nu h - \frac{\lambda}{4} h^4 \right]$$

Conformal transformation

Introduce the auxiliary scalaron field with EoM $s = R_J$, the inflaton ϕ, and the conformal transformation given by

$$g_{\mu\nu} = \Omega^2 g_{J\mu\nu} \, , \; \Omega^2 = 1 + \frac{s}{3M^2 M_P^2} = e^{\sqrt{\frac{2}{3}} \frac{\phi}{M_P}} \, ,$$

$$R_J = \Omega^2 \left[R - 3\Box \ln \Omega^2 + \frac{3}{2} g^{\mu\nu} \partial_\mu \ln \Omega^2 \partial_\nu \ln \Omega^2 \right]$$

SM + Starobinsky inflation in the Einstein frame

- Absorb the exp. factors via the field redefinition $\tilde{h} = e^{-\frac{1}{2}\sqrt{\frac{2}{3}}\frac{\phi}{M_P}} h$.
- Calculate loop correction in dS (ϕ: constant) and RGI with

$$\Delta V_{\rm loops} = \frac{1}{64\pi^2} \sum_{i=1}^{31} \left\{ n_i \tilde{\mathcal{M}}_i^4 \left[\log\left(\frac{|\tilde{\mathcal{M}}_i^2|}{\mu^2} \right) - d_i \right] + \frac{n_i'}{144} R^2 \log\left(\frac{|\tilde{\mathcal{M}}_i^2|}{\mu^2} \right) \right.$$

- Eliminate the mixed kinetic term through the FD $\phi = \tilde{\phi} + f(\tilde{h})$.
- Diagonalize the Higg's kinetic term with the FD $\rho = g(\tilde{h})\tilde{h}$.

$$\mathcal{L} \approx \frac{M_P^2}{2} R + \frac{1}{2}\partial_\mu \tilde{\phi} \partial^\mu \tilde{\phi} + \frac{1}{2}\partial_\mu \rho \partial^\mu \rho - U(\tilde{\phi}, \rho, \mu_*) ,$$

where $U(\tilde{\phi}, \rho, \mu_*) = V_{\rm I}(\tilde{\phi}) + V_{\rm H}^{\rm RGI}(\rho, \mu_*, \tilde{\phi}) + \mathcal{O}(\frac{\rho^6}{M_P^2})$.

The RGI effective Higgs potential in the Einstein frame

$$U(\tilde{\phi}, \rho, \mu_*) = V_{\rm I}(\tilde{\phi}) + V_{\rm H}^{\rm RGI}(\rho, \mu_*, \tilde{\phi})$$

Starobinsky inflation: $V_{\rm I}(\tilde{\phi}) = \frac{3M^2 M_P^4}{4}\left(1 - e^{-\sqrt{\frac{2}{3}}\frac{\tilde{\phi}}{M_P}}\right)^2$

$$V_{\rm H}^{\rm RGI}(\rho, \mu_*, \tilde{\phi}) = m_{\rm eff}^2(\tilde{\phi}, \mu_*)\frac{\rho^2}{2} + \lambda_{\rm eff}(\tilde{\phi}, \mu_*)\frac{\rho^4}{4} + \frac{\alpha(\mu_*)}{144}R^2(\tilde{\phi})$$

$$m_{\rm eff}^2 = \xi R + 3\alpha^2 M_P^2 \Xi \left(1 - e^{-\sqrt{\frac{2}{3}}\frac{\tilde{\phi}}{M_P}}\right) e^{-\sqrt{\frac{2}{3}}\frac{\tilde{\phi}}{M_P}} + \frac{\Xi}{M_P^2}\partial_\mu\tilde{\phi}\partial^\mu\tilde{\phi}\,,$$

$$\lambda_{\rm eff} = \lambda + 3\alpha^2\Xi^2 e^{-2\sqrt{\frac{2}{3}}\frac{\tilde{\phi}}{M_P}} + \frac{4\left[\xi R + \Delta m_1^2\right]\Xi^2}{M_P^2} + \frac{4\Xi^3}{M_P^4}\partial_\mu\tilde{\phi}\partial^\mu\tilde{\phi}\,,$$

with $\Xi(\mu_*) = \xi(\mu_*) - \frac{1}{6}$.

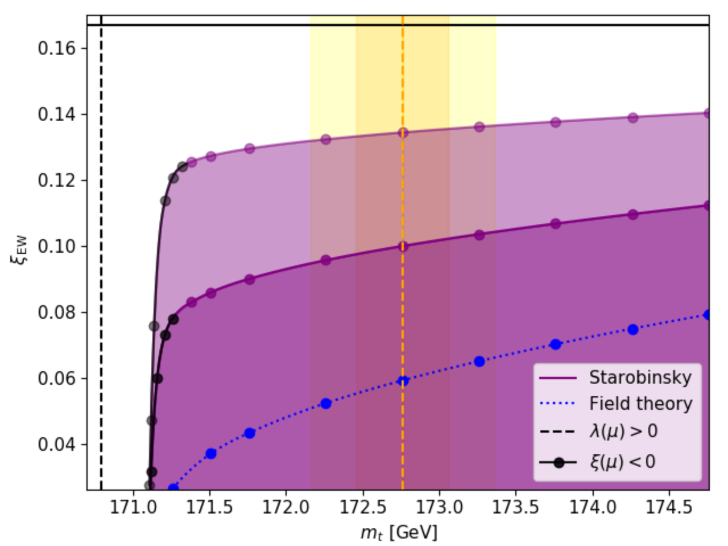
Overview of computation

1. Calculate ΔV_{H} and plug it in Γ.
2. Cosmological quantities according to the inflationary model $V_{\mathrm{I}}(\tilde{\phi})$.
3. Complete calculation of $\langle \mathcal{N} \rangle$ imposing the condition $\langle \mathcal{N} \rangle \leq 1$.

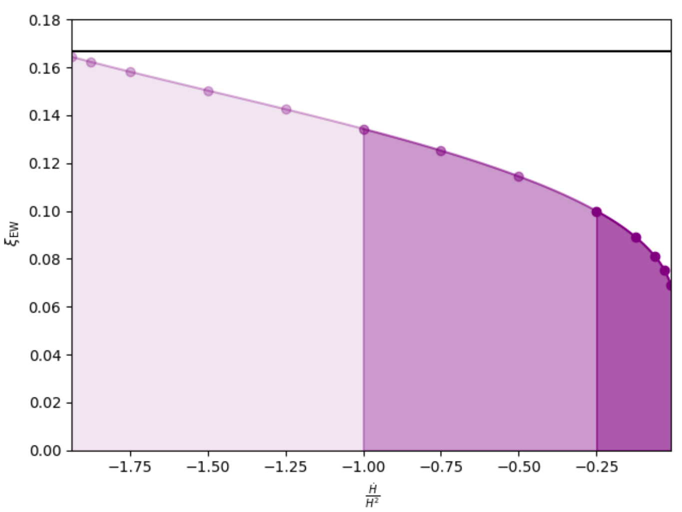
$$\langle \mathcal{N} \rangle = \frac{4\pi}{3} \int_0^{N_{\mathrm{start}}} dN \left(\frac{a_{\mathrm{inf}} \left(\eta_0 - \eta\left(N\right)\right)}{e^N} \right)^3 \frac{\Gamma(N)}{H(N)} \leq 1$$

4. Result: constraints on $\xi \geq \xi_{\langle \mathcal{N} \rangle = 1}$.

Results: Lower ξ-bounds for varying top quark mass



ξ-bounds with varying definition for the end of inflation



Conclusions

- Minimal model of the early universe: SM $+$ Starobinsky inflation (observationally favoured) from modification of gravity $R + R^2$.

- Vacuum decay constraints on the Higgs-curvature coupling, with state-of-the-art $V_{\rm H}^{\rm RGI}$ (3-loop couplings, 1-loop dS corrections):

$$\xi_{\rm EW} \gtrsim 0.1 > 0.06\,,$$

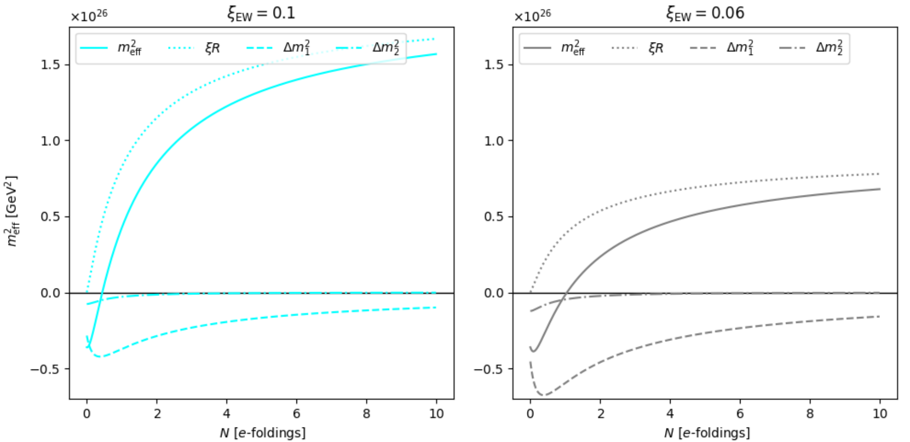
  give stricter ξ-bounds from extra negative terms in $V_{\rm H}^{\rm RGI}$.

- Bubble nucleation in the last moments of inflation: breakdown of dS approximations and necessity to consider the dynamics of reheating.

- Possibly hints against eternal inflation (again).

Additional slides

Factors of the potential's quadratic term



HM bounce in Starobinsky Inflation and Field Theory