SUPERSYMMETRY
Problems: set 2

December 20, 2008

1. Define differential operators $P_\mu,\ \tilde{P}_\mu,\ Q_\alpha,\ \bar{Q}_{\dot\alpha},\ D_\alpha,\ \bar{D}_{\dot\alpha}$ in superspace coordinates $\{x,\,\theta,\,\bar\theta\}$ as follows:

$$
\begin{aligned}
P_\mu &= i\partial_\mu,\ \tilde{P}_\mu = -i\partial_\mu,\\
Q_\alpha &= -i\tfrac{\partial}{\partial\theta^\alpha} + \sigma^\mu_{\alpha\dot\alpha}\bar\theta^{\dot\alpha}\partial_\mu,\\
\bar{Q}^{\dot\alpha} &= -i\tfrac{\partial}{\partial\bar\theta_{\dot\alpha}} + (\theta\sigma^\mu\epsilon)^{\dot\alpha}\partial_\mu,\\
D_\alpha &= -i\tfrac{\partial}{\partial\theta^\alpha} - \sigma^\mu_{\alpha\dot\alpha}\bar\theta^{\dot\alpha}\partial_\mu,\\
\bar{D}^{\dot\alpha} &= -i\tfrac{\partial}{\partial\bar\theta_{\dot\alpha}} - (\theta\sigma^\mu\epsilon)^{\dot\alpha}\partial_\mu.
\end{aligned}
$$

   Show that

$$
\begin{gathered}
\{Q_\alpha, \bar{Q}_{\dot\alpha}\} = 2\sigma^\mu_{\alpha\dot\alpha}P_\mu,\ \{D_\alpha, \bar{D}_{\dot\alpha}\} = 2\sigma^\mu_{\alpha\dot\alpha}\tilde{P}_\mu,\\
\{Q_\alpha, \bar{D}_{\dot\alpha}\} = 0 = \{Q_\alpha, D_\alpha\}.
\end{gathered}
$$

2. Given a general scalar superfield $S(x,\,\theta,\,\bar\theta) = \phi(x) + \theta\psi(x) + \bar\theta\bar\chi(x) + \theta^2 M(x) + \bar\theta^2 N(x) + \theta\sigma^\mu\bar\theta A_\mu(x) + \theta^2\bar\theta\bar\lambda(x) + \bar\theta^2\theta\rho(x) + \theta^2\bar\theta^2 D(x)$ find the supersymmetry transformations of the component fields. Remember that $\delta_\xi S = (i\xi Q + i\bar\xi\bar{Q})S$.

3. Given a chiral superfield Φ find the component expansion of $\log(\Phi)$ and Φ^n, where n is a natural number.

4. For a general real superfield $V = V^\dagger$ find the D-component of V^n, where n is a natural number.

5. Define a "spurion" superfield $Z = a + b\theta^2 + b^*\bar{\theta}^2 + c\theta^2\bar{\theta}^2$, where a, b, c are complex numbers. Does Z behave like a proper superfield under supersymmetry transformations? Find the component expansion for the Lagrangian density

$$\int d^4\theta\, Z\Phi^\dagger\Phi,$$

where Φ is a chiral superfield. Is the theory described by this Lagrangian density supersymmetric?

6. Find the component expansion of the Lagrangian density

$$\int d^4\theta \left(\Phi_1^\dagger e^{-V}\Phi_1 + \Phi_2^\dagger e^{+V}\Phi_2 + 2\xi V\right)$$
$$\int d^2\theta \left(m\Phi_1\Phi_2 + \frac{1}{M}(\Phi_1\Phi_2)^2 - \frac{1}{4}W^\alpha W_\alpha\right) + h.c.$$

Use the Wess-Zumino gauge for the vector superfield, $V = \theta\sigma^\mu\bar{\theta}A_\mu(x) + i\theta^2\bar{\theta}\bar{\lambda}(x) - i\bar{\theta}^2\theta\lambda(x) + \frac{1}{2}D(x)$. The $\Phi_{1,2}$ are chiral superfields with charges -1, $+1$ respectively.

7. Show that one can define components of a chiral superfield Φ as follows:

$$\mathcal{A} = \Phi|_0,\ \chi_\alpha = \frac{i}{\sqrt{2}}D_\alpha\Phi|_0,$$
$$\mathcal{F} = \frac{1}{4}D^2\Phi|_0,$$

where $|_0$ means $\theta_\alpha = \bar{\theta}_{\dot{\beta}} = 0$.

8. Consider an $SU(2)$, $N = 1$ supersymmetric theory with three chiral superfields on the adjoint representation of $SU(2)$: $\Phi_{1,2,3}$. The superpotential is given by

$$W = \epsilon_{ijk} Tr\, \Phi_i[\Phi_j, \Phi_k],$$

where $\epsilon_{123} = 1$. The Kähler potential is the minimal renormalizable one. Is supersymmetry broken in this model? Next, add to the superpotential

$$\delta W = \sum_i m_i Tr\, \Phi_i^2.$$

What about supersymmetry breaking in the presence of these terms? Show that the equations of motion become $[\Phi_i, \Phi_j] = \epsilon_{ijk}m_k\Phi_k$. Which matrices satisfy these equations?