Summary of Professional Accomplishments

1. Name

Marta Wacławczyk

2. Diplomas, degrees conferred in specific areas of science or arts

- **Doctor of technical sciences in the field of mechanics**
  Date: the 21st of June 2007
  Title: “Modelling of near-wall turbulence by means of Probability and Filtered Density Function methods”
  Supervisor: Prof. Jacek Pozorski
  Organization: Institute of Fluid Flow Machinery, Polish Academy of Sciences (PAS), Gdańsk
- **Master of science in physics,**
  Date: the 29th of March 2001
  Title: “Statistical analysis of turbulent flow field”
  Supervisor: Prof. Jacek Pozorski
  Organization: Faculty of Technical Physics and Applied Mathematics, Gdańsk University of Technology, Gdańsk

3. Information on employment in research institutes or faculties/departments

Position	Time period	Institution
Assistant professor (research and teching)	since X.2019	Institute of Geophysics, Faculty of Physics, University of Warsaw
Assistant professor (research)/ senior specialist	IV.2016–IX.2019 health leave of absence: III.2018 – I.2019	Institute of Geophysics, Faculty of Physics, University of Warsaw
senior specialist	IX.2015–XII.2015	Institute of Fluid Flow Machinery (PAS), Gdańsk
Research associate “Wissenschaftliche Mitarbeiterin”	I.2012–VII.2012 VII.2012–X.2015	Center of Smart Interfaces Chair of Fluid Dynamics, Dept. of Mechanical Engineering, Technical University (TU) of Darmstadt (Germany)
Postdoctoral researcher, scholarship of TU Darmstadt	XII.2010–XII.2011	Chair of Fluid Dynamics, Dept. of Mechanical Engineering, TU Darmstadt (Germany)
Postdoctoral researcher, Alexander von Humboldt scholarship	III.2008–VII.2009 maternal leave: VIII.2009 – XI.2010	Chair of Fluid Dynamics, Dept. of Mechanical Engineering, TU Darmstadt (Germany)
Assistant professor (research)	IV.2007–II.2008	Institute of Fluid Flow Machinery (PAS), Gdańsk
Research assistant, specialist DAAD scholarship	IX.2000–III.2007 maternal leave: VIII.2005 – III.2006 VIII.2003–VII.2004	Institute of Fluid Flow Machinery (PAS), Gdańsk Dept. of Civil Engineering TU Darmstadt (Germany)

4. Description of the achievements, set out in art. 219 para 1 point 2 of the Act

As scientific achievements, the habilitant, pursuant to Article 219 (14) of the Law on Higher Education and Science from the 20th of July 2018[1], indicates a series of thematically related publications entitled:

Symmetries and scaling in turbulence: from theoretical analyses to applications in the studies of atmospheric flows.

The series of thematically related publications consists of 9 papers published between 2014 and 2024. Corresponding author of each work is indicated by the index *:

<table>
<tr><th>Authors</th><th>Title</th><th>Journal</th></tr>
<tr><td>H1. M. Wacławczyk*,
N. Staffolani
M. Oberlack
A. Rosteck
M. Wilczek
R. Friedrich</td><td>Statistical symmetries
of the Lundgren-Monin-Novikov Hierarchy</td><td>Physical Review E
vol. 90, 013022, (2014)

(IF=2.3) MNiSW 140
number of citations: 29</td></tr>
<tr><td colspan="3">Individual contribution:
derivation of statistical symmetries for the probability density functions of velocity, linking statistical symmetries to the description of intermittent (laminar-turbulent) flows, preparation and proofreading of the manuscript.</td></tr>
<tr><td>H2. M. Wacławczyk*,
V. N. Grebenev
M. Oberlack</td><td>Lie symmetry analysis of the
Lundgren-Monin-Novikov equations
for multi-point probability density
functions of turbulent flow</td><td>Journal of Physics A:
Mathematical and Theoretical
Vol. 50, 175501 (2017)
(IF=2.0) MNiSW 100
number of citations: 10</td></tr>
<tr><td colspan="3">Individual contribution:
development of research hypothesis, performing analytical calculations, (equal contribution with Dr. V. N. Grebenev), preparation and proofreading of the manuscript.</td></tr>
<tr><td>H3. M. Wacławczyk*,
Y.F. Ma
J.M. Kopeć
S.P. Malinowski</td><td>Novel approaches to estimating
the turbulent kinetic energy dissipation
rate from low- and moderate-resolution
velocity fluctuation time series</td><td>Atmospheric Measurement
Techniques
Vol. 10, 4573–4585 (2017)
(IF=3.1) MNiSW 140
number of citations: 8</td></tr>
<tr><td colspan="3">Individual contribution:
design of the methods to calculate dissipation rate from low and moderate-resolution velocity fluctuation time series based on signal zero-crossings, contribution to development of numerical code, data analysis, preparation and proofreading of the manuscript.</td></tr>
</table>

[1] Law on Higher Education and Science, Art. 219 (2018), Chapter 3, pages 121-122

<table>
<tr><td>H4. E.O. Akinlabi,
M. Wacławczyk*
J.P. Mellado
S.P. Malinowski</td><td>Estimating Turbulence Kinetic Energy Dissipation Rates in the Numerically Simulated Stratocumulus Cloud-Top Mixing Layer: Evaluation of Different Methods</td><td>Journal of the Atmospheric Sciences
Vol. 76, 1471–1488 (2019)
(IF=3.2) MNiSW 140
number of citations: 17</td></tr>
<tr><td colspan="3">Individual contribution:
conceptualization, design of method to calculate dissipation rate from low-resolution velocity fluctuation time series (based on the variance of derivatives) and method to calculate intermittency factor, contribution to the development of numerical code (as co-supervisor of E.O. Akinlabi), correction and proofreading of the manuscript.</td></tr>
<tr><td>H5. M. Wacławczyk*
V. N. Grebenev
M. Oberlack</td><td>Conformal invariance of characteristic lines in a class of hydrodynamic models</td><td>Symmetry
Basel
Vol. 12, 1482 (2020)
(IF=2.6) MNiSW 70
number of citations: 10</td></tr>
<tr><td colspan="3">Individual contribution:
development of research hypothesis, performing analytical calculations, extension of previous findings to a broader class of hydrodynamic models including the quasi-geostrophic model, analysis of the viscous and friction terms, writing and proofreading of the manuscript.</td></tr>
<tr><td>H6. M. Wacławczyk
V. N. Grebenev*
M. Oberlack</td><td>Conformal invariance of the 1-point statistics of the zero-isolines of 2d scalar fields in inverse turbulent cascades</td><td>Physical Review Fluids
Vol. 6, 084610 (2021)
(IF=2.5) MNiSW 70
number of citations: 5</td></tr>
<tr><td colspan="3">Individual contribution:
development of research hypothesis, performing analytical calculations, formulating the example with possible applications of the conformal invariance to the prediction of statistics of non- homogeneous turbulence field based on solutions for the homogeneous field, writing of the manuscript.</td></tr>
<tr><td>H7. M. Wacławczyk*
J. Nowak
H. Siebert
S.P. Malinowski</td><td>Detecting Nonequilibrium States in Atmospheric Turbulence</td><td>Journal of the Atmospheric Sciences
Vol. 79, 2757–2772 (2022)
(IF=3.2) MNiSW 140
number of citations: 7</td></tr>
<tr><td colspan="3">Individual contribution:
conceptualization, methodology, data analysis, writing of the manuscript.</td></tr>
<tr><td>H8. J-I Yano*
M. Wacławczyk</td><td>Symmetry Invariant Solutions in Atmospheric Boundary Layers</td><td>Journal of the Atmospheric Sciences
Vol. 81, 263–277 (2024)
(IF=3.2) MNiSW 140
liczba cytowań: 1</td></tr>
<tr><td colspan="3">Individual contribution:
both authors contributed equally, MW worked on methodology, derivation of solutions and contributed to writing.</td></tr>
<tr><td>H9. M. Wacławczyk*
J-I Yano
G. Florczyk</td><td>Local similarity theory as the invariant solution of the governing equations</td><td>Boundary Layer Meteorology
Vol. 190, 23 (2024)
(IF=4.3) MNiSW 100
liczba cytowań: 0</td></tr>
<tr><td colspan="3">Individual contribution:
conceptualization, derivation of solutions, data analysis, writing of the manuscript.</td></tr>
</table>

4.1 Introduction

Turbulence is a common phenomenon in nature. In particular, *atmospheric flows* are to a substantial extent turbulent.
The Latin word "turbulentum" translates as: violently disturbed, stormy, unruly, violent and riotous.
The related word "turbidus", on the other hand, describes, among others, something confusing, unclear and troublesome. These two meanings anticipate the difficulties one encounters when *describing turbulent flows* in the statistical sense. The purpose of this description is to characterize the apparent disorder and chaos by means of universal *scaling laws* for ensemble averaged quantities (statistics of turbulence). These laws can take a simple form and often are related to the invariance of statistics with respect to certain transformations of variables, or in other words: *the symmetries*.

The phenomenon of turbulence is usually associated with symmetry breaking. An example is the laminar-turbulent transition in a pipe flow. An ordered laminar flow is characterized by axial symmetry. When the pressure difference between the ends of the pipe is sufficiently large, the flow becomes turbulent. The velocity of the fluid changes in time and space, axial symmetry is therefore broken. Turbulence, however, is characterized by a tendency to restoring symmetries in the statistical sense [1]. This is usually the case for large Reynolds numbers and sufficiently far from the boundaries. For example, in turbulent flow in a pipe, the mean velocity is again axisymmetric. In the description of turbulence, an important role is played by the solutions of equations that remain invariant with respect to symmetries. The best example are the Kolmogorov scaling laws, which are a consequence of the scale invariance. They are commonly used in the analysis of measurement data of (among others) atmospheric turbulence.

The works presented as the scientific achievement are based on the common assumption that the mathematical analysis of the structure of the underlying equations is key to improve description of turbulence, even if these equations are not solved explicitly. The analysis leads to the applications of the theory in the description of atmospheric flows, including flows with stratification and flows in the quasi-geostrophic approximation. The goal is to propose new methods to parameterize these phenomena.

The works H1, H2, H5, H6 concerned the determination of the symmetries of the equations describing turbulent flows in a statistical sense. In H1 and H2, one of the scaling groups was linked to the phenomenon of the external intermittency, where turbulent flow can alternate with laminar one. **The papers H8 and H9 present an application of this analysis to describe flows in the atmospheric boundary layer (ABL) with stable stratification that occurs due to the rapid cooling of the Earth's surface after sunset [2]**. In this case, turbulent flow is suppressed and locally laminar areas can be formed.
The works H5 and H6 deal with two-dimensional turbulence, which approximates flow of the largest scales in the atmosphere. Such flows are characterized by distinctive physical properties. In the three-dimensional turbulence, the classical Richardson-Kolmogorov model of the energy cascade assumes that the production of turbulence kinetic energy occurs at the the largest eddies, followed by a transfer of energy in the space of scales, from the large eddies to the smallest ones, where it is converted into internal energy in a process of dissipation [3]. On the other hand, in two-dimensional flows an inverse energy transfer is observed, from small to large scales. This leads to the formation of large vortex structures.
The rate of dissipation of kinetic energy is one of the basic quantities characterizing turbulence. The dissipation occurs at the the smallest eddies, in the atmospheric boundary layer they are of the order of millimeters or centimeters, therefore it is difficult to determine the rate of dissipation on the basis of direct measurements. The disspation is estimated indirectly, using the scaling law of the velocity structure functions. **In works H3 and H4 alternative ways of determining the dissipation rate were proposed, based on the time series of velocity fluctuations, measured with low resolution. The methods use telegraphic approximation of the signal and the variance of the signal derivatives.**

In H7 the dissipation scaling in strongly non-stationary flows was addressed. Therein, it was shown based on theoretical analysis, that the so-called dissipation factor can be used as an indicator to assess turbulence states. The dissipation coefficient has a certain fixed value in stationary turbulence, increases when turbulence decays, and decreases when turbulence becomes stronger due to intense production. The method has been applied in the analysis of measurement data in ABL.

In the following section, methods of statistical description of turbulence will be presented in Section 4.2 and the concept of invariance with respect to symmetry is addressed in Section 4.3. Section 4.4 deals with the symmetries of the transport equations for the probability density function and Section 4.5 with the application of symmetries to derive invariant solutions in the atmospheric boundary layer. Section 4.6 is devoted to the scaling of turbulence kinetic energy dissipation rate. This is followed by the conclusions and perspectives (Section 4.7).

4.2 Statistical description of turbulence

Turbulent flows are described mathematically by the closed system of the Navier-Stokes equations. This system can be solved using appropriate numerical methods, however the problem is the enormous computational cost of such simulations and the high sensitivity of the solutions to small changes in the initial and boundary conditions. Because of this, turbulent fields can be treated as random fields [3]. For their complete description in the statistical sense, we need the knowledge of the mutual correlations of velocity, temperature and other variables describing the the flow at its various points.
After applying the averaging operator to the Navier-Stokes equations, one obtains the Reynolds equations which contain mutual correlations of two velocity components. These correlations become new unknowns. The transport equations for these correlations can be derived in the analogous way, by using the Navier-Stokes equations, performing further manipulations and applying the averaging operation. However, the derived equations will contain further unknowns - the third-order correlations and two-point correlations. Proceeding further, one finally obtains an infinite system of equations, the so-called Friedmann-Keller hierarchy [4].
Alternatively, on the basis of the Navier-Stokes equations, it is possible to derive the transport equation for the probability density function of velocity [5]. Analogously to the previous case, it will contain the unknown two-point probability density function. By deriving the subsequent equations for two-, three- point etc. functions one obtains an infinite system of Lundgren-Monin-Novikov (LMN) transport equations [5, 6, 7].
A third way, which provides the full description of turbulence in the statistical sense, by E. Hopf [8], uses a single transport equation for the characteristic functional. All multipoint statistics of the turbulent velocity can be derived by calculating the functional derivatives of the Hopf function. However, this concise way of describing turbulence is difficult to use in practical applications. Although more than seventy years have passed since the seminal paper by E. Hopf, numerical methods to solve the derived functional equation have been proposed only recently [9].
Due to the large number of independent variables, it is also problematic to solve equations for multi-point probability density functions and multi-point statistics. Therefore, in most of the Reyolds-Averaged-Navier-Stokes (RANS) turbulence closures in use, only the single-point statistics are considered. Unknowns in the equations are replaced by proper functions (models) of the known quantities [10]. Single-point statistics relevant to turbulence modelling include: the mean velocity vector $\langle \mathbf{u} \rangle$, where $\mathbf{u} = [u, v, w]$, or in the index notation $\mathbf{u} = [u_1, u_2, u_3]$, and the components of the Reynolds stress tensor which can be written using index notation as

$$\langle u'_i u'_j \rangle, \quad i, j = 1, 2, 3.$$

Above, $\langle \cdot \rangle$ denotes the ensemble-average operator and terms with the upper index $'$ stand for the fluctuations, e.g. $u'_i = u_i - \langle u_i \rangle$. The mean kinetic energy is defined as the half of the trace of the Reynolds-stress tensor

$$k = \frac{1}{2}\langle u'^2 + v'^2 + w'^2 \rangle = \frac{1}{2}\langle u'_i u'_i \rangle,$$

where the Einstein index summation convention is used.
Apart from this, in RANS turbulence models additional equation for the dissipation rate of the turbulence kinetic energy is solved. The dissipation rate is defined as

$$\epsilon = 2\nu\langle s_{ij}s_{ij}\rangle, \tag{1}$$

where ν is the kinematic viscosity and the gradient tensor s_{ij} is defined as

$$s_{ij} = \frac{1}{2}\left(\frac{\partial u_i'}{\partial x_j} + \frac{\partial u_j'}{\partial x_i}\right).$$

Due to the presence of fluctuating derivatives in the above formula for ϵ, information on fine-scale structure of turbulence, where the process of dissipation takes place, is needed to determine this quantity. The size of the smallest eddies can be estimated with the use of dimensional analysis, by using the kinematic viscosity ν and the dissipation rate ϵ

$$\eta = \left(\frac{\nu^3}{\epsilon}\right)^{1/4}. \tag{2}$$

Another useful length scale is the Taylor microscale defined as

$$\lambda = \sqrt{10\frac{\nu}{\epsilon}k} = \sqrt{15\frac{\nu}{\epsilon}}\,\mathcal{U}, \tag{3}$$

where $\mathcal{U}^2 = 2/3\,k$.
Having k and ϵ, or $\mathcal{U}$ and ϵ one can determine the characteristic size of large structures $\mathcal{L}$. The three variables are connected by the Taylor's law [44], which serves as the cornerstone for many turbulence models

$$\epsilon = C_\epsilon\frac{\mathcal{U}^3}{\mathcal{L}}, \tag{4}$$

where C_ϵ is the dissipation coefficient.
In spectral models of turbulence, transport equations for the spectral energy density or the second-order structure function defined as

$$\langle\delta u_i^2\rangle = \langle(u_i(\mathbf{x}+\mathbf{r},t) - u_i(\mathbf{x},t))^2\rangle, \quad i = 1,2,3.$$

are solved. Both functions are the two-point statistics that provide information on what portion of the total kinetic energy is contained in eddies of certain sizes. However, even this does not provide the full description of turbulence in the statistical sense. For a complete description, it would be necessary to know the Hopf functional, from which it is possible to determine all multipoint statistics, including the structure functions of any order.

4.3 Invariance, symmetries and scaling laws

The concept of invariance is used in the solutions of many problems in physics, meteorology and engineering, even without direct reference to the symmetry theory. An example is the well-known work by Monin and Obukhov [11] in which, based on the invariance of statistics with respect to transformations of variables, a logarithmic profile for the mean velocity near the atmospheric surface layer was derived.
Let us assume that the independent variables z and t, where z denotes the distance to the surface of the Earth, are transformed such that new independent variables z^* and t^* are formed. Additionally, the dependent variable $\theta(z,t)$ is transformed into a new variable $\theta^*(z^*,t^*)$. An invariant is a function $C(\theta,z,t)$, which conserves its form after the change of variables

$$C(\theta,z,t) = C(\theta^*,z^*,t^*).$$

Monin and Obukhov [11] argued that differences of the wind speeds in the atmospheric surface layer are invariant with respect to the following scaling

$$x^* = \lambda x, \quad y^* = \lambda y, \quad z^* = \lambda z, \quad t^* = \lambda t. \tag{5}$$

The velocity difference at the two heights is a function of both z_1 and z_2, but because it should be invariant with respect to the scaling transformation (5), it must be a function of z_2/z_1

$$\frac{\langle u(z_2)\rangle - \langle u(z_1)\rangle}{u_\tau} = f\left(\frac{z_2}{z_1}\right), \tag{6}$$

where u_τ is a velocity scale. Relation (6) is satisfied if the profile of the mean wind speed in the atmospheric surface layer is logarithmic.

In Ref. [11] the invariance of statistics was not linked to the underlying system of equations which describes flow in the atmospheric surface layer. This link can be found by analysing symmetries of the equations, i.e. such transformations of independent and dependent variables which do not change the functional form of the equation. The transformations can be called a Lie group if they form a set with an operation which is associative, every element in the set has an inverse and an identity element exists. [12].

All properties of the group are satisfied by the scaling (5), which can be presented in the following exponential form $z^* = \lambda z = \mathrm{e}^{\epsilon} z$, where $\epsilon \in \mathrm{R}$. Combination of two transformations $z^{**} = \mathrm{e}^{\epsilon_1}(\mathrm{e}^{\epsilon_2} z)$ is a new transformation from the same set $z^* = \mathrm{e}^{\epsilon_1+\epsilon_2} z$. The unitary element is $\epsilon = 0$. The element inverse to $\exp(\epsilon)$ is $\exp(-\epsilon)$, such that $z^* = \mathrm{e}^{\epsilon}\mathrm{e}^{-\epsilon} z = z$. The associativity is also satisfied, as $z^* = (\mathrm{e}^{\epsilon_1}\mathrm{e}^{\epsilon_2})\mathrm{e}^{\epsilon_3} z = \mathrm{e}^{\epsilon_1}(\mathrm{e}^{\epsilon_2}\mathrm{e}^{\epsilon_3})z$.

The Lie group analysis allows to derive the set of symmetries of equations under consideration, invariants and invariant solutions. It has found numerous applications in the analysis of processes described by partial differential equations [12].

The Navier-Stokes equations are invariant with respect to time and space translations

$$t^* = t + t_0, \tag{7}$$

$$\mathbf{x}^* = \mathbf{x} + \mathbf{f}(t), \quad \mathbf{u}^* = \mathbf{u} + \frac{\mathrm{d}\mathbf{f}(t)}{\mathrm{d}t}, \quad p^* = p - \mathbf{x} \cdot \frac{\mathrm{d}^2\mathbf{f}(t)}{\mathrm{d}t^2}, \tag{8}$$

where $\mathbf{f}(t)$ is an arbitrary vector function of time, with respect to rotations of the coordinate system and the pressure translations $\bar{p}^* = \bar{p} + g(t)$, where $g(t)$ is an arbitrary function of time. In the limit of negigible viscosity ($\nu \to 0$) two independent scaling groups of space and time are found

$$t^* = t, \quad z^* = \mathrm{e}^{a_z} z, \quad \mathbf{u}^* = \mathrm{e}^{a_z}\mathbf{u}, \quad \bar{p}^* = \mathrm{e}^{2a_z} p \tag{9}$$

$$t^* = \mathrm{e}^{a_t} t, \quad z^* = z, \quad \mathbf{u}^* = \mathrm{e}^{-a_t}\mathbf{u}, \quad \bar{p}^* = \mathrm{e}^{-2a_t} p, \tag{10}$$

which are of particular importance for the derivation of the scaling laws. With non-zero viscosity the scaling groups (9) and (10) are reduced to one scaling symmetry, with $a_t = 2a_z$.

After the ensemble averaging, the aforementioned symmetries become transformations of mean variables. The viscosity ν can be assumed negligible e.g. in a transport equation for structure function, in a restricted range of scales. In this case one can consider invariance with respect to both scaling groups (9) and (10). As an example, the transformation (5), discussed in the paper by Monina and Obukhov [11], is a combination of the scaling groups (9) and (10) under the assumption $a_t = a_z$, which imposes certain kind of self-similarity of statistics in the atmospheric surface layer. In this case we obtain $\lambda = \mathrm{e}^{a_z} = \mathrm{e}^{a_t}$.

A. N. Kolmogorov is the author of the best-known turbulence theory [13]. The first hypothesis of Kolmogorov states that, under the assumption of local isotropy, there exists a certain range of scales, much smaller than the characteristic size of large, energy-containing eddies, where statistics (in particular, the structure functions) take a self-similar form and depend on the turbulence kinetic energy dissipation rate ϵ and viscosity ν. The second Kolmogorov's hypothesis assumes that within the aforementioned range of scales one can identify a subrange (called the inertial subrange), where flow statistics depend only on the dissipation rate ϵ, but not on the viscosity ν.

U.Frisch [1], paraphrased Kolmogorov's hypotheses using the concept of symmetry of the Navier-Stokes equations. The first hypothesis, according to U. Frisch, assumes that in the limit of infinitely large Reynolds numbers all the symmetries of the Navier-Stokes equations, usually broken by mechanisms of turbulence production, are restored in a statistical sense and at sufficiently large distances from the boundaries of the flow region. For example, the statistical homogeneity of the flow is due to the invariance of statistics with respect to translations (7), isotropy is related to invariance with respect to rotations. In the case of isotropy, structure functions and other two-point statistics will depend only on the distance between points r. Invariance with respect to scaling (9) and (10) results in the following form of the velocity structure function of order p

$$\langle \delta u_i^p \rangle^* = \mathrm{e}^{p(a_z - a_t)} \langle \delta u_i^p \rangle, \quad i = 1, 2, 3. \tag{11}$$

U. Frisch assumed that at sufficiently small scales turbulent flow is self-similar, which also implies that there exist a relation between parameters a_z and a_t. Another hypothesis states that the rate of dissipation ϵ in turbulent flow has a finite, non-zero value even for $\nu \to 0$. Dimensional analysis then leads to the following relation:

$$\langle \delta u_i^p \rangle \propto (\epsilon r)^{p/3}, \quad i = 1, 2, 3, \tag{12}$$

which together with Eq. (11) determines the value of the ratio $a_t/a_z = 2/3$.
In turbulent flows Eq. (12) is satisfied only for low-order functions. As p increases, it becomes apparent that the scaling symmetry is broken, which is related to the phenomenon of internal intermittency. The velocity field generated by the smallest eddies is highly inhomogeneous. Areas of high activity occur alternately with "smooth" areas. In determining the structure function, the share of "active" areas increase with the order of the function p. In addition, at small scales, viscosity plays an important role, so the assumptions made to determine the function (12) are not met. Equation (12) is, however, satisfied with a good approximation for second- and third-order structure functions, which is used extensively, among others, in the analysis of atmospheric measurement data.
The Lie group theory and, particularly, derived invariant solutions were applied in numerous studies of turbulent flows. A series of scaling laws for near-wall flows was derived in [14]. Similar approach was used in Refs. [15, 16]. In Ref. [17] an infinite Friedmanna-Keller hierarchy of equations for turbulence statistics was considered. The authors showed that due to their linearity, the equations are invariant with respect to additional scaling and translation symmetries. This allowed to derived new solutions, cf. the review work [18]. The studies on symmetry invariant solutions in near-wall turbulence were continued, among others, in Refs. [19, 20]. Another methodology was used in Refs. [21, 22]. Therein, authors introduced the concept of a random scaling coefficient, which allows e.g. to scale the mean flow differently than the turbulence intensity. The subject of symmetries in the statistical approach is also addressed in the works included in the habilitation achievement, presented in the next section.

4.4 Symmetries of transport equations for probability density functions

Works H1 and H2, included in the habilitation achievement, concern the symmetries of a system of equations for the multi-point probability density functions of velocity. In this approach turbulent field is treated as the stochatic field. Its n-point probability density function is denoted as

$$f_n = f_n(\{\mathbf{v}_i, \mathbf{x}_i\}, t) = f_n(\mathbf{v}_1, \ldots, \mathbf{v}_n; \mathbf{x}_1, \ldots, \mathbf{x}_n, t), \tag{13}$$

where $\mathbf{v}_i$, $i = 1, \ldots, n$ are variables from the sample space of velocity at points $\mathbf{x}_i$ and in time t: $\mathbf{u}(\mathbf{x}_i, t)$. Based on f_n we can determine arbitrary n-point statistics of velocity, by multiplying f_n by a function of the arguments $\{\mathbf{v}_i\}$ and integrating over the sample space. Transport equation for f_n, derived from the Navier-Stokes equations, is written symbolically as [23]

$$\frac{\partial f_n}{\partial t} + \sum_{i=1}^{n} \mathbf{v}_i \cdot \nabla_i f_n = \mathcal{H}_{n+1} f_{n+1}, \tag{14}$$

where ∇_i is the gradient operator at point $\mathbf{x}_i$, and $\mathcal{H}_{n+1}$ is an integro-differential operator. The right hand side of Eq. (14) contains $n+1$-point probability density function. A transport equation for the $n+1$-point probability density functions contains terms with $n+2$-point probability density function. Hence, the LMN system contains infinite number of equations.
The motivation for undertaking the research topic was connected with results of previous works on the symmetries of the Friedmann-Keller system [17]. These equations are invariant with respect to additional translation and scaling groups, which are not symmetries of the Navier-Stokes equations. In the following description, they will be referred to as "statistical transformations". The question was whether it is possible to find their counterparts in the system of LMN equations and how invariance under these transformations can be interpreted.
First, in H1, invariance of the LMN equations under symmetries of the Navier-Stokes equations was shown. Next, the applicant derived transformations corresponding to the additional, statistical translation and scaling transformations of the Friedmann-Keller hierarchy. For this, the probability density function f_1 was expressed in terms if its characteristic function Φ_1 as follows

$$f_1(\mathbf{v};\mathbf{x},t) = \frac{1}{(2\pi)^3}\int \Phi_1(\mathbf{s};\mathbf{x},t)\mathrm{e}^{-i\mathbf{v}\cdot\mathbf{s}}\mathrm{d}\mathbf{v}. \tag{15}$$

n-th order velocity statistics are calculated as derivatives of the characteristic function Φ_1 at $\mathbf{s}=0$. Hence, the statistics form coefficients of the Taylor series expansion of the function Φ_1 around $\mathbf{s}=0$. With this representation it was possible to calculate the transformed characteristic function Φ^* by substituting the transformed velocity statistics, known from Ref. [17], into its Taylor series expansion. Next, the corresponding transformed probability density function f_1^* was calculated using the relation (15). This transformed function is of the form

$$f_n^* = \mathrm{e}^{a_s} f_n + (1-\mathrm{e}^{a_s})\delta(\mathbf{v}_1-\mathbf{u}_0)\cdot\ldots\cdot\delta(\mathbf{v}_n-\mathbf{u}_0), \tag{16}$$

where $a_s \leq 0$ is the scaling parameter. Due to its restriction to non-positive numbers the scaling transformation forms a semi-group. The term $\delta(\mathbf{v}_n-\mathbf{u}_0)$ is a Dirac-delta, and $\mathbf{u}_0$ is a given, constant velocity. The probability density function of the form (16) is used to describe the external intermittency, i.e. alternating laminar/turbulent flows, see e.g. Ref. [3], Eq. (5.301) therein. The probability density function of non-fluctuating, laminar velocity field is described by the Dirac delta function. Hence, the parameter $\mathrm{e}^{a_s}=\gamma$ can be interpreted as the intermittency factor, which expresses the probability of appearance of turbulent flow. In H1, an example of plane channel flow was discussed. In a somewhat simplified approach (by ignoring intermediate states) it can be assumed, that the flow can be either laminar or turbulent, with certain probability which depends on the Reynolds number.
The condition (16) expresses the fact that the Navier-Stokes equations can have two distinct types of solutions: ordered (laminar) and turbulent, which appear with probailities $1-\gamma$ and γ, respectively. This gives statistical transformations an interpretation, even though there are no corresponding symmetries of the Navier-Stokes equations. The problem of laminar-turbulent transition and the dependence of the coefficient γ on the Reynolds number were recently investigated experimentally, e.g. in [24].
The form (15) satisfies the normalisation condition of the probability density function, however the separation condition, which states that velocity fluctuations at very distant points are statistically independent, is not satisfied by (15). According to the separation condition, e.g. the two-point probability density function should be a product of two one-point functions $f_2(\mathbf{v}_1,\mathbf{v}_2;\mathbf{x}_1,\mathbf{x}_2,t) = f_1(\mathbf{v}_1;\mathbf{x}_1,t)f_1(\mathbf{v}_2;\mathbf{x}_2,t)$ for $|\mathbf{x}_1-\mathbf{x}_2|\to\infty$. However, in case of intermittent flows, consider conditional averages should be considered. The separation condition is satisfied separately for turbulent field and for laminar field (with the Dirac delta probability density function), but not for their weighted average. The applicant used this argument in the discussion in Ref. [25]. Statistical transformations follow from the linearity of the transport equations for turbulence statistics. It should be noted that the functional Hopf equation [8], which fully describes the turbulent field in the statistical sense, is also linear.

Despite the derivation of the transformations corresponding to the symmetries of the Friedmann-Keller equations in H1, an open question was whether the set of symmetries is complete and whether the equations under consideration are invariant with respect to yet other transformations. Symmetries of equations can be derived with the Lie group analysis. This procedure is well established in case of partial differential equations and the details can be found in textbooks, e.g. [12]. Less standard is the Lie group analysis of integro-differential equations. A formal problem is also caused by the incompleteness of each of the LMN equations due to the presence of higher-order probability density function. **To find transformation which do not change the form of the LMN system, in H2 its systematic analysis was performed.** (The applicant and Dr. V. N. Grebenev contributed equally to this analysis.) For this the first and the second equation from the infinite hierarchy were presented as the following conservation laws

$$\frac{\partial f_1}{\partial t} + \frac{\partial J_i}{\partial y_i^{(1)}} = 0, \quad i = 1, \ldots, 6 \tag{17}$$

$$\frac{\partial f_2}{\partial t} + \frac{\partial I_i}{\partial y_j^{(2)}} = 0, \quad i = j, \ldots, 12, \tag{18}$$

where $y_i^{(1)}$ and $y_j^{(2)}$ are independent variables. Components of the vectors J_i and I_j were determined by relations, part of which are integral relations. Symmetries of the conservation laws (17) and (18) are known, see e.g. [26], hence, their form was a starting point of the analysis. We used the methodology of Lie-Bäcklund operators, introduced in Ref. [27]. During the analysis, the integral relations which determine the components J_i and I_i were taken into account one by one. This led to the final set of symmetry transformations of the first LMN equation. It included all symmetries of the Navier-Stokes equations and the statistical transformations already known from H1. Apart from this, the Lie symmetry analysis of LMN equations for velocity did not reveal any additional symmetries.

The same methodology was used in work [28] (coauthored by the applicant) and therein it was shown that under particular conditions the first equation from the LMN system for vorticity is invariant under conformal transformation of space. This subject was continued in works H5 and H6, which concerned two-dimensional turbulence, which serves as an approximation of turbulent flow in thin layers. It is believed that large synoptic scales (of the order of thousands of kilometers) in the Earth's atmosphere may also exhibit certain properties of 2D turbulence. When transport equations for vorticity are reduced from three to two dimensions, the so-called "vortex stretching" term cancels out. This fact is usually interpreted as the reason for substantial differences in the dynamics of turbulent flows in two and three dimensions. The vortex stretching mechanism contributes to the formation of the energy cascade, from large to small scales. In 2D turbulence the energy is transferred in the inverse cascade, from small to large eddies. As a result of this process, either the largest scales, or the mean flow, become enhanced.

In 2D turbulence one observes also the direct enstrophy cascade (i.e. towards small scales), where the enstrophy is defined as the mean square of the vorticity $Z = \langle \omega^2 \rangle$. The existence of the double energy and enstrophy cascade was predicted analytically by R. Kreichnan [30]. The theory was also confirmed in later numerical experiments by [31]. As viscosity does not affect the inverse cascade, a perfect Kolmogorov scaling (12) is observed in 2D turbulence, even for higher-order structure functions. The existence of inverse energy cascade in the atmosphere is a subject of debate, e.g. in Ref. [32] only the direct enstrophy cascade towards smaller scales was confirmed, whereas in the recent paper [33], the mechanism of inverse energy transfer, possible at scales as small as 15km, was described.

In Ref. [29] it was suggested that some statistics of turbulence in 2D remain invariant with respect to a transformation more general than the scaling (8), namely the conformal transformation. It is a scaling which depends on the position

$$\mathbf{x}^* = \lambda(\mathbf{x}), \tag{19}$$

which changes distances but preserves angles. In Ref. [29] an analysis of a numerical experiment was performed. It turned out that zero vorticity lines bounding large vortex structures are invariant to conformal transformations. However, this property was not exhibited by the lines bounding the fine structures of the turbulence field. Hence the conjecture that invariance with respect to conformal mappings is related to the inverse energy cascade. A question which remained open was whether the observed properties of turbulence in 2D could be explained by analyzing the structure of underlying equations.

The question was addressed in works H5 and H6, where procedures developed in H2 and Ref. [28] were further applied. In H5 the applicant conducted the Lie group analysis of a transport equation for the probability density function of a scalar Φ. The evolution in time and space of this scalar is described by hydrodynamic models of the form

$$\frac{\partial \Phi}{\partial t} + \boldsymbol{u} \cdot \nabla \Phi = \kappa \nabla^2 \Phi - \gamma \Phi, \tag{20}$$

where $\boldsymbol{x} = (x, y)$ and $\boldsymbol{u} = (u, v)$, κ denotes molecular diffusivity, and the component $-\gamma\Phi$, where $\gamma > 0$ was introduced to model the large-scale friction, which removes energy from the system. The velocity components in Eq. (20) read

$$u = \beta \int \mathrm{d}\boldsymbol{x}' \Phi(\boldsymbol{x}', t) \frac{(y - y')}{|\boldsymbol{x} - \boldsymbol{x}'|^m}, \tag{21}$$

$$v = -\beta \int \mathrm{d}\boldsymbol{x}' \Phi(\boldsymbol{x}', t) \frac{(x - x')}{|\boldsymbol{x} - \boldsymbol{x}'|^m}, \tag{22}$$

where β is a constant, which depends on a model and the exponent $m > 1$. When $m = 2$, the scalar Φ is the vorticity in 2D turbulence, for $m = 3$, Eq. (20) describes buoyancy in the surface quasi-geostrophic model, which is used to mimic dynamics of synoptic scales in mid-latitudes [34, 35]. This model takes into account the stratification and the Coriolis force, due to the Earth's rotation. Physical interpretation can also found for the case $m = 6$, [36].

In H5, the transport equation for the probability density function $f_1(\phi; x, y, t)$ was considered, where ϕ denotes a variable from the sample space of the scalar $\Phi(x, y, t)$. This equation is of hyperbolic type and its charcteristics can be interpreted as Lagrangian trajectories of particles moving in a conditionally-averaged velocity field. In H5 it was show that, for arbitrary $m > 1$, the probability measure $f_1(0; x, y, t)\mathrm{d}\phi$ that the scalar is contained within $0 < \Phi(x, y, t) < \mathrm{d}\phi$, is invariant with respect to conformal transformations.

A link to the work [29] is that therein the conformal symmetry was observed for the zero-vorticity lines. In H5 it was further shown that the conformal invariance was retained for $\gamma \neq 0$, but broken for non-zero diffusivity $\kappa \neq 0$. This could explain the fact that in Ref. [29] the conformal invariance was only present at large scales, which are not affected by molecular diffusivity.

The conformal invariance is present only if the two-point probability density function $f_2(\phi_1, \phi_2; \mathbf{x}_1, \mathbf{x}_2, t)$ transforms in a particular way. It was further shown in H5 that because of this condition statistics calculated based on the conformal transformations could be only approximations of real solutions, up to the first order term of Taylor series expansion. To assess accuracy of this approximation, detailed analysis of numerical or experimental data would be necessary. Possibly, statistics of large synoptic scales in the atmosphere could also be used for this purpose. The conjecture, that they can be conformally invariant was stated in Ref. [37].

A different interpretation of the derived transformations was discussed in paper H6. Therein, the applicant presented examples of two-point and one-point probability density functions, which satisfied the conditions derived in H5 and assured the conformal invariance of the considered transport equation. This held only for a certain form of two-point correlations of the variable Φ. It was argued that invariance with respect to conformal transformations implies that certain statistics of a non-homogeneous vorticity field could be obtained by transforming the statistics determined for a homogeneous field, without having to solve the equations again. The question whether the derived form of the two-point correlations is indeed observed in the considered classes of turbulent flows requires further research and detailed analysis of numerical or experimental data.

4.5 Invariant solutions in the stable atmospheric boundary layers

Works H8 and H9 concerned application of the Lie symmetry theory in the analysis of atmospheric boundary layer flows, under stable stratifications. Since turbulence is characterized by a tendency to restore symmetries in the statistical sense, the mean velocity and Reynolds stress components can be expected to take some characteristic form close to invariant solutions in a given class of flows. An example is the well-known logarithmic profile of mean velocity near the wall. It is observed in various types of flows, including atmospheric surface layer turbulence at neutral conditions (when temperature changes are small enough and do not affect the velocity field). The logarithmic profile can be derived as an invariant solution taking into account scaling symmetries and translation of the mean velocity. The latter has no counterpart in the symmetries of the Navier Stokes equations, yet the Friedmann-Keller equations are invariant with respect to this translation. The translation of the mean velocity was indirectly used in the aforementioned work by Monin-Obukhov [11]. Its authors considered velocity differences that were invariant to the translation of the mean velocity.

Buoyancy forces play an important role in the ABL flows. ABLs take on two characteristic forms: the typical, "convective" daytime state caused by solar heating, and the typical night-time state, with density decreasing with altitude, leading to the formation of a "stable" layer. The latter is particularly dominant over the ice-covered regions of the Arctic Ocean, especially during the long polar night. The heat budget of the Arctic is a key component of climate predictions. At the same time, the boundary layer is so thin that typical atmospheric models do not resolve the details of its structure. Therefore, the physical description of the ABL must rely on statistical approaches. The basic statement of the Monin-Obukhov theory is that the complex vertical structure of the ABL can be characterized by a single length scale, called the Obukhov scale. This theory remains one of the guiding principles in the studies of ABLs. Specifically, for a stable atmospheric layer, the theory predicts that the fluxes of momentum $\langle u'w'\rangle$ and the potential temperature $\langle w'\theta'\rangle$ are approximately constant, and the gradients of mean velocity and mean temperature are expressed by the formulas

$$\phi_m = \frac{\kappa z}{u_*}\frac{\partial\langle u\rangle}{\partial z} = 1 + \beta\frac{z}{L}, \qquad \phi_h = \frac{\kappa z}{\theta_*}\frac{\partial\langle \theta\rangle}{\partial z} = 1 + \beta\frac{z}{L}, \tag{23}$$

where $u_*^2 = -\langle u'w'\rangle = const$, $\theta_* = -\langle w'\theta'\rangle/u_* = const$, $\beta = 5$ is a constant, and the Obukhov scale L is defined as $L = u_*^2\bar{\theta}_0/(\kappa g\theta_*)$, where $\bar{\theta}_0$ is a reference temperature (e.g. surface temperature or vertically-averaged temperature). Small L (hence large $z/L \gg 1$) indicates strong stratification. On the other hand, large L mean that the temperature does not affect the velocity field in any significant manner. In this case Eqs. (23) describe approximately logarithmic profiles of mean velocity and temperature.

The assumptions behind Monin-Obukhov theory are satisfied only for weak stratifications. However, as the degree of stratification increases, an increasing discrepancy between the predictions and measurement data is observed. In particular, in a stable ABL turbulence can locally collapse, such that the flow becomes intermittent (laminar-turbulent). Given the high degree of complexity of flows in the stable AWG, its parameterisation remains a current and open research topic.

The work H8 used the methodology of Lie group theory to describe statistics in ABL.

The applicant's contribution was the derivation of invariant solutions based on the symmetries of the equations, and her participation in the interpretation of the results. For neutral flows, i.e., those in which temperature is a passive scalar and does not affect the velocity field, the temperature transport equation is invariant with respect to the temperature scaling group with coefficient a_θ. On the other hand, when temperature affects the velocity field, the Boussinesque approximation is used. The temperature appears in the momentum transport equation, which leads to the breaking of the independent scaling group, so that the parameter a_θ is related to the scaling parameters of velocity, i.e. a_z and a_t in Eqs. (9), (10).

In H8, invariant solutions for the velocity, temperature, and the fluxes of momentum and temperature were derived. They contain the scaling parameters a_z and a_t, a_θ and the a_s parameter associated with statistical scaling (16). The assumptions for the fluxes determine the scaling of the of mean wind speed and mean temperature. Thus, in the neutral case when one assumes that the fluxes in the surface layer are approximately constant with height and $a_s = 0$, logarithmic solutions are obtained, as predicted by the Monin-Obukhov theory [11]

$$\frac{\kappa z}{u_*}\frac{\partial \langle u\rangle}{\partial z} = C_u, \quad \frac{\kappa z}{\theta_*}\frac{\partial \langle \theta\rangle}{\partial z} = C_\theta, \tag{24}$$

where C_u and C_θ are constants. When the temperature affects the velocity field, the same condition (on constant fluxes) leads to the linear solutions

$$\frac{\kappa z}{u_*}\frac{\partial \langle u\rangle}{\partial z} = C'_u\frac{z}{L}, \quad \frac{\kappa z}{\theta_*}\frac{\partial \langle \theta\rangle}{\partial z} = C'_\theta\frac{z}{L}. \tag{25}$$

where again C'_u and C'_θ are constants. Hence, equations (23) proposed by Monin and Obukhov [11] are weighted sums of the invariant solutions (24) and (25).

Deviations from linear scaling at large stratifications are observed in numerous experiments, e.g. [38]. They are caused by locally vanishing turbulence. In the context of scaling groups, this means that the statistical scaling factor (associated with intermittency) must be different from zero. **In another paper H9, the applicant derived the so-called local similarity theory as an invariant solution. This theory accounts for the variability of fluxes and Obukhov length with height, cf. Ref. [39]. In H9, the dependence of turbulence statistics on time was additionally considered. The derived solutions include the scaling factor $a_s \neq 0$ related to the intermittency.** It was shown in H9 that its role increases in the case of strong stratification.
Classical Monin-Obukhov theory assumes constancy of statistics over time. It also follows from the formula (23) that the turbulent Prandtl number $Pr_t = \phi_h/\phi_m$ should be constant in the stable ABL. In H9, another relation for the turbulent Prandtl number was derived, where Pr_t was presented as a function of the ratio of the components of the Reynolds stresses $\overline{uw}/\overline{w^2}$. Analysis of the experimental data showed that, indeed, the Prandtl number changes (decreases) with increasing stratification. The values of Pr_t under strong stratification were well correlated with the values of the argument $\overline{uw}/\overline{w^2}$.

4.6 Scaling of the turbulence kinetic energy dissipation rate

In the works H3, H4 the Kolmogorov scaling laws (11) was applied to estimate the dissipation rate of turbulence kinetic energy in atmospheric turbulence. In H7 the problem of non-stationarities, which cause deviations from the law (11), was addressed.
The first hypothesis of Kolmogorov, mentioned in Section 4.1, assumes the existence of a range of scales, where turbulence statistics take a universal form, which depends on the rate of dissipation ϵ and viscosity ν. In particular, the second order structure function in this range of scales takes the following form

$$\langle \delta u_i^2\rangle = \epsilon^{2/3} r^{2/3} F_i\left(\frac{r}{\eta}\right), \quad i = 1, 2, 3 \tag{26}$$

where F_i is a function of its argument r/η, and η is the Kolmogorov microscale, defined in Eq. (2). The same can be expressed in terms of the spectral energy density $E(\kappa)$. It follows from Kolmogorov's first hypothesis and from dimensional analysis that there is a certain range of scales where it takes the universal form

$$E(\kappa) = \epsilon^{2/3}\kappa^{-5/3} f(\eta\kappa), \tag{27}$$

where κ denotes the wavenumber and f is a function, which is assumed to be universal. According to the second hypothesis of Kolmogorov, within the considered range of scales one can identify an inertial subrange, where turbulence statistics depend only on the rate of dissipation ϵ. In the inertial subrange, equations (26) and (27) are reduced to the form

$$\langle \delta u_i^2\rangle = C_i \epsilon^{2/3} r^{2/3}, \quad E(\kappa) = C_k \epsilon^{2/3}\kappa^{-5/3}, \quad i = 1, 2, 3. \tag{28}$$

The rate of dissipation of the turbulence kinetic energy ϵ is described by the formula (1). In order to estimate it directly from measurements, one would need to use high frequency sensors, able to record the smallest scales of turbulence. In case of airborne measurements in the atmosphere, this is practically impossible. Wind speeds are measured with a resolution of metres or tens of centimetres, while the smallest eddies are of the order of millimetres. For this reason, the dissipation rate is estimated indirectly, usually using the scaling laws for the second-order structure function and the spectral energy density (28), under the assumption that the coefficients C_i and C_k are universal.
Indirect methods for determining the dissipation rate are not perfect. First of all, the assumptions behind them about stationarity and local isotropy of small scales are not always fulfilled, and the question of the universality of the constants C_i and C_k remains open. Errors also result from the low resolution of the signal under study and relatively short time series. Therefore, the estimated values of ϵ may vary depending on the method used, although the structure functions and frequency spectra are mathematically equivalent. **Using several different methods allows a more accurate estimate of ϵ. This was the motivation for the research in H3. Therein, the applicant proposed new methods for determining the dissipation rate on the basis of the so-called telegraphic approximation of low- and moderate-resolution time series of velocity fluctuations**. The telegraphic approximation determines the number of intersections of the signal with the zero level. It turns out that such information is sufficient to estimate the rate of dissipation of turbulence kinetic energy.
K. Sreenivasan [40] derived the following relation between the number of zero-crossings and the value of ϵ:

$$\epsilon = 15\pi^2 \nu \langle u'^2 \rangle N^2, \tag{29}$$

where N stands for the number of zero-crossings per unit length of the longitudinal (i.e. parallel to the direction of measurements) component of velocity fluctuations. Apart form the dissipation rate, other turbulence statistics can be estimated based on N, e.g. the length scale of large eddies or information on eddy clustering.
Equation (29) is satisfied exactly only for velocity time series of high resolution. In spite of this, it was observed in Ref. [41] that zero-crossing statistics calculated from low-resolution airborne measurements were correlated with estimated values of the dissipation rates. This suggested that Eq. (29) could be modified appropriately and allow to determine ϵ from such signals.
This problem was addressed in H3. The first method proposed in H3 involves filtering the time series of velocity fluctuations several times with a low-pass filter. The ϵ-value is estimated using a scaling law for the number of zero-crossings in the inertial range. The second method is based on the first hypothesis of Kolmogorov (27). It reconstructs the unmeasured part of the spectral energy density by assuming the form of the function $f(\kappa\eta)$ in equation (27). This function describes the contribution of the smallest viscosity-affected eddies. In the proposed method, an initial value of the dissipation rate ϵ_0 is assumed, and then the Kolmogorov microscale η and the shape of the missing part of the spectrum $f(\kappa\eta)$ are determined. On this basis, a correction factor $\mathcal{C}_F$ to Eq. (29) can be calculated

$$\epsilon = 15\mathcal{C}_F \pi^2 \nu \langle u'^2 \rangle N_{cut}^2, \tag{30}$$

where N_{cut} is the number of zero-crossings of a signal of low-resolution. The dissipation rate calculated from Eq. (30) is next used to correct the value of η in the following iteration. After a few iterations the method converges to the final value of the dissipation rate, independent of the initial condition ϵ_0.
The new methods were tested in the work H4, where data from numerical experiment of stratocumulus-topped boundary layer [42], were investigated. The characteristic Reynolds number of the simulations was around 300 times smaller than the typical Reynolds numbers of atmospheric flows.
Nevertheless, on the basis of data analysis it is possible to draw conclusions about the processes taking place inside the clouds. Since smallest turbulence scales were resolved in the numerical simulations, it was possible to determine the value of the dissipation from its definition (1) and compare it with the ϵ values estimated by indirect methods. In H4, data were analysed from a perspective of a “virtual” aircraft

flying through a cloud and measuring velocity along its trajectory. The dissipation rate was estimated from the frequency spectrum, second- and third-order structure functions and the methods proposed in H3. Additionally, the applicant proposed a modified version of the iterative method, in which instead of the number of zero-crossings, the mean square of the velocity fluctuation gradient was used. The dissipation was then calculated from the formula

$$\epsilon = 15\nu \left\langle \left(\frac{\partial u'}{\partial x} \right)^2 \right\rangle \mathcal{C}_F, \tag{31}$$

where x denotes direction along the flight and the correcting factor $\mathcal{C}_F$ is calculated analogously as in H3. Having the detailed numerical data at hand, it was also possible to compare different models for the high-wavenumber part of the energy spectrum $f(\eta\kappa)$. The model proposed in Ref. [3] proved to be the most universal one.

The work H4 showed that deviations from Kolmogorov scaling (28) are present in the upper part of the startocumulus cloud, where the turbulent flow was strongly inhomogeneous. The use of indirect methods based on the assumption of local isotropy is not justified in such a case and leads to a significant underestimation of the dissipation rate. The values of ϵ determined from the one-dimensional spectra of vertical velocity inside the cloud were, in turn, overestimated, even though a clear inertial range with Kolmogorov scaling was present. The reason for the discrepancy was the anisotropy of the flow which possibly changed the value of a constant in Eq. (28).

Breaking of the scaling symmetry (12) was addressed in H7 in the context of strongly non-stationary turbulence in a statistical sense. According to the classical energy scenario of the eddy cascade, the rate of energy transfer in the space of scales is constant and equal to both the turbulence energy production at the largest scales and the rate of dissipation at the smallest scales.

Hence, statistics of the largest scales, that is their characteristic velocity scale $\mathcal{U}$ and the length scale $\mathcal{L}$ are related to the dissipation rate ϵ through the Taylor's law (4). It further follows from Eq. (4) that the ratio of the integral $\mathcal{L}$ and the Taylor length scale λ (defined in Eq. (3)) can be expressed in terms of C_ϵ and the Reynolds number

$$\frac{\mathcal{L}}{\lambda} = \frac{C_\epsilon}{15} Re_\lambda, \quad C_\epsilon = 0.5, \tag{32}$$

where $Re_\lambda = \mathcal{U}\lambda/\nu$. Until recently, it was assumed that the value of the coefficient $C_\epsilon = 0.5$ is universal. However, this has been contradicted by a number of recent research papers (see review article [45]). In particular, it was found that in the case of decaying turbulence coefficient C_ϵ increases, reaching the limit value $C_\epsilon = 1$. In the initial stages of turbulence decay C_ϵ varied inversely proportional to the local Reynolds number Re_λ. This suggested the existence of a different, non-classical, although universal scaling law describing such states. Authors of the works [46, 47] linked the observed changes in the dissipation coefficient with deviations from Kolmogorov's scaling law. The states in which such deviations were observed were called "non-equilibrium" states. They typically occur after sudden changes in the forcing term, before the turbulent flow reaches a new equilibrium state.

The authors of Ref. [46] derived the following relations for the ratio $\mathcal{L}/\lambda$ and the dissipation coefficient C_ϵ in non-equilibrium states

$$\frac{\mathcal{L}}{\lambda} = \frac{C_{\epsilon 0}}{15} Re_{\lambda 0}^{15/14} \left(\frac{1}{Re_\lambda} \right)^{1/14}, \quad C_\epsilon = C_{\epsilon 0} \left(\frac{Re_{\lambda 0}}{Re_\lambda} \right)^{15/14}, \tag{33}$$

where $C_{\epsilon 0} = 0.5$ and $Re_{\lambda 0}$ are equilibrium values, before the change in forcing. They can also represent the initial conditions. Relationships close to the derived equations (33) have been observed in controlled laboratory and numerical experiments.

Flows in the atmosphere are highly dynamic, however, sensors placed on an aircraft or on a platform hung below it measure only the current state of turbulence. It does not provide direct information about time changes.

The idea of the applicant was to use the results of the theoretical works discussed above to evaluate states of the atmospheric turbulence. Equations (33) are qualitatively different from their corresponding classical counterparts (32). Hence, they can be used to detect non-equilibrium states.

Analysis of atmospheric measurement data is difficult due to their limited amount. In H7, it was shown that the resolution and the length of the investigated time series of velocity fluctuations was sufficient for estimating ϵ, the kinetic energy, length scale $\mathcal{L}$ and Taylor scale λ. This allowed to determine the coefficient C_ϵ from the formula (4), the ratio $\mathcal{L}/\lambda$, and to study their variability with the Reynolds number Re_λ. This was the first application of this new methodology in the study of turbulence in the atmosphere.

In H7, data from the ACORES measurement campaign conducted over the North Atlantic area near Graciosa Island were analysed [48]. The purpose of the campaign was to study the atmospheric properties of a stratocumulus-topped boundary layer. Low stratocumulus clouds play an important role in the Earth's radiation balance, as they cover about 20% of the sky above our planet and at the same time reflect a significant portion of the solar radiation reaching Earth. Despite the greenhouse effect of clouds related to the absorption of the Earth's infrared radiation, their resultant effect on the Earth's energy balance is cooling [49]. Data from the ACORES campaign were analysed in Ref. [50]. In particular, its authors studied the phenomenon of the "decoupling" of stratocumulus cloud associated with an increase in its altitude, cooling and a change in the nature of the organization of convection. It was speculated in [50] that turbulence in the "decoupled" cloud may decay in some areas. The phenomenon of "decoupling" and the simultaneous decrease in the temperature of the cloud top also changes its radiation balance and may lead to a change in the resultant effect to a warming one.

The goal of the work H7 was to analyse measurements from the ABL topped by two types of stratocumulus clouds - "coupled" with strong exchange of heat and water vapor with the ocean surface through a convection system and the "decoupled" one, and to investigate differences between turbulent states in the two cases.

It follows from relations (33) that during the decay of turbulence, when Re_λ decreases, $C_\epsilon/C_{\epsilon 0} > 0$, on the other hand $C_\epsilon/C_{\epsilon 0} < 1$ indicate states with strong turbulence production. By analysing the dependence of C_ϵ and $\mathcal{L}/\lambda$ on the Reynolds number Re_λ it is possible to asses whether turbulence is in its equilibrium state, according to Eqs. (32) or the non-equilibrium one, governed by Eqs. (33). The latter case would indicate rapid changes in flow conditions (e.g., due to changes of energy production). Sample results from the decoupled stratocumulus-topped ABL are presented in Fig. 1. It turned out that the smallest values of $C_\epsilon/C_{\epsilon 0}$ in both types of ABL corresponded to flight sections made closest to the Earth's surface. In this area, the production of turbulence kinetic energy by buoyancy and the mechanical production due to the shear were the largest. $C_\epsilon/C_{\epsilon 0}$ were larger at altitudes of 500–600m, which correspond to half of the ABL, where rising thermals or descending currents of cool air lose kinetic energy. In contrast, when analyzing data from flights through clouds, we found that areas of high turbulence production and areas of turbulence decay alternated, indicating strong flow intermittency.

In H7 it was found that for all corresponding flight segments, values of $C_\epsilon/C_{\epsilon 0}$ in the decoupled stratocumulus-topped BL were higher than in the coupled one. This suggested that turbulence in the decoupled BL is weaker.

Moreover, in the "decoupled" cloud, most of the data indicated the presence of non-equilibrium turbulence. This phenomenon may have been related to a change in the organization of convection during the process of decoupling [50]. The analysis in H7, performed by the present applicant, provided additional information on the processes taking place in the ABL's under study. This was possible by applying the theoretically derived scaling laws in the analysis of atmospheric measurement data.

4.7 Conclusions and perspectives

The presented series of works is based on the mathematical analysis of equations describing turbulent flow in the statistical sense. The problem of this description is the lack of closure, associated with the presence of unknown higher-order statistics in the equations under study. Despite this, the very structure of the equations and resulting invariance with respect to the transformation of variables, provide important infor-

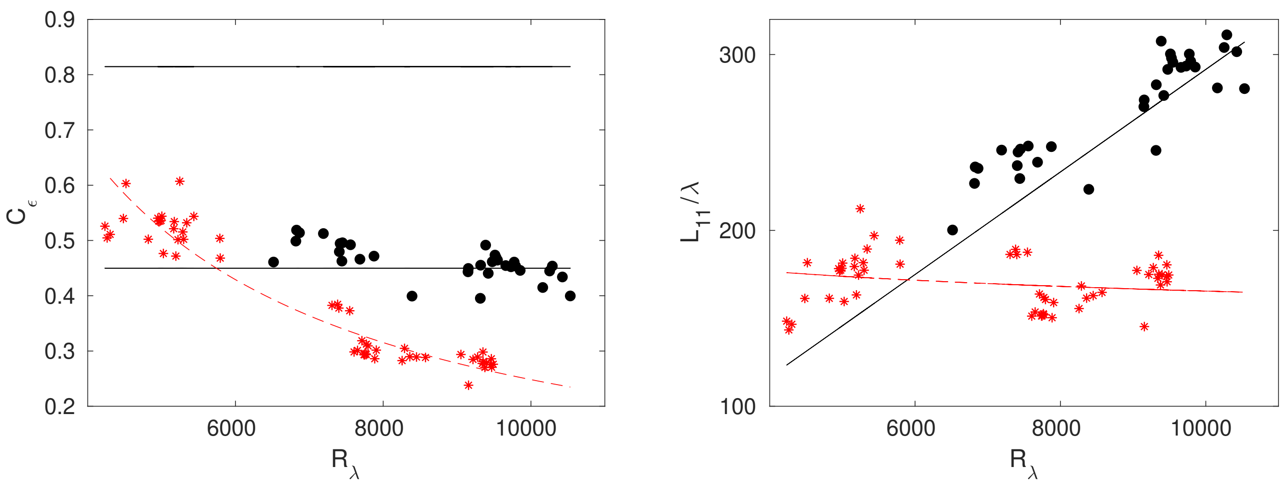

Figure 1: (Left panel) C_ϵ as a function of Re_λ in the decoupled stratocumulus-topped ABL. (Right panel): Ratio of the longitudinal integral length scale $L_{11} = \mathcal{L}$ and the Taylor microscale. Data from experiment: black and red symbols, equilibrium relations (32): black solid lines, non-equilibrium relations (33): red, dashed lines.

mation about the investigated phenomenon and allow to derive some characteristic relationships (scaling laws).

The best-known example is Kolmogorov's scaling law of the structure function in the inertial range. In this range, turbulent flow is characterized by self-similarity - regardless of the observed scales, it has similar statistical properties. For example, it follows from the theory of Kolmogorov, that the energy transfer rate from large to small eddies should be constant across the scales.

The symmetry analysis of the transport equations for the probability density function of velocity performed in papers H1 and H2 showed that they are invariant with respect to the symmetries of the Navier-Stokes equations and, because of their linearity, with respect to the additional scaling and translation. The proposed interpretation links this invariance to the phenomenon of intermittency, understood as the alternations of turbulent and laminar flow. It occurs, for example, in a stable atmospheric boundary layer that forms after sunset. The air near the cooled surface of the Earth becomes denser, so that turbulent fluctuations become suppressed.

The symmetries of the equations were used in paper H8 to derive the vertical profiles of the mean wind velocity, mean temperature and turbulent fluxes as invariant solutions. The paper H9, in turn, derives a local similarity theory for a stable atmospheric layer, where an additional scaling associated with flow intermittency plays a crucial role. It introduces the dependence of statistics on the height of the boundary layer. In H9 the non-stationarity of the flow was also taken into account. This work is currently being continued within a project financed by the Polish National Science Centre. An interesting further perspective is to consider the role of horizontal transport and account for the Coriolis force effects in the analysis. It is known from experimental studies that they affect flow statistics in the surface layer at very strong stratifications, however, the relationships were derived mostly through dimensional analyses. Symmetry analysis allows a more methodical derivation of relationships between variables describing the flow.

The works H5 and H6 were also devoted to symmetry analysis. They concern a class of hydrodynamic models in two dimensions. Flows of large synoptic scales in the Earth's atmosphere have some features of two-dimensional turbulence. The papers H5 and H6 prove that the transport equations for the probability density functions of the scalar are invariant to the conformal transformation of space, under certain conditions. If these conditions are satisfied, then it would be possible, for example, to determine certain statistics of inhomogeneous turbulence based on solutions for statistically homogeneous flow. The interpretation of the derived properties and their practical applications require further research, including analysis

of observational or numerical data.
Kolmogorov's scaling law is widely used in the analysis of atmospheric measurements. It can be used to estimate the turbulence kinetic energy dissipation rate, even if the available instruments record only large eddy motions of the size of meters or tens of meters. Typically, second-order structure functions or the frequency spectrum of the measured time series are used for this purpose. Alternative methods have been proposed in papers H3 and H4. They are based on the scaling law for the number of signal's zero-crossings, or on determining the variance of the derivatives of the measured wind speed. Although the methods should ideally give the same results, in practice they can respond differently to errors related to, for example, insufficient signal resolution or too short averaging time windows. The use of several different methods simultaneously allows to increase the accuracy of the estimates.
Another source of problems in determining the rate of dissipation is that assumptions of stationarity and local isotropy are not always met. This was shown in the work H4 based on the analysis of numerical data of a startocumulus cloud. Its upper layers are stably stratified and turbulence therein is strongly inhomogeneous. The values of ϵ determined by indirect methods were underestimated in this region. In the area of strong convection, on the other hand, the rate of dissipation was overestimated. The same problem applies to measurements in the atmosphere. Dissipation rate values determined in different parts of the atmospheric boundary layer can be overestimated or underestimated depending on the degree of stratification. An interesting perspective is to consider effects of buoyancy force on velocity structure functions. Work related to this topic is currently underway, as part of a research collaboration with Prof. J. C. Vassilicos of the University of Lille (France).
The spectral energy density of turbulence is also modified by non-stationarity of the flow. This issue was addressed in paper H7. The non-stationarity can be assessed by analysing the value of the dissipation coefficient C_ϵ. Small values of C_ϵ indicate high production of turbulence kinetic energy. It can grow in time or be transported in physical space to other areas of the flow. Large values of this parameter, on the other hand, mean that turbulence is locally decaying. Such analysis provides important information about the processes taking place in the ABL on the basis of very sparse measurement data. Further applications of this method concern the analysis of turbulence shortly before sunset. The rapid decay of convection leads to changes in kinetic energy and dissipation rate. These changes can be described by new scaling laws of non-equilibrium turbulence, which can provide a better parameterization of processes occurring shortly before and after sunset.

4.8 Description of selected additional works, not included in the habilitation achievement

In addition to the papers described in Sections 4.1–4.7, after completing my Ph.D., I was the co-author of other articles. Selected additional works are listed at the end of the Literature section (below) as papers [D1]–[D12]. This chapter provides a brief description of these papers together with a description of my contribution.

One of the topics was the modelling of the dynamics of large eddies (so-called coherent structures), using the Proper Orthogonal Decomposition (POD) method. In this method, the optimal functional basis is determined from the results of laboratory or numerical experiments. The velocity fluctuation field is then developed in series using the determined basis. The dynamics of coherent structures is modelled by numerically solving a system of equations for time-varying coefficients of expansion. In Ref. [D1], the near-wall turbulence velocity field was modelled numerically using the POD method and the temperature field was modelled using the Lagrangian particle method. With this, the problem of the coupled heat transfer between the solid wall and the fluid was solved. My contribution to this work was to write the numerical code, perform the calculations and analyses, and contribute to the editing of the paper. In Ref. [D2], the POD method was used to simulate dispersed near-wall flows containing small particles. Together with Dr. Cyrille Allery, I participated in the development of the numerical code for the velocity field.

Another research topic was the parameterisation of turbulence in two-phase flows with a surface separation (water-air flows). In the presence of turbulent eddies, the position of the surface is disturbed; on the other hand, gravity and surface tension forces have a stabilizing role and prevent the fluctuations. The interaction of the two mechanisms leads to the development of an intermittent zone, where the probability of finding water or air is non-zero. Within the work [D3], I proposed a model in which the width of the intermittent region was determined by the interaction of two components in the equation for the intermittency factor. One of them was diffusive and was responsible for the increase in the thickness of the intermittent layer due to the presence of turbulent eddies disturbing the surface. The other term of the equation was responsible for the contraction, i.e. the decrease in the thickness of the layer due to the stabilizing action of the gravity and surface tension forces. The coefficients of the model have been related to the results of experimental observations, classifying flows with a separation surface into several characteristic types, (including the case of a flat undeformed surface, a surface with fine capillary waves, and a surface with gravity waves). My contributions to the paper [D3] were also the numerical calculations and the editing of the text. In yet another article [D4], in collaboration with Dr. Tomasz Wacławczyk we proposed together a different, conservative model of turbulence-interface interactions. In addition, I performed analysis of the numerical data, compared results with the model predictions and contributed to the editing of the text.

After obtaining my doctoral degree, I also worked on the topic of statistical description of turbulence. The work [D5], which I carried out together with Prof. Martin Oberlack, was devoted to the symmetry analysis of the Hopf equation for the characteristic functional, which contains information about all multi-point velocity statistics. My contribution was to perform the calculations and write the text.
In addition to the articles included in the habilitation achievement, I was a co-author of the papers [D6]–[D9] on the symmetry theory. I contributed to the editing of the review article [D6] and performed a significant part of the calculations in the papers [D7]–[D9].

A new direction of research I undertook at the Faculty of Physics, University of Warsaw, concerned the estimation of the turbulence kinetic energy dissipation rate. Three of the published papers on this topic are part of the habilitation achievement. In addition, an error analysis and comparison of different methods for the estimation of the dissipation rate were carried out in the paper [D10]. I was the initiator of this research and I supervised the work of students.
As co-supervisor of Mr Emmanuel O. Akinlabi, I collaborated with him and with Prof. Szymon Malinowski on the modelling of sub-grid (unresolved) scales in the Large Eddy Simulation (LES) method. The LES method is commonly used in cloud turbulence simulations. Because the available computer power is not sufficient to track the dynamics of all turbulent structures, in LES the Navier-Stokes equations are filtered (spatially averaged). The equations are then solved on a computational grid with a characteristic size 2 to 4 orders of magnitude larger than the Kolmogorov scale. The effect of unresolved scales on the dynamics of large eddies is took into account by means of an appropriate closure assumptions. The work [D11] concerned the fractal reconstruction model of subgrid scales. I proposed the research direction and participated in the development of the methods and in the editing of the text.

The recent research direction I have undertaken connects methods of theoretical analysis with the description of turbulence in the stable atmospheric boundary layer. In addition to papers H8 and H9 included in the habilitation achievement, together with Dr. Jun-Ichi Yano we worked on the method of non-dimensionalization of the transport equations, cf. Ref. [D12]. The purpose was to determine the characteristic scales based on it, including the Obukhov scale. I contributed to the discussion and interpretation of the results and participated in the editing of the article.

5 Presentation of significant scientific or artistic activity carried out at more than one university, scientific or cultural institution, especially at foreign institutions

After receiving my doctoral degree in the Institute of Fluid Flow Machinery in Gdańsk, I continued to work with the supervisor of my PhD thesis, Prof. Jacek Pozorski. I also cooperated with Dr. Cyrille

Allery and Dr. Claudine Béghein from the University of La Rochelle (France). I was involved in turbulence modeling using the probability density function method and modeling of large vortex structures in turbulent flow based on the Proper Orthogonal Decomposition (POD) method. The results of the work have been published in [D1, D2].
In 2008, I was awarded the Alexander von Humboldt Scholarship, which I held at the Technical University of Darmstadt (Germany) in the group of Prof. Martin Oberlack. During my stay in Darmstadt (until 2015), I worked on several topics. The first was the modelling of turbulence in two-phase air-water flows with a separation surface (papers [D3] and [D4]). I continued this topic within a university scholarship and next as the principal investigator (PI) of the project:

„*Modelling of turbulence-interface interaction in two fluid systems*", financed by
DFG (Deutscher Forschungsgemeinschaft), projekt number 220504256.

From January 2015 to September 2015, I was a leader of the research group "Turbulence and symmetries" at the Chair of Fluid Dynamics, Faculty of Mechanical Engineering, Technical University of Darmstadt. I also collaborated with Dr. Vladimir N. Grebenev (Federal Research Center for Information and Computational Technologies, Russian Academy of Sciences, Novovibirsk, Russia). I continued the work after returning to Poland as the principal investigator of the project

„*Description of turbulence as a stochastic field and its symmetry-based modelling.* financed by the National Science Center, Poland (project number 2014/15/B/ST8/00180)

carried out first at the Institute of Fluid Flow Machinery of the Polish Academy of Sciences in Gdańsk, and since April 2016 at the Faculty of Physics, University of Warsaw. This collaboration resulted in a number of papers, some of them are included in the habilitation achievement, others are described in section 4.8, as supplementary papers [D5]–[D9].
In 2017–2020, I was the co-supervisor of the PhD work of Mr. Emmanuel O. Akinlabi, employed at the University of Warsaw within the COMPLETE - Cloud MicroPhysics Turbulence Telemetry project (programm Horizon 2020). His doctoral thesis, "Analysis and Modelling of Small-Scale Turbulence" concerned the estimation of the turbulence kinetic energy dissipation rate and modeling of sub-grid scales in the Large Eddy Simulation (LES) method. After completing his Ph.D. in 2020, Dr. Emmanuel O. Akinlabi was a postdoctoral fellow at Boston University (USA), where he is currently continuing his research.
In 2020 I started a cooperation with Dr. Jun-Ichi Yano from Météo-France, (Tuluse, France) which is currently continiued within the scinentific project, of which I am the principal investigator:

"*Stable atmospheric boundary layer: beyond Monin-Obukhov theory*"
National Sciance Centre, Poland, project number 2020/37/B/ST10/03695

The ongoing work concerns the parameterization of turbulence in the stable atmospheric layers. Involved in the project were (or are) the following students and co-investigators: Mr. Jackson Nzotungishaka, M.Sc. Paweł Jędrejko, who started his doctoral studies at the Faculty of Physics in October 2023, M.Sc. Grzegorz Florczyk and Dr Jakub Nowak.
After the publication of the article H7, together with Prof. Szymon Malinowski, Dr. Jakub Nowak and M.Sc. Stanisław Król, I was invited to participate in the Lille Turbulence Program 2023 at the University of Lille (France). Since then, we have been continuing our cooperation with Prof. John Christos Vassilicos on the scaling of velocity and temperature structure functions in atmospheric turbulence.
Apart from the above-mentioned works, I was involved, as a co-investigator, in the following projects "Turbulent dynamics and microphysics in a Stochastic Lagrangian Cloud Model" (National Science Center, Poland), "Next Generation Earth Modelling Systems (NextGEMS)" (European Commission, HORIZON 2020), both led by Prof. Hanna Pawłowska and "Particle fluxes in urban environment with remote sensing (PURER-SENS)" led by Dr. Pablo Ortiz Amezcua. Within this latter project I cooperated with the group of Prof. Iwona Stachlewska on the analysis of data of decaying ABL turbulence short before the sunset.

References

[1] Frisch U. *Turbulence: The Legacy of A. N. Kolmogorov*, Cambridge University Press, 2006

[2] Łobocki L. *Podstawy dynamiki atmosfery*, Oficyna wydawnicza Politechniki Warszawskiej, Warszawa, 2018

[3] Pope S. B. *Turbulent flows*, Cambridge University Press, 2000

[4] Keller L. and Friedmann A., Differentialgleichungen für die turbulente Bewegung einer kompressiblen Flüssigkeit, Proc. First. Int. Congr. Appl. Mech. str. 395–405, 1924.

[5] Lundgren T. S. Distribution functions in the statistical theory of turbulence, Phys. Fluids, **10**, str. 969–975, 1967

[6] Monin A. S., Equations of turbulent motion, Prikl. Mat. Mekh, **31**, str. 1057–1967, 1967.

[7] Novikov E. A., Kinetic equations for a vortex field, Soviet Physics-Doklady, **12**, str. 1006–1008, 1968

[8] Hopf E. Statistical hydromechanics and functional calculus, J. Rational Mech. Anal., **1**, str. 87–122, 1952

[9] Venturi D., The numerical approximation of nonlinear functionals and functional differential equations, Physics Reports, **732**, str. 1–102, 2018

[10] Pozorski J., Zagadnienia turbulencji w mechanice płynów, Zeszyty Naukowe Instytutu Maszyn Przepływowych Polskiej Akademii Nauk w Gdańsku, **515**, str. 1–43, 2000.

[11] Monin, A. S., and Obukhov, A. M., Basic laws of turbulent mixing in the surface layer of the atmosphere (in Russian). Tr. Geofiz. Inst., Akad. Nauk SSSR, **24**, 163–187, 1954

[12] Bluman, G. W., Kumei S., *Symmetries and Differential Equations.* Elsevier, 1989

[13] Kolmogorov, A. N., Dissipation of energy in the locally isotropic turbulence. Proc. Roy. Soc. London, **A434**, 15–17, 1941

[14] Oberlack M., A unified approach for symmetries in plane parallel turbulent shear flows, J. Fluid Mech., **427**, str. 299–328, 2001.

[15] Lindgren, B., Osterlund, J.M. Johansson A.V., Evaluation of scaling laws derived from Lie group symmetry methods in zero-pressure-gradient turbulent boundary layers. J. Fluid Mech. **502**, 127–152, 2004

[16] Marati, N., Davoudi, J., Casciola, C. M. Eckhardt, B. Mean profiles for a passive scalar in wall-bounded flows from symmetry analysis. Journal of Turbulence **7**, N61, 2006

[17] Oberlack M., Rosteck A., New statistical symmetries of the multi-point equations and its importance for turbulent scaling laws, Discrete Contin. Dyn. Sys., Ser. S, **3**, str. 451–471, 2010.

[18] Oberlack, M., Wacławczyk M., Rosteck A., Avsarkisov V., Symmetries and their importance for statistical turbulence theory. Mech. Eng. Rev., **2**, 15–00157, 2015

[19] Oberlack M., Hoyas S., Kraheberger S.V., Alcántara-Ávila F., and Laux J. Turbulence Statistics of Arbitrary Moments of Wall-Bounded Shear Flows: A Symmetry Approach, Phys. Rev. Lett. **128**, 024502, 2022

[20] Alcántara-Ávila F., García-Raffi L.M., Hoyas S., Oberlack M., Validation of symmetry-induced high moment velocity and temperature scaling laws in a turbulent channel flow, Phys. Rev. E **109**, 025104, 2024

[21] She Z.-S., Chen X., Hussain F., Quantifying wall turbulence via a symmetry approach: a Lie group theory. Journal of Fluid Mechanics **827**, 322–356, 2017

[22] She Z.-S., Zou H.-Y., Xiao M.-J., Chen X., Hussain F. Prediction of compressible turbulent boundary layer via a symmetry-based length model. Journal of Fluid Mechanics. **857**, 449–468, 2018

[23] Friedrich R., Daitche A., Kamps O., Lüff J., Voßkuhle M., Wilczek M., The Lundgren-Monin-Novikov hierarchy: Kinetic equations for turbulence, Com. Ren. Phy. **13**, str. 929–953, 2012.

[24] Klotz Ł., Lemoult G., Avila K., Hof B. Phase Transition to Turbulence in Spatially Extended Shear Flows Phys. Rev. Lett. **128**, 014502, 2022

[25] Wacławczyk, M., Oberlack M., Reply to "Comment on 'Statistical symmetries of the Ludgren-Monin-Novikov hierarchy.'" Phys. Rev. E, **92**, 067002, 2015

[26] Boyko V. On Galilei invariance of continuity equation. Proc. Inst. Math. NAS Ukraine **30** 99–102, 2000

[27] Grigoriev Y. N., Ibragimov N. H., Kovalev V. F. and Meleshko S. V. Symmetries of Integro-Differential Equations: With Applications in Mechanics and Plasma Physics. Dordrecht:Springer, 2010

[28] Grebenev V.N., Wacławczyk M., Oberlack M. Conformal invariance of the Lungren–Monin–Novikov equations for vorticity fields in 2D turbulence. Journal of Physics A: Mathematical and Theoretical, **52**, 335501, 2017

[29] Bernard D., Boffetta G., Celani A., Falkovich G. Conformal invariance in two-dimensionalturbulence Nat. Phys. 2 124–8, 2006

[30] Kreichnan K. H. Inertial Ranges in Two Dimensional Turbulence. Physics of Fluids **10**, 1417, 1967

[31] Tabeling P., Two-dimensional turbulence: a physicist approach, Physics Reports, **362**, 1–62, 2002

[32] Cho J. Y. N., Lindborg E., Horizontal velocity structure functions in the upper troposphere and lower stratosphere: 1. Observations, J. Geophys. Res., **106**(D10), 10223–10232, 2001

[33] Alexakis A., Marino R., Mininni P.D., van Kan A., Foldes R., Feraco F. Large-scale self-organization in dry turbulent atmospheres. Science **383**, 1005–1009, 2024

[34] Bouchet F., Venaille A., Statistical mechanics of two-dimensional and geophysical flows. Phys. Rep. 515, 227–295, 2009

[35] Held I. M., Pierrehumbert R. T., Garner S. T., Swanson K. L. Surface quasi-geostrophic model, J. Fluid Mech. **282**, 1–20, 1995

[36] Horton W., Hasegawa A., Quasi-two-dimensional dynamics of plasmas and fluids. Chaos: Interdiscip. J. Nonlinear Sci., **4**, 227, 1994

[37] Fallah B., Saberi A. A., Sodoudi S., Emergence of global scaling behaviour in the coupled Earth-atmosphere interaction, Sci Rep. **6** 34005, 2016

[38] Grachev A.A., Andreas E.L., Fairall C.W. et al. SHEBA flux–profile relationships in the stable atmospheric boundary layer. Boundary-Layer Meteorol **124**, 315–333, 2007.

[39] Nieuswtadt F.T.M., The turbulent structure of the stable, nocturnal boundary layer. J. Atmos. Sci. **41**, 2202–2216, 1984

[40] Sreenivasan, K., Prabhu, A., and Narasimha, R.: Zero-crossings in turbulent signals, J. Fluid Mech., **137**, 251–272, 1983.

[41] Kopeć, J. M., Kwiatkowski, K., de Haan, S., and Malinowski, S. P., Retrieving atmospheric turbulence information from regular commercial aircraft using Mode-S and ADS-B, Atmos. Meas. Tech., **9**, 2253–2265, 2016

[42] Schulz, B., Mellado J. P., Wind shear effects on radiatively and evaporatively driven stratocumulus tops. J. Atmos. Sci., **75**, 3245–3263, 2018

[43] Wacławczyk M., Gozingan A. S., Nzotungishaka J., Mohammadi M., Malinowski S. P., Comparison of Different Techniques to Calculate Properties of Atmospheric Turbulence from Low-Resolution Data, Atmosphere , **11** str. 2073–4433, 202

[44] Taylor, G. I., Statistical theory of turbulence. Proc. Roy. Soc. London, **A151**, 421–444, 1935

[45] Vassilicos, J. C., Dissipation in turbulent flows. Annu. Rev. Fluid Mech., 47, 95–114, 2015

[46] Bos, W. J. T., Rubinstein R., Dissipation in unsteady turbulence. Phys. Rev. Fluids, **2**, 022601, 2017

[47] Obligado M., Vassilicos J. C., The non-equilibrium part of the inertial range in decaying homogeneous turbulence, European Physical Letters **127**, 64004, 2019

[48] Siebert H., and Coauthors, Observations of aerosol, cloud, turbulence, and radiation properties at the top of the marine boundary layer over the eastern North Atlantic Ocean: The ACORES campaign. Bull. Amer. Meteor. Soc., **102**, E123–E147, 2021

[49] Wood R. Stratocumulus clouds. Mon. Weather Rev. **140**, 2373–2423, 2012

[50] Nowak J. L., Siebert H., Szodry K.-E., Malinowski S. P., Coupled and decoupled stratocumulus-topped boundary layers: Turbulence properties. Atmos. Chem. Phys., **21**, 10965–10991, 2021

[D1] Wacławczyk M., Pozorski J., Minier J.-P.: Filtered density function modelling of near-wall scalar transport with pod velocity modes, International Journal of Heat and Fluid Flow, vol. 30, pp. 7687 (2009)

[D2] Béhein C., Allery C., Wacławczyk M., Pozorski J.: Application of POD-based dynamical sys- tems to dispersion and deposition of particles in turbulent channel flow, International Journal of Multiphase Flow, vol. 58, pp. 97-113 (2014)

[D3] Wacławczyk M., Oberlack M.: Closure proposals for the tracking of turbulence-agitated gas-liquid interface in stratified flows International Journal of Multiphase Flow, vol. 37(8), pp. 967-976, (2011)

[D4] Wacławczyk M., Wacławczyk T.: A priori study for the modelling of ensemble-averaged velocity-interface correlations in the stratified air-water flows, International Journal of Heat and Fluid Flow, vol. 52, pp. 40–49 (2015)

[D5] Wacławczyk M., Oberlack, M.: Application of the extended Lie group analysis to the Hopf functional formulation of the Burgers equation, Journal of Mathematical Physics, vol. 54, pp. 072901, (2013)

[D6] Oberlack, M., Wacławczyk M., Rosteck, A., Avsarkisov, V.: Symmetries and their impor- tance for statistical turbulence theory, Bulletin of the JSME: Mechanical Engineering Reviews, vol. 2(2), pp. 1-72, (2015); (invited review article)

[D7] Janocha, D.D., Wacławczyk M., Oberlack, M.: Lie symmetry analysis of the Hopf functional- differential equation for turbulence, Symmetry, vol. 7, pp. 1536-1566, (2015)

[D8] Grebenev, V., Wacławczyk M., Oberlack, M.: Conformal invariance of the LungrenMonin- Novikov equations for vorticity fields in 2D turbulence, Journal of Physics A: Mathematical and Theoretical, Vol. 50, 435502 (2017)

[D9] Grebenev V. N., Wacławczyk M., Oberlack, Conformal invariance of the zero-vorticity La- grangian path in 2D turbulence, Journal of Physics A: Mathematical and Theoretical, vol. 52, 225501, (2019)

[D10] Wacławczyk M., A. S. Gozingan; J. Nzotungishaka, M. Mohammadi; S P. Malinowski: Comparison of Different Techniques to Calculate Properties of Atmospheric Turbulence from Low- Resolution Data, Atmosphere, vol. 11, 199, (2020)

[D11] Akinlabi E. O., Wacławczyk M., Mellado J. P., Malinowski S. P.: Fractal reconstruction of sub-grid scales for large eddy simulation, Flow, Turbulence and Combustion Vol. 103, pp. 293–322, (2019)

[D12] Yano, J., Wacławczyk, M. Nondimensionalization of the Atmospheric Boundary-Layer System: Obukhov Length and Monin–Obukhov Similarity Theory. Boundary-Layer Meteorol 182, 417–439 (2022). https://doi.org/10.1007/s10546-021-00657-7.

6 Presentation of teaching and organizational achievements as well as achievements in popularization of science or art

6.1 Teaching

After completing my PhD I thaught the following classes:

Dept. Mechanical Engineering, Technical University of Darmstadt, Germany:

- **Summer semester 2014/15**
  - Fundamentals of Turbulence – *Excercises*
- **Summer semester 2012/13 – Summer semester 2014/15**
  - Student seminar – *Individual student projects.*

Faculty of Physics, University of Warsaw:

- **Winter semester 2017/18**
  - Turbulence and atmospheric boundary layer – *Lecture*
- **Winter semester 2019/20**
  - Turbulence and atmospheric boundary layer – *Lecture: 30h*
  - Mathematics I – *Tutorial: 90h*
  - Statistical Physics A – *Tutorial: 45h*
- **Winter semester, 2020/21**
  - "Fizyka na start"–advanced course – *Tutorial: 30h*
  - Mathematics I – *Tutorial: 90h*
  - Statistical Physics A – *Tutorial: 45h*
- **Summer semester 2020/21**
  - Turbulence and atmospheric boundary layer – *Lecture: 30h*
- **Winter semester 2021/22**
  - Statistical Physics A – *Tutorial: 45h*
  - Mathematics III for opticians – *Tutorial: 30h*
- **Summer semester 2021/22**
  - Turbulence and atmospheric boundary layer – *Lecture: 30h*
  - Thermodynamics with elements of statistical physics – *Tutorial: 45h*
- **Winter semester, 2022/23**
  - Statistical Physics A – *Tutorial: 45h*

  - Mathematics III for opticians – *Tutorial: 30h*
- **Summer semester 2022/23**
  - Turbulence and atmospheric boundary layer – *Lecture: 30h*
  - Thermodynamics with elements of statistical physics – *Tutorial: 45h*
  - Geophysical Laboratory – *Laboratory*
- **Winter semester 2023/24**
  - Statistical Physics A – *Tutorial: 45h*
  - Mathematics III for opticians – *Tutorial: 30h*
  - Geophysical Laboratory – *Laboratory*
- **Summer semester, 2023/24**
  - Turbulence and atmospheric boundary layer – *Lecture: 30h*
  - Thermodynamics with elements of statistical physics – *Tutorial: 45h*
  - Geophysical Laboratory – *Laboratory*
  - Selected topics in fluid mechanics – *Tutorial: 30h*

I was a supervisor or co-supervisor of the following master theses

1. **D. Ciesielski**: "*LES simulations of turbulent jets*", Faculty of Technical Physics and Applied Mathematics, Gdańsk University of Technology, (2007)
2. **S.V. Kraheberger**: "*Numerical Study of the Intermittency Region in Two-Fluid Turbulent Flow*", Dept. Mechanical Engineering, Technical University of Darmstadt, Germany (2014)
3. **D. Janocha**: "*Lie symmetry analysis of the Hopf functional-differential equation for turbulence*", Dept. Mechanical Engineering, Technical University of Darmstadt, Germany, (2015)
4. **P. Jędrejko**: "*Formation of thermal vortex rings*", Interdisciplinary Centre for Mathematical and Computational Modelling, University of Warsaw, (2023)

and the co-supervisor of the PhD thesis

1. **E. O. Akinlabi** "*Analysis and Modelling of Small-Scale Turbulence*", Faculty of Physics, University of Warsaw (2020)

6.2 Organizatinal activity

- Member of the "Local Organising Committee" of the Interdisciplinary Turbulence Conference (iTi), Bertinoro/Italy, 2014
- Associate Member of the Fluid Mechanics Section, Commitee of Mechanics, Polish Academy of Sciences, from November 2018 till now
- Member of the local organizing commitee of the 3rd Workshop of COMPLETE (Cloud-MicroPhysics-Turbulence-Telemetry) ITN - ETN Network 04-08.02.2019
- Member of the Faculty Council, at the Faculty of Physics, UW 2020–2024
- Member of the selection commitee for the Doctoral School of Exact and Natural Sciences at the Faculty of Physics, University of Warsaw, in 2022

6.3 Dissemination of science

In 2017 and 2019–2022 within the Science Festival at the University of Warsaw I prepared:

- A workshop for children titled "What kind of cloud is it?"
- Lessons for school children "Are turbulences dangereous?", "Why it's so difficult to predict the weather?"
- Short movies (online on the youtube platform) "Van Gogh and others: physics meets art", "What kind of cloud is it?", "Short movie about sailing".

After the 2020 Science Festival, I was invited to Kampus Radio, where I gave an interview about the relationship between physics with art. I also wrote popular science articles for the journal "Fizyka w Szkole" (Physics at School):

- Wacławczyk M., Wacławczyk Z., Van Gogh i inni, czyli fizyka spotyka sztukę. Fizyka w Szkole z Astronomią: czasopismo dla nauczycieli. no. 1, pp. 26–27, 2021.
- Wacławczyk M., Czy turbulencje są niebezpieczne? Fizyka w Szkole z Astronomią: czasopismo dla nauczycieli. no. 3, pp. 15–17, 2021.
- Wacławczyk M., O fizyce żeglowania. Fizyka w Szkole z Astronomią: czasopismo dla nauczycieli. no. 6, pp. 28–30, 2021.

In cooperation with the "Ask a Physicist" website, I provided answers to several questions from "everyday physics".

In 2023, I participated, together with Dr. Jakub Nowak, in a promotional video for the article H7, included in the habilitation achievement, published within the NextGems project:

- Wacławczyk M., Nowak J. L., *How Can We Detect Non-Stationarities of Turbulence in the Atmosphere?*, Latest Thinking, 2023 https://doi.org/10.21036/LTPUB101137

---

7 Apart from information set out in 1-6 above, the applicant may include other information about his/her professional career, which he/she deems important.

Funding obtained (as principal investigator)

- Projekt "Modelling of turbulence-interface interaction in two fluid systems", financed by the DFG (Deutscher Forschungsgemeinschaft) in years 2012–2014, project number 220504256
- "Description of turbulence as a stochastic field and its symmetry-based modelling" financed by the National Science Center, Poland in years 2015–2019 (project number 2014/15/B/ST8/00180)
- "Stable atmospheric boundary layer: beyond Monin-Obukhov theory" financed by the National Science Center, Poland in years 2021–2024 (project number 2020/37/B/ST10/03695)

Invited lectures on scientific conferences/workshops:

- Interdisciplinary Turbulence Conference 2014, Bertinoro/Włochy organizers: Prof. M. Oberlack, Prof. J. Peinke, Prof. A. Talamelli
- Summer School of Multiphase Flows 2015, Jantar/Poland organizor: Prof. J. Pozorski
- Symposium "Perspectives on turbulence and wind energy research", 2017, Oldenburg/Niemcy, organizors: Prof. M. Kühn, Prof. J. Peinke,
- Workshop "Opening Workshop on turbulent flows" within Lille Turbulence Program 2023, Lille/Francja, organizor: prof. J. C. Vassilicos

Member of the Advisory Commitee of the conferences:

- "Eleventh International Symposium on Turbulence and Shear Flow Phenomena" (TSFP11), 2019
- "Twelfth International Symposium on Turbulence and Shear Flow Phenomena (TSFP12), 2022

In 2024 I was asked to be a member of the Programme Board of the conference

- Sympozjum Młodych Naukowców Wydziału Fizyki, Warsaw 2024

and to co-organize the Session S10-Turbulence and Reactive Flows on the conference

- 95th Annual Meeting of the International Association of Applied Mathematics and Mechanics (GAMM), which will take place in Poznań, in April 2025.

Reviews of PhD theses:

- Mina Golshan Kovi "Cloud Turbulence Microphysics At Interfaces: A DNS model with phase change and droplet" Politecnico di Milano, Italy, 2023.

Scholarships and awards

- Individual Third Degree Award of the Rector of the University of Warsaw, 2023
- Status IOP Trusted Reviewer, awarded by the Institute of Physics, 2019
- Prime Minister's Award for Outstanding Doctoral Dissertation, 2009
- Scholarship of TU Darmstadt "Wiedereinstiegsstipendium" 2010-2011
- Alexander von Humboldt scholarship 2008-2009
- Scholarship of Foundation for Polish Science for young researchers in 2005
- Scholarship Deutcher Akademischer Austauschdienst (DAAD) 2003-2004

....

(Applicant's signature)