

POLS fellowship

Conformal and CR methods in general relativity

Abstract for the general public

More than a century after its formulation, Albert Einstein's General Theory of Relativity remains a robust model of the large-scale structure of our universe, and a number of its predictions have now been tested experimentally, the latest being the detection of gravitational waves. In its mathematical formulation, each physical phenomenon often takes the form of a *geometric structure*. For instance, gravitation causes our spacetime to *curve*, and particles move along *geodesics*, the analogues of straight lines in curved space. Here, the laws of motion are governed by the *Einstein field equations*, which are notoriously difficult to solve explicitly. Yet, the presence of additional structures may have the remarkable effect of simplifying them.

The aim of this project is to investigate two geometric structures that played a central rôle in the description of light rays in the context of solutions to Einstein's equation. One is the notion of a *conformal structure*, that is, a means to measuring angles and relative lengths. This is intrinsically connected to the propagation of light along geodesics of 'zero length'.

The other one is the notion of a *Cauchy-Riemann (CR) structure*, which underlies very special families of light rays. These are said to be *non-shearing* and feature in many important gravitational objects such as black holes.

These two concepts came to shape the development of mathematical relativity in the 1960ies and 1970ies through the work of the Polish relativist Andrzej Trautman and his English collaborator Ivor Robinson. In this project, we shall take their approach further by drawing on contemporary techniques of differential geometry, and apply our results to the study of black holes and related geometries known as *horizons*.

These gravitational objects are significant by virtue of the fact they feature a *gravitational singularity*. There, the differential geometric fabric of spacetime breaks down and quantum theory, which describes the small-scale structure of the universe, takes over. They thus constitute a fertile playground on which predictions regarding quantum gravity can be put forward.

The results expected from this research project will be a more conceptual understanding of the geometric structures behind gravitational phenomena such as found around black holes and their horizons. A thorough analytic understanding of these will allow an invariant description of physical quantities and facilitate their computation. This will in turn provide new solutions to the Einstein field equation in dimension four and higher.