Chair of Condensed Matter Physics
Institute of Theoretical Physics
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Semester Zimowy 2011/2012

Wykład

Modelowanie Nanostruktur

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Coherent Transport

- ✓ **2D-Electron Gas**
- ✓ **Coherent quantum transport: Conductance from Transmission**
  General expression for the current
- ✓ **Tunneling in semiconductors**
- ✓ **Resonant Tunneling Diode**
  Negative Differential Resistance

Recommended reading:
Supriyo Datta, "Quantum Transport, Atom to Transistor"
Cambridge University Press 2005

Supriyo Datta, "Electronic Transport in Mesoscopic Systems"
Cambridge University Press 1995

Modelowanie Nanostruktur, 2011/2012
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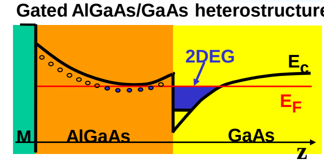
Wykład 5 – *8 XI 2011*

Coherentny Transport w Nanostrukturach

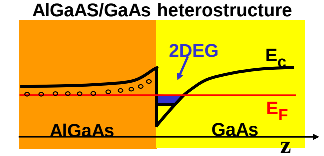
- **Przewodnosc poprzez transmisje**
- **Tunelowanie w nanostrukturach**

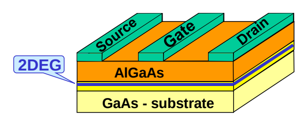
2D-Electron Gas

Gated AlGaAs/GaAs heterostructure



AlGaAS/GaAs heterostructure





Electron mobility

- Bulk GaAs at T=300 K – 9 000 $cm^2 V^{-1} s^{-1}$
- 2DEG at T=300 K in HEMT – 10 000-12 000 $cm^2 V^{-1} s^{-1}$

Classical in-plane transport

- **Electron mobility in 2DEG at low temperatures (< 1 K) can reach 20 000 000 $cm^2 V^{-1} s^{-1}$!!!**

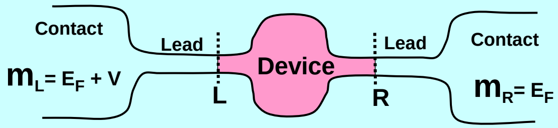
Ballistic, Coherent Quantum Transport

Mesoscopic devices – macroscopic with quantum effects present

Conductance from Transmission

The Landauer approach – very useful in describing mesoscopic transport

The current through a conductor (device) is expressed in terms of ***probability that an electron can transmit through it***



$f_F(E,\mu_L) = f_F(E, E_F + V)$ $\quad f_F(E,\mu) \equiv \frac{1}{\exp[\beta(E-\mu)]+1}$ $\quad f_F(E,\mu_R) = f_F(E,E_F)$

V – external bias

E_F – Fermi energy (chemical potential in the absence of bias)

m_L - electrochemical potential = chemical potential + voltage

- Leads are reservoirs of electrons in which energy- and momentum relaxation processes are so effective that the electron system remains in equilibrium even under a given applied voltage bias
- The electron concentration in the leads is so high that the electrostatic potential in each lead is taken to be constant (as for the case of metal)

Conductance from Transmission

Time-independent transport; Inelastic processes are negligible

Schrödinger Equation: $\left[-\frac{\hbar^2\nabla^2}{2m^*} + V(x,y,z)\right]\psi(x,y,z) = E\psi(x,y,z)$

The potential energy $V(x,y,z) = V_1(x) + V_2(y,z)$ ⟺ $\psi(x,y,z) = \psi(x)\psi_T(y,z)$, $E = E_\parallel + E_T$

$\psi(x), E_\parallel$ **Scattering states**
Transport in x-direction

$\psi_T(y,z) = \psi_{n,m}(y,z)$ $E_T = E_{n,m}$
Transverse modes

Asymptotic solutions for energy $E_\parallel$ $V_1(x)$, $x \to \pm\infty$

The wave function of electrons incoming from the left lead

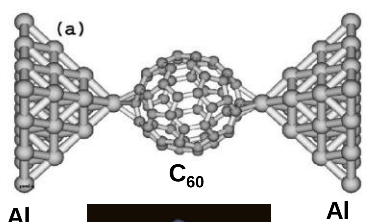
Incoming wave — Reflected wave — Transmitted wave

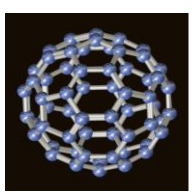
$$\psi^{(+)} = \begin{cases} e^{ik_l x} + r_l e^{-ik_l x} & x \approx x_L \\ t_r e^{ik_r x} & x \approx x_R \end{cases}$$

I, R, T

Fullerene-based molecular nanobridges

Ab-initio studies of the electrical transport in nanocontacts





Conductance from Transmission

The wave function of electrons incoming from the right lead

Incoming wave — Reflected wave — Transmitted wave

$$\psi^{(-)} = \begin{cases} e^{-ik_r x} + r_r e^{-ik_r x} & x \approx x_R \\ t_l e^{-ik_l x} & x \approx x_L \end{cases}$$

T, I, R

$\psi^{(+)}$ & $\psi^{(-)}$ Two linearly independent solutions for energy $E_\parallel$

Continuity equation for current (particle flow) $div\vec{j} = 0 \Rightarrow \vec{j} = const \Rightarrow j_x = const$

$j(x_L) = j(x_R)$ $\qquad j := -\frac{i\hbar}{2m^*}\left(\psi^*\frac{d\psi}{dx} - \psi\frac{d\psi^*}{dx}\right)$

Continuity of current for scattering states

$\psi^{(+)} \Rightarrow k_l(1 - r_l r_l^*) = k_r t_r t_r^*$

$\psi^{(-)} \Rightarrow k_r(1 - r_r r_r^*) = k_l t_l t_l^*$

Conductance from Transmission

Proof of the relation $k_l(1-r_l r_l^*) = k_r t_r t_r^*$ **for** $\psi \equiv \psi^{(+)}$

Left lead

$$\psi_L = \exp(ik_l x) + r_l \exp(-ik_l x) \qquad \frac{d\psi_L}{dx} = ik_l \exp(ik_l x) - ik_l r_l \exp(-ik_l x)$$

$$\psi_L^* = \exp(-ik_l x) + r_l^* \exp(ik_l x) \qquad \frac{d\psi_L^*}{dx} = -ik_l \exp(-ik_l x) + ik_l r_l^* \exp(ik_l x)$$

$$\psi_L^* \frac{d\psi_L}{dx} = ik_l - ik_l r_l \exp(-2ik_l x) + ik_l r_l^* \exp(2ik_l x) - ik_l r_l r_l^*$$

$$\psi_L \frac{d\psi_L^*}{dx} = -ik_l - ik_l r_l \exp(-2ik_l x) + ik_l r_l^* \exp(2ik_l x) + ik_l r_l r_l^*$$

$$j := -\frac{i\hbar}{2m^*}\left(\psi^* \frac{d\psi}{dx} - \psi \frac{d\psi^*}{dx}\right) \Rightarrow j(x_L) = -\frac{i\hbar}{2m^*}(2ik_l - 2ik_l r_l r_l^*) = \frac{\hbar}{m^*} k_l (1 - r_l r_l^*)$$

Right lead

$$\psi_R = t_r \exp(ik_r x) \qquad \frac{d\psi_R}{dx} = ik_r t_r \exp(ik_r x)$$

$$\psi_R^* = t_r^* \exp(-ik_r x) \qquad \frac{d\psi_R^*}{dx} = -ik_r t_r^* \exp(ik_r x)$$

$$j(x_R) = -\frac{i\hbar}{2m^*}\left(\psi_R^* \frac{d\psi_R}{dx} - \psi_R \frac{d\psi_R^*}{dx}\right) = \frac{\hbar}{m^*} k_r t_r t_r^*$$

$$k_l(1 - r_l r_l^*) = k_r t_r t_r^*$$

Conductance from Transmission

More relations for transmission and reflection coefficients

$$1 = r_l r_l^* + t_r^* t_l \quad (*) \qquad r_l^* t_r + t_r^* r_r = 0 \;\Downarrow\; r_l t_r^* + t_r r_r^* = 0 \;\Downarrow\; r_l = \frac{-t_r r_r^*}{t_r^*}$$

$$1 = r_r r_r^* + t_l t_l^* \quad (**) \qquad r_l t_l^* + t_l r_r^* = 0 \;\Downarrow\; r_l = \frac{-t_l r_r^*}{t_l^*}$$

$$\frac{t_r}{t_r^*} = \frac{t_l}{t_l^*} \Rightarrow t_r t_l^* = t_l t_r^*$$

(*) & (**) ⇓

$$r_l r_l^* = r_r r_r^* \Rightarrow |r_l|^2 = |r_r|^2$$

Previously it was proved

$$1 - r_l r_l^* = \frac{k_r}{k_l}|t_r|^2$$

$$1 - r_r r_r^* = \frac{k_l}{k_r}|t_l|^2$$

$$\frac{k_r}{k_l}|t_r|^2 = \frac{k_l}{k_r}|t_l|^2 \Rightarrow k_r^2 |t_r|^2 = k_l^2 |t_l|^2$$

Conductance from Transmission

More relations for transmission and reflection coefficients

$\psi^{(+)}$ & $\psi^{(-)}$ Two linearly independent solutions for energy $E_\parallel$

$\psi^{(+)*}$ & $\psi^{(-)*}$ Are also solutions of the Schrödinger Equation for energy $E_\parallel$

Schrödinger equation – equation of the 2nd order

⇒ ONLY TWO LINEARLY INDEPENDENT SOLUTIONS

(I) $\psi^{(+)*} = A\psi^{(+)} + B\psi^{(-)}$ (II) $\psi^{(-)*} = C\psi^{(+)} + D\psi^{(-)}$

$$\psi^{(+)*}(L) = A\psi^{(+)}(L) + B\psi^{(-)}(L)$$

$$\exp(-ik_l x) + r_l^* \exp(ik_l x) = A\exp(ik_l x) + Ar_l \exp(-ik_l x)) + Bt_l \exp(-ik_l x)$$

$$1 = Ar_l + Bt_l \qquad A = r_l^*$$

$$\psi^{(+)*}(R) = A\psi^{(+)}(R) + B\psi^{(-)}(R)$$

$$t_r^* \exp(-ik_r x) = At_r \exp(ik_r x) + B\exp(-ik_r x) + Br_r \exp(ik_r x)$$

$$0 = At_r + Br_r \qquad B = t_r^*$$

$$1 = r_l r_l^* + t_r^* t_l \qquad r_l^* t_r + t_r^* r_r = 0$$

From Eq. (II) one obtains

$C = t_l^*$, $D = r_r^*$

$$1 = r_r r_r^* + t_l t_l^* \qquad r_l t_l^* + t_l r_r^* = 0$$

Conductance from Transmission

Definition of transmission coefficients

Components of state $\psi^{(+)}$ $\psi_{in} = \exp(ik_l x)$ $\psi_{out} = t_r \exp(ik_r x)$ $\psi_{refl} = r_l \exp(-ik_l x)$

Incoming and outgoing electron flows

$$j := -\frac{i\hbar}{2m^*}\left(\psi^* \frac{d\psi}{dx} - \psi \frac{d\psi^*}{dx}\right)$$

$$j_{in} = \hbar k_l / m^* = v_l \qquad j_{out} = \frac{\hbar k_r}{m^*}|t_r|^2 = v_r |t_r|^2$$

Transmission coefficient through the device for state $\psi^{(+)}$

ratio of outgoing to incoming flow

$$T_{L\to R} = \frac{j_{out}}{j_{in}} = \frac{k_r}{k_l}|t_r|^2 = \frac{v_r}{v_l}|t_r|^2$$

Transmission coefficient through the device for state $\psi^{(-)}$

$$T_{R\to L} = \frac{k_l}{k_r}|t_l|^2$$

$$T_{R\to L} = T_{L\to R} = T(E_\parallel)$$

$E_\parallel = \frac{\hbar^2 k_\parallel^2}{2m^*}$ Kinetic energy

Reflection coefficient=the ratio of reflected and incoming flows

$$R(E) = \frac{j_{refl}}{j_{in}} = |r_l|^2 = |r_r|^2$$

$$T(E) + R(E) = 1$$

Conductance from Transmission

Electric current through the device

The number of electrons in state $\left|k_\parallel, n, m\right\rangle$ per unit length $2f(E(k_\parallel, n, m) - \mu_L)/L_c$

The total contribution to the electric current from the electrons entering from the left

$$I_L = \frac{-2e}{L_c}\sum_{n,m}\sum_{k_\parallel>0} v_\parallel T(E_\parallel) f(E(k_\parallel, n, m) - \mu_L)$$

Similarly, for the electrons entering from the right

$$I_R = \frac{-2e}{L_c}\sum_{n,m}\sum_{k_\parallel<0} v_\parallel T(E_\parallel) f(E(k_\parallel, n, m) - \mu_R)$$

$$\mu_L = E_F + V$$
$$\mu_R = E_F$$

The total electric current through the device

$$I = I_L + I_R = \frac{2e}{L_c}\sum_{n,m}\sum_{k_\parallel>0} v_\parallel T(E_\parallel)[f(E(k_\parallel, n, m) - \mu_R) - f(E(k_\parallel, n, m) - \mu_L)]$$

It is convenient to introduce the function $F(E_\parallel - \mu) := 2\sum_{n,m} f(E(k_\parallel, n, m) - \mu)$

$$E = E(k_\parallel, n, m) = E_\parallel + E_T(n, m)$$

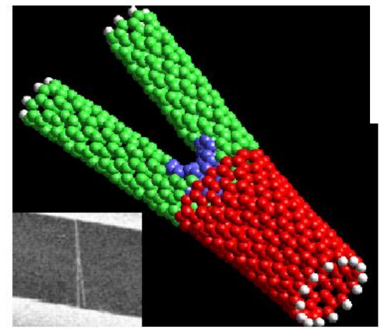
$$\sum_{k_\parallel}\{\ldots\} = L_c\int\frac{dk_\parallel}{2\pi}\{\ldots\} = L_c\int\frac{dE_\parallel}{2\pi\hbar v_\parallel}\{\ldots\}$$

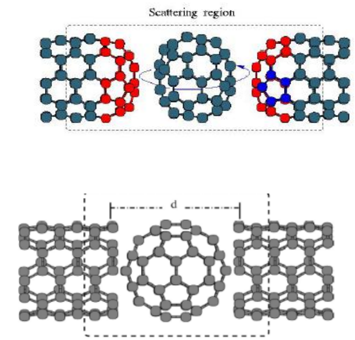
The total electric current

independent on $v_\parallel$

$$I = e\int\frac{dE_\parallel}{2\pi\hbar}T(E_\parallel)[F(E_\parallel - \mu_R) - F(E_\parallel - \mu_L)]$$

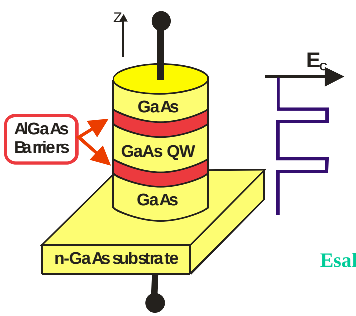
A





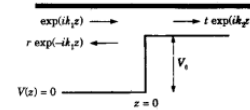
Example of Quantum Transport: Coherent Tunneling

Resonant Tunneling Diode (RTD)



Esaki, Chang, Tsu (IBM, 1974)

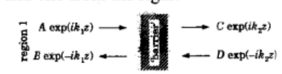
Potential step



r = <u>reflection amplitude</u>; t = <u>transmission amplitude</u>

In general we will have two incoming wave, one from the left and one from the right:

$$k_1^2 = \frac{2mE}{\hbar^2}\ ;\ k_2^2 = \frac{2m(E - V_0)}{\hbar^2}$$



The solutions of Schroedinger equation are

$$\psi(z) = \begin{cases} A\exp(ik_1z) + B\exp(-ik_1z) & , z < 0 \\ C\exp(ik_2z) + D\exp(-ik_2z) & , z > 0 \end{cases}$$

Boundary conditions at z=0 ⟹

$$\begin{cases} A + B = C + D \\ k_1(A - B) = k_2(C - D) \end{cases}$$

For a wave incident from the left (A=1, B=r, C=t, D=0) we have

$$t = \frac{2k_1}{k_1 + k_2}\ ;\ r = \frac{k_1 - k_2}{k_1 + k_2}$$

$$T = \left(\frac{C}{A}\right)^2$$
$$R = \left(\frac{B}{A}\right)^2$$

or, in terms of the *flux transmission and reflection coefficients* T, R:

$$T = \frac{4k_1k_2}{(k_1 + k_2)^2}\ ;\ R = \left(\frac{k_1 - k_2}{k_1 + k_2}\right)^2$$

Theoretical Semiconductor Physics - Quantum devices

Tunneling current

Once $T(E)$ is known, the current voltage characteristics can be calculated under the following assumptions: *i)* electrons on the right and left of the barrier are described by Fermi distributions with appropriate quasi Fermi levels μ_R and μ_L, respectively; *ii)* the applied potential V shifts the quasi Fermi level with respect to one another by $\mu_R - \mu_L = eV$

The current due to electrons from the left is

$$I_L = 2e\int_0^\infty f\left[E(k),\mu_L\right]v(k)\,T(k)\frac{dk}{2\pi} = \frac{2e}{h}\int_{U_L}^\infty f\left[E(k),\mu_L\right]\,T(E)\,dE$$

$$E = \frac{\hbar^2}{2m}k^2 \qquad dE = \frac{\hbar^2}{2m}2k\,dk \qquad v = \frac{1}{\hbar}\frac{\partial E}{\partial k} = \frac{\hbar k}{m}$$

and for the electrons from the right

$$I_R = -\frac{2e}{h}\int_{U_R}^\infty f\left(E(k),\mu_R\right)T(E)\,dE$$

$$f = \frac{1}{1+e^{\beta(E(k)-\mu)}} \qquad \beta = \frac{1}{k_B T}$$

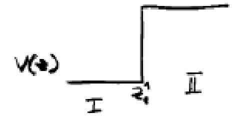
so that the total current is (for $U_L > U_R$)

$$I = \frac{2e}{h}\int_{U_L}^\infty \left[f\left(E(k),\mu_L\right) - f\left(E(k),\mu_R\right)\right]\,T(E)\,dE$$

which is in general a complicated non linear function of the bias

Theoretical Semiconductor Physics - Quantum devices

TRANSFER MATRIX APPROACH



$V(z)$ is constant in a given region

$$\left[-\frac{\hbar^2}{2m}\frac{d^2}{dz^2} + V(z) - E\right]\psi(z) = 0$$

$$\psi(z) = Ae^{ikz} + Be^{-ikz}$$

$$\frac{\hbar^2 k^2}{2m} = E - V \qquad \begin{array}{l} E-V>0 \Rightarrow k \text{ real} \\ E-V<0 \Rightarrow k \text{ imaginary} \end{array}$$

Boundary conditions

$$\left.\begin{array}{l} \psi(z_1^-) = \psi(z_1^+) \\ \frac{1}{m_1}\frac{d\psi}{dz}\Big|_{z_1^-} = \frac{1}{m_2}\frac{d\psi}{dz}\Big|_{z_1^+} \end{array}\right\} \Rightarrow \begin{bmatrix} A_1 \\ B_1 \end{bmatrix} = M \begin{bmatrix} A_2 \\ B_2 \end{bmatrix}$$

M ↑ TRANSFER MATRIX

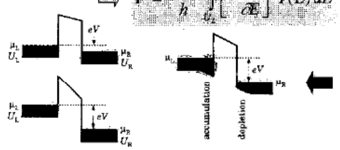
Tunneling currents

Limiting cases:

$$f(E-E_F-V) = f(E-E_F) - V\frac{\partial f}{\partial E} + \cdots$$

ohmic behavior

low bias ⇨ $I = \frac{2e^2V}{h}\int_{U_L}^\infty\left[-\frac{\partial f}{\partial E}\right]T(E)\,dE$ ⇨ $G = \frac{I}{V} \propto \frac{e^2}{h}$



Realistic case: quasi equilibrium regions defining the "leads" only where the potential is flat

high bias ⇨ $I = I_L$ only left electrons contribute

low temperature ⇨ $I = \frac{2e}{h}\int_{\mu_R}^{\mu_L} T(E)\,dE$ ⇨ $G = \frac{2e^2}{h}T(\mu)$

$$f(E-\mu) = \theta(\mu - E)$$

∴ in a perfect wire containing no obstruction T=1 and $G=2e^2/h$

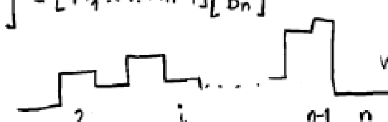
Theoretical Semiconductor Physics - Quantum devices

Transfer Matrix Approach (2)

$C := k_1 m_2 + k_2 m_1$ & $D := k_1 m_2 - k_2 m_1$

$$M = \frac{1}{2k_1 m_2}\begin{bmatrix} C\exp[i(k_2-k_1)z_1] & D\exp[-i(k_2+k_1)z_1] \\ D\exp[i(k_2+k_1)z_1] & C\exp[-i(k_2-k_1)z_1] \end{bmatrix}$$

$$\begin{bmatrix} A_1 \\ B_1 \end{bmatrix} = [M_1 \ldots M_{n-1}]\begin{bmatrix} A_n \\ B_n \end{bmatrix}$$

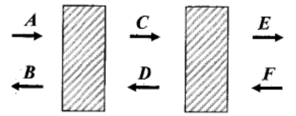


$V_i \rightarrow k_i$

m_i

T-matrices

The transmission and reflection coefficients for a generic step-like potential profile can be expressed in terms of transfer (or T) matrices



$$\begin{pmatrix} C \\ D \end{pmatrix} = T^{(21)} \begin{pmatrix} A \\ B \end{pmatrix} = \begin{pmatrix} T_{11}^{(21)} & T_{12}^{(21)} \\ T_{21}^{(21)} & T_{22}^{(21)} \end{pmatrix} \begin{pmatrix} A \\ B \end{pmatrix} \; ; \; \begin{pmatrix} E \\ F \end{pmatrix} = T^{(32)} \begin{pmatrix} C \\ D \end{pmatrix}$$

⇨ $T^{(31)} = T^{(32)} T^{(21)}$

For a potential step at z=0: $T^{(21)} = \frac{1}{2k_2}\begin{pmatrix} k_2+k_1 & k_2-k_1 \\ k_2-k_1 & k_2+k_1 \end{pmatrix} = T(k_2,k_1)$ (with k_2=iκ_2 for $E<V_0$)

The transmission and reflection amplitudes are given, in terms of the elements of T, by

$$t = \frac{T_{11}T_{22} - T_{12}T_{21}}{T_{22}} \; ; \; r = -\frac{T_{21}}{T_{22}}$$

In general: $T_{11} = T_{22}^* \; ; \; T_{12} = T_{21}^* \; ; \; \det|T| = 1$

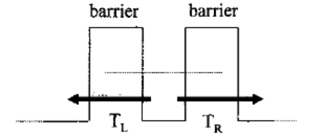
Given T at the **origin, T(0)**, it is possible to write T(d) for the step at z=d through the similarity transformation:

$$T(d) = \begin{pmatrix} e^{-ik_2 d} & 0 \\ 0 & e^{ik_2 d} \end{pmatrix} T(0) \begin{pmatrix} e^{ik_1 d} & 0 \\ 0 & e^{-ik_1 d} \end{pmatrix}$$

Theoretical Semiconductor Physics - Quantum devices

Resonant tunneling

The quantum well inside the two barriers has a resonant or quasi-bound state. The energy of the state cannot be precisely defined, but is spread into a range $\hbar/\tau$, where τ is the lifetime of an electron in the well before it tunnels away.



Usually the transmission probability of the two barriers is roughly the product of the values for the two individual barriers. Near a resonance, T rise dramatically to values close to unity.

$$T_R = \begin{pmatrix} 1/t_R^* & -r_R^*/t_R^* \\ -r_R/t_R & 1/t_R \end{pmatrix} \; ; \; T_L = \begin{pmatrix} 1/t_L^* & r_L/t_L \\ r_L^*/t_L^* & 1/t_L \end{pmatrix}$$

$$T = \begin{pmatrix} e^{-ika/2} & 0 \\ 0 & e^{ika/2} \end{pmatrix} T_R \begin{pmatrix} e^{ika/2} & 0 \\ 0 & e^{-ika/2} \end{pmatrix} \begin{pmatrix} e^{ika/2} & 0 \\ 0 & e^{-ika/2} \end{pmatrix} T_L \begin{pmatrix} e^{-ika/2} & 0 \\ 0 & e^{ika/2} \end{pmatrix}$$

$$= \begin{pmatrix} (1 - r_L^* r_R^* e^{-2ika})/t_L^* t_R^* & (r_L e^{ika} - r_R^* e^{-ika})/t_L t_R^* \\ (r_L^* e^{-ika} - r_R e^{ika})/t_L^* t_R & (1 - r_L r_R e^{2ika})/t_L t_R \end{pmatrix}$$

⇨ Transmission amplitude $t = \frac{t_L t_R}{1 - r_L r_R \exp(2ika)}$

Theoretical Semiconductor Physics - Quantum devices

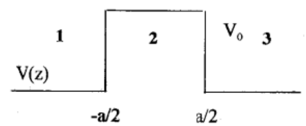
Transmission through a barrier

For a rectangular potential barrier

$$T^{(31)} = T^{(32)}(a/2)\,T^{(21)}(-a/2)$$

$$= \begin{pmatrix} e^{-ik_1 a/2} & 0 \\ 0 & e^{ik_1 a/2} \end{pmatrix} T(k_1,k_2) \begin{pmatrix} e^{ik_2 a/2} & 0 \\ 0 & e^{-ik_2 a/2} \end{pmatrix}$$

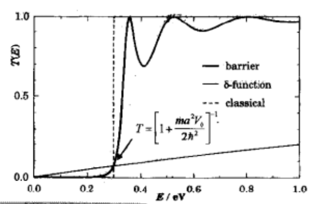
$$\times \begin{pmatrix} e^{ik_2 a/2} & 0 \\ 0 & e^{-ik_2 a/2} \end{pmatrix} T(k_2,k_1) \begin{pmatrix} e^{-ik_1 a/2} & 0 \\ 0 & e^{ik_1 a/2} \end{pmatrix}$$



from which we have

$$T_{21}^{(31)} = \frac{i(k_1^2 - k_2^2)\sin(k_2 a)}{2k_1 k_2}$$

$$T_{22}^{(31)} = \frac{2k_1 k_2 \cos(k_2 a) - i(k_1^2 + k_2^2)\sin(k_2 a)}{2k_1 k_2} e^{ik_1 a}$$



and

$$T = \frac{4k_1^2 k_2^2}{4k_1^2 k_2^2 + (k_1^2 - k_2^2)\sin^2(k_2 a)} = \left[1 + \frac{V_0^2}{4E(E - V_0)}\sin^2(k_2 a)\right]^{-1} \approx \frac{16E}{V_0}\exp(-2\kappa_2 a)$$

for $E<V_0$ and $\kappa_2 a \gg 1$

Theoretical Semiconductor Physics - Quantum devices

Resonant tunneling

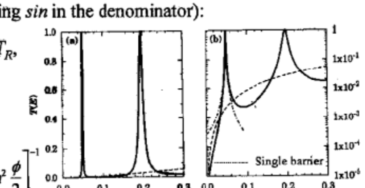
By expressing the complex coefficient in polar form, e.g. r_L=$|r_L|\exp(i\rho_L)$, the flux transmission coefficient i given by:

$$T = |t|^2 = \frac{T_L T_R}{1 + R_L R_R - 2\sqrt{R_L R_R}\cos(2ka + \rho_L + \rho_R)} = \frac{T_L T_R}{\left(1 - \sqrt{R_L R_R}\right)^2 + 4\sqrt{R_L R_R}\sin^2(\phi/2)}$$

Near a resonance, the most rapid variation as a function of energy is due to the change in phase ϕ of the wave between the barriers, $2ka$, thus we can assume that the other term vary slowly ⇒**maximum T for ϕ=$2\pi n$** (vanishing *sin* in the denominator):

$$T_{pk} = \frac{T_L T_R}{\left(1 - \sqrt{R_L R_R}\right)^2} \approx \frac{4T_L T_R}{(T_L + T_R)^2} \quad ; T_{pk}=1 \text{ if } T_L=T_R,$$

The condition $\phi = 2ka + \rho_L + \rho_R = 2\pi n$ requires constructive interference within the well.



$$T \approx \frac{T_L T_R}{(T_L + T_R)^2/4 + 4\sqrt{R_L R_R}\sin^2(\phi/2)} = T_{pk}\left[1 + \frac{16\sqrt{R_L R_R}}{(T_L + T_R)^2}\sin^2\frac{\phi}{2}\right]^{-1}$$

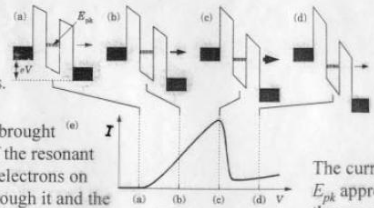
Near the peak: $T \approx T_{pk}\left[1 + \frac{4(\delta\phi)^2\sqrt{R_L R_R}}{(T_L + T_R)^2}\right]^{-1} = \frac{T_{pk}}{1 + (\delta\phi/\phi_0)^2}$ ⇨ Resonant peak has Lorentzian shape

Theoretical Semiconductor Physics - Quantum devices

Resonant tunneling diode

For small bias (a), E_{pk} is above the sea of incoming electrons, and little current flows.

In (b), the bias has brought down the energy of the resonant level, so the sea of electrons on the left can pass through it and the current increases



A further bias increase (c) pulls E_{pk} down so much that the resonant state is no longer available to electrons and the current decreases.

The current grows linearly as E_{pk} approaches the bottom of the sea of electrons (d)

In 1D, for sufficiently low temperatures (Γ=width of the resonant peak):

$$I=\frac{2e}{h}\int_{\mu_R}^{\mu_L}T(E)\,dE \approx \frac{2e}{h}\int_{-\infty}^{\infty}T(E)\,dE=\frac{2e}{h}T_{pk}\int_{-\infty}^{\infty}\left[1+\left(\frac{E-E_{pk}}{\Gamma/2}\right)^2\right]^{-1}dE=\frac{2e}{h}\frac{\pi}{2}\Gamma T_{pk}$$

In 3D, the additional factor μ_L-E appears and

$$J=\frac{e}{h}\frac{m}{\pi\hbar^2}\left(\mu_L-E_{pk}\right)\frac{\pi}{2}\Gamma T_{pk}$$

The maximum J corresponds to the bottom of the range of incoming energies, because there we have the maximum number of states with such energies

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Comparison of theory with experiment

Double-barrier tunneling

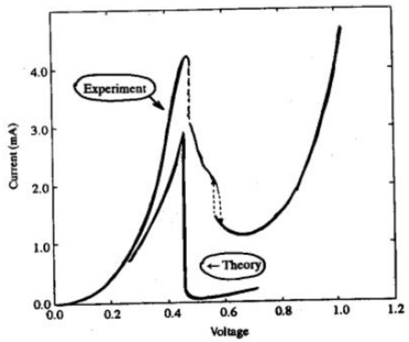

Fig. 6.1.3. *I–V* characteristics of a resonant tunneling diode 8 μm in diameter. Reprinted with permission of MIT Lincoln Laboratory, Lexington, Massachussetts, from E. R. Brown, C. D. Parker, T. C. L. G. Sollner, C. I. Huang and C. E. Stutz (1989), *Proceedings of the OSA Topical Meeting on Picosecond Electronics and Optoelectronics*, March 1989, Salt Lake City, Utah.