Self-presentation

1. Name:

   Paweł Jakubczyk

2. Degrees:

   (a) **Master of Science** – Faculty of Physics, University of Warsaw, 2002, thesis: "Filling transition in a non-symmetric wedge", Advisor: prof. Marek Napiórkowski

   (b) **Doctor of Philosophy** – Faculty of Physics, University of Warsaw, 2005, thesis: "Line tension in adsorption at inhomogeneous substrates", Advisor: prof. Marek Napiórkowski

3. Employment in academic institutions:

   a) 2002-2005 **Ph.D. studies**, Faculty of Physics, University of Warsaw

   b) since 2006 **Assistant professor (adiunkt)**, Faculty of Physics, University of Warsaw

   c) 2007 - 2009 **postdoctoral researcher**, Max Planck Institute for Solid State Research, Stuttgart, prof. W. Metzner's group

4. Indication of achievement under Art. 16.2 of the Act of 14 March 2003 on Academic Degrees and Title and Degrees and Title in Art (Journal of Laws No. 65, item 595, as amended.): series of 10 publications

   a) title

   Functional renormalization group for thermal and quantum phase transitions

   b) Publications of the series:

   [H1] **P. Jakubczyk**, P. Strack, A.A. Katanin, W. Metzner, *"Renormalization group for phases with broken discrete symmetry near quantum critical points"*, Phys. Rev. B **77**, 195120 (2008).

   [H2] **P. Jakubczyk**, *"Renormalized ϕ^6 model for quantum phase transitions in systems of itinerant fermions"*, Phys. Rev. B **79**, 125115 (2009).

   [H3] P. Strack, **P. Jakubczyk**, *"Phase boundary and finite temperature crossovers of the quantum Ising model in two dimensions"*, Phys. Rev. B **80**, 085108 (2009).

   [H4] **P. Jakubczyk**, W. Metzner, H. Yamase, *"Turning a First Order Quantum Phase Transition Continuous by Fluctuations: General Flow Equations and Application to d-Wave Pomeranchuk Instability"*, Phys. Rev. Lett **103**, 220602 (2009).

   [H5] **P. Jakubczyk**, J. Bauer, W. Metzner, *"Finite temperature crossovers near quantum tricritical points in metals"*, Phys. Rev. B **82**, 045103 (2010).

[H6] H. Yamase, **P. Jakubczyk**, W. Metzner, *"Nematic quantum criticality without order"*, Phys. Rev. B **83**, 125121 (2011).

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[H8] **P. Jakubczyk**, *"Capillary-wave models and the effective-average-action scheme of functional renormalization group"*, Phys. Rev. E **84**, 021124 (2011).

[H9] **P. Jakubczyk**, *"Quantum interface unbinding transitions"*, Phys. Rev. B **86**, 075142 (2012).

[H10] **P. Jakubczyk**, M. Napiórkowski *"Critical Casimir forces for O(N) models from functional renormalization"*, Phys. Rev. B **87**, 165439 (2013).

c) Discussion of the scientific goal of the above works and achieved results together with discussion of their applications.

1 Introduction

To bridge the gap between a microscopic system defined by a Hamiltonian, and its equilibrium macroscopic properties (such as the phase diagram and the equations of state) poses a central task of equilibrium statistical physics. A framework to achieve this goal is provided by the Gibbs ensemble theory, within which thermodynamics is extracted by means of computing the partition function. This is a purely technical problem, though usually not a very easy one. Its difficulty stems from the fact that the calculation involves a large number of coupled degrees of freedom. Quite contrary, most approximate methods of theoretical physics work only for problems involving just one degree of freedom. These methods heavily rely on extensivity, and involve approximations allowing for treating the original system as a one-body problem with some effective field created by the remaining degrees of freedom. These methods, referred here as mean-field type, are often successful in delivering at least qualitatively correct description of the system in question (and often much more!) provided not too many of the original degrees of freedom reside in a spatial region of the size of the correlation length. Systems close to a second-order phase transitions are one example case which certainly does not fall into this category, since the correlation length at the critical point is infinite. The renormalization group (RG) approach to the equilibrium many-body problem [1, 2] aims at going around the difficulties encountered within mean-field (or any perturbation theory around a soluble model) by refraining from summing fluctuations corresponding to different energy scales in one step. Its basic philosophy is to organize this summation according to a decreasing scale. It consists in building an effective theory for a subset of degrees of freedom (the low-energy modes) by integrating out the others (high-energy modes). If the rapid-mode integration could be performed exactly, one might iterate this step to arrive at an exact solution to the original problem. This is however not the case in the most interesting situations, where a practical construction of the effective theory is approximate.

1.1 Non-perturbative renormalization group

In the most common formulation one constructs the RG transformation perturbatively and follows the flow of a finite number of couplings under the scale reduction. Within the framework of field theory, another possible approach is to consider an exact equation governing the flow of a generating functional for a family of correlation functions [3]. In any practical calculation one subsequently makes an ansatz on the allowed form of this flowing functional. This way of proceeding is usually referred to as "functional RG" (fRG), "non-perturbative RG" (NPRG), or "exact RG" (ERG). There are different implementations of the NPRG. The so-called Wilson-Polchinski

formulation [1, 3, 4] relies on an exact flow equation for the generator of connected correlation functions. This early variant of NPRG led to a number of important formal results [3]. However, practical implementations of this idea (even for simple systems) were developed only in the last 20 years within the framework of the (so called) one-particle irreducible (1PI) variant of NPRG [3, 5], also known as the effective action method. The present author's contribution takes this formulation as the starting point.

1.2 The 1PI scheme

In the present context of equilibrium statistical physics we are confronted with the task of computing the partition function

$$Z = \int D\chi e^{-S[\chi]} . \tag{1}$$

We have assumed that the partition function can be cast in the form of a functional integral. Standard procedures provide routes to formulate numerous problems in this way [6, 7]. Condensed-matter models are always valid up to a certain lengthscale (like lattice spacing). The theory we discuss below is therefore always supplemented with an upper cutoff Λ_{UV} in momentum space.

The fluctuating field $\chi(x)$ depends on the spatial coordinates in d dimensions and is here taken to be single-component and real for the sake of simplicity of the presentation. The following reasoning easily generalizes to $\chi(x)$ being multicomponent, complex, or Grassmann-valued. By adding the source term $\int d^d x \chi(x) J(x)$ and taking the logarithm, one obtains the generating functional for connected correlation functions

$$W[J] = \log \int D\chi e^{-S[\chi] + \int d^d x \chi(x) J(x)} . \tag{2}$$

We will consider the Legendre transform

$$\Gamma[\phi] = -W[J] + \int d^d x \phi(x) J(x) , \tag{3}$$

where

$$\phi(x) = \langle \chi(x) \rangle = \frac{\delta W}{\delta J(x)} . \tag{4}$$

The quantity $\Gamma[\phi]$ corresponds to the thermodynamic potential, whose natural variable is the average value of the fluctuating field χ (which in a particular setup may for example be magnetization). Following standard literature, we shall refer to $\Gamma[\phi]$ as free energy or the (full) effective action.

The scale-dependent effective action $\Gamma_\Lambda[\phi]$ is a generalization of this notion with the difference that one includes only fluctuations with momenta $q > \Lambda$ in the partition function summation. By lowering the momentum cutoff scale Λ from Λ_{UV}, successive modes are incorporated. By definition $\Gamma_{\Lambda=0} = \Gamma$. On the other hand, we associate $\Gamma_{\Lambda=\Lambda_{UV}}$ with $S[\phi]$ (the "microscopic" action).

An explicit construction can be carried out as follows [3]: We define

$$W_\Lambda[J] = \log \int D\chi e^{-S[\chi]-\Delta S_\Lambda[\chi]+\int d^d x J(x)\chi(x)} ,$$ (5)

where the infrared (IR) cutoff term $\Delta S_\Lambda[\chi]$ has been added, with

$$\Delta S_\Lambda[\chi] = \frac{1}{2}\int \frac{d^d q}{(2\pi)^d} R_\Lambda(q)\chi(q)\chi(-q) .$$ (6)

The cutoff function $R_\Lambda(q)$ is assumed to vanish for $\Lambda \to 0$ and diverge for $\Lambda \to \Lambda_{UV}$. We also impose that for $\frac{q}{\Lambda} \ll 1$ we have $R_\Lambda(q) \sim \Lambda^2$. With this choice adding the cutoff term $\Delta S_\Lambda[\chi]$ to the action in Eq. (5) results in giving an artificial mass $\sim \Lambda$ to all modes with $q < \Lambda$, while leaving unaffected the modes with $q > \Lambda$.

Subsequently one defines the scale-dependent effective action $\Gamma_\Lambda[\phi]$ via

$$\Gamma_\Lambda[\phi] = -W_\Lambda[J] + \int d^d x J(x)\phi(x) - \Delta S_\Lambda[\phi] .$$ (7)

The modification of the standard Legendre transform by subtracting the term $\Delta S_\Lambda[\phi]$ assures that [3]

$$\Gamma_{\Lambda_{UV}}[\phi] = S[\phi] .$$ (8)

One now investigates the variation of $\Gamma_\Lambda[\phi]$ upon an infinitesimal change of the scale Λ. Differentiation of Eq. (7) and algebraic manipulations lead to the following equation [5]:

$$\frac{\partial}{\partial \Lambda}\Gamma_\Lambda[\phi] = \frac{1}{2}\mathrm{Tr}\left\{ \frac{\partial R_\Lambda(q)}{\partial \Lambda}\left[\Gamma_\Lambda^{(2)}[\phi] + R_\Lambda(q) \right]^{-1} \right\} ,$$ (9)

where (in the present context) the trace sums over momenta:

$$\mathrm{Tr} = \int \frac{d^d q}{(2\pi)^d} ,$$ (10)

and the inverse propagator $\Gamma_\Lambda^{(2)}[\phi]$ is given by the second functional derivative of $\Gamma_\Lambda[\phi]$:

$$\Gamma_\Lambda^{(2)}[\phi](q_1, q_2) = \frac{\delta^2\Gamma_\Lambda[\phi]}{\delta\phi(-q_1)\delta\phi(q_2)} .$$ (11)

Equation (9) is recognized in literature as the Wetterich equation. It describes the flow of the effective action $\Gamma[\phi]$ upon reducing the IR cutoff scale Λ, and interpolates between the bare action (the "microscopic action") $S[\phi]$ at $\Lambda = \Lambda_{UV}$ and the full effective action (the free energy) $\Gamma[\phi]$ at $\Lambda = 0$. This equation is the starting point for all of the analysis of the sequence of publications that the present habilitation comprises. A few remarks are in place before we proceed.

- An exact solution to Eq. (9) implies expressing the free energy in terms of the parameters of $S[\phi]$ and therefore yields the partition function as given by Eq. (1). Reformulating the basic problem of statistical physics as a functional differential equation opens a way for very different approximation strategies as compared to those applied within the traditional formulation via a path integral.

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- Even though Eq. (9) was first presented in Ref. [5], a very similar formulation of the problem (up to a Legendre transform) dates back to the 1970s and is known as Wilson-Polchinski equation. It considers the flow of $W_\Lambda[J]$ rather than $\Gamma_\Lambda[\phi]$ and led to a number of formal results. It however completely failed in practical calculations whenever the anomalous scaling of the propagator had to be considered. The reasons for this may be thought of as technical.

- In an approximate solution to Eq. (9), the quantity $\Gamma_\Lambda[\phi]$ does not need to be parametrized by a finite number of couplings. Quite contrary, a standard approximation strategy (the derivative expansion) computes the flow of an infinite number of parameters. This makes the present approach by far more powerful as compared to the perturbative RG constructions. On the other hand, one may make contact with the perturbative RG structures by imposing an appropriate truncation of Eq. (9).

- The most delicate point of any practical calculation based on Eq. (9) is the parametrization of $\Gamma_\Lambda[\phi]$. Even though typically no small parameter can be devised to expand in, a number of systematic procedures have been developed (see below). There is also no easy way of calculating the errors arising due to truncation, but the accuracy may be estimated by gauging the sensitivity of the results to the choice of the cutoff function $R_\Lambda(q)$.

The results summarized in Section 2 were obtained relying on two approximation schemes briefly discusses below.

1.2.1 Derivative expansion

The truncation of Eq. (9) known as derivative expansion exploits the symmetry of the studied model and classifies the invariants by the number of derivatives. For the simple case of a real scalar field ϕ and Z_2 symmetry, it amounts to the following expansion

$$\Gamma_\Lambda[\phi] = \int d^d x \left[U_\Lambda(\rho) + \frac{1}{2} Z_\Lambda(\rho)(\nabla\phi)^2 + ... \right], \tag{12}$$

where $\rho = \frac{1}{2}\phi^2$, and the neglected terms involve invariants with more than two derivatives. At this approximation level, Eq. (9) casts onto a closed set of two nonlinear partial differential equations for the local effective potential $U_\Lambda(\rho)$ and the field-dependent Z-factor. These equations are susceptible to numerical analysis. We explicitly write down the flow equation for $U(\rho)$

$$\frac{\partial}{\partial\Lambda} U_\Lambda(\rho) = \frac{1}{2} \int \frac{d^d q}{(2\pi)^d} \frac{\partial R_\Lambda(q)}{\partial\Lambda} \left[Z_\Lambda(\rho)q^2 + R_\Lambda(q) + U'_\Lambda(\rho) + 2\rho U''_\Lambda(\rho) \right]^{-1}. \tag{13}$$

The lowest (zeroth) order in the derivative expansion neglects the flow of $Z(\rho)$ whatsoever; Eq. (13) is then closed.

A suitable transformation of variables brings Eq. (13) to a scale-invariant form, which allows for existence of fixed points. This way one makes contact with the standard formulations of Wilsonian RG, which involve momentum-shell integration followed by the variables' rescaling.

For the simplest case of the Ising universality class in $d = 3$, the derivative expansion has been pushed to the order ∂^6 and the critical indices obtained this way are in excellent agreement with Monte-Carlo simulations. It is also worth noting that the approach very accurately reproduces the exactly known values in $d = 2$, where the perturbative RG fails.

1.2.2 Vertex expansion

By taking subsequent derivatives of Eq. (9), one derives the flow equations for the 1-particle-irreducible vertex functions. For example, acting on Eq. (9) with the operator $\frac{\delta^2}{\delta\phi_{q_1}\delta\phi_{q_2}}$ leads to a flow equation for the inverse propagator $\Gamma^{(2)}$. This equation involves the vertices $\Gamma^{(3)}$ and $\Gamma^{(4)}$. In general, an exact flow equation for the n-th vertex function involves $\Gamma^{(n+1)}$ and $\Gamma^{(n+2)}$. This hierarchy of flow equations has a simple interpretation in terms o Feynman diagrams, where the terms contributing to the flow of $\Gamma^{(n)}$ involve (exclusively) 1-loop, 1-particle-irreducible diagrams with n external legs. One may now devise truncations of this hierarchy, where sufficiently high order vertices are neglected. This approach is complementary to the derivative expansion in that it allows for an accurate calculation of the momentum dependencies. These are crucial, for example, in treating the problem of competing instabilities in interacting Fermi systems.

The derivative expansion is a powerful computational tool. It however inevitably leads to the problem of numerical integration of non-linear partial differential equations. In many cases one may simplify the problem by expanding the effective potential $U_\Lambda(\phi)$ in powers of the field ϕ. Effectively, this combines derivative expansion with the vertex expansion and projects the flow of the potential onto flow of the masses and a finite number of interaction couplings. In the simplest case of the scalar ϕ^4 model, this leads to a coupled set of two ordinary differential equations for the mass and the interaction coupling. If necessary, one may also compute the flow of the Z-factor by another suitable projection. Note that the flow equations obtained this way still involve terms of arbitrary order in the mass and interaction parameters and may therefore be thought of as non-perturbative. This reduction of complexity simplifies numerical computations, but also leads to analytical insights. The price to pay is the reduced accuracy of the obtained numerical values. One should also keep in mind that the existance of a well-behaved field expansion of $U(\phi)$ is not always guaranteed, as discussed in Section 2.

2 Summary of key results

The present habilitation comprises 10 publications centered around applications of the functional RG (in the 1PI formulation) to systems involving a first- or second-order phase transition, amenable to a description in terms of a (bosonic) order-parameter action. The publications [H1-H3] and [H5-H7] refer to interacting Fermi systems displaying a quantum critical point. Publication [H4] addresses quantum

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criticality of a magnetic system of the quantum Ising model type. In the papers [H8-H9] we study interface unbinding transitions of classical [H8] and quantum [H9] nature. In [H10] we calculate Casimir forces for the $O(N)$ models with periodic boundary conditions, varying dimensionality between $d = 2$ and $d = 3$. The present summary is divided into three topical sections.

2.1 Quantum criticality in itinerant Fermi systems

Interacting Fermi systems can exhibit different types of symmetry-breaking leading to various types of ordering (for example magnetic, superconducting, nematic, or of the charge-density-wave type). The relevant order parameter is usually a composite bosonic quantity, bilinear in the original fermionic variables. Considering the ground-state (at temperature $T = 0$), the transition between different phases may be tuned by varying chemical composition, pressure, or other types of a non-thermal control parameter [8]. One should recognize at the outset that the fermionic excitations above the Fermi surface are massless at $T = 0$ and therefore the system hosts generically soft modes. If the transition at $T = 0$ is continuous, strong scattering between these fermionic modes and the soft order-parameter fluctuations gives rise to unusual (non Fermi liquid) behavior in the vicinity of the quantum critical point, in the so-called quantum-critical regime spanning into finite T above the quantum critical point. In the recent years quantum criticality has become one of the leading paradigms in the quantum many-body theory. In addition to providing a fascinating arena for theoretical and experimental investigations of exotic quantum matter, it is also of fundamental importance due to its relevance to high-temperature superconductors, where soft magnetic fluctuations play a crucial role in inducing effective attraction between electrons, leading to the formation of Cooper pairs.

The standard approach to quantum criticality in itinerant Fermi system is known as Hertz-Millis theory [9, 10]. Within this approach one introduces an order-parameter field ϕ via a Hubbard-Stratonovich transformation. This decouples the original degrees of freedom at the cost of introducing an extra functional integral in the partition function. The original fermionic degrees of freedom can then be integrated out. This leads to an exact representation of the partition function as a functional integral over the order-parameter field. In the subsequent approximation the order-parameter action is expanded (usually to quartic order) and only dominant momentum q an frequency ω dependencies in the propagator are kept. Such dependencies are neglected whatsoever in the interaction vertices. The effect of damping of the order-parameter fluctuations by the soft fermionic bath occurs via the dependence of the propagator on the Matsubara frequencies. The obtained action takes the form,

$$S[\phi] = \frac{1}{2} \int_q \phi_q \left(Z_\omega \frac{|\omega_n|}{|\mathbf{q}|^{z-2}} + Z\mathbf{q}^2 \right) \phi_{-q} + \mathcal{U}[\phi] \,, \qquad (14)$$

where ω_n are the (bosonic) Matsubara frequencies, $q = (\mathbf{q}, \omega_n)$, z is the dynamical

critical exponent, and $\int_q = T \sum_n \int \frac{d^d q}{(2\pi)^d}$. The quantity

$$\mathcal{U}[\phi] = \int_0^{1/T} d\tau \int d^d x \, U(\phi(x, \tau)) \tag{15}$$

(with τ denoting the imaginary time), and the bare effective potential is expanded as:

$$U(\phi) = a_2 \phi^2 + a_4 \phi^4 + \dots . \tag{16}$$

In the original paper by Hertz [9] the action defined by Eq. (14-16) was subject to a renormalization procedure. The analysis by Hertz may be viewed as largely incorrect at $T > 0$ and the work by Millis [10] delivers a correction. The motivation for our study contained in Ref. [H1] was related to a number of approximations made in the analysis by Millis. We provided a refinement to Ref. [10] in the respects listed below:

- The Millis procedure neglects the renormalization of the interaction coupling (except for terms coming from rescaling of the variables). It therefore does not capture the RG Wilson-Fisher fixed point, governing critical behavior at $T > 0$. In diagrammatic language, it only takes account of the mass renormalization via the tadpole contribution.
- The Millis analysis is restricted to the disordered phase, and the shape of the phase boundary at $T > 0$ is in fact identified with the Ginzburg line (the boundary of the critical region).
- The Millis analysis neglects the anomalous dimension, which may in principle give a correction to the universal power laws derived in the vicinity of the quantum critical point.

Our study performed in Ref. [H1] is an improvement of the Millis theory in all these respects. As compared to Ref. [10], it departs from a completely different RG formulation, taking the Wetterich equation (9) as a starting point. It relies on the vertex expansion built on top of the derivative expansion, and treats quantum and classical fluctuations on equal footing. It also takes into account the flow of the Z-factor and the accompanying anomalous dimension η. The approach is valid also in the phase with broken (discreet) symmetry. The derived RG flow equations are solved numerically and this allows for an accurate computation of the transition line (at $T \geq 0$) of the model defined by the action (14) and comparison to the Ginzburg line. It also provided a flexible framework for the extensions and generalizations contained in the following sequence of papers.

In the publication [H2] the framework of Ref. [H1] was extended by including a term of the order ϕ^6. We then investigated the possibility of changing the order of a quantum phase transition by order-parameter fluctuations. Indeed, as turns out, the renormalization of the quartic coupling a_4 due to non-zero hexatic coupling a_6 creates (in particular for two-dimensional systems) a sizable tendency of increasing the value of a_4 and may lead to changing its sign from negative to positive (but never the other way round). This suggests that quantum phase transitions, which are first order at mean field level, may be of second order in a more complete theory taking the fluctuations into account. We also addressed the fine-tuned situation,

where a line of thermal phase transitions of second order terminates at $T = 0$ with a quantum tricritical point; or the system is in a state close to such a scenario and both quantum critical and quantum tricritical scaling are observed. There are indications that a number of realistic compounds is indeed not far (in the parameter space) to this scenario [11]. Relying on scaling theory, we finally provided a classification of all the possible values of the shift exponent for quantum multicriticality.

The publication [H3] is a detour in the series devoted to itinerant Fermi systems. It applies the framework of Ref. [H1] to the effective theory describing the low energy physics of the two-dimensional quantum Ising model in a transverse magnetic field and in the vicinity of the quantum critical point. It demonstrates the power and convenience of the approach developed in Ref. [H1] in a case, where the low-energy physics is governed by non-Gaussian fixed points both at $T = 0$ and at $T > 0$. The system shows scaling behavior characteristic to the 2-dimensional, 3-dimensional, and 5-dimensional (mean field) classical Ising model, and the crossover between the different scaling regimes may be obtained either by varying the thermodynamic parameters (temperature and the transverse magnetic field), or as a function of the cutoff scale Λ. These crossovers might be much harder to obtain within the more conventional perturbative RG constructions.

The publications [H1-H3] take a ϕ^4 (or ϕ^6) -type action as the starting point for the RG analysis. In Ref. [H4] we address a specific microscopic Hamiltonian (the so-called f-model), displaying an electronic-nematic phase, where the symmetry of the two-dimensional Fermi surface becomes reduced with respect to the symmetry of the lattice. Transitions of this type are established in the compound $Sr_3Ru_2O_7$ [12], but are also discussed for example in the context of superconducting cuprates [13]. As turns out, the procedure of deriving the Hertz action leads in this system to the form given in Eq. (14) with $z = 3$ and yields a robustly first-order transition to the nematic phase at sufficiently low T. However, contrary to the assumptions underlying the Hertz-Millis type approach, the effective potential cannot be expanded in powers of the order-parameter field. Namely, an expansion of $U(\phi)$ around one of the minima is not able to capture the other ones. For example, when the expansion around $\phi = 0$ is enforced, one obtains all the interaction couplings $a_4, a_6, \ldots$ negative. On the other hand, the potential $U(\phi)$ behaves as ϕ^2 at large ϕ. These facts invalidate the vertex expansion and enforce keeping the full functional dependence $U(\phi)$ in the RG study. The 1PI scheme of fRG is a tool perfectly suited to address this problem. The performed computation employed the derivative expansion adapted to the form of Eq. (14). We took the flow of the Z-factor into account in a way analogous to the procedure of Ref. [H1]. As a conclusion we obtained that for a range of parameters, the transition is driven second-order by including the order-parameter fluctuations.

The publication [H5] is an extension of Ref. [H2]. Upon linearization of the flow equations around the Gaussian fixed point at $T = 0$ we were able to solve the flow equations analytically. We studied crossover behavior between quantum critical and tricritical behavior with main focus on the quantum critical region in dimensionality $d > 2$. The work provided a more complete analytical understanding of the results of Ref. [H2], in particular of the conclusion that order-parameter fluctuations can

turn first-order phase transitions continuous, but not vice-versa.

In Ref. [H6] we revisited the model studied in [H4] and investigated the possibility of completely annihilating the order occurring within mean-field description by order-parameter fluctuations. Such phenomena are well-known to occur in two-dimensional systems exhibiting continuous symmetry-breaking (for example the Heisenberg model), by the Mermin-Wagner theorem [14] but the system under interest here displays only breaking of a discrete symmetry. We have established that such a possibility indeed occurs for a certain range of microscopic parameters. Interestingly, scaling typical to quantum criticality may still be observed at sufficiently high temperatures even if there is no ordering at $T = 0$. In addition, we studied the (fine-tuned) scenario, where there is no order at $T \geq 0$, but the system hosts a quantum critical point. In standard Landau theory language this corresponds to choosing the parameters so that the ϕ^2 coefficient is not proportional to $\delta - \delta_c$ (δ being the control parameter, and δ_c its critical value), but to $(\delta - \delta_c)^2$.

Ref. [H7] is another refinement of the Hertz-Millis theory. We focused on the case of two-dimensional systems with the aim of solving the flow equations analytically and comparing the Ginzburg $T_G(\delta)$ and transition $T_c(\delta)$ lines. As a conclusion we obtained that the region with strong both thermal and quantum fluctuations occupies a sizable region of the phase diagram and approximating $T_c(\delta)$ with $T_G(\delta)$ is not always justified. This fact was already recognized in Ref. [H1], but the contribution [H7] delivered a more complete analytical understanding of this fact.

2.2 Interface unbinding transitions

The field of interfacial phase transitions is a well-established branch of classical condensed-matter physics. The RG theory of these phenomena was developed years ago, and it is well-recognized that the RG phenomenology is quite distinct from that relevant to bulk criticality [15]. In particular, there is no possibility of treating the flowing local interfacial interaction potential by a parametrization with just the mass and the quartic coupling (or any other finite set of flowing couplings). In the cases, where mean-field theory is not a sufficient description, one has to resort to a functional RG treatment. On the other hand, the problems encountered by the early functional RG formulations are irrelevant here, since the anomalous dimension is identically zero. The present habilitation makes two contributions to the theory of interfacial phase transitions (Refs. [H8] and [H9]), approaching these phenomena from the point of view of the 1PI framework.

In Ref. [H8] we revisited the functional renormalization group theory of classical interface unbinding transitions [16], and showed how it can be recovered applying the leading-order derivative expansion. The virtue of the proposed approach is the clarity of all the approximations (contrasting it to the previous contributions). Subsequently, we investigated the sensitivity of the results to the cutoff scheme (the choice of the cutoff function $R_\Lambda(q)$). We showed that the key parameter ω of a standard approximate linearized treatment of wetting transitions (the linearized functional RG [17]) is not a strictly well-defined quantity for dimensionalitites $d < 3$, meaning that

it does depend on the choice of $R_\Lambda(q)$. Importantly, this dependence ceased at the physical dimension $d = 3$, which also happens to be the upper bound on the allowed values of the upper critical dimension. The study showed that in the physical case $d = 3$ the capillary parameter ω is an absolutely robustly defined quantity (unlike the case of $d < 3$).

In Ref. [H9] we considered interfacial phenomena accompanying bulk quantum phase transitions in the presence of surface fields. By extending a general argument due to Cahn [18], we argued that interface unbinding transitions generically accompany quantum critical phenomena. Importantly, the boundary transitions may occur (and be of second order) also if the bulk transitions are discontinuous. This part of the analysis is valid in spatial dimensionality $d \geq 2$. We subsequently generalized the linear RG theory of classical interface unbinding transitions, accounting for quantum effects, extracted the relevant scaling regimes in the interfacial phase diagram, and computed the critical singularities. The latter part of the analysis applies for $d = 3$, and sufficiently short-ranged intermolecular interactions. From the theoretical point of view this is one of the most interesting cases, where exotic nonuniversal critical singularities occur, and where the interfacial correlation length may exhibit either power-law divergence or an essential singularity depending on the value of the capillary parameter ω.

2.3 Casimir forces

The last contribution to the habilitation (Ref. [H10]) contains a calculation of critical Casimir amplitudes for the $O(N)$ models with periodic boundary conditions, focusing on the cases of the number of order-parameter components $N = 1, 2$ and continuously varying spatial dimensionality d between 2 and 3. According to the author's knowledge, this is the first application of functional renormalization group methodology to this problem. The proposed fRG truncation is an adaptation of the 1st order derivative expansion to the case of a system finite in one direction. For the Ising universality class in $d = 2$, the method accurately reproduced the exact results. In $d = 3$ one may compare the obtained numbers with Monte-Carlo simulations as well as earlier perturbative RG calculations. It turns out that (both for $N = 1$ and $N = 2$) the obtained Casimir amplitudes come out almost halfway between the best estimates of perturbative RG and the Monte Carlo results.

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5. Discussion of the other scientific achievements

In course of my master studies, I analyzed the filling transition in a non-symmetric wedge [P. Jakubczyk and M. Napiórkowski, Phys. Rev. E **66**, 041107 (2002)].

The three papers published in the years 2004-2005 were the core of my Ph.D. thesis. These are: P. Jakubczyk and M. Napiórkowski, Physica A **334**, 173 (2004); P. Jakubczyk and M. Napiórkowski, J. Phys.: Cond. Matter **16**, 6917 (2004); and P. Jakubczyk and M. Napiórkowski, Phys. Rev. E **72**, 011603 (2005). They were all concerned with the influence of chemical impurities and substrate curvature on adsorption phenomena. The presence of impurities breaks translational invariance along the substrate and gives rise to line tension (and/or point tension) whose properties were the main topic of the Ph.D. thesis of mine. The substrate curvature modifies the structure of the capillary-wave Hamiltonian and may be considered as an additional (relevant) scaling field for interfacial phase transitions. Parts of the analysis remained in the framework of Landau theory, the other parts employed effective capillary-wave type models solved in mean-field approximation.

After my Ph.D. we extended the analysis of the impurity impact on adsorption focusing on the case of two-dimensional systems. These turn out to be susceptible to exact analytical treatment via transfer-matrix methods. One important result is that (in $d = 2$), for certain parameter ranges, the point tension is not a well-defined quantity, meaning that (due to thermal fluctuations) it does not converge to a finite value in the thermodynamic limit where both the system and the substrate impurity become macroscopically large; but instead shows a logarithmic divergence. Another interesting result was a calculation of the scaling shapes of the adsorbed layer, which turn out to exhibit highly universal properties. The analysis is published in the two contributions: P. Jakubczyk, M. Napiórkowski, and A. O. Parry, Phys. Rev. E **74**, 031608 (2006) and P. Jakubczyk and M. Napiórkowski, J. Phys. A: Math. Theor. **40**, 2263 (2007).

In the publication H. Yamase and P. Jakubczyk, Phys. Rev. B **82**, 155119 (2010) we analyzed a specific system featuring a transition to a state with spontaneously broken symmetry of the Fermi surface. Of our particular focus was its response to the non-ordering fields (more specifically - magnetic fields), i.e. fields which do not couple directly to the order parameter. Within the analyzed model, we have found a jump of the longitudinal susceptibility at the second-order transition to the state with reduced Fermi surface symmetry. The magnitude of this jump diverges at the tricritical point. We discussed the implications of these results to a specific compound hosting an electronic nematic phase.

In the last two papers published in 2013 [M. Napiórkowski, P. Jakubczyk, and K. Nowak, J. Stat. Mech. 06015 (2013); and P. Jakubczyk and M. Napiórkowski J. Stat. Mech. 10019 (2013)] we analyze the properties of the d-dimensional imperfect Bose gas, i.e. a gas of interacting bosons in the Kac scaling limit. The analysis can be performed exactly. By means of calculating the critical exponents at Bose-Einstein condensation, we have identified the universality class of the system to be the same as for the classical spherical model. We also analyzed the scaling function for the Casimir forces varying dimensionality. The scaling function takes a trivial (constant) shape above $d = 4$. We

also analyzed the phase diagram in the vicinity of the quantum critical point, and found the scaling regimes in full agreement with renormalization-group predictions in $d > 2$. For $d = 2$ we have found that the correlation length displays an essential singularity in the limit $T \to 0$ at sufficiently large chemical potential.

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