Class problems #7

- 1. Find relativistic and non-relativistic limits for equilibrium number density (n), energy density (ρ) and pressure (p) as a function of temperature for fermions and bosons.
- 2. Derive an exact expression for the number of relativistic degrees of freedom and reproduce (discuss) qualitatively fig. 1 from the lecture notes #5, in particular show that for $T \gg 100 \text{ MeV}$ $(T \gtrsim 10^6 \text{ MeV}) g_{\star} = 106\frac{3}{4}$, for $T \ll 1 \text{ MeV} (T \lesssim 10^{-1} \text{ MeV}) g_{\star} = 3.36$, while for $10^2 \text{ MeV} \gtrsim T \gtrsim 1 \text{ MeV} g_{\star} \simeq 10\frac{3}{4}$. Verify tables 1 and 2 from the lecture notes.
- 3. Derive

$$p_i(T) = g_i \int \frac{|\vec{p}|^2}{3E_i(\vec{p})} f_i(\vec{p}, T) \frac{d^3p}{(2\pi)^3},$$

which is eq.(4) of the lecture notes.