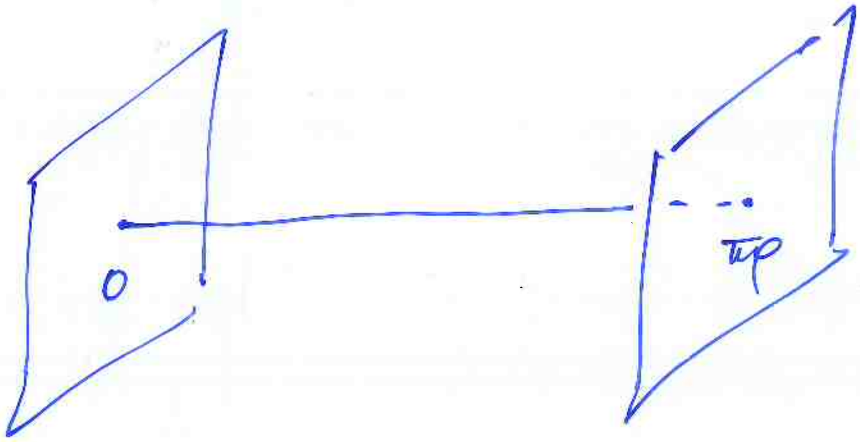
RANDALL – SUNDRUM MODEL



S^1/Z_2

$$S = \int d^5x \sqrt{-G}\,(R + V_0) +$$

$$+ \int_{i=1,2} d^5x \sqrt{-g}\, \lambda_i \delta_i$$

Ansatz:

$$ds^2 = e^{2A(x^5)} \gamma_{\mu\nu} dx^\mu dx^\nu + (dx^5)^2$$

$\gamma_{\mu\nu} = \eta_{\mu\nu}$ (Minkowski), $R^{(4)} = 0$

$\gamma_{\mu\nu} = \mathrm{diag}(-1, e^{2Lt}, e^{2Lt}, e^{2Lt})$

(dS_4),

$R^{(4)} = +12L^2$

$\gamma_{\mu\nu} = \mathrm{diag}(-e^{2Lx^3}, e^{2Lx^3}, e^{2Lx^3}, 1)$

AdS_4,

$R^{(4)} = -12L^2$

$$\delta \int R \sqrt{-g} =$$

$$\int E_{ik}\, \delta g^{ik} \sqrt{-g},$$

$$E_{ik} = R_{ik} - \frac{1}{2} R g_{ik},$$

$$\delta \sqrt{-g} = -\frac{1}{2} \sqrt{-g}\, \delta g^{ik} g_{ik}$$

$$\delta S_m = \frac{1}{2} \int T_{ik}\, \delta g^{ik} \sqrt{-g}$$

EINSTEIN EQ^{ns} :

$$E_{00} - \frac{1}{2} g_{00} \left(V_0 + \lambda_1 \delta_1 + \lambda_2 \delta_2 \right) = 0$$

$$E_{55} - \frac{1}{2} V_0 = 0$$

take AdS_4 , $V_0 \geq 0 \leftarrow$ 'AdS_5'

$$A'^2 + A'' = \frac{1}{12} V_0$$

$$e^{-2A} L^2 + A'^2 = \frac{1}{12} V_0$$

put $a = e^A \Rightarrow \frac{a''}{a} = \frac{1}{12} V_0$

$$\Rightarrow \quad a = \alpha e^{k|x^5|} + \underline{e^{-k|x^5|}}$$

localization

$$k^2 = \frac{V_0}{12} , \qquad \alpha = \frac{L^2}{4k^2} \Rightarrow L = 0$$

jump conditions on branes

$$a'_{i+} - a'_{i-} = \frac{\lambda_i}{6} a_i$$

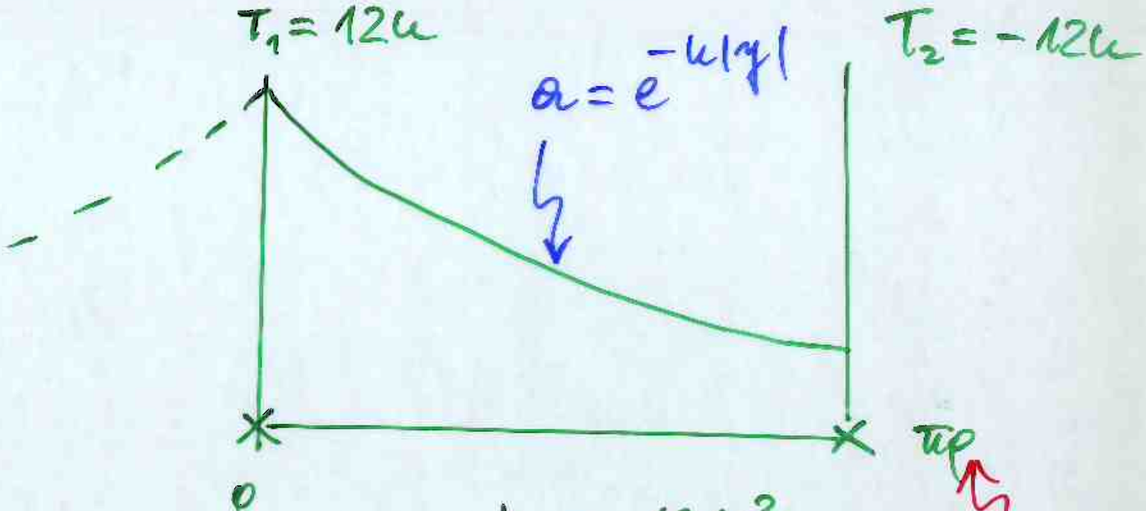
1st brane +ve tension $T_1 = -\lambda_1 > 0$

$$\alpha = \frac{1 - |\lambda_1|/12k}{1 + |\lambda_1|/12k}$$

$$-\lambda_1 = 12k \equiv \lambda_{crit}$$

$$\lambda_2 = (-12k)\,\frac{\delta e^{\pi\rho k} - e^{-\pi\rho k}}{\delta e^{\pi\rho k} + e^{-\pi\rho k}}$$

$$\delta = 0 \implies \lambda_2 = -\lambda_1 = \lambda_{\text{const}}$$



$V_0 = 12k^2$

ρ as a modulus

$$\langle R \rangle = 8k\left(\delta(x^5) - \delta(x^5 - \pi\rho)\right) - 10k^2$$

$$-M_{Pl}^2 = M^3 \int_{-\pi\rho}^{\pi\rho} e^{-2k|x^5|} = \frac{M^3}{k}\left[e^{-2k\pi\rho} - 1\right]$$

hierarchy of scales

$$m(x^5) = m\, e^{-k|x^5|}$$

VACUUM ENERGY = ?

$$E = \int \sqrt{-g}\, e^{-4k|x^5|} \times \Big\{ V_0 +$$

$$+ 2M^3 \left[8k(\delta_1 - \delta_2) - 10k^2 \right] - V_1\delta_1 - V_2\delta_2 \Big\}$$

$$= 0 \text{ when } \Lambda^2 = 0$$

i.e. $V_4(\rho) \equiv 0$

this result follows from 2 finetunings:

$\lambda_1 = -\lambda_{crit}$, $\lambda_2 = \lambda_{crit}$

MASS HIERARCHY

$$ds^2 = e^{-2kT(x^\mu)|\varphi|}\,\bar{g}_{\mu\nu}(x^\mu)\,dx^\mu dx^\nu + $$
$$+ T^2(x^\mu)(d\varphi)^2$$

$$\langle T \rangle = \rho$$

$$\bar{g}_{\mu\nu} = \eta_{\mu\nu} + \bar{h}_{\mu\nu}(x)$$

this gives

- $R_{\mu\nu} = \bar{R}_{\mu\nu}(x) + \frac{1}{\rho^2} e^{-2k\rho|\varphi|} \times$
  $\times \left(2\rho k \left(\delta(\varphi) - \delta(\varphi - \pi)\right) - 4\rho^2 k^2\right)\eta_{\mu\nu}$
- $R_{55} = 8\rho k \left[\delta(\varphi) - \delta(\varphi - \pi)\right] +$
  $- 4\rho^2 k^2$

$$\Rightarrow R = G^{NM} R_{MN} = \frac{1}{\rho^2}\left[16\rho k\left(\delta(\varphi) + \right.\right.$$
$$\left.\left. - \delta(\varphi - \pi)\right) - 20\rho^2 k^2\right] + e^{2k\rho|\varphi|}\bar{R}$$

$$S_{eff} \supset + \frac{1}{2} M^3 \int_{-\pi}^{\pi} \rho\, d\varphi \int d^4x \sqrt{-\bar{g}}\, e^{-2k\rho|\varphi|} \times$$
$$\times \bar{R}$$

$$\Rightarrow M_P^2 = \frac{M^3}{k}\left[1 - e^{-2k\pi\rho}\right]$$

$$S_\phi(\pi\rho) = \int_{x^5=\pi\rho} d^4x \sqrt{-g}\, \{ g^{\mu\nu} \partial_\mu H^\dagger \partial_\nu H + \\ - \lambda \left(|H|^2 - v_0^2 \right)^2 \}$$

substituting the ansatz

$$\int_{\pi\rho} d^4x \sqrt{-\bar{g}}\, \{ \bar{g}^{\mu\nu} \partial_\mu H^\dagger \partial_\nu H e^{-2k\pi\rho} + \\ - \lambda e^{-4k\pi\rho} \left(|H|^2 - v_0^2 \right)^2 \}$$

after $H \to e^{k\rho\pi} H$

the kinetic term is canonical wrt $\bar{g}_{\mu\nu}$ and

$$v_0 \to v = e^{-k\pi\rho} v_0$$

This can be generalized to any classical and dynamical (like $\Lambda_{\text{condensate}}$) mass scale

$$m_0 \to e^{-k\pi\rho} m_0 = a(\pi\rho) m_0$$

If $kr_c \gg 1 \Rightarrow M \sim k \sim M_P$

if $m_0 = M_P \to M_{EW}(\pi\rho) = a\, m_0 = 1\,\text{TeV}$

If $k\rho \sim 12$

KK MODES

$$G_{MN} = \begin{pmatrix} e^{-2k|x^5|}(\eta_{\mu\nu}+h_{\mu\nu}) & \phi \\ \phi & 1 \end{pmatrix}$$

$$h = \phi_m(x)\,\psi_m(x^5)$$

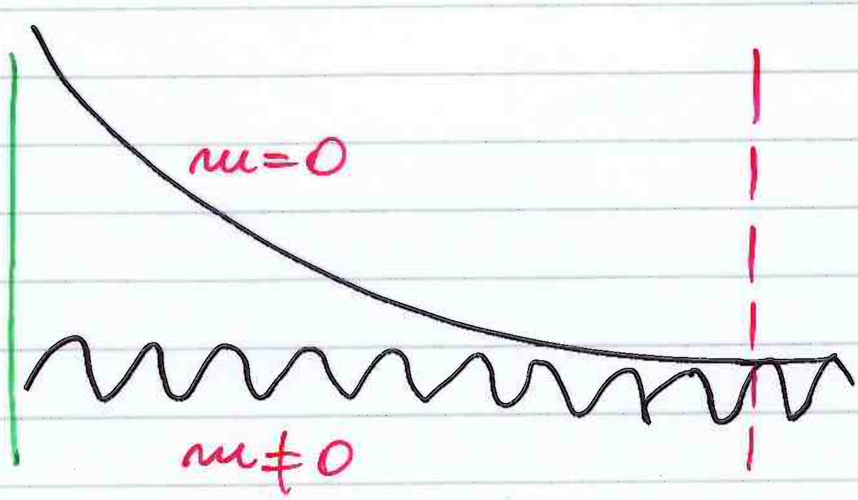
$$\phi_m = e^{ipx} \quad , \; p^2 = -m^2$$

* masses :

$$m_n = \pi\left(n+\frac{1}{4}\right) k\, e^{-k\pi\rho}$$

* amplitudes :

$$\frac{\psi_m(0)}{\psi_0(0)} \sim a(\pi\rho) \quad ; \quad \frac{\psi_m(\pi\rho)}{\psi_0(\pi\rho)} \sim a^{-1}(\pi\rho)$$



coupling of heavy gravitons to matter

$$\delta S = -\frac{1}{2\Lambda^{(n)}} \int \sqrt{-g}\, T^{\mu\nu}_{SM}\, \tilde{h}^{(n)}_{\mu\nu}$$

$$\Lambda^{(n)}(y) = M \frac{\psi^{0}(y)}{\psi^{(n)}(y)}$$

$$\Rightarrow \Lambda^{(n)} = \underbrace{M\, a(y)}_{M_{GUT}} \sim 1\,\mathrm{TeV}$$

* processes to watch –
  – same as in the ADD scenario

SKRZYWIONE KOMPAKTYFIKACJE

* Na 'skrzywionej' ściance

$$M_{GUT} = \mu = M_* \, a(\pi\rho) \ll M_*$$

ciężkie grawitony $m_{KK} \geq \mu \frac{k}{M_*}$

* przy energiach $E \geq \mu \frac{M_*}{k}$

mody KK i mody SM

łączą się w silnie sprzężoną teorię pola

* Zasada Holograficzna

• słabo sprzężona grawitacja ze ściankami w 5d

jest dualna

• do silnie sprzężonej CFT (konforemnej teorii pola) w 4d

CFT sprzęga się do 4d grawitacji poniżej M_P, jej symetrie złamane przy $\mu = M_* \, a(\pi\rho)$