The Boltzmann equation for a Planck gas

M. Kozlowski and J. Marciak-Kozlowska

We define Planck gas - a gas of massive particles (black holes) with masses $m = M_P = \left(\frac{\hbar c}{G}\right)^{\frac{1}{2}}$. To the description of transport processes, e.g. thermal process we apply the quantum Heaviside heat transport equation [1]

$$\frac{\lambda_B}{v_h}\frac{\partial^2 T}{\partial t^2} + \frac{\lambda_B}{\lambda}\frac{\partial T}{\partial t} = \frac{\hbar}{M_P}\nabla^2 T. \tag{1}$$

In Eq. (1) M_P is the Planck mass, λ_B the de Broglie wavelength and λ mean free path.

Recently the dissipation of the thermal energy in the cosmological context (e.g. viscosity) was described in the frame of EIT (Extended Irreversible Thermodynamics [2] and it was shown that $v_h = c$ (c = light velocity). Considering that the relaxation time τ is defined as [1]

$$\tau = \frac{\hbar}{M_P v_h^2} \tag{2}$$

one obtains

$$\tau = \frac{\hbar}{M_P c^2} = \left(\frac{\hbar G}{c^5}\right)^{\frac{1}{2}} = \tau_P \tag{3}$$

i.e. the relaxation time is equal to the Planck time. For particles with mass $m = M_P$ the Boltzmann equation reads [3]

$$\left[\frac{\partial}{\partial t} + \vec{v}\nabla_{\vec{r}} + \frac{F}{m}\nabla_{\vec{v}}\right]\vec{f}(\vec{r},\vec{v},t) = \left(\frac{\partial f}{\partial t}\right)_{\text{scattering}} \tag{4}$$

In Eq. (4) $f(\vec{r},\vec{v},t)$ is the distribution function for Planck gas. In the relaxation approximation one obtains from Eq. (4)

$$\left[\frac{\partial}{\partial t} + \vec{v}\nabla_{\vec{r}} + \frac{F}{m}\nabla_{\vec{v}}\right](f^0 + g) = -\frac{g}{\tau_P} \tag{5}$$

where

$$f(\vec{r}, \vec{v}, t) = f_{\mathrm{Maxwellian}}(\vec{r}, \vec{v}, t) + g$$

and

$$\frac{g}{f_{\mathrm{Maxwellian}}(\vec{r}, \vec{v}, t)} = -\frac{\left(\frac{\hbar G}{c^3}\right)^{\frac{1}{2}}}{L} \tag{6}$$

In Eq. (6) L denotes the distance where $f(\vec{r}, \vec{v}, t) \neq f_{\mathrm{Maxwellian}}(\vec{r}, \vec{v}, t)$.

When the distribution function for Planck gas can be described as formula (6) started then one obtains for heat conductivity K

$$K = \frac{5}{2}\rho k T \left(\frac{\hbar G}{c^5}\right)^{\frac{1}{2}} \tag{7}$$

and for viscosity, μ

$$\mu = \rho k T \left(\frac{\hbar G}{c^5}\right)^{\frac{1}{2}} \tag{8}$$

In formulae (7) and (8) ρ denotes the density of Planck gas and k is Boltzmann constant and T is temperature (in Kelvins)

It will be very interesting to investigate the thermodynamics of the Planck gas with transport coefficients described by formulae (7) and (8).

References

[1] M. Kozlowski, J. Marciak-Kozlowska, *From Quarks to Bulk Matter*, Hadronic Press, USA, 2001.

[2] D. Jou, J. Casas-Váskez and G. Lebon, *Extended Irreversible Thermodynamics*, Springer, 2001.

[3] Kerson Huang, *Statistical Mechanics*, Wiley, 1963.