The smearing out of the thermal initial conditions created in a Planck Era

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Abstract

In this paper the quantum heat transport in a Planck gas in the presence of the potential (other than the thermal one) is investigated. The new quantum heat transport equation which generalizes our potential free QHT (*Foundations of Physics Letters* (vol. **10**, No 3 (1997)) is developed. The thermal wave solution of QHT for a Planck gas is obtained and condition for distortionless propagation of thermal wave is formulated. It is argued that the initial conditions of the Beginning (i. e. for $t = 0$) are smeared in the time scale of the Planck time.

Key words: Thermal properties; Planck gas; Distortionless thermal wave

1 Introduction

In paper [1] the QHT (quantum hyperbolic heat transport equation) for a Planck gas was obtained. Planck gas, i.e. gas of the massive particles all with the masses equal Planck mass, $M_p = (\hbar c/G)^{1/2}$ was created at the Planck era i. e. for time period equals Planck time $t_p = (\hbar G/c^5)^{1/2}$.

On time scales of Planck time, black holes of the Planck mass spontaneously come into existence. Via the process of Hawking radiation, the black hole can then evaporate back into energy. The characteristic time scale for this to occur happens to be approximately equal to Planck time. Thus the Universe at $t_p = 10^{-43}$ s in age was filled with Planck gas.

As it was shown in paper [1] the thermal properties of the Planck gas can be described by the QHT:

$$t_p \frac{\partial^2 T}{\partial t^2} + \frac{\partial T}{\partial t} = \frac{\hbar}{M_p} \nabla^2 T, \tag{1}$$

where T – is the temperature, $\hbar$ – Planck constant and M_p – Planck mass. Equation (1) can be written as:

$$\frac{\partial^2 T}{\partial t^2} + \left(\frac{c^5}{\hbar G}\right)^{1/2} \frac{\partial T}{\partial t} = c^2 \nabla^2 T. \tag{2}$$

Quantum hyperbolic heat transport equation as a hyperbolic equation shed a light on the time arrow in Planck gas. For time period shorter than t_p we have preserved time reversal for thermal process, viz

$$\frac{1}{c^2} \frac{\partial^2 T}{\partial t^2} = \nabla^2 T, \tag{3}$$

and for $t \gg t_p$

$$\frac{\partial T}{\partial t} = \left(\frac{\hbar G}{c}\right)^{1/2} \nabla^2 T \tag{4}$$

the time reversal symmetry is broken. These new properties of QHT open new possibilities for the interpretation of Planck time t_p. Before t_p thermal processes in Planck gas are symmetrical in time. After t_p the time symmetry is broken. Moreover gravitation is activated after t_p and this fact creates time arrow (formula (4)).

In the present paper we develop the generalized quantum heat transport equation for Planck gas, which includes the potential energy term. The condition for the conserving the shape of the thermal wave created at the Planck time will be developed and investigated.

2 The generalized quantum heat transport equation for a Planck gas

For a long time the analogy between the Schrödinger equation and diffusion equation was recognized [2]. Let us consider, for the moment the parabolic heat transport equation i. e. equation (4)

$$\frac{\partial T}{\partial t} = \frac{\hbar}{M_p}\nabla^2 T. \tag{5}$$

When the real time $t \to \frac{it}{2}$ and $T \to \Psi$ equation (5) has the form of the free Schrödinger equation

$$i\hbar\frac{\partial \Psi}{\partial t} = -\frac{\hbar^2}{2M_p}\nabla^2 \Psi. \tag{6}$$

The complete Schrödinger equation has the form

$$i\hbar\frac{\partial \Psi}{\partial t} = -\frac{\hbar^2}{2M_p}\nabla^2 \Psi + V\Psi, \tag{7}$$

where V denotes the potential energy. When we go back to real time $t \to -2it$ and $\Psi \to T$ the new parabolic quantum heat transport is obtained

$$\frac{\partial T}{\partial t} = \frac{\hbar}{M_p}\nabla^2 T - \frac{2V}{\hbar}T. \tag{8}$$

Equation (8) describes the quantum heat transport in a Planck gas for $\Delta t > t_p$. For heat transport in the period $\Delta t < t_p$ one obtains the generalized hyperbolic heat transport equation [1] with potential term added

$$t_p \frac{\partial^2 T}{\partial t^2} + \frac{\partial T}{\partial t} = \frac{\hbar}{M_p} \nabla^2 T - \frac{2V}{\hbar} T. \tag{9}$$

Considering that $t_p = \hbar / M_p c^2$ [1] equation (9) can be written as:

$$\frac{1}{c^2} \frac{\partial^2 T}{\partial t^2} + \frac{M_p}{\hbar} \frac{\partial T}{\partial t} + \frac{2V M_p}{\hbar^2} T = \nabla^2 T. \tag{10}$$

In formula (10) c – is the light velocity in the vacuum.

In the following we consider the one dimensional heat transport phenomena with constant potential energy $V = V_0$:

$$\frac{1}{c^2} \frac{\partial^2 T}{\partial t^2} + \frac{M_p}{\hbar} \frac{\partial T}{\partial t} + \frac{2V_0 M_p}{\hbar^2} T = \frac{\partial^2 T}{\partial x^2}. \tag{11}$$

For quantum heat transport equation (11) we seek solution in the form

$$T(x,t) = e^{-t/t_p} u(x,t). \tag{12}$$

After substitution equation (12) to equation (11) one obtains

$$\frac{\partial^2 u(x,t)}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2} + b^2 u(x,t), \tag{13}$$

where

$$b = \sqrt{\left(\frac{M_p c^2}{2\hbar}\right)^2 - \frac{2V_0 M_p}{\hbar^2} c^2}. \tag{14}$$

The general solution of equation (14) for Cauchy initial conditions:

$$u(x,0) = f(x), \qquad \left.\frac{\partial u(x,t)}{\partial t}\right|_{t=0} = F(x), \tag{15}$$

has the form [3]

$$u(x,t) = \frac{f(x-ct) + f(x+ct)}{2} + \frac{1}{2} \int_{x-ct}^{x+ct}, \Phi(x,t,z) dz \tag{16}$$

where

$$\Phi(x,t,z) = \frac{1}{c}F(z)J_0\left(\frac{b}{c}\sqrt{(z-x)^2 - c^2t^2}\right) + btf(z)\frac{J_0'\left(\frac{b}{c}\sqrt{(z-x)^2 - c^2t^2}\right)}{\sqrt{(z-x)^2 - c^2t^2}}, \tag{17}$$

and $J_0(z)$ denotes the Bessel function of the first kind. Considering formulae (12 – 15) the solution of equation (11) describes the propagation of the initial state of the Planck gas, $f(x)$ as the thermal wave with velocity c. It is quite interesting to formulate the condition at which these waves propagates without the distortion, i. e. conserve the shapes. The conditions for conserving the shape can be formulated as:

$$b = \sqrt{\left(\frac{M_p c^2}{2\hbar}\right)^2 - \frac{2V_0 M_p}{\hbar^2}c^2} = 0. \tag{18}$$

When the equation (18) holds then the equation (13) has the form:

$$\frac{\partial^2 u(x,t)}{\partial t^2} = c^2\frac{\partial^2 u}{\partial x^2}. \tag{19}$$

Equation (19) is the wave equation with the solution (for Cauchy boundary conditions (15))

$$u(x,t) = \frac{f(x-ct) + f(x+ct)}{2} + \frac{1}{2c}\int_{x-ct}^{x+ct} F(z)dz. \tag{20}$$

Equation (18) — the distortionless condition can be written as:

$$V_0 t_p = \frac{\hbar}{8} < \hbar. \tag{21}$$

We can conclude that in the presence of the potential V_0 one can "observe" the undisturbed quantum thermal wave (created at $t = 0$) only when the *Heisenberg uncertainty* relation (21) is fulfilled.

Combining equation (12) and (20) the complete solution of equation (11) (for $b = 0$) can be written as

$$T(x,t) = e^{-t/2t_p}\left[\frac{f(x-ct)+f(x+ct)}{2} + \frac{1}{2c}\int_{x-ct}^{x+ct} F(z)dz\right]. \qquad (22)$$

One can say that the formula (22) is a very pessimistic because the initial conditions (which operate at the Beginning) are smeared in the time scale of the order of the Planck time.

3 Conclusion

In the paper the quantum heat transport for a Planck gas in the presence of the potential energy was obtained and solved for Cauchy boundary conditions [4]. The condition for propagation of the undisturbed thermal wave (created at the Beginning) was formulated. It was shown that the initial conditions which operates at the Beginning are smeared in the time scale of the order of a Planck time.

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References

[1] M. Kozłowski, J. Marciak-Kozłowska, *Foundations of Physics Letters* **10**, 295 (1997).

[2] E. Nelson, *Phys. Rev.* **150**, 1079 (1966).

[3] M. S. Carslaw, J. C. Jaeger, *Operational Methods in Applied Mathematics*, Oxford University Press, Oxford 1953.

[4] The classical hyperbolic heat transport equation has been discussed also in: D. You, J. Casas, and G. Lebon, *Extended Irreversible Thermodynamics* (Springer, New York, 1993); D. Y. Tzou, *Macro- to Microscale Heat Transfer* (Taylor & Francis, Washington, 1997).