Klein-Gordon thermal equation for a Planck gas

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Abstract

In this paper the quantum hyperbolic equation formulated in our earlier paper [Found. Phys. Lett. **10**, 599 (1997)] is applied to the study of the propagation of the initial thermal state of the Universe. It is shown that the propagation depends on the barrier height. The Planck wall potential is introduced, $V_P = \hbar/8t_P = 1.125\ 10^{18}$ GeV where t_P is a Planck time. For the barrier height $V < V_P$ the master thermal equation is *the modified telegrapher's equation*, and for barrier height $V > V_P$ the master equation is *the Klein-Gordon equation.* The solutions of both type equations for Cauchy boundary conditions are discussed.

Key words: Thermal properties, Planck gas, Planck wall, Distorted thermal waves

1 Introduction

In this paper the thermal behaviour of a Planck gas in the presence of the potential barrier is investigated. The generalized quantum hyperbolic heat transport equation formulated in paper [1] is applied to the study of the propagation of the initial thermal state of the Universe. It will be shown that the propagation depends on the barrier height. The thermal information on the Beginning is carried through the distorted thermal waves. But the undistorted thermal information is completely diminished for the time of the order of a Planck time.

In paper the possibility of the motion "up the stream of time" (in spirit of W. Thompson and J. C. Maxwell) is discussed [2]. It will be shown that only hyperbolic heat transport equation guarantees the possibility of this motion. This possibility does not exists with Fourier type (parabolic) heat transport equation.

2 Klein-Gordon equation for a Planck gas

On time scales of Planck time, black holes of the Planck mass spontaneously come into existence. Via the process of Hawking radiation, the black hole can then evaporates back into energy. The characteristic time scale for this to occur happens to be approximately equal to Planck time. Thus the Universe at 10^{-43} seconds in age was filled with Planck gas i.e. gas of massive particles all with masses equal Planck mass M_p. In the following we will describe the thermal properties of the Planck gas in the field of the potential V.

As was shown in paper [1] the thermal properties of the Planck gas can be described by hyperbolic quantum heat transport equation, viz:

$$t_p \frac{\partial^2 T}{\partial t^2} + \frac{M_p}{\hbar}\frac{\partial T}{\partial t} + \frac{2VM_p}{\hbar^2}T = \nabla^2 T. \tag{1}$$

In equation (1) t_p denotes Planck time, M_p is the Planck mass and V denotes the potential energy.

For the uniform Universe it is possible to study only one-dimensional heat transport phenomena. In the following we will consider the thermal properties of a Planck gas in constant potential $V = V_0$. In that case the one dimensional quantum heat transport equation has the form:

$$\frac{1}{c^2}\frac{\partial^2 T}{\partial t^2} + \frac{M_p}{\hbar}\frac{\partial T}{\partial t} + \frac{2V_0 M_p}{\hbar^2}T = \frac{\partial^2 T}{\partial x^2}, \tag{2}$$

where formula for $t_p = \hbar/M_p c^2$ was used [1]. In equation (2) c - denotes the light velocity. As $c \neq \infty$ we can not omit the second derivative term and consider only Fokker-Planck equation:

$$\frac{M_p}{\hbar}\frac{\partial T}{\partial t} + \frac{2V_0 M_p}{\hbar^2}T = \frac{\partial^2 T}{\partial x^2}, \tag{3}$$

for heat diffusion in the potential energy V_0, or free heat diffusion:

$$\frac{\partial T}{\partial t} = \frac{\hbar}{M_p}\frac{\partial^2 T}{\partial x^2}. \tag{4}$$

It occurs that only if we retain the second derivative term we have the chance to study the conditions in the Beginning.

Some implications of the forward and backward properties of the parabolic heat diffusion equation were beautifully described by J. C. Maxwell [2]:

> *"Sir William Thompson has shown in a paper published in the Cambridge and Dublin Mathematical Journal in 1844 how to deduce, in certain cases the thermal state of a body in past time from its observed conditions at present.*

If the present distribution of temperature is such that it may be expressed in a finite series of harmonics, the distribution of temperature at any previous time may be calculated but if (as in generally case) the series of harmonics is infinite, than the temperature can be calculated only when this series is convergent. For present and future time it is always convergent, but for past time it becomes ultimately divergent when the time is taken at a sufficiently remote epoch. The negative value of t for which the series becomes ultimately divergent, indicates a certain date in past time such that the present state of things can not be deduced from any distribution of temperature occurring previously to the date, and becoming diffused by ordinary conduction. Some other event besides ordinary conduction must have occurred since that date in order to produce the present stage of things".

As can be easily seen the second derivative term in equation (1) carriers the memory of the initial state which occurred at time $t = 0$. If we pass with $c \to \infty$ we lost the possibility to study the influence of the initial conditions at the present epoch as it is explained above by J. C. Maxwell. It means that by limiting procedure $c \to \infty$ we cut off the memory of the Universe.

For hyperbolic quantum heat transport equation (2) we seek a solution of the form:

$$T(x,t) = e^{-t/2t_p} u(x,t). \tag{5}$$

After substitution of equation (5) to equation (2) one obtains:

$$\frac{1}{c^2}\frac{\partial^2 u}{\partial t^2} - \frac{\partial^2 u}{\partial x^2} + qu = 0, \tag{6}$$

where

$$q = \frac{2V_0 M_p}{\hbar^2} - \left(\frac{M_p c}{2\hbar}\right)^2. \tag{7}$$

In the following we shall consider positive values of V_0, $V_0 \geq 0$, i.e. we shall consider the potential barriers and steps.

The structure of the Eq. (6) depends on the sign of the parameter q. Let us define the Planck wall potential, i.e. potential for which $q = 0$. From equation (7) one obtains:

$$V_P = \frac{\hbar}{8t_P} = 1.125\ 10^{18}\ \text{GeV}, \tag{8}$$

where t_P is a Planck time. In Fig. 1 the parameter q is calculated as the function of V_0. For $q < 0$, i.e. when $V_0 < V_P$ Eq. (6) is *the modified telegrapher equation* (MTE) [3]. For the Cauchy initial condition:

$$u(x,0) = f(x), \qquad \frac{\partial u(x,0)}{\partial t} = g(x), \tag{9}$$

and the solution of Eq. (5) has the form [3]:

$$\begin{aligned} u(x,t) &= \frac{f(x-ct)+f(x+vt)}{2} \\ &+ \frac{1}{2c}\int_{x-ct}^{x+ct} g(\zeta) I_0\left[\sqrt{-q(c^2t^2-(x-\zeta)^2)}\right] d\zeta \\ &+ \frac{(c\sqrt{-q})t}{2}\int_{x-ct}^{x+ct} f(\zeta)\frac{I_1\left[\sqrt{-q(c^2t^2-(x-\zeta)^2)}\right]}{\sqrt{c^2t^2-(x-\zeta)^2}} d\zeta. \end{aligned} \tag{10}$$

In equation (10) I_0, I_1 denotes the Bessel modified function of the zero and first order respectively.

When $q > 0$, i.e for $V_0 > V_P$ equation (6) reduces to *the Klein-Gordon Equation* (K-GE) well known from its application in elementary particle and nuclear physics.

For the Cauchy initial condition (9) the solution of K-GE can be written as [3]:

$$\begin{aligned} u(x,t) &= \frac{f(x-ct)+f(x+ct)}{2} \\ &+ \frac{1}{2c}\int_{x-ct}^{x+ct} g(\zeta)J_0\left[\sqrt{q(c^2t^2-(x-\zeta)^2)}\right]d\zeta \\ &- \frac{(c\sqrt{q})t}{2}\int_{x-ct}^{x+ct}\frac{J_1\left[\sqrt{q(c^2t^2-(x-\zeta)^2)}\right]}{\sqrt{c^2t^2-(x-\zeta)^2}}d\zeta. \end{aligned} \tag{11}$$

The case for $q = 0$ was discussed in paper [1] and it describes the distortionless quantum thermal waves. Both solutions (10) and (11) exhibit the domains of dependence and influence for the modified telegrapher's equation and Klein-Gordon equation. These domains, which characterize the maximum speed, c, at which the thermal disturbance travels are determined by the principal terms of the given equation (i.e. the second derivative terms) and do not depend on the lower order terms. It can be concluded that these equations and the wave equation have identical domains of dependence and influence. Both solutions (10) and (11) represents the distorted thermal waves in the field of potential barrier or steps V.

3 Conclusions

In the paper the thermal behaviour of a Planck gas in the presence of a potential barrier is investigated. It was argued that the hyperbolic quantum heat transport equation offers the possibility for the study of the thermal history of the Universe up to the Beginning, but the information is transmitted through the distorted thermal waves. It was shown that for a barrier height $V < V_P$ the quantum heat transport equation is the *modified telegrapher's* *equation.* For a barrier height $V > V_P$ the quantum heat transport equation is *the Klein-Gordon equation.* It is quite interesting to observe that only for $t_P \neq 0$ the Planck wall has the finite height.

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