

 Chair of Condensed Matter Physics
Institute of Theoretical Physics
Faculty of Physics, University of Warsaw

Semester Zimowy 2012/2013

Wykład

Modelowanie Nanostruktur

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SZ 2012/2013

Modelowanie Nanostruktur

Jacek A. Majewski

- Wykład dla  z podstawową znajomością **Mechaniki Kwantowej**
- **Wykład --- 30 h**
- **Ćwiczenia praktyczne – 45 h**
 - Poniedziałek - Przemek Trędak
 - Środa – Przemek Trędak
 - Czwartek – Maciek Marchwiany

SZ 2012/2013

Modelowanie Nanostruktur

- Sposób zaliczenia = zaliczenie na **ocenę**
- Punkty = pkt. z wykładu (test) + pkt. z ćwiczeń
- Punkty z wykładu = 1/2 punktów z ćwiczeń, na podstawie testu na koniec semestru
- Punkty z ćwiczeń – 14 pkt.
- Punkty z wykładu – 7 pkt.
- Punkty total = **21 pkt**
- **Ocena:** Zalicza > 10 Pkt. , **bdb** > 17 Pkt

SZ 2011/2012

Modelowanie Nanostruktur

Program wykładu

- **Podstawy method atomistycznych:**
 - Metody z pierwszych zasad (DFT)
 - Metody pół-empiryczne
 - metoda ciasnego wiązania = tight-binding
 - metoda pseudopotencjału
- **Metody ciągłe**
- **Zasady dynamiki molekularnej**
- **Metody Monte Carlo**



Modelowanie Nanostruktur, 2012/2013

Jacek A. Majewski

Wykład 1 – 2 X 2012

Introduction to

Computational Science,

(Computer Simulations, Computer Modeling),

Computational Materials Science

Modeling of Nanostructures

Why should we bother with it?

Computational Science

- a relatively new discipline
- involves using computers to study *scientific problems*
- complements the areas of theory and experimentation in traditional scientific investigation
- seeks to gain understanding of science principally through the use and analysis of mathematical models on (high performance) computers
- emerged as a powerful and indispensable method of analyzing a variety of problems in research, product and process development, and manufacturing.

➔ **Computational Simulations**
Changing the way we do Science ?

Computational Simulations

- *Computer simulations* provide both qualitative and quantitative insights into many phenomena that are too complex to be dealt with by analytical methods or too expensive or dangerous to study by experiments.
- Many experiments and investigations that have traditionally been performed in a laboratory, a wind tunnel, or the field are being augmented or replaced by computer simulations.
- Some studies, such as nuclear repository integrity and global climate change, involve time scales that preclude the use of realistic physical experiments.

Computational Simulations

The availability of

- high performance computers,
- graphic workstations,
- and high speed networks,

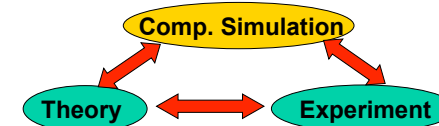
coupled with major advances in algorithms and software, has brought about a revolution in the way scientific and engineering investigations are carried out.

Computational Science vs. Computer Science

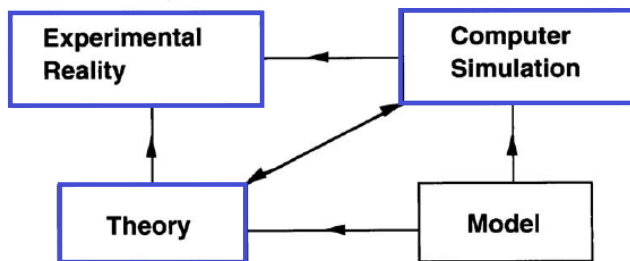
- These two things should not be confused !
- **Computational science** focuses on a scientific or engineering problem and draws from computer science and mathematics to gain an improved understanding of the problem.
- **Computer science** focuses on the computer itself.
- **However,**
Even though the areas are quite distinct, many of the topics typically considered to be in the domain of computer science are of much value in computational science.

Computational Sciences

- Computational Physics
- Computational Astrophysics and Cosmology
- Computational Geophysics
- Computational Chemistry
- Computational Biology
- Computational Engineering
- Computational Materials Science
- **Computational Nanoscience (Modeling of Nanostructures)**

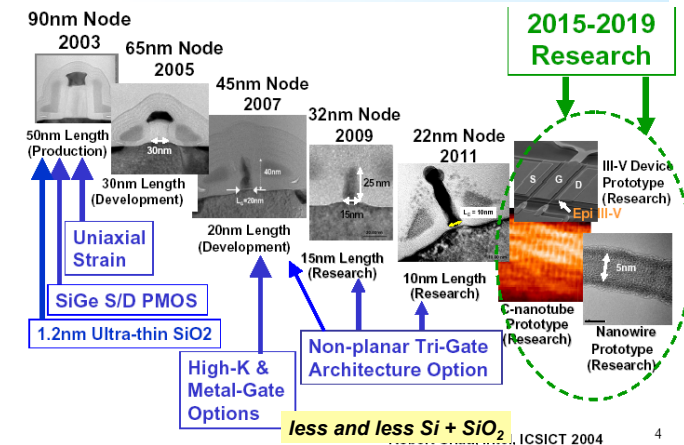


Relationship between modeling, theory and experiment

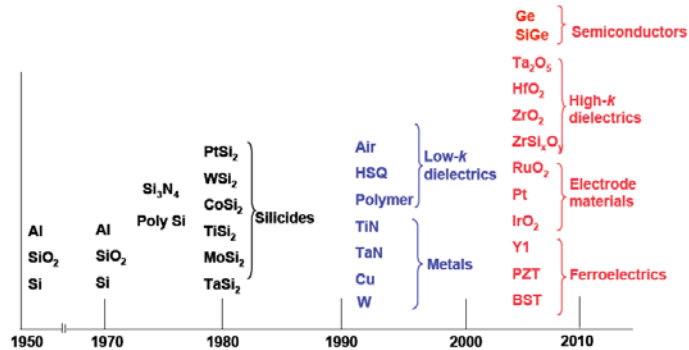


Dr P. D. Bristowe, University of Cambridge,
Department of Materials Science and Metallurgy

Information Technology of the Future



New Materials for Si- Technology



Moore's Law increasingly relies on material innovations

Importance of Materials for Society

The computer is possibly the most visible example of a high tech product that depends critically upon advanced materials

There are many other similar examples that also profoundly affect our lives,

- new lightweight alloys,
- polymers,
- composites, etc.

➡ *Materials Science*

Computational Materials Science

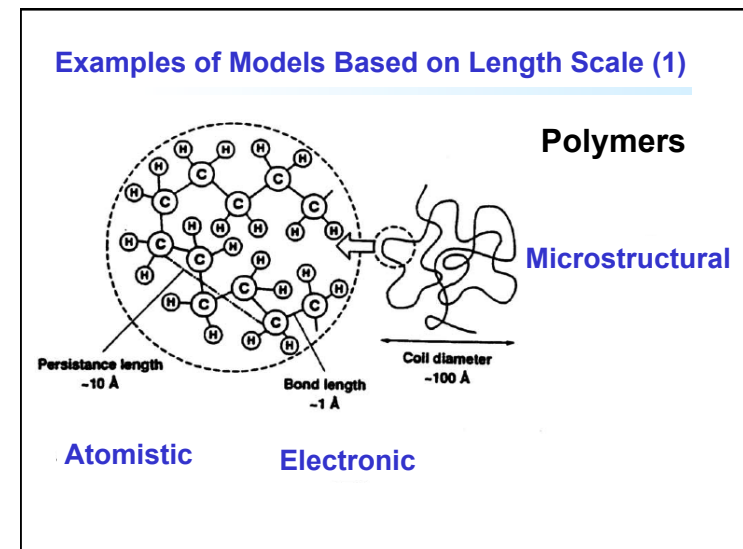
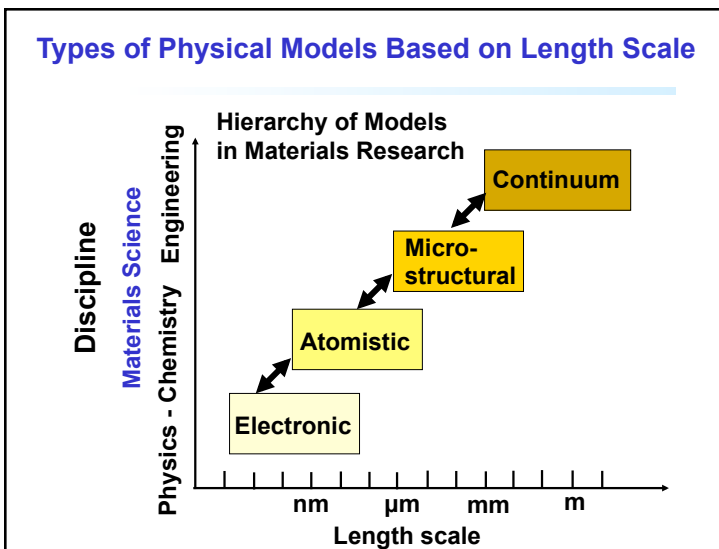
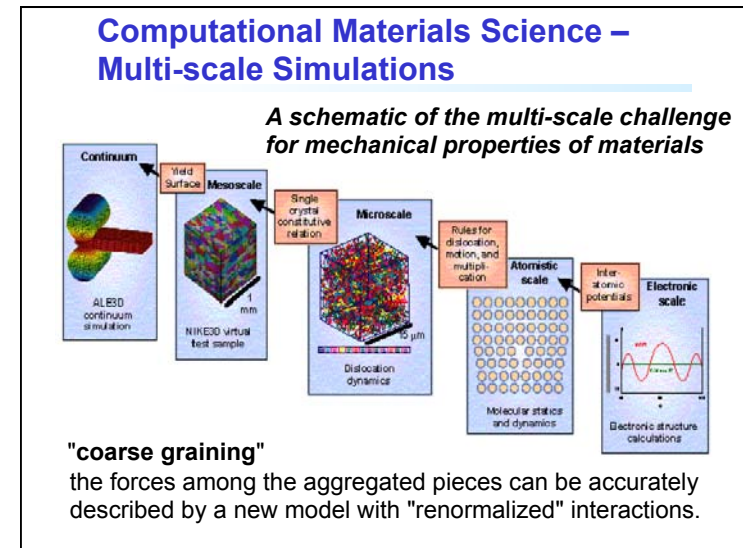
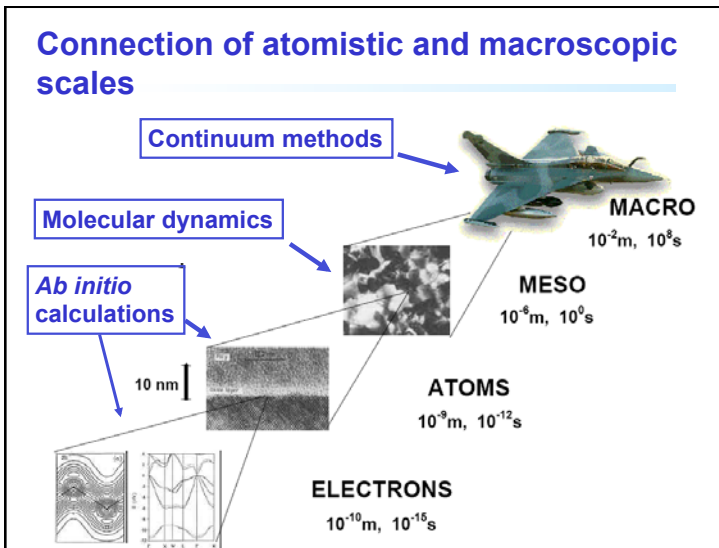
- From the Bronze Age to the silicon-driven Information Age, civilization has defined itself—and advanced itself—by mastering new materials.
- The ability to identify, improve and implement materials -- whether stone, iron, paper, steel or silicon -- has profoundly shaped human societies.
- Today, thanks to increasingly powerful computers, the materials science community finds itself on the verge of another revolution.

➡ **extensive computational modeling will complement and sometimes even replace traditional methods of trial-and-error experimentation.**

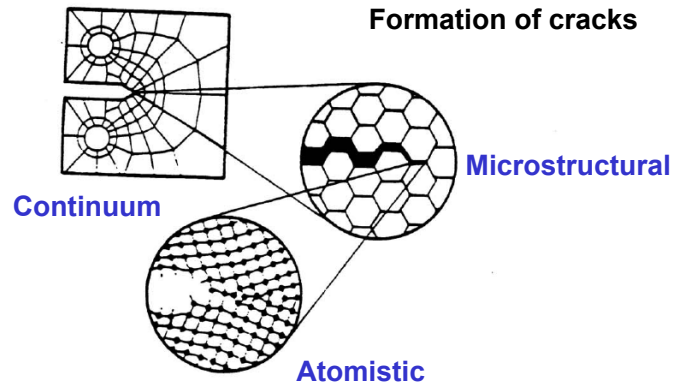
➡ **better understanding and design of new materials**

Computational Materials Science - Scales

- Scientists have a handle on the smallest length scale (relevant to materials), which cannot be seen with a microscope, and the largest length scale, which can be seen with the naked eye.
- In between is an intermediate length scale where there are particularly exciting new frontiers.
- The primary scientific challenge is to uncover the elusive connections in the hierarchy of time and length scales and to unravel the complexity of interactions that govern the properties and performance of materials.



Examples of Models Based on Length Scale (2)



Examples of Computational Simulations

Crack propagation

Computational Simulations of Crack Propagation

- Cracks happen! ▶

Cracks happen! (1)

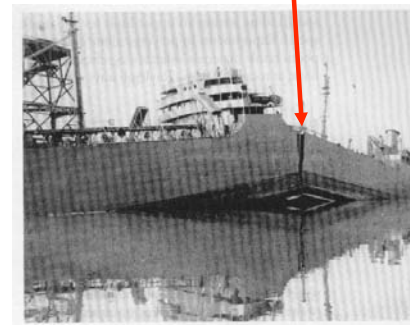


Figure 1. Photograph of the T-2 tanker that failed at its pier: "On January 16, 1943, a T-2 tanker lying quietly at her fitting-out pier at Portland, Oregon, suddenly cracked in a brittle manner... without warning and with a report that was heard for at least a mile..."³ (Photograph courtesy of John Wiley & Sons and the National Academy of Sciences.) ▶

Cracks happen! (2)



The nucleation and growth of cavities in ductile fracture of metallic materials

Experiment !

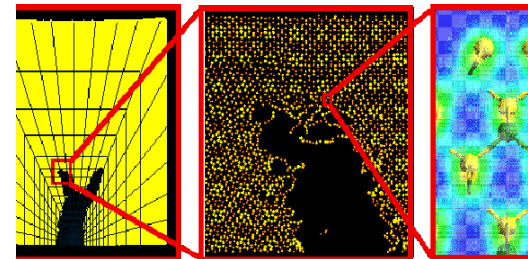
Computational Simulations of Crack Propagation

- **Cracks happen!** ▶
- Fracture of materials causes structural damage, loss of productivity, and at times, loss of life.
- However, predicting how and when a material or structure will fail is a difficult task for scientists.
- Why and how things break involve complex fundamental processes and phenomena occurring on a wide range of length and time scales.

Computational Simulations of Crack Propagation

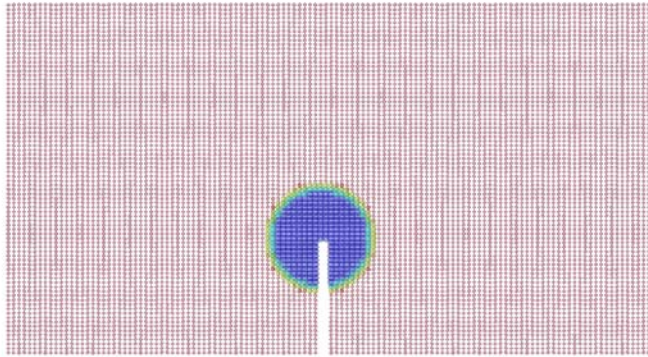
- Scientists are turning to the largest computers available in order to uncover new clues in solving the mystery of catastrophic materials failure.
- Laboratory testing of materials can reveal fracture strengths and explore the macroscopic reasons for fracture.
- Material scientists know that, ultimately, *fracture involves breaking bonds between the atoms that make up the solid.*
- ➡ Computers are ideal for studying these local, atomistic processes.

Three scales of crack simulations – Schematic of a multi-scale simulation approach



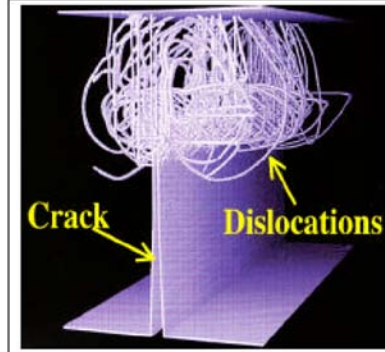
- *Electronic structure method* is combined with *molecular dynamics (MD)* to describe properly bond breakage and bond formation phenomena.
- *The MD*, in turn, is embedded in the *finite-element method* to take into account nonlinearities in the system.

Computational Simulations of Crack Propagation



Computational Simulations of Crack Propagation

The simulation illustrates some of the complex events that occur as a crack moves in a ductile metal.



Atomistic model of crack propagation in ductile metal

At first, the crack moves very rapidly and local bonds break in a "brittle" manner,

- At first, the crack moves very rapidly and local bonds break in a "brittle" manner,
- but at some point the crack-tip begins to emit dislocations and stops propagating.

➡ **blunted crack**

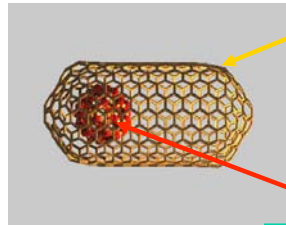
Computational Simulations of Crack Propagation

- **A blunted crack** can cause intense local deformation, but it does not cause failure.
 - **Cracks that can emit dislocations usually stop before complete structural failure occurs.**
 - However, when the material is unable to emit dislocations, crack propagation does not terminate, which eventually leads to failure and irreversible damage of the material.
 - The fundamental phenomena that determine whether a material is able to emit the beneficial dislocations are yet to be elucidated.
- ➡ **design of new materials and composites**

Examples of Computational Simulations

- **Future nano-electronics**
- **Chemical reactions**
- **Computational biology**

Simulation of a nanotube-based memory element

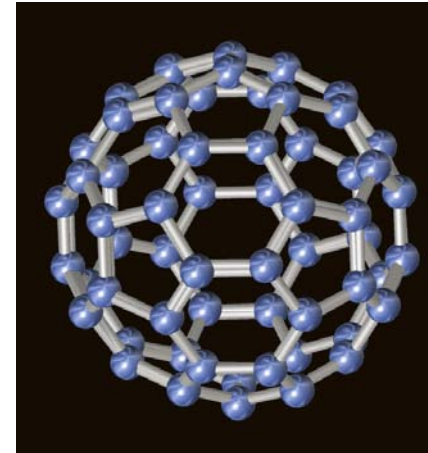


The outer capsule - short segment of (10,10) carbon nanotube with diameter 1.4 nm terminated at both ends by perfect fullerene caps.

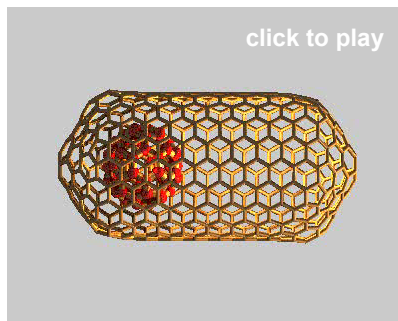
The capsule is large enough to hold a **C₆₀** buckyball molecule inside.

- The buckyball carries a net charge if it contains an alkali atom in the hollow cage.
- The **C₆₀** molecule can be shifted from one end to the other by applying an *electric field* between the ends of the capsule

Fullerene (buckyball)



Simulation of a nanotube-based memory element



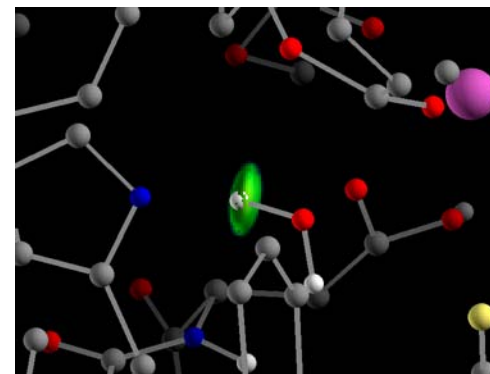
The **two energy minima** of this system, with the buckyball bonded to either end of the capsule, can be associated with **bit 0** and **bit 1**.

The simulation has been performed by Young-Kyun Kwon

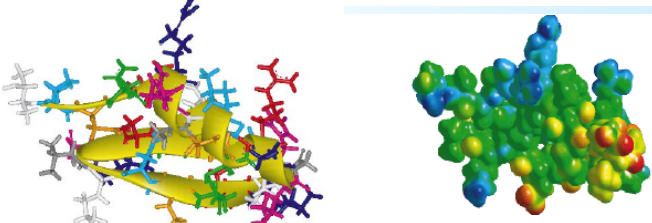
This structure has been patented as a non-volatile memory element and awarded U.S. Patent No. 6,473,351

Ab initio MD Simulations of Chemical Reaction

Dissociation of water molecule



DFT as a new tool for computational biology



Structure of the 1PNH protein, a scorpion toxin ($\text{PO}_5\text{-NH}_2$)

- It consists of 31 amino acid Residues (ca. 500 atoms)
- Different colors are used to represent different amino acids

Electrostatic potential for electrons

- Zero value – green
- Repulsive regions – red and yellow
- Attractive regions - blue

Role of the protein charge state on the geometry – change of the charge state of the protein does not destabilize the local energy minima

Computational Materials Science: A Scientific Revolution about to Materialize

- The materials science community is on the verge of a paradigm shift in the way it does science with the promise of building a sturdy bridge across the "valley of death" between basic science and technological impact.
- A useful construct for thinking about this potential paradigm is "**Pasteur's Quadrant.**"

D. E. Stokes, "Pasteur's Quadrant, Basic Science and Technological Innovation,, The Brookings Institution, Washington D.C., 1997

Computational Materials Science: A Scientific Revolution about to Materialize

- Due to the complexity of materials systems, progress has necessarily proceeded either within the Bohr quadrant or Edison's quadrant

Pasteur's Quadrant

Understanding Driven	Bohr	Pasteur
		Edison
	Use driven	

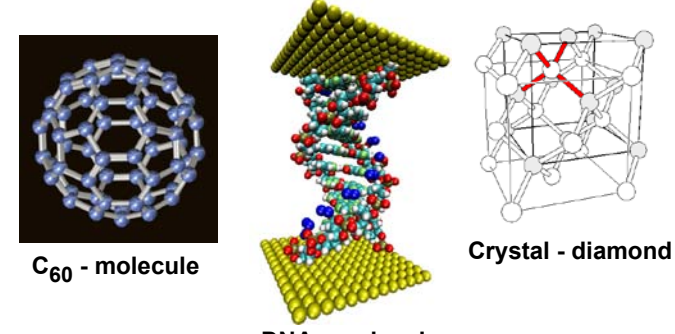
experiment and theory done on model systems

research and development by trial and error

Realistic simulation is the vehicle for moving materials research firmly into Pasteur's quadrant.

Fundamental problem in materials science

A fundamental problem in materials science is the prediction of condensed matter's electronic structure

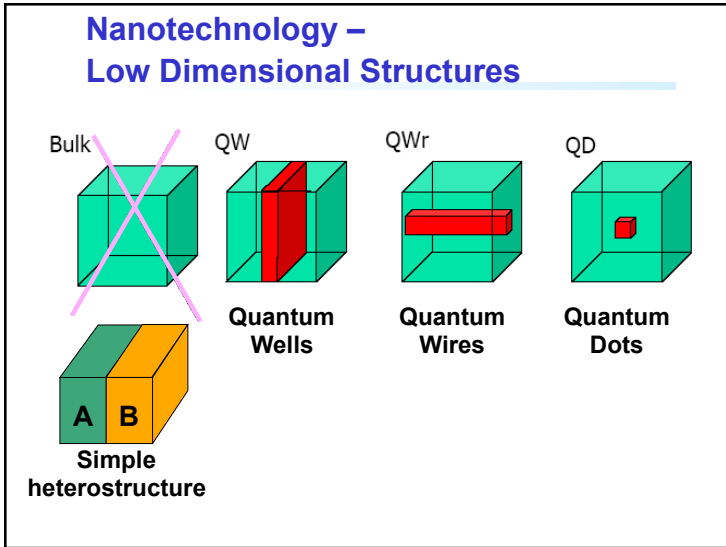
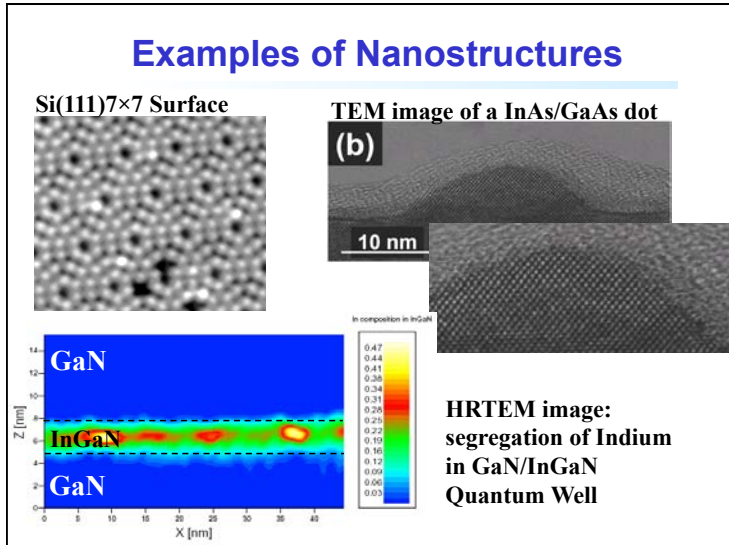


C₆₀ - molecule

DNA - molecule

Crystal - diamond

Modeling Nanostructures



What about realistic nanostructures ?

Inorganics

- 3D (bulks)** : 1-10 atoms in the unit cell
- 2D (quantum wells)**: 100 – 1000 atoms in the unit cell
- 1D (quantum wires)**: 1 K-10 K atoms in the unit cell
- 0D (quantum dots)**: 100K-1000 K atoms in the unit cell

Organics

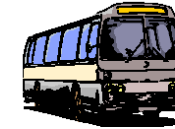
- Nanotubes, DNA**: 100-1000 atoms (or more)

Atomistic methods for modeling of nanostructures

- *Ab initio* methods (up to few hundred atoms)
- Semiempirical methods (up to 1M atoms)
 - Empirical Pseudopotential
 - **Tight-Binding Methods**

Materials Science: Why *ab-initio* approach is needed?

- A model as simple as possible
- A model non-empirical and realistic



- Explanation and extrapolation of experimental results
- Physical insight
- Qualitative physics
- Reliable predictions of matter's properties
- Design of new materials
- Unexpected phenomena and unusual conditions (e.g., extreme pressures)

Materials Science: Examples of Schrödinger Equation?

- Materials are composed of nuclei $\{Z_\alpha, M_\alpha, \vec{R}_\alpha\}$ and electrons $\{\vec{r}_i\}$
 → the interactions are known

$$H = -\sum_{\alpha} \frac{\hbar^2 \nabla_{\alpha}^2}{2M_{\alpha}} - \sum_i \frac{\hbar^2 \nabla_i^2}{2m} + \frac{1}{2} \sum_{\alpha, \beta} \frac{Z_{\alpha} Z_{\beta} e^2}{|\vec{R}_{\alpha} - \vec{R}_{\beta}|} - \sum_{i, \alpha} \frac{Z_{\alpha} e^2}{|\vec{R}_{\alpha} - \vec{r}_i|} + \frac{1}{2} \sum_{i, j} \frac{e^2}{|\vec{r}_i - \vec{r}_j|}$$

Kinetic energy of nuclei Nucleus-Nucleus interaction Electron-Electron interaction
 Kinetic energy of electrons Electron-Nucleus interaction

$$H\Psi = E\Psi$$

***Ab-initio* (first principles) Method – ONLY Atomic Numbers $\{Z_i\}$ as input parameters**

Spectrum of Electronic Hamiltonian: What *ab initio* methods do we have?

$$H\Psi = E\Psi$$

$$H = -\sum_{\alpha} \frac{\hbar^2 \nabla_{\alpha}^2}{2M_{\alpha}} - \sum_i \frac{\hbar^2 \nabla_i^2}{2m} + \frac{1}{2} \sum_{\alpha, \beta} \frac{Z_{\alpha} Z_{\beta} e^2}{|\vec{R}_{\alpha} - \vec{R}_{\beta}|} - \sum_{i, \alpha} \frac{Z_{\alpha} e^2}{|\vec{R}_{\alpha} - \vec{r}_i|} + \frac{1}{2} \sum_{i, j} \frac{e^2}{|\vec{r}_i - \vec{r}_j|}$$

- ☹ There is no exact solution for even two electrons

→ Approximations needed

- Hartree – Fock + correction
- reformulate the problem → density functional theory (DFT)
 → Use approximations to DFT (e.g., LDA)

DFT for silicon nanostructures

Silicon nanoparticles (clusters, dots)

- > optoelectronic materials on silicon basis
- > biosensors to detect biological and chemical warfare agents

71 Si atoms 'passivated' by hydrogens

Electrons are in the center of the dot

Dramatic change of the optical properties (wavelength) of the silicon nanostructure

G. Gali & F. Gygi, Lawrence Livermore National Laboratory

Computational Materials Science – The Era of Applied Quantum Mechanics

The properties of new and artificially structured materials can be predicted and explained

- *entirely by computations,*
- *using atomic numbers as the only input .*

Tight-Binding methods

Tight-Binding Formalism

$$\varphi_n(\vec{r}) = \sum_{\alpha,i} c_n^{\alpha i} \chi_{\alpha i}(\vec{r})$$

$\{\chi_{\alpha i}\}$ orthogonal set of functions
 → NOT ATOMIC ORBITALS !

index of orbital α, i

index of atom

$$\varepsilon_n = \langle \varphi_n | H | \varphi_n \rangle = \sum_{i\alpha} \sum_{j\beta} (c_n^{\alpha i})^* c_n^{\beta j} \langle i\alpha | H | j\beta \rangle$$

Tight-binding Hamiltonian

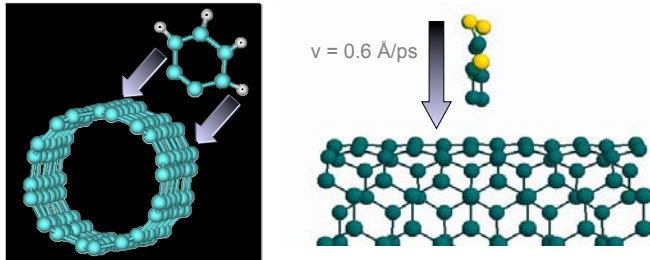
$H = \sum_{i\alpha} \varepsilon_{i\alpha} |i\alpha\rangle\langle i\alpha| + \sum_{i\alpha, j\beta} t_{i\alpha, j\beta} |i\alpha\rangle\langle j\beta|$

on-site

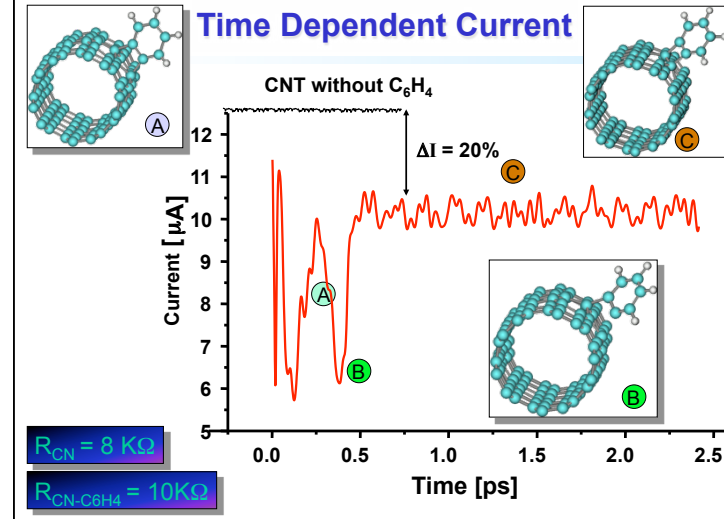
hoping TB-parameters

Green's Function + Molecular dynamics Carbon Nanotubes

- **Molecular Dynamics** simulations of a reactive collision of a biased nanotube ($V=100\text{mV}$) and benzene
- **Current** flowing in the nanotube calculated at each MD step

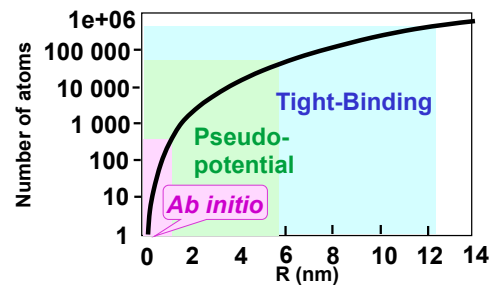


Time Dependent Current



Conclusions

- Microscopic approaches can be applied to calculate properties of realistic nanostructures



Number of atoms in a spherical Si nanocrystal as a function of its radius R . Current limits of the main techniques for calculating electronic structure. Nanostructures commonly studied experimentally lie in the size range 2-15 nm.

Continuum theory-
Envelope Function Theory

Electron in an external field

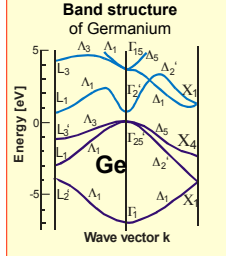
$$\left[\frac{\hat{p}^2}{2m} + V(\vec{r}) + U(\vec{r}) \right] \psi(\vec{r}) = \epsilon \psi(\vec{r})$$

Periodic potential of crystal

Non-periodic external potential

● Strongly varying on atomic scale
● Slowly varying on atomic scale

$U(\vec{r}) = 0 \Rightarrow \epsilon_n(\vec{k}) \Rightarrow$



Band structure of Germanium

Envelope Function Theory – Effective Mass Equation

J. M. Luttinger & W. Kohn, Phys. Rev. B **97**, 869 (1955).

$$[\epsilon(-i\vec{\nabla}) + U(\vec{r}) - \epsilon] F_n(\vec{r}) = 0 \quad \text{(EME)}$$

EME does not couple different bands

$$\psi(\vec{r}) = F_n(\vec{r}) u_{n0}(\vec{r})$$

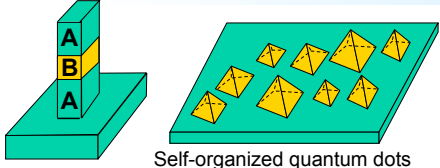
"True" wavefunction

Envelope Function

Periodic Bloch Function

- Special case of constant (or zero) external potential
 $U(\vec{r}) = 0 \Rightarrow F_n(\vec{r}) = \exp(i\vec{k} \cdot \vec{r}) \Rightarrow \psi(\vec{r})$ Bloch function
- $U(z) \Rightarrow F_n(\vec{r}) = \exp[i(k_x x + k_y y)] F_n(z)$

Electron States in Quantum Dots



Electrons confined in all directions

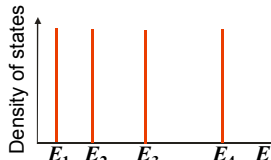
$U(x, y, z)$

Self-organized quantum dots

$$-\frac{\hbar^2}{2m^*} \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right) F_n(x, y, z) + U(x, y, z) F_n(x, y, z) = E_n F_n(x, y, z)$$

Density of states for zero dimensional (0D) electrons (artificial atoms)

$$G^{(0D)}(E) = \sum_{\nu} \delta(E - E_{\nu})$$



Density of states

Calculation of the strain tensor

Elastic energy $E = \frac{1}{2} \int C_{ijkl}(\vec{x}) e_{ij}(\vec{x}) e_{kl}(\vec{x}) d\vec{x}$

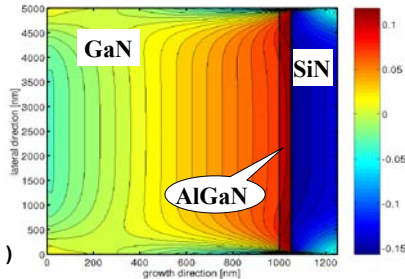
Minimization of elastic energy gives the strain distribution $e_{kl}(\vec{x})$

It corresponds to $\frac{\partial s_{ij}}{\partial x_i} = 0$

$s_{ij} = C_{ijkl} e_{kl}$
Hook's Law

Strain Map

(for GaN/AIGaN HEMT)



lateral direction [nm] vs growth direction [nm]

3D nano-device simulator - nextnano³

Simulator for 3D semiconductor nano-structures:

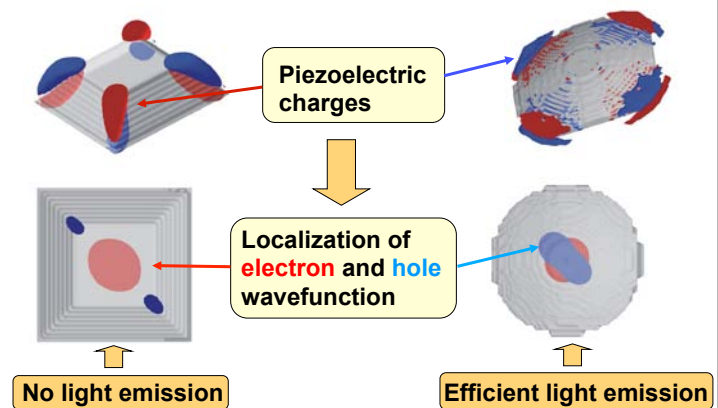
- Si/Ge and III-V materials
- Flexible structures & geometries
- Fully quantum mechanical
- Equilibrium & nonequilibrium

Calculation of **electronic structure** :

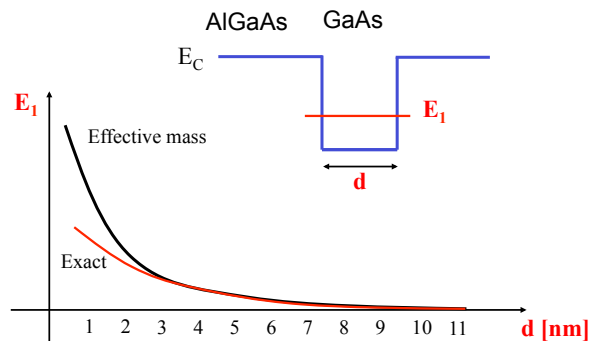
- 8-band kp-Schrödinger+Poisson equation
- Global strain minimization
- Piezo- and pyroelectric charges
- Exciton energies, optical matrix elements,...

Calculation of **current** only close to **equilibrium** with **new** approach

Dot shape and piezoelectric charges



How good is effective mass approx. ?



- **Atomistic details sometimes matter !**

Software for nanostructures modeling

<http://www.facebook.com/pages/List-of-software-for-nanostructures-modeling/144056828940389>

Modeling of **nanostructures** at classical and quantum levels.

Ascalaph Designer
Atomistix ToolKit & Virtual NanoLab
CST STUDIO SUITE(TM)
CoNTub

Nanohub allows simulating geometry, electronic properties and electrical transport phenomena in various nanostructures

Nanorex

NEMO 3-D enables multi-million atom electronic structure simulations in empirical tight binding. It is open source. An educational version is on nanoHUB as well as [Quantum Dot Lab](#)

.....

facebook

Usage of NanoHub (<http://nanohub.org>)



Thank you!

Modeling of Nanostructures: Literature:

- Werner Krauth, ***Statistical Mechanics, Algorithms and Computations***, Oxford University Press, 2006
Includes codes for over 100 algorithms
- Kalman Varga & Joseph A. Driscoll, ***Computational Nanoscience, Applications for Molecules, Clusters, and Solids***, Cambridge University Press, 2011
The computer codes and examples used in this book are available on www.cambridge.org/9781107001701

Computational Materials Science: Literature

- Gonis A., ***Theoretical Materials Science, Tracing the Electronic Origin of Materials Behavior*** (Materials Research Society, 2000)
- D. Raabe, ***Computational Materials Science***, (Wiley, 1992)
- K. Ohno, K. Esfarjani, and Y. Kawazoe, ***Computational Materials Science, From Ab Initio to Monte Carlo Methods*** (Springer, 1999).
- Robert G. Parr and Weitao Yang, ***Density-Functional Theory of Atoms and Molecules*** (Oxford University Press, 1989)
- Richard M. Martin, ***Electronic Structure: Basic Theory and Practical Methods*** (Cambridge University Press, 2004).
- Z. H. Barber (ed), ***Introduction to Materials Modelling***, (Maney, 2005)
- J. M. Haile, ***Molecular Dynamics Simulation*** (Wiley 1992)