Class problems #1

1. Introduce the natural physical units
$$\hbar = c = k_B = 1,$$
the Planck length ($l_{\rm Pl}$), the Planck time ($t_{\rm Pl}$) and the Planck mass ($M_{\rm Pl}$) in the SI unit system. Compare $M_{\rm Pl}$ with the mass of a bacteria e.g. E.coli (in kg).

2. Derive the Friedmann equations from the cosmological principle (homogeneity and isotropy) using non-relativistic Newtonian dynamics [1, 2].

3. Find the orbital velocity of stars in a galaxy far from a massive core (stellar bulge). Assume isotropy.

4. Find the mass distribution in a galaxy that would explain flat rotational curves. Assume isotropy.

5. Estimate the orbital speed of the solar system knowing that the distance to the center of the Galaxy is about 8 kpc(define pc). The Milky Way contains about 2×10^{11} stars.

6. The estimated energy (mass) density of dark matter (DM) in the solar neighborhood is around 0.3 GeV/ cm^3. Suppose that the DM is made of WIMPs (Weakly Interacting Massve Particle) of mass $\sim$ 100 GeV.
   - How many WIMPs are roughly inside your body at any time?
   - What is the DM flux, i.e. the number of particles per cm^2, per s, if they move with the typical galactic velocity $v \sim 200$ km/sec?

7. Assume that the Milky Way is a typical galaxy containing about 10^{11} stars and that averaged distance between galaxies is about 1 Mpc. Estimate the Universe density in SI units.

8. Spectral energy density for a black body radiation is
$$\varepsilon(\nu) = \frac{8\pi h}{c^3} \frac{\nu^3}{e^{\frac{h\nu}{k_B T}} - 1}, \qquad [\varepsilon] = \frac{\rm J}{\rm m^3 Hz}$$
The binding energy of the hydrogen atom is 13.6 eV. CMB has a spectrum of black body at $T = 2.728$ K, find:
   - the average energy $\langle \varepsilon \rangle$ and the number density $n_{\rm tot}$ of photons in eV and compare to the hydrogen atom binding energy,
   - the frequency and the wavelength of photon for the maximum of the sppectrum,
   - the total photon energy density in J m^{-3}, eV m^{-3}.

9. Derive the following table

	CMB		Barions	
$\langle \varepsilon \rangle$	10^{-22} J	$6 \cdot 10^{-4}$ eV	$1.5 \cdot 10^{-10}$ J	10^9 eV
$\varepsilon_{\rm tot}$	$4 \cdot 10^{-14}$ J m^{-3}	$2 \cdot 10^5$ eV m^{-3}	$1.5 \cdot 10^{-10}$ J m^{-3}	10^9 eV m^{-3}
$n_{\rm tot}$	$4 \cdot 10^8$ m^{-3}		1 m^{-3}	

References

[1] Andrew Liddle, "An Introduction to Modern Cosmology".

[2] Joan Arnau Romeu, "Derivation of Friedman equations", Barcelona, June 2014.