Mateusz Wolniewicz 370 397 $\varphi \vee 1$ $\boxed{5/5}$

Zad 1 $m, q, \vec{B} = B\hat{e}_z = (0,0,B)$

$V = -\frac{k}{2}(x^2+y^2) \quad k>0$

$\det \begin{vmatrix} e_x & e_y & e_z & e_x & e_y \\ V_x & V_y & V_z & V_x & V_y \\ 0 & 0 & B & 0 & 0 \end{vmatrix} = V_yBe_x - V_xBe_y$

$F_{odv} = -kx - ky$

$$\begin{cases} m\ddot{x} = +kx + q\dot{y}B \\ m\ddot{y} = +ky - qB\dot{x} \\ m\ddot{z} = 0 \end{cases} \qquad \begin{matrix} x(0) = x_0 & V_x(0) = 0 \\ y(0) = 0 & V_y(0) = V_0 \\ z(0) = 0 & V_z(0) = 0 \end{matrix}$$

Brak przyspieszeń i sił w kierunku z więc ruch w 2D

$$\begin{cases} m\ddot{x} = +kx + q\dot{y}B \ /m \\ m\ddot{y} = +ky - qB\dot{x} \ /m \end{cases} \rightarrow \begin{cases} \ddot{x} = +\omega_0^2 x + \omega_B \dot{y} & (1) \\ \ddot{y} = +\omega_0^2 y - \omega_B \dot{x} & (2) \end{cases} \quad \omega_0 = \sqrt{\frac{k}{m}} \quad \omega_B = \frac{qB}{m}$$

~~$(1)\ \ddot{x} + \omega_0^2 x - \omega_B \dot{y} = 0 \rightarrow \frac{d}{dt}(\dot{x} + \omega_0^2 - \omega_B y) = 0$~~

~~$\dot{x} + \omega_0^2 - \omega_B y = const \rightarrow \dot{x} = -\omega_0^2 + \omega_B y + C$~~

~~$(2)\ x = e^{\chi t} \quad y = e^{\Upsilon t} \quad \dot{x} = \chi x \quad \ddot{x} = \chi^2 x \quad \ddot{y} = \Upsilon^2 y$~~

~~$\begin{cases} \chi^2 = -\omega_0^2 + \omega_B \Upsilon \\ \Upsilon^2 = -\omega_0^2 - \omega_B \chi \end{cases}$~~

$W = x + iy \qquad \dot{W} = \dot{x} + i\dot{y} \qquad \ddot{W} = \ddot{x} + i\ddot{y}$

$(1) + (2)\cdot i \rightarrow \quad \ddot{x} + i\ddot{y} = +\omega_0^2(x+iy) + (\dot{y} - i\dot{x})\omega_B$

$\dot{y} - i\dot{x} = -i\dot{W}$

$\ddot{W} = +\omega_0^2 W - i\dot{W}\omega_B$

$$\ddot{W} + i\omega_B \dot{W} - \omega_0^2 W = 0 \longleftarrow W = Ae^{i\lambda t}$$

$$\dot{W} = i\lambda A e^{i\lambda t}$$

$$\ddot{W} = -\lambda^2 A e^{i\lambda t}$$

$$\cancel{-\lambda^2 - \lambda\omega_B + \omega_0^2 = 0}$$

$$\cancel{\lambda^2 + \lambda\omega_B - \omega_0^2 = 0}$$

$$\cancel{\Delta = \omega_B^2 - 4(-\omega_0^2)} \qquad \cancel{\sqrt{\Delta} = \sqrt{\omega_B^2 + 4\omega_0^2}}$$

$$\cancel{\lambda_1 = \frac{-\omega_B + \sqrt{\omega_B^2 + 4\omega_0^2}}{2}} \qquad \cancel{\lambda_2 = \frac{-\omega_B - \sqrt{\omega_B^2 + 4\omega_0^2}}{2}}$$

$$\cancel{W(t) = Ae^{i\lambda_1 t} + Be^{i\lambda_2 t} = Ae^{i\frac{-\omega_B + \sqrt{\omega_B^2 + 4\omega_0^2}}{2}t} + Be^{i\frac{-\omega_B - \sqrt{\omega_B^2 + 4\omega_0^2}}{2}t}}$$

$$\cancel{W(0) = x_0 + 0i \qquad \dot{W}(0) = 0 + iV_0}$$

z 1 warunku

$$\cancel{A + B = x_0}$$

z ~~2 warunku~~

$$-\lambda^2 - \lambda\omega_B - \omega_0^2 = 0 \qquad \lambda^2 + \lambda\omega_B + \omega_0^2 = 0$$

$$\Delta = \omega_B^2 - 4\omega_0^2$$

$$\lambda_1 = \frac{-\omega_B + \sqrt{\omega_B^2 - 4\omega_0^2}}{2} \qquad \lambda_2 = \frac{-\omega_B - \sqrt{\omega_B^2 - 4\omega_0^2}}{2}$$

$W = Ae^{i\lambda_1 t} + Be^{i\lambda_2 t}$ gdz. $Re(W) = x \quad Im(W) = y$

$\text{I} \; W(0) = x_0 + 0i \qquad \text{II} \; \dot{W}(0) = 0 + iV_0$

z I warunku

$$A + B = x_0 \qquad \cancel{A = (x_0 - B)}$$

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2 $\underline{\overset{\frown}{u}}$ warunku

$$\dot{w} = Ai\lambda e^{i\lambda_1 t} + iB\lambda e^{\lambda t}$$

$$\dot{w}(0) = Ai\lambda_1 + Bi\lambda_2 = iV_0$$

$$A\lambda_1 + B\lambda_2 = V_0 \rightarrow \cancel{A} \quad B = \frac{V_0 - A\lambda_1}{\lambda_2}$$

$$\cancel{\lambda x_0 - B\lambda_1 + B\lambda = V_0}$$

$$A + \frac{V_0 - A\lambda_1}{\lambda_2} = x_0 \rightarrow \lambda_2 A + V_0 - A\lambda_1 = x_0\lambda_2$$

$$A(\lambda_2 - \lambda_1) + V_0 = x_0\lambda_2 \rightarrow A = \frac{x_0\lambda_2 - V_0}{(\lambda_2 - \lambda_1)} \wedge B = x_0 - \frac{x_0\lambda_2 - V_0}{(\lambda_2 - \lambda_1)}$$

Nie uciekną gdy λ_1 i $\lambda_2 \in \mathbb{R}$ czyli

$$\sqrt{\omega_B^2 - 4\omega_0^2} \in \mathbb{R} \Leftrightarrow \omega_B^2 - 4\omega_0^2 \geq 0$$

$$\omega_B \geq 2\omega_0$$

$$\frac{qB}{m} \geq 2\sqrt{\frac{k}{m}}$$