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Description of the achievement (autoreferat)

A. **Name:** Tomasz Pawłowski

B. **Scientific titles/diplomas:**

1. MSc – theoretical physics, 2000, Faculty of Physics, University of Warsaw.
2. PhD – physics sciences, 2005, Faculty of Physics, University of Warsaw, *Isolated horizons – a quasi local black hole theory.*

C. **Employment in scientific institutions:**

1. Institute for Gravitational Physics and Geometry, the Pennsylvania State University, USA, Jan 2005 – Aug 2007, postdoc.
2. Instituto de Estructura de la Materia, CSIC, Spain, Sep 2007 – Aug 2010, postdoc (program I3P-JAE).
3. Department of Mathematics and Statistics, University of New Brunswick, Canada, Oct 2010 – Jan 2012, postdoc.
4. Division for Mathematical Methods of Physics, University of Warsaw, Poland, Feb 2012 – Feb 2013, assistant professor.
5. Departamento de Ciencias Fisicas, Universidad Andres Bello, Chile, Mar 2013 – Dec 2015 assistant professor.
6. Center for Theoretical Physics, PAS, PAN, Poland, from Aug 2016, assistant professor.

D. **Scientific achievement:**

1. Title: **Dynamics of Early Universe in Loop Quantum Cosmology**
2. **Works contributing to the achievement:**
    (1) Ashtekar A, Pawłowski T, Singh P, 2006, *Quantum Nature of the Big Bang* Phys.Rev.Lett. **96**, 141301.
    (2) Ashtekar A, Pawłowski T, Singh P, 2006, *Quantum Nature of the Big Bang: An Analytical and Numerical Investigation* Phys.Rev. **D73**, 124038.
    (3) Ashtekar A, Pawłowski T, Singh P, 2006, *Quantum Nature of the Big Bang: Improved dynamics* Phys.Rev. **D74**, 084003.
    (4) Martín-Benito M, Mena Marugán G A, Pawłowski T, 2008 *Loop Quantization of Vacuum Bianchi I Cosmology* Phys.Rev. **D78**, 064008.
    (5) Kamiński W, Lewandowski J, Pawłowski T, 2009, *Physical time and other conceptual issues of QG on the example of LQC* Class.Quant.Grav. **26**, 035012.
    (6) Brizuela D, Mena Marugán G A, Pawłowski T, 2010, *Big Bounce and inhomogeneities* Class. Quant.Grav. **27**, 052001.
    (7) Martín-Benito M, Mena Marugán G A, Pawłowski T, 2009 *Physical evolution in Loop Quantum Cosmology: The example of vacuum Bianchi I* Phys.Rev. **D80**, 084038.
    (8) Kamiński W, Lewandowski J, Pawłowski T, 2009, *Quantum constraints, Dirac observables and evolution: group averaging versus Schroedinger picture in LQC* Class.Quant.Grav. **26**, 245016.
    (9) Kamiński W, Pawłowski T, 2010 *The LQC evolution operator of FRW universe with positive cosmological constant* Phys.Rev. **D81**, 024014.

Tomasz Pawłowski

(10) Kamiński W, Pawłowski T. 2010 *Cosmic recall and the scattering picture in Loop Quantum Cosmology* Phys.Rev. **D81**, 084027.

(11) Brizuela D, Mena Marugán G A, Pawłowski T, 2011 *Effective dynamics of the hybrid quantization of the Gowdy T3 universe* Phys.Rev **D84**, 124017.

(12) Mena Marugán G A, Olmedo J, Pawłowski T, 2011 *Prescriptions in Loop Quantum Cosmology: A comparative analysis* Phys.Rev **D84**, 064012.

(13) Husain V, Pawłowski T. 2011 *Dust reference frame in quantum cosmology* Class.Quant.Grav. **28**, 225014.

(14) Pawłowski T, Ashtekar A, 2012 *Positive cosmological constant in loop quantum cosmology* Phys.Rev. **D85**, 064001.

(15) Artymowski M, Dapor A, Pawłowski T, 2013 *Inflation from non-minimally coupled scalar field in loop quantum cosmology* JCAP **1306**, 010.

(16) Barbero F, Pawłowski T, Villasenor E, 2014, *Separable Hilbert space for loop quantization* Phys.Rev. **D90**, 067505.

(17) Pawłowski T, 2015, *Observations on interfacing loop quantum gravity with cosmology* Phys.Rev. **D92**, 124020.

(18) Pawłowski T. 2016, *Universe's memory and spontaneous coherence in loop quantum cosmology* Int.J.Mod.Phys. **D25**, 2016, no.08, 1642013.
invited contribution to special issue

(19) Martín de Blas D, Olmedo J, Pawłowski T, 2017 *Loop quantization of the Gowdy model with local rotational symmetry* Phys.Rev. **D96**, 106016.

3. **Description of the contributing works:**

Modern theoretical physics is based on two main pillars: on one hand General Relativity (GR) accurately describing reality at large (astronomical) scales and strong gravitational fields, on the other hand the high energy and short scale phenomena are accurately captured by Quantum Physics. While both theories are based on mutually exclusive principles, correct description of certain phenomena in observed reality requires taking into account both relativistic and quantum aspects of the matter and the spacetime itself. Such unified description (known under the name of quantum gravity) is believed to be necessary in capturing the correct physics of a very early universe, as well as in the interiors of black holes. There are several approaches to construct such theory, string theory being the most famous. One of the leading approaches is the so called Loop Quantum Gravity (LQG) [1, 2]. Most of its defining principles is taken from GR with the requirement of strict background independence and coordinate choice invariance being the most emphasized [3]. One of mathematical consequences of this requirement is the need to use a quantum representations distinct from the standard (Schrödinger) one and known as the polymer representation. This in turn significantly changes the properties of space (geometry) at the smallest scales. In particular, at Planck scales the geometry attains a discrete structure. Such drastic (with respect to the continuous geometry assumption in standard field theories) change is in turn expected to significantly alter the physical processes at highest (Planckian) energies. However due to the extreme complication level of LQG getting really robust and general predictions out of it has eluded the researchers so far.

In an attempt to sidestep the enormous technical difficulties of the full theory, its simpler version has been constructed: a theory applying the methods of LQG (as well as some results of the latter as a heuristic input) to quantize highly symmetric spacetimes, usually (but not only) cosmological models. Created about 2000 by M. Bojowald, it is known as Loop Quantum Cosmology [4, 5]. Its aim is making predictions regarding the physics of a very early Universe, especially near the classical big bang singularity where General Relativity fails and quantum nature of reality is expected to significantly alter the properties of spacetime and matter on it. It is actually simple enough to provide robust physical predictions even on a genuine quantum level. Habilitant's research, in particular the set of works listed above as (1)-(19) has played the critical role in reaching by LQC this level of maturity.

The scientific achievement presented for habilitation is the candidate's contribution to the development of LQC to the level, where solid physical predictions could be made and to actually making those predictions, some of which provided a qualitative change to our (standard) picture of the Universe evolution.

The results contributing to the achievement can be divided into the following aspects:

1. **Probing the genuine quantum dynamics of isotropic Universe in LQC framework and the big bounce:**

For the initial couple of years since its birth at the beginning of the century the development of LQC focused almost entirely on its mathematical structure [6]. While there ware attempts to make predictions regarding the physical properties of early Universe, they were based almost entirely on the observed qualitative mathematical properties of the components of the formalism, in particular the quantized Hamiltonian constraint [7]. The flag example of such was the regularity of the abovementioned constraint at the point of vanishing volume (representing in classical theory the big bang singularity) [8] which led to the heuristic conclusion, that in LQC framework the singularity is "passable" and the evolution of the universe (parametrized by a scale factor) leads it straight through it [9]. Going beyond the heuristics was however impossible as neither a precise notion of the evolution of a quantum states nor the observables necessary to describe it were included in the framework. That situation changed with start of the (led by A. Ashtekar) project aimed towards giving LQC a form of a solid quantum mechanical (QM) model at least in the simplest setting (the one describing isotropic Universe with a single homogeneous matter field) and to probing its physical properties via tools of a conservative quantum mechanics. In this project habilitant played a crucial role.

The precise model considered in the initial stage of the project was the one of the isotropic flat Universe (so called Friedman-Robertson-Walker Universe, or FRW) of which matter content was the massless scalar field. While sufficiently simple to handle precisely, it shared sufficiently many properties with models featuring more realistic matter content to give useful insights. Due to the symmetries of spacetime the model features just few global degrees of freedom with general structure resembling that of 1-dimensional quantum mechanical systems[1]. Thanks to that, it was possible to perform a very detailed analysis of it and to confront it against almost textbook examples of simple QM systems. On the other hand, the necessity to use an exotic (polymer) quantum representation have made even this simple model quite challenging technically, as most of standard developments of quantum mechanics could not be applied here. Yet, by a combination of analytical and numerical methods (some of them built specifically for this task) it was possible to analyze the dynamics of quantum universe in a robust and unambiguous manner. In particular the physical Hilbert space of the system has been constructed precisely (a task nontrivial in theories with constraints instead of a true Hamiltonian) by group averaging techniques [10] and the notion of the evolution has been defined by means of the so called partial observables (originally introduced for full LQG) [11]. These components allowed to evolve backward in time a set of "initial data" – semiclassical states representing in the model the (radical simplification of the) expanding Universe we observe. this in turn has been done by a dedicated numerical toolbox. The numerical analysis of the dynamics focused in particular on the early universe epoch, where the classical theory predicts a big bang singularity. The results were qualitatively different than both the predictions of GR and the existing (at that time) heuristic predictions of LQC. Instead of terminating in an initial singularity or transiting through it, the early universe have shown an unexpected feature: the discreteness of quantum geometry have made a gravity repulsive at high matter energy densities. As a consequence, instead of big bang, the quantum universe have undergone through a quantum bounce connecting deterministically two epochs (pre-bounce contracting and post-bounce expanding) during which Universe evolved according to the laws of General Relativity. While at the time there existed proposals of the universe evolution scenaros featuring a bounce [12], they were either postulated or were a consequence of including in the model exotic matter (i.e. phantom scalar field). By contract in LQC this phenomenon was a prediction of a quantum evolution. This result (published in (1)-(3)) not only presented a paradigm shift (replacing initial singularity by a bounce) but also by predicting the previous pre-bounce epoch it indicated, that the structure of physical reality may be much richer than previously expected. Its impact reached outside of its particular area, having an effect not only on general physics community bat also leaving some imprint among philosophers. It was further substanitally covered in popular science and public press.

The particular contribution of the habilitant to this project included development of the numerical toolkit dedicated to probing dynamics in LQC, majority of the analysis and interpretation of the results of the simulations as well as contribution to the derivation/improvement of the mathematical structure of the models. The latter appeared to be necessary, as the in

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[1] After the so called deparametrization where on of dynamical degrees is freedom was used as a clock.

the original formulation of LQC the models were showing inconsistencies (visible only after systematic methods of probing the dynamics were used). One of the flag problems was lack of well defined infrared regulator removal limit[2]. This particular aspect have led to construction of the so called improved dynamics of LQC, published in (3), which up to date exceeded 500 ISI citations.

The original results obtained for flat FRW model with scalar fields were subsequently extended to more complicated isotropic models, in particular:

- Flat FRW universe with positive cosmological constant (14):

  Since a small positive value of cosmological constant is actually observed in our Universe, its inclusion into LQC models was a necessary step in making them more realistic. It presented however a significant challenge. With the scalar matter clock used in the previous works the evolution operator induced by the Hamiltonian constraint admits many self-adjoint extensions, each leading to inequivalent unitary evolution of the quantum system. Here again the role of the habilitant were all the numerical aspects of the studies and analysis of the dynamics as well as building the mathematical structure necessary to probe the properties of abovementioned self-adjoint extensions and possible differences in predictions following from their use. The model again featured the quantum bounce, however (quite surprisingly) the evolution was quasi-periodic. The expanding universe reached infinite volume for a finite value of the internal matter clock and subsequently transited (through asymptotic future infinity, or scri) into a contracting one, thus starting the next cycle.

- FRW universe with dust (13):

  In full LQG the only viable (back in 2011) method of describing the dynamics of the system was the deparametrization with respect to matter reference frames. The use of standard matter (rotating dust/scalar field) have led to systems featuring a true Hamiltonian, however using it to actually probe the dynamics was impossible due to its complicated mathematical structure (square root of a complicated combinatorial operator). However, a synthesis of a particular matter clock choice (irrotational dust) and the diffeomorphism invariant formulation of LQG (performed with significant contribution of the habilitant (30)) allowed to define a loop quantum gravity dynamics in general setting (without symmetries) as generated by a Hamiltonian acting on a domain in precisely known Hilbert space and of which action could be evaluated numerically. This brought the task of probing the dynamics of full LQG to within technical reach. One of the elements of the studies was testing the effects of using the chosen matter field as time in simpler setting – the one of isotropic LQC. It was shown, that this choice has cured a series of previously encountered difficulties: (*i*) the evolution operator now remains essentially self-adjoint (thus generates the evolution uniquely) for all non-exotic matter, including positive cosmological constant, (*ii*) the elements of the so called semiclassical effective analysis [13] (based on capturing the quantum dynamics in the equations of motion for the moments in the so called Hamburger decomposition of the semiclassical state) have much better properties, which in turn enlarges the domain of applicability of the method and (*iii*) the so called modified Friedman equation capturing essential phenomenological aspects of LQC dynamics could be derived presicely. Here the habilitant performed most of the analytical studies (no numerics was involved).

  This work was further complemented by studies of the flat FRW universe filled with radiation (33). While neither dust nor massless scalar field were observed (as fundamental fields) or predicted in standard model of particle physics, the electromagnetic fields are commonly observed. Therefore, to probe the possibility of using the electromagnetic radiation as an internal clock a model considering the isotropic distribution of photons was considered. As the gravitational backreaction of any electromagnetic plane wave can be mimicked by a homogeneous magnetic field, the whole system could be represented by an isotropic universe admitting a specific configuration of just three fields. The (equal for all three fields) potential of these fields could serve as an internal clock. Here, the only difficulty was the need to significantly improve the numerical methods used to identify the asymptotics of the Hilbert space basis functions as they are used to calculate the correct normalization of the basis (which in turn is needed to correctly assemble semiclassical physical states).

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[2] In defining Hamiltonian constraint and mamenta the densities have to be integrated over the whole space. Due to homogeneity and noncompactness of spatial constant time slices in flat models these integrals are infinite. To regulate the infinities the integrals were restricted to a finite region, then took the limit of the region being expanded to infinity.

2. **Extension to dynamics of anisotropic homogeneous models in LQC:**

   The probing of the genuine quantum dynamics in isotropic LQC, while providing many interesting results and insights into what to expect out of realistic cosmological models, could be treated only as the initial step in development of the theory and needed to be generalized. The natural next step was probing the anisotropic homogeneous models, as they still feature global degrees of freedom. The natural starting point for these extensions was flat Bianchi I model. While first attempts of completing the LQC quantization program for it followed the development of isotropic LQC very quickly [14, 15] they suffered from the same inconsistencies (lack of the correct infrared regulator removal limit) as the early isotropic LQC quantization (see [16] for the discussion of the problem). Implementing the improved dynamics scheme, that cured the isotropic models, here proven to be a difficult task. Due to technical ambiguities in implementing the scheme for couple of years two independent prescriptions were considered as viable by the community.

   The first of them, introduced in (4), based on the principle of the separability of the Hamiltonian (constraint) with respect to configuration variables natural for LQC, gave relatively simple description. However, even there the standard methods of building the dynamical sector were failing [17] due to numerical instabilities. Only by explicit implementation of the group averaging techniques with heavy use of numerical methods developed earlier for isotropic LQC it was possible to properly identify the space of physical states. Habilitant's contribution to this area of research consisted in performing this analysis for vacuum Bianchi I toroidal model, which in turn required a novel approach to define meaningful physical observables. The construction of such was provided (and subsequently used to probe the dynamics of the system) in (7) and involved building a set of unitarily related observables, which were precise implementations of Rovelli's partial observables idea, but of which physical interpretation was precise only asymptotically. Nonetheless the developed tools were sufficient to confirm the bounce picture and semiclassicality properties found for isotropic models also in setting. The results of this work were published in (4) and (7). Habilitant contributed to it with numerical aspects of the studies, developing the construction of the observables and performing part of the mathematical analysis (in particular implementing the group averaging procedure).

   The approach discussed above still suffered form incorrect infrared regulator removal limit when applied to flat Bianchi I, thus not reproducing the correct low energy classical theory there. The consistent description (reproducing the correct low energy limit, see [18]) was constructed [19] only, when the existing ambiguities were fixed by probing (back then just on the quasi heuristic level) the relation between the degrees of freedom of LQC with those of full LQG. However, the constructed model have proven to be mathematically very complicated and controlling its dynamical sector have eluded the researchers effort for almost a decade. The completion of the quantization program sufficient for the robust analysis of its dynamics has been achieved only recently (also by habilitant) [20] and required a qualitative improvement in several aspect of LQC methodology: both significant extension of the numerical methods and novel constructions of the physical Hilbert space and observables.

3. **Quantum properties of Universe in LQC (semiclassicality, coherence):**

   Appearance in dynamical predictions of LQC of a phenomenon of quantum bounce has caused at the beginning a bit of controversy among some more conservative researchers in the field. In particular, since the origin of the bounce was purely quantum, it was alleged, that due to its nature it causes a decoherence (in this context a loss of semiclassicality) of the once semiclassical universe, which thus may be semilcassical in one evolution epoch while losing this property in another [21]. Subsequent studies in turn have defended the semiclassicality preservation [22]. In order to settle the issue, an adaptation of the scattering picture was applied in LQC. The global evolution of an LQC universe (from distant past to distant future) has been cast as a transition (scattering) of an ever contracting geometrodynamical (known as Wheeler DeWitt – WDW)[3] universe into an ever expanding one. The precise analysis of relation between preferred (energy) bases in both LQC and WDW formulations allowed to identify a precise scattering matrix for the model of isotropic flat FRW universe with massless scalar field. Thanks to that, by combination of the numerical and analytical methods it was possible to identify strict triangle inequalities constraining the variances (uncertainties) of relevant physical observables of considered model in a distant past and future. This in turn allowed to unambiguously

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[3] A model of an isotropic universe of which degrees of freedom have been quantized using standard Schrödinger representation.

prove the semiclassicality preservation through the bounce. In this work, published in (10) the habilitant was responsible for the formulation of the scattering picture, all the numerical and significant portion of the analytical studies.

The topic of the coherence and semiclassicality preservation has been subsequently studied in more complicated models featuring quasi-cyclic evolution (infinite chain of bounces and either recollapses or transitions through asymptotic future/past infinity – the scri). The results of these studies for the particular model of flat FRW universe with (massless scalar field and) positive cosmological constant have been published in an invited contribution (18). These results cover two aspects of the topic:

- Long term decoherence and semiclassicality preservation:

  While the classically recollapsing models (like universe of spherical topology or with negative cosmological constant) or models admitting positive cosmological constant feature qualitatively cyclic behavior, the cycles of evolution in an infinite chain are not exactly the same. This behavior is captured in the model by slight deviation of the energy spectrum of the evolution operator[4] from uniformity. Combined numerical and analytical studies of this deviation allowed to establish the bounds on the growth of variances of observables used to describe the system along given number of the universe evolution cycles. It was shown that this increase, while present (nonvanishing) is so small, that for physically relevant range of parameters characterizing the universe it takes enormous number of cycles (more than 10^{60}) for it to significantly affect the semiclassicality.

  Some, much less precise estimates on the variance growth have been performed (mainly by numerical means) also for other models, in particular the spherical one (28) and the one admitting negative cosmological constant (29).

- Spontaneous coherence in quasi-cyclic LQC models:

  The increase of variances of observables between cycles of universe's evolution implies, that the universe semiclassical at given epoch (in given cycle) will loose the semiclassicality after sufficiently large number of cycles. In such situation one can ask an inverse question: given a generic quantum state of the universe, will it admit at some point in its evolution a semiclassical epoch? This question was again addressed on the example of a flat FRW universe with massless scalar field and positive cosmological constant. With use of certain elements of number theory it was possible to show, that, provided an otherwise generic state is at some time already sharply peaked about some scalar field momentum (which is the constant of motion for that system) it will always admit in its future an epoch when it remains semiclassical for many cycles. The results of this study were included in (18).

4. **Physical properties of inhomogeneous models in LQC:**

   While the isotropic and homogeneous anisotropic cosmological model can already capture some crucial properties of the observed universe evolution, making contact with observation requires inclusion of inhomogeneities be it on perturbative or nonperturbative level. In context of LQC the latter case was explored on the example of the so called Gowdy models [23] describing spacetimes of compact spatial slices and admitting two spacelike Killing fields (symmetries of space). For these models the so called hybrid quantization was introduced [24]. There, the inhomogeneities have been captured as Fourier modes of a certain scalar, subsequently quantized a la Fock, while for the remaining (homogeneous) degrees of freedom the loop quantization methods originally developed for Bianchi I (i.e. flat homogeneous anisoropic) universe (4), (7) were applied. The hybrid quantization program has been developed to the level, where it was possible to explicitly determine the action of the Hamiltonian constraint [25], although up to date no successful prediction regarding the dynamics on the genuine quantum level was made.

   An alternative approach to (loop) quantization of the inhomogeneous models on the nonperturbative level are the so called midisuperspaces, originally devised for the spherically symmetric spacetimes [26]. That technique relies on splitting of the space akin to Geroch reduction: the symmetric space is represented by the lower dimensional reduced one (where the subspaces closed with respect to action of symmetries are points) admitting additional fields capturing the degreed of freedom of the original space which are no longer present in the reduced geometry. That lower dimensional geometry is then quantized by methods of full LQG whereas for the objects originally intrinsic to the surfaces preserved by the symmetry (generalized Killing orbits) the techniques of LQC are applied. By combining this technique with appropriate modification

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[4] The dynamics of the system is captured by a certain operator playing the role of the square of the Hamiltonian. The spectrum of its square root can be interpreted as the energy spectrum of the system.

to the classical GR constraint algebra, making the Hamiltonian constraint ultralocal in the inhomogeneous direction and called abelianization. [27] it was possible to complete the quantization program for the spherically symmetric spacetimes and get some insights into structure of (spherically symmetric) black holes in loop quantization [28].

Habilitant's contribution to this field focused on Gowdy cosmological models and was twofold:

- Semiclassical analysis of the dynamics of Gowdy models in hybrid quantization:

  While even for the simplest models of hybrid-quantized Gowdy cosmologies the operator generating the evolution of the system was too complicated for the genuine quantum analysis of the dynamics, one could resort to the so called classical effective dynamics: a classical theory heuristically devised do mimic the genuine quantum evolution [29] and extensively tested in the sector of LQC where comparing it against the genuine quantum dynamics was available (see (1)-(3),(7), (14), (28) and (29)). This technique was applied to the toroidal vacuum Gowdy universe with the aim of testing, how the amplitudes of inhomogeneities change in the process of transiting through the bounce. The systematic numerical probing revealed, that, while below certain threshold, these amplitudes were amplified, while above it they evolved through the bounce statistically unchanged. In this research (published in (6), (11)) habilitant's role (besides formulation of the problem and selection of methodology) was mainly supervisory with significant involvement in part of the simulation data analysis.
- Physical sector of the abelianized midisuperspace model of Gowdy universe:

  The synthesis of the abelianization procedure and the midisuperspace approach to loop quantization of the symmetric spacetimes originally used in [27] was applied to the model of T^3 vacuum Gowdy universe with local rotational symmetry in order to test the methodology on relatively well known example. Here, it was possible to precisely identify the physical Hilbert space and sufficiently rich set of observables through group averaging. The subsequent analysis of the dynamics of physical states in the asymptotic future and past revealed a serious deficiency of the approach. Due to combining the abelianization with specific to LQG treatment of the diffeomorphism constraint (averaging of group of finite diffeomorphism instead of finding the kernel of the quantum counterpart of the diffeomorphism constraint) the model built in the framework features too many degrees of freedom, thus appropriate restriction of GR (to considered cosmological model) does not emerge uniquely as the low energy limit of the quantum model. In this work (published in (19)) habilitant participated mainly in applying the group averaging techniques as well as in the analysis of the asymptotic past/future behavior of the physical states.

5. **Development of the mathematical structure of the theory:**

   While the mathematical foundations for LQC were formulated back at the beginning of the century [6], the task of completing the quantization program and probing the dynamics on a genuine quantum level in a robust manner required their substantial extension. There habilitant either contributed significantly or led the research in several crucial aspects. The most relevant of these are:

   - Time reparametrization vs dynamical sector structure in LQC:

     For most types of nonexotic matter the models of homogeneous LQC lead to the unitary quantum evolution generated uniquely by the self-adjoint evolution operator (playing the role of the square of the Hamiltonian), however inclusion of either massive scalar field or positive cosmological constant presented a nontrivial challenge. While for the deparametrization using the scalar field (or the choice of the group averaging procedure analogous to it) the evolution operator involves multiple selfadjoint extensions (9), each generating nonequivalent unitary evolution, the natural choice of lapse function in canonical formalism (lapse $N = 1$) leads to a selfadjoint Hamiltonian constraint. Further studies of the dynamical sector generated by this constraint shows, that it features a unique unitary evolution. This apparent paradox has been investigated in the context of flat isotropic FRW universe with massless scalar field and positive cosmological constant. The comparison (published in (8))) of two scenarios listed above revealed, that the Hilbert space emerging via group averaging procedure in the lapse $N = 1$ case is an integral over a family of Hilbert spaces corresponding to the nonunique selfadjoint extensions of the evolution operator in the other scenario. These extensions introduce a natural fibration of the space corresponding to $N = 1$ case, although these fibers are not superselection sectors, as the standard LQC observables do mix them.

- Properties of observables in loop quantization:

  Precise and robust probing of the quantum dynamics in LQC models discussed in previous points required: (*i*) extending existing constructions of the physical observables and (*ii*) investigating the detailed properties of these (and other, more standard) observables. This involved in particular:

  - Construction of the unitarily related families of partial observables. which provided a precise notion of unitary evolution for vacuum Bianchi I model. This construction was based on one of the crucial properties of the physical Hilbert space: the whole physical state could be determined just by data on one slice (corresponding to a freely chosen fixed value of one of configuration variables). Combining it with the splitting of the energy eigenbasis later associated with in/outgoing geometrodynamical components in the scattering picture (10) allowed for building partial observables measuring values of configuration variables with respect to one of them (used as an evolution parameter). Due to use of the assymptotics of the Hamiltonian (constraint) eigenfunctions in identifying the correct physical interpretation of these observables, that interpretation was precise only in asymptotic future/past of the Universe's history.
  - Detalied analysis of spectral properties of the operators, in particular energy density. Once the consistent formulation of LQC quantization has been found (the so called improved dynamics) the probing of the dynamical trajectories have shown a crucial role of the matter (or balancing it gravitational) energy density as its specific critical value determined the point of the bounce. The detailed analytical and numerical studies of its spectral properties have revealed that its continuous spectrum is compact and bounded by zero and the identified earlier critical value (of the Planck order). While the discrete spectrum was nontrivial (depending on the particular details of the quantization prescription and factor ordering) it corresponded to the eigenvectors peaked about the point of classical singularity, thus not influencing the large semiclassical universe (5).

- Hilbert space and superselection sectors:

  In isotropic LQC the Hamiltonian constraint and consequently the evolution operator have a structure of a 2nd order difference operator (of regular step), thus naturally dividing both kinematical and physical Hilbert space (both nonseparable) onto separable superselection sectors consisting of those states which are supported on the lattices preserved by action of the mentioned operators. Since the relevant observables also preserve those sectors it is enough to work with just one of them instead of the whole nonseparable space. For the simplest models (isotropic FRW universe) it was checked explicitly that the physics emerging from the models has a minuscule dependence on the choice of the sector (1)-(3), (28), (29). The situation complicates in the anisotropic case as for example in Bianchi I flat model the support of a single sector is dense in some of the configuration variables. Also, in the cases, where the separation in the difference operator becomes a nontrivial function of the phase space variables (like for example in certain form of polymer quantization of the scalar field [30]) considering single superselection sectors does not reproduce correct physics (31).

  Following this observation and the nonseparability of the original spaces an alternative was developed in research led by habilitant. There, the structure of "superselection sectors" was still used, but they were considered just a fibration of a biger *integral* Hilbert space. That space occurred to be separable and the tests for the polymeric quantization of the scalar field in an isotropic cosmological model (31) and in the case of Bianchi I model [20] have proven to lead to dynamics reproducing the correct physics at low energies. The construction, applicable also in full LQC was discussed in detail in (16).

6. **Aspects of inflation in LQC:**

   While the presence of the bounce could solve certain problems of modern cosmology (like the horizon problem) on its own, it is a general belief in the community, that in order to reproduce the observational data it would need to be complemented by inflation. That sparked a research on inflationary scenarios in LQC, leading for example to estimates on the inflation probability [31, 32]. One of particular points of interests was probing the models featuring the nonminimally coupled scalar field, as at the time in standard cosmology this type of matter offered slightly better fit to the observational data. In this context, the flat FRW universe with nonminimally coupled scalar field admitting ϕ^4 type potential was studied. The dynamics of the system was probed via the effective classical dynamics (the classical model mimicking the quantum evolution but neglecting the second and higher order quantum corrections) in the so called Einstein frame, where the system has been conformally transformed to the one, where the field was coupled minimally. The transformed geometry has then been loop-quantized. The distinct feature of the resulting dynamics were the "mexican hat" trajectories featuring two bounces with a recollapse between them. The results of the studies were published in (15). Habilitant's contribution to the work was the numerical analysis and significant part of the data analysis as well as some analytical studies of the model.

7. **Numerical tools of LQC:**

   One of critical factors allowing for a progress in probing the dynamics of the cosmological scenarios in LQC were the numerical tools developed specifically for it. These tools at the beginning developed solely by habilitant, later with contribution of younger researchers (J. Olmedo, D. Martin De Blas, recently M. Kisielowski) has been steadily expanded since 2005 and at present consists of an objective oriented C++ library handling all the numerical aspects of studies listed in points above. It is mainly focused on performing the spectral analysis of the LQC operators, evolving the physical states in various models and evaluating the quantum trajectories, however it also handles the auxiliary tasks, like (for example) evaluating scattering matrix (10), probing the structure of selfadjoint extensions (9, 14) probing the dynamics of geometrodynamical systems or the classical effective approximations. Its core abilities were presented in part in (12). Together with the actual programs using it in solving specific numerical problems of LQC it exceeds 17K lines of code. It was used in almost all of works listed in point D.2 as well as in works by other authors (for example [33]). Till this day it is the most universal numerical tool in the field.

8. **Relation of LQC with full LQG:**

   The models of LQC have provided a series of interesting results, however part of their role was being a test bed for full LQG. In particular the results obtained in LQC could not be considered as the final answers of full theory. At most they provide a qualitative insights into what one can expect there. With the dynamical sector of the full theory still unavailable for physically interesting scenarios a lot of effort has been dedicated towards making a connection between LQC and LQG in a way allowing to extrapolate the results of the former. This involved investigating possible embeddings [34], role of symmetries in diffeomorphism invariant theories [35] and detailed studies of simplifications of LQG retaining its crucial properties [36, 37].

   Habilitant contribution to this field focused on probing the relation between the basic quantities representing degrees of freedom in LQC in context of full theory. The work, published in (17), was dedicated to identification of objects, which were well defined in both theories and using them to provide the dictionary between LQG and LQC. This dictionary was next used to determine the restrictions on the initial heuristic assumptions of LQC coming purely from consistency in relating the dictionary element. Furthermore the remnant of the diffeomorphism group in the isotropic LQC was investigated in detail in the context of relations discussed above, providing in particular a correction to critical energy density indicating a bounce.

E. Other scientific achievements.

The scientific works not classified as part of the achievement presented for habilitation are listed below. They belong to three groups: works within LQC not included in the habilitation achievement due to formal requirements, studies of dynamical sector of full loop quantum gravity and the works in the context of classical black hole theory (using the isolated horizon framework). The research within the last group was mainly contribution to the PhD thesis of habilitant.

1. **Publications:**

   (20) Lewandowski J, Pawłowski T, 2002, *Geometric Characterizations of the Kerr Isolated Horizon*, Int.J.Mod.Phys. **D11**, 739-746.

   (21) Lewandowski J, Pawłowski T, 2003, *Extremal Isolated Horizons: A Local Uniqueness Theorem*, Class.Quant.Grav. **20**, 587-606.

   (22) Pawłowski T, Lewandowski J, Jezierski J, 2004, *Spacetimes foliated by Killing horizons*, Class.Quant.Grav. **21**, 1237-1252.

   (23) Ashtekar A, Engle J, Pawłowski T, Van Den Broeck C, 2004, *Multipole Moments of Isolated Horizons*, Class.Quant.Grav. **21**, 2549-2570.

   (24) Lewandowski J, Pawłowski T, 2005, *Quasi-local rotating black holes in higher dimension: geometry*, Class.Quant.Grav. **22**, 1573-1598.

(25) Korzyński M. Lewandowski J. Pawłowski T. 2005. *Mechanics of multidimensional isolated horizons*. Class.Quant.Grav. **22**. 2001-2016.

(26) Lewandowski J. Pawłowski T. 2006. *Symmetric non-expanding horizons*. Class.Quant.Grav. **23**. 6031-6058.

(27) Ashtekar A. Pawłowski T. Van Den Broeck C. 2007. *Mechanics of higher-dimensional black holes in asymptotically anti-de Sitter space-times*. Class.Quant.Grav. **24**. 625-644.

(28) Ashtekar A. Pawłowski T. Singh P. Vandersloot K. 2007 *Loop quantum cosmology of K=1 FRW models* Phys.Rev. **D75**. 024035.

(29) Bentivegna E. Pawłowski T. 2008. *Anti-deSitter universe dynamics in LQC*. Phys.Rev. **D77**. 124025.

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(31) Kreienbuehl A. Pawłowski T. 2013. *Singularity resolution from polymer quantum matter*. Phys.Rev. **D88**. 043504.

(32) Lewandowski J. Pawłowski T. 2014. *Neighborhoods of isolated horizons and their stationarity*. Class.Quant.Grav. **31** 175012.

(33) Pawłowski T. Pierini R. Wilson-Ewing E. 2014. *Loop quantum cosmology of a radiation-dominated FLRW universe*. Phys.Rev **D90**. 123538.

2. **Dynamical sector of full LQG:**

The theory of General Relativity, in consequence its polymeric quantization – Loop Quantum Gravity is a theory with constraints. In particular it is time reparametrization invariant: the dynamics is generated by the Hamiltonian constraint instead of true Hamiltonian. In consequence, identifying its dynamical sector is a nontrivial (and difficult) task. The most promising program aimed towards achieving it is an adaptation of the Dirac program [1]: the theory is first quantized on the kinematical level (ignoring the constraints), then the physical sector is identified via group averaging as diffeomorphism invariant kernel of the operator representing the Hamiltonian constraint quantized in the kinematical level. Unfortunately, due to complicated nature this operator (complicated combinatorial operator acting on the domain in Hilbert space spanned by spin networks – graphs equipped with quantum labels) finding its kernel has not been possible so far. To circumvent this problem researchers focused on the deparametrization of the theory with respect to suitably chosen matter fields [38]. These fields then provided an operational clock, Hamiltonian constraint became an evolution equation and its gravitational part (possibly with some other matter components) became the true Hamiltonian. Initially two types of matter have been used for that purpose: dust [39, 40] and the massless scalar field [41]. Unfortunately, in both these cases the (now true) Hamiltonian took the form of a square root of a complicated combinatorial opeartor, thus its action could be written down only on a formal level. No actual calculations of its action on the state of initial data could be performed.

To overcome this problem an alternative deparametrization has been introduced in (30). There the synthesis of (*i*) deparametrization with respect to *irrotational dust* [39], (*ii*) preferred gauge choice distinguished by the dust field and (*iii*) diffeomorphism invariant formulation of LQG allowed to formulate the theory, where the physical Hilbert space is explicitly known, the evolution is generated by the analog of Schrödinger equation featuring time-independent true Hamiltonian, of which action on the physical states can be explicitly computed. Later, the condition (*ii*) was relaxed by recasting the deparametrization in diffeomorphism invariant mathematical framework [42]. These approaches brought the possibility of calculating the time evolution of the states at least in simplest settings to within technical reach. Although there exist new proposals involving the deparametrization with respect to the scalar field [43], till present the discussed technique remains the singular one allowing for any practical computations in LQG [44] on nonperturbative level.

3. **Quasi-local black hole theory:**

In the standard formulation of black hole theory [45] a Black Hole (BH) is defined as a complement of the domain of outer communication [46], thus to describe a BH one needs an information about the entire spacetime geometry. On the other hand, modern investigations in this area (for example determining the gravitational waves produced by a black hole merger) require a description, which allows to treat a BH as an object "in a lab" – without the need to include the information about distant objects into its description. One of attempts to construct such is a framework of non-expanding/isolated horizons (NEH/IH) [47, 48] and its generalization to fully dynamical situations known as dynamical horizons (DH) [49]. This framework is quasi-local: a BH in equilibrium is represented by its surface: a non-expanding horizon. The physical data is encoded in the horizon's intrinsic geometry featuring local free degrees of freedom. While in most situations one considers horizons embedded in a (usually 4-dimensional) spacetime, they can be treated as abstract standalone objects. In the case of horizons embedded in 4-dimansional spacetime their geometric structure [50] as well as mechanics [51] (including the generalization of the BH thermodynamics laws) has been investigated in detail.

Habilitant's contribution to this area can be summarized within the following set of tasks:

- Geometric characterization of the Kerr horizon:

  The standard black hole uniqueness theorems [52] rely on the global properties of a given spacetime containing a black hole. In isolated horizon formalism only the intrinsic geometry of the horizon is available for BH characteristics, thus identifying geometric/physical features distinguishing known BH solutions is a nontrivial task. A study being the core of habilitant's MSc thesis have shown, that for nonextremal horizon (that is that of nonvanishing surface gravity) it is enough to consider the restriction of the symmetry group of the horizon and the Petrov classification of the horizon as the surface embedded in 4-dimansional spacetime. As a result, a quasi-local version of a uniqueness theorem for Kerr horizon was formulated and proven (20): the only non-extremal horizons which are isolated to the 2nd order (that is admit null symmetry preserving metric up to 2nd order in derivatives with respect to direction transversal to the horizon), axially symmetric and of Petrov type D are the horizons of the geometry data corresponding to that of a Kerr BH horizon.

- Extremal black hole horizons:

  The extremal black holes are distinguished class as being of the "zero temperture" in context of the Hawking radiation. A NEH representing such BH horizon features a geometry which (unlike the non-extremal ones) is subject to strong constraints, of which structure suggests, that this class of the horizons may be characterized by a finite number of global quantities. The potential validity of this suspicion has been studied in context of (some classes of) the NEHs embedded in electro-vacuum spacetimes, with the following results:

  - Uniqueness theorem for extremal horizons:

    In geometric characterization of the extremal NEHs the specification of Petrov type can be dropped, allowing to formulate the quasi-local version of the uniqueness theorem for extremal Kerr-Newman horizon (21) as follows: the only axisymmetric and extremal horizon geometries in electrovac spacetime are those of the horizon of a Kerr-Newmann black hole spacetime.

  - Spacetimes foliated by NEHs:

    Investigating the class of the so called Kundt spacetimes (see for example [53]) it was possible to construct the class of spacetimes foliated by NEHs (22). Further studies of their geometry have shown, that the leaves of foliation (for this construction) are actually Killing horizons, further intersected by yet another Killing horizon, transversal to the leaves of foliation and admitting a geometry of the extremal one.

- Multipole moments of a NEH/DH geometry:

  In electrodynamics (in particular electrostatics), a useful tool in describing the electric field is the decomposition of either charge density distribution of the sources or the field far away from them onto a set of multipole moments. Due to linearity of the theory there is a 1-1 correspondence between these two decompositions. In GR, due to its nonlinearity no such correspondence can be established, however several independent constructions (for both types of multipoles) have been considered in context of black hole theory [54, 55]. The quasi-local formalism of NEH have given a chance to build an analog of source multipoles using NEHs intrinsic geometry. In (23) a precise technique for constructing such source multipoles was proposed and studied in detail in context of axisymmetric horizons. This construction is based on a decompositon of a specific complex scalar (in the so called Newman Penrose formalism denoted as $\Psi_2^{-1/3}$) composed of the Gauss curvature of spatial cuts of the horizon and the rotation form of the horizon in terms of spherical harmonics in specific coordinate system, distinguished in turn in a geometric way by the axial symmetry. In particular, for the Kerr horizon only two lowest moments are nonvanishing: the monopole encodes area whereas the dipole encodes the angular momentum of a black hole. The properties of this decomposition have proven to be useful for description of settling down of the numerically identified dynamical horizon to a Kerr isolated horizon in late stages of black hole mergers.

- BH horizons in higher dimension:

  The formalism of NEH/IH originally has been developed for the surfaces of dimension 3 embeddable in 4-dimensional spacetime. Due to heavy use of the Newman-Penrose formalism (specific to spacetimes of dimension 4), extending the formalism do a different dimension was highly nontrivial and required reformulation of basic description. While adapting the formalism in spacetime dimension 3 was not particularly difficult [56], not much has been done for higher dimension. On the other hand, the progress in string theory and the approaches related to it or inspired by it has sparked a lot of interest in formulating a robust description of black holes in spacetime dimension higher than 4. Such description has been provided in context of NEH/IH in (24) and (25), where the analogs/extensions of the results of [50] and [51] regarding, respectively, the geometry and mechanics of a NEH have been provided. In particular the zeroth and first law of black hole thermodynamics has been generalized to arbitrary dimension. These results provided a framework subsequently applied in the studies of black holes in asymptotically anti-de Sitter spacetimes of higher dimension (27).

- Symmetries of NEHs:

  In standard black hole theory the known black hole spacetimes are distinguished by their symmetries. Therefore it was also important to analyze the role of symmetries in quasi-local setting. In this framework the symmetry is a transformation generated by a vector field of which flow preserves the horizon's intrinsic metric and which commutes with intrinsic covariant derivative. The properties of these symmetries on the maximal analytic extensions of NEHs were studied in detail in arbitrary dimension in (26). The most relevant results obtained there are:

  - The analysis of the helical symmetry at the horizon of arbitrary dimension allowed to formulate and prove a quasi-local version of the Hawking rigidity theorem [57].
  - For spacetime dimension 4 the full classification of symmetric NEHs (their possible symmetry groups) has been provided.

- Spacetime neighbourhood of a NEH:

  Since the formalism of NEH uses for BH description the intrinsic geometry of the horizon (which in turn does not need to be embedded in a spacetime at all), its relation with the spacetime geometry at its neighbourhood is again nontrivial and requires careful studies. In particular the geometry of the horizon is insufficient to determine the metric at its spacetime neigbourhood. In order to uniquely determine the metric at portion of the neighbourhood one needs to supplement it by the data on the 2nd null surface transversal to the horizon [58]. The details of the relation of a NEH with its neighbourhood have been studied in (32). The basic tool for the analysis was the construction of a distinguished (invariant) coordinate system being an analog of a Bondi coordinate system at null infinity and defined in the spacetime neighborhood of a NEH of arbitrary dimension via geometry invariants of the horizon. With its use, in spacetime dimension 4 it was possible to define a radial expansion of a spacetime metric about the horizon and to identify the free data needed to specify it up to a given order. For the case of an electro-vacuum horizon in four-dimensional spacetime, it was further possible to determine the necessary and sufficient conditions for the existence of a Killing field at its neighborhood. These conditions took the form of differential conditions for the horizon data and data for the null surface transversal to the horizon.

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