

# Załącznik 3. Autoreferat (English)

## 1 First and Last name.

Rafał Demkowicz-Dobrzański

## 2 Scientific degrees.

- 2007. PhD degree in Physics (honours degree), University of Warsaw  
PhD thesis: *The role of correlations in quantum state estimation and the possibility of decorrelating quantum states*  
Advisor: prof. dr hab. Marek Kuś
- 2003. Master degree in Theoretical Physics, Quantum Optics (honours degree), University of Warsaw  
Thesis: *Cloning of quantum states*  
Advisor: prof. dr hab. Krzysztof Wódkiewicz

## 3 Employment in academic institutions.

- 2011-2013.  
Assistant professor, Institute for Theoretical Physics, Faculty of Physics, University of Warsaw  
Postdoc under the TEAM project financed by the Foundation for Polish Science
- 2010.  
Scientific and technical specialist, Faculty of Physics, University of Warsaw  
Postdoc under the TEAM project financed by the Foundation for Polish Science
- 2007-2009.  
Assistant professor at the Institute of Physics, Nicolaus Copernicus University, Toruń
- 2003-2007.  
Assistant at the Center for Theoretical Physics, Polish Academy of Sciences, Warsaw

## 4 Scientific accomplishment.

### a) Title of the scientific accomplishment

publication cycle: *Fundamental precision bounds in quantum metrology*

**b) (Authors, publication titles, publication year, journal name),**

- [1] R. Demkowicz-Dobrzański, J. Kołodyński, M. Guta, *The elusive Heisenberg limit in quantum enhanced metrology*, Nat. Commun. **3**, 1063 (2012)
- [2] M. Jarzyna, R. Demkowicz-Dobrzański, *Quantum interferometry with and without an external phase reference*, Phys. Rev. A **85**, 011801(R) (2012)
- [3] R. Demkowicz-Dobrzański, *Optimal phase estimation with arbitrary a priori knowledge*, Phys. Rev. A **83**, 061802(R) (2011)
- [4] J. Kołodyński, R. Demkowicz-Dobrzański, *Phase estimation without a priori knowledge in the presence of loss*, Phys. Rev. A **82**, 053804 (2010)
- [5] M. Kacprowicz, R. Demkowicz-Dobrzański, W. Wasilewski, K. Banaszek, I. A. Walmsley, *Experimental quantum-enhanced estimation of a lossy phase shift*, Nature Photon. **4**, 357 (2010)
- [6] R. Demkowicz-Dobrzanski, *Multi-pass classical vs. quantum strategies in lossy phase estimation*, Laser Phys. **20**, 1197 (2010)
- [7] K. Banaszek, R. Demkowicz-Dobrzanski, I. A. Walmsley, *Quantum states made to measure*, Nature Photon. **3**, 673 (2009)
- [8] R. Demkowicz-Dobrzanski, U. Dorner, B. J. Smith, J. S. Lundeen, W. Wasilewski, K. Banaszek, I. A. Walmsley, *Quantum phase estimation with lossy interferometers*, Phys. Rev. A **80**, 013825 (2009)
- [9] U. Dorner, R. Demkowicz-Dobrzanski, B. J. Smith, J. S. Lundeen, W. Wasilewski, K. Banaszek, I. A. Walmsley, *Optimal Quantum Phase Estimation*, Phys. Rev. Lett. **102**, 040403 (2009)
- [10] P. Kolenderski, R. Demkowicz-Dobrzanski, *Optimal state for keeping reference frames aligned and the Platonic solids*, Phys. Rev. A **78**, 052333 (2008)

**c) Description of the scientific goal of the above mentioned works, obtained results and prospects of applications.**

In what follows, references [1–10] refer to the cycle of publications being the basis of the present habilitation application, references [11–26] cover the remaining papers of the applicant not belonging to the cycle, whereas all the remaining references are included in order to provide a proper background for the research described. Sections 4.1, 4.2 have introductory character and are the starting point for the further discussion of the original research results. Original results are summarized in section 4.3 and are described in more detail in the subsequent sections 4.4–4.8.

## 4.1 Introduction and motivation of the research

Precision with which we are able to measure basic physical quantities such as time, mass, length etc., determines the range of physical phenomena which may in principle be put under scientific investigation. It also defines our abilities to test basic physical theories and may be critical in realizing ambitious technological projects. Among some of the most spectacular examples are spectroscopic measurements enabling tests of quantum electrodynamics predictions up to 12 significant digits, gravitational wave detectors capable of detecting gravitational waves with amplitudes below the size of the proton or the GPS system which operation is crucially dependent on precise atomic clocks that require calibration of only 10 ns per day.

All measurements involve a certain degree of uncertainty, which may be due to the dynamics of the measured system, noise resulting from the interaction with the environment or imperfection of the measuring devices themselves. In the classical physics framework, none of these contributions to the final uncertainty has a fundamental character, and could in principle be decreased to an arbitrary low level. This is, however, an illusion that arise when neglecting the fundamental quantum nature of physical systems. In the most precise measurements, when „classical” sources of noise (technical noise etc.) are negligible, quantum effects become the dominant contribution to the overall measurement uncertainty. In optical measurements, the quantized nature of light leads to the so called shot noise in intensity measurements, which is ultimately linked with the probabilistic behavior of a single photon in the detection process. Similarly, in atomic clock calibration, discrete structure of energy levels manifest itself in probabilistic nature of the detection process—projection noise—when the number of excited atoms is to be determined in order to obtain a feedback information necessary for clock calibration.

It is possible to reduce the impact of the shot or projection noise by increasing the light power, or the number of atoms involved. This can be intuitively understood, taking a simplified (though usually adequate) approach were in typical experiments photons or atoms evolve independently of each other. Consequently, an experiment involving  $n$  photons or atoms may be regarded as  $n$  independent single particle experiments. In this case, according to basic laws of statistics, parameter estimation uncertainty will decrease in general as  $1/\sqrt{n}$ , analogously as when estimating the mean of a random variable based on its  $n$  independent realizations. Precision bound of the  $1/\sqrt{n}$  form is usually referred in the literature as the Standard Quantum Limit (SQL).

The SQL limit, however, does not have a fundamental character, since it is a consequence of the assumption of independent behavior of quantum systems (atoms, photons) in the measurement process. Under the same resource constraints (the same  $n$ ), it is in principle possible to obtain better precision when preparing quantum systems in an appropriately correlated (entangled) state. In this more general approach it is in theory possible to reach the precision scaling of  $1/n$ , which is referred to as the Heisenberg limit (HL). Taking into account large numbers  $n$  in typical metrological setups, employing entangled states could lead to a qualitative (quadratic) improvement in precision, and result in an unimaginable progress in the whole field of metrology [27].

For practical implementation of these ideas, it is, however, necessary to consider more realistic models that take into account omnipresent effects of quantum decoherence, such

as e.g. photon loss, spontaneous emission or fluctuations of external electromagnetic fields. Entangled states that are optimal when analyzing idealized models are typically extremely fragile under even very modest levels of decoherence. Because of that, it is crucial for the whole field of quantum metrology to derive fundamental bounds for precision in entangled based protocols in realistic models that take into account decoherence. In particular, one of the most important questions is whether the quadratic gain achievable in idealized scenarios thanks to the use of entangled states is present also in models taking into account decoherence.

The cycle of publications which is the core of this habilitation application provides detailed answers to the above questions. It has been demonstrated that even an infinitesimally small level of decoherence makes it impossible to achieve the quadratic gain in precision in the asymptotic limit (large  $n$ ) and the use of entangled states allows asymptotically only for a constant factor improvement in precision. Moreover, mathematical tools have been developed that enable effective calculation of this factor and as a result obtain quantitative bounds on maximal precision enhancement achievable with the help of entangled states. Knowledge of such bounds is crucial for a fair assessment of the usefulness of entangled states in particular implementations of metrological protocols and shed light on the prospects of the whole field of quantum metrology. It also enables one to judge how closely the present day implementation of quantum metrological protocols approach the fundamental limits, and what is the space for further improvement possible thanks to the use of e.g. more sophisticated entangled states.

## 4.2 Idealized quantum interferometry

One of the basic measurement techniques in metrology is interferometry. Optical interferometers such as: Michelson interferometer, Mach-Zehnder interferometer, Fabry-Perot interferometer and others, allow for an extremely precise measurements of distances between optical elements that constitute the interferometers, and are commonly employed in engineering, basic science as well as for the calibration of other measurement devices. The essence of the quantum enhancement problem in interferometry is easiest to analyze with an example of the Mach-Zehnder interferometer.

In the standard configuration, one input port is fed with a „classical” state of light, mathematically described as a coherent state with amplitude  $\alpha$ . After propagating through the interferometer: splitting on a balanced beam-splitter, propagation through the interferometer arms with relative phase delay  $\varphi$  and finally interfering on the second balanced beam splitter, the amplitudes at the two output ports read respectively:  $\alpha_1 = \alpha \sin(\varphi/2)$ ,  $\alpha_2 = \alpha \cos(\varphi/2)$  (Figure 1a). Average number of photons registered at the output detectors read respectively  $\bar{n}_1 = |\alpha_1|^2 = \bar{n} \sin^2(\varphi/2)$ ,  $\bar{n}_2 = |\alpha_2|^2 = \bar{n} \cos^2(\varphi/2)$ , where  $\bar{n} = |\alpha|^2$  is the average input number of photons. In order to estimate phase  $\varphi$  based on the photon numbers registered by the two detectors  $n_1, n_2$ , one needs to specify a function  $\tilde{\varphi}(n_1, n_2)$  called the estimator. A natural choice for an estimator (which is apparently optimal in this case) is  $\tilde{\varphi}(n_1, n_2) = \arccos\left(\frac{n_2 - n_1}{n_2 + n_1}\right)$ . Due to Poissonian fluctuations of the number of photons registered, characteristic for classical sources of light, the value of the estimated phase will also fluctuate from shot to shot. This results in the already mentioned phase estimation

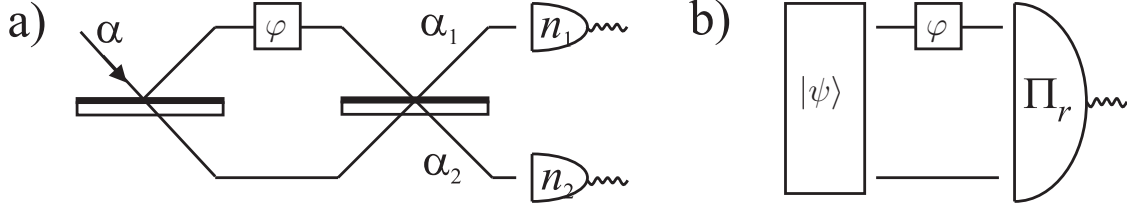


Figure 1: a) Standard configuration of the Mach-Zehnder interferometer with a coherent state sent into one of the input ports. b) General scheme of phase delay estimation with arbitrary input state  $|\psi\rangle$  and with the most general quantum measurement  $\Pi_r$ .

uncertainty

$$\Delta\varphi = 1/\sqrt{\bar{n}} \quad (1)$$

referred to as the shot noise or the SQL.

When thinking of employing non-classical states of light in order to improve the precision one might hit upon the idea to replace the input coherent state with a state of light with definite photon number  $n$  (a Fock states). In this case the joined probability distribution  $p(n_1, n_2)$  for the number of photons detected will become a binomial rather than a Poissonian distribution. Nevertheless, in this case the behavior of estimation precision as function of number of photons used is exactly the same as for the coherent states,  $\Delta\varphi = 1/\sqrt{\bar{n}}$ , which simply stems from the fact that effectively one can still regard the process in a way that each photon “interferes only with itself” and we basically deal with  $n$  independent single photon experiments. Only the use of more general input states of light, e.g. photons sent into both input ports simultaneously, may offer a chance of exploiting quantum correlations (entanglement) in order to enhance the precision.

Looking for the optimal estimation strategy, it is convenient to consider a setup, in which a general  $n$  photon two-mode state of light

$$|\psi\rangle = \sum_{k=0}^n c_k |k, n-k\rangle, \quad (2)$$

senses phase delay  $\varphi$ , and is subsequently measured using a general quantum measurement  $\Pi_r$ , yielding measurement result  $r$  (Figure 1b)<sup>1</sup>. Probability of getting a result  $r$  depends on  $\varphi$  and is given by  $p(r|\varphi) = \text{Tr}(|\psi_\varphi\rangle\langle\psi_\varphi|\Pi_r)$ , where  $|\psi_\varphi\rangle = \sum_{k=0}^n c_k e^{ik\varphi} |k, n-k\rangle$  is the final state of light after experiencing the phase shift  $\varphi$ .

Finding the optimal states of light, i.e. optimal amplitudes  $c_k$ , optimal measurements and estimators, which allow for the maximal quantum enhancement as compared with the SQL, is one of the fundamental problems of theoretical quantum metrology. Solution to this problem provides an information on maximally precision enhancement allowed within the quantum mechanics framework, at therefore leads to the fundamental quantum precision bound.

<sup>1</sup>Mathematically, a general quantum measurements is described with a set of positive semi-definite operators  $\Pi_r \geq 0$ , such that  $\sum_r \Pi_r = \mathbb{1}$

Useful bounds may be derived using the Craméra-Rao inequality[28, 29], which states that irrespectively of the choice of a measurement and an estimator (assuming it is unbiased), having at ones disposal a  $\varphi$  dependent quantum state  $\rho_\varphi$  (we assume the most general case of a mixed state), it is not possible to obtain better estimation precision than:

$$\Delta\varphi \geq \frac{1}{\sqrt{F(\rho_\varphi)}}, \quad (3)$$

where  $F(\rho_\varphi)$  is the so called Quantum Fisher Information (QFI). In the case of ideal interferometer, considered above,  $\rho_\varphi = |\psi_\varphi\rangle\langle\psi_\varphi|$  is a pure state which makes the explicit formula for the QFI relatively simple:

$$F(|\psi_\varphi\rangle) = 4 \left( \langle \dot{\psi}_\varphi | \dot{\psi}_\varphi \rangle - |\langle \dot{\psi}_\varphi | \psi_\varphi \rangle|^2 \right), \quad (4)$$

where  $|\dot{\psi}_\varphi\rangle = \frac{d|\psi_\varphi\rangle}{d\varphi}$  and leads to an equality  $F(|\psi_\varphi\rangle) = 4\Delta^2\hat{n}_1$ , where  $\Delta^2\hat{n}_1$  is the variance of the number of photons traveling in the upper arm of the interferometer. In the above approach, the quest for finding the optimal state amounts to finding a state that maximizes the above variance. Such states are called the n00n [30] states, because of their simple structure:

$$|\psi_\varphi\rangle = \frac{1}{\sqrt{2}} (|n, 0\rangle + |0, n\rangle), \quad (5)$$

and the corresponding QFI,  $F(|\psi_\varphi\rangle) = n^2$ , leads to the Craméra-Rao inequality:

$$\Delta\varphi \geq 1/n, \quad (6)$$

which at least in principle gives hope for reaching the Heisenberg limit (HL).

### 4.3 Summary of the key results

From the point of view of practical interferometry, the above considerations are not satisfactory for many reasons. The cycle of publication, which is the basis of the habilitation application, is a reaction to these oversimplified considerations.

The basic drawback of the analysis presented in Sec.4.2 is the fact that effects of decoherence, present in all experimental implementations, are not taken into account. In the case of optical implementations, the main factor limiting the potential of non-classical states of light, such as e.g. the n00n states, is the photon loss. This problem has been analyzed systematically for the first time employing the concept of the QFI in [8, 9]. Estimation strategies based on multiple-passes of light through the phase delaying sample have been analyzed in [6], and their efficiency has been compared with the optimal ones. Moreover, an experimental realization of the optimal phase estimation in the presence of loss has been reported in [5]. Results obtained, as well as the discussion of perspectives for the future development of the field of quantum enhanced metrology have been published in a short review article [7]. The above papers are discussed in detail in Sec. 4.4.

Analysis of achievable precision based solely on the concept of QFI may be insufficient in many situations. While the Craméra-Rao inequality is satisfied for all unbiased estimators it is not obvious that it can always be saturated. Moreover, QFI is a tool allowing for a precise

judgement of sensitivity of only the so-called *local* parameter estimation problems, where it is assumed that the value of the estimated parameter fluctuates relatively weakly around a *known* value  $\varphi_0$ . In some situations, it is therefore more practical to use an alternative Bayesian approach to estimation, where one explicitly assumes certain a priori probability distribution of  $\varphi$  and subsequently minimizes the average estimation cost. The Bayesian approach does not suffer from saturability issues nor it requires the assumptions of locality which is not always justified. This approach applied to the problem of phase estimation in the presence of loss and the flat a priori distribution, has been proposed and solved for the first time in [4]. Moreover, in [3] the solution to a more general problem where the a priori distribution is *arbitrary* has been derived. Results obtained in the Bayesian approach to estimation are presented in detail in Sec. 4.5.

Research on the role of decoherence on the achievable precision in quantum metrology has been culminated in [1], where general tools have been developed which allow for a direct and computationally efficient derivation of useful bounds on precision for general decoherence models. In this way it has also been proven that generically even infinitesimally small amount of noise makes the Heisenberg scaling, characteristic for idealized models, unachievable asymptotically. Results are presented in Sec. 4.6.

Apart from the single parameter estimation problems, which were addressed by the above mentioned publications, two models of multi-parameter estimation have also been analyzed. In [2] a general approach to optical quantum interferometry has been presented taking into account the role of an additionally phase reference beam, which led effectively to a two-parameter estimation problem. In [10] the concept of Fisher matrix has been employed to find  $n$ -qubit states optimal from the point of view of estimation of the rotation group elements, being an example of a three-parameter estimation problem. Results obtained for multi-parameter models are presented in Sec. 4.7.

#### 4.4 Optimal local phase estimation in the presence of loss

Idealized analysis presented in Sec. 4.2 has been generalized in [8, 9] to cover the case of the optical interferometer *with* loss. A general model, presented in Fig. 1b, has been considered where loss was taken into account by inserting additional virtual beam-splitters of transmissivities  $\eta_1, \eta_2$  in the two arms of the interferometer respectively. Due to loss, the output state if light  $\rho_\varphi$  is mixed and it is no longer possible to use the simple formula for QFI, Eq. (4), valid in the pure state case. The output state reads explicitly:

$$\rho_\varphi = \Lambda_\varphi(|\psi\rangle\langle\psi|) = U_\varphi \Lambda(|\psi\rangle\langle\psi|) U_\varphi^\dagger, \tag{7}$$

where  $U_\varphi$  represents the unitary evolution resulting solely from the relative phase delay in the interferometer, while  $\Lambda$  is a non-unitary operation describing the loss process (decoherence). Moreover, as shown in [8, 9], the order in which the loss and the phase delay transformation are placed does not influence the form of the final state. In order to perform rigorous calculations, it is necessary to use a general formula for the QFI, which involves computing the so called symmetric logarithmic derivative. This in general requires performing the diagonalization of  $\rho_\varphi$ , which makes analytical search for the optimal state as well as the numerical optimization for large number of photons  $n$  impossible. In [9] it has been shown,

however, that in the case of single-arm losses  $\eta_1 = \eta$ ,  $\eta_2 = 1$ , the formula for the QFI simplifies to the following weighted average:

$$F(\rho_\varphi) = \sum_{l=0}^n p_l F(|\psi_\varphi^{(l)}\rangle), \quad (8)$$

where  $p_l$  is the probability of a loss of  $l$  photons while  $|\psi_\varphi^{(l)}\rangle$  is the pure state conditioned on the loss of  $l$  photons. Thanks to the above explicit form it was possible to perform numerical optimization over  $c_k$  parameters for a general input  $n$  photon states, Eq. (2), for  $n < 80$ , and as a result a numerical bound on the maximal achievable precision as a function of loss parameter  $\eta$  has been found. Moreover, it has been proven that the optimization amounts to a maximization of a concave function over a convex set, and as such guarantees that any local maximum will be the global one. In the case when losses are present in both arms, analogous procedure does not lead to a strict equality as before. Nevertheless, it still provides a useful upper bound on the QFI in the form:

$$F(\rho_\varphi) \leq \sum_{l_1=0}^n \sum_{l_2=0}^{n-l_1} p_{l_1, l_2} F(|\psi_\varphi^{(l_1, l_2)}\rangle), \quad (9)$$

where  $p_{l_1, l_2}$  is the probability of losing  $l_1$  photons in the upper and  $l_2$  photons in the lower arm, whereas  $|\psi_\varphi^{(l_1, l_2)}\rangle$  is the corresponding conditional state of light. Analysis of the upper bound is sufficient in order to draw conclusions regarding the bounds on the maximal achievable precision. Moreover, it was shown numerically [9] that the above approximation was actually very precise and did not result in appreciable deviations from the QFI calculated rigorously. Further analysis and optimization of the above quantities over input states led to first numerical results suggesting that in the presence of loss the Heisenberg limit is not achievable and the asymptotic scaling of precision is most likely to have a classical,  $1/\sqrt{n}$ , character. Additionally, the structure of optimal states has been analyzed in [8] and suboptimality of the strategies based on the use of non Fock states has been demonstrated. It has also been proven that neither distinguishability of photons nor investigates states of light with indefinite photon number would lead to an increase in estimation precision and as such are not relevant from the point of view of looking for the fundamental bounds.

In [6] the above results have been generalized in the model where the possibility of multiple passes of light through the phase delaying element has been taken into account. Optimal quantum strategies with fixed photon number  $n$  have been compared with one-photon multiple-pass strategies in the presence of loss. It has been proven that non-classical states of light are indispensable to reach the optimal precision enhancement and it is not possible to replace them with multiple-pass strategies employing single-photon or classical light sources. The analysis has been carried out both under the assumption that multiple passes are treated as a resource (included in  $n$ ), and in the case where they are not included in the total resources consumed.

Theoretical analysis of the optimal local phase estimation strategies has been accompanied by an experimental work where the optimal phase estimation strategy in the presence of loss based on the use of two-photon states has been implemented [5]. Advantage of the



optimal two-photon states, found using the tools from [8, 9], both over the classical strategies and the quantum strategies based on the use of the  $n00n$  states has been demonstrated. A detailed statistical analysis of the obtained results has been carried out and it has been proven that the precisions achieved agree with the fundamental theoretical bounds calculated for the model with relevant experimental parameters such as loss and interferometer visibility taken into account.

In [7], which is a Comment written for the Nature Photonics journal, recent achievements both in theoretical and experimental quantum metrology have been revised, with a special focus on optical interferometry and atomic spectroscopy. The role of appropriate quantification of the resources that are used to compare classical and quantum strategies has been highlighted. In particular, it has been pointed out, that the most of the so called proof-of-principle experiments based on post-selection which are supposed to demonstrate the quantum precision enhancement, are not capable of proving unambiguously the practical usefulness of the quantum estimation protocols implemented.

## 4.5 Bayesian phase estimation in the presence of loss

An alternative approach to quantum phase estimation which is not based on the QFI is the Bayesian approach. Let  $p(\varphi)$  be a probability distribution describing a priori knowledge on the value of phase  $\varphi$  before any measurements haven taken place—in particular  $p(\varphi) = 1/2\pi$  represents complete ignorance on the value of  $\varphi$ . Assume that a measurement  $\Pi_r$  is performed on the output state  $\rho_\varphi = \Lambda_\varphi(|\psi\rangle\langle\psi|)$ , being the final result of the  $\varphi$ -dependent evolution, yielding result  $r$ . Based on the result, a phase is estimated according to the estimator function  $\tilde{\varphi}(r)$  which leads to an average estimation cost given by

$$\bar{C} = \int_0^{2\pi} d\varphi p(\varphi) \text{Tr}(\rho_\varphi \Pi_r) C(\varphi, \tilde{\varphi}(r)), \quad (10)$$

where  $C(\varphi, \tilde{\varphi})$  is a cost function. Looking for the optimal Bayesian strategies amounts to determining the optimal measurements  $\Pi_r$ , estimators  $\tilde{\varphi}(r)$  and input states  $|\psi\rangle$  ( $c_k$  coefficients in Eq. (2)) which minimize the above formula. In general, this is an extremely complex mathematical problem and it is not possible to arrive at an analytical solution.

Problem of Bayesian phase estimation in the presence of loss and in the absence of a priori knowledge,  $p(\varphi) = 1/2\pi$ , has been addressed in [4], where a natural cost function  $C(\varphi, \tilde{\varphi}) = 4 \sin^2[(\varphi - \tilde{\varphi})/2]$  has been chosen—a simple cost function which for small deviations  $\varphi - \tilde{\varphi}$  approximates the variance, but at the same time respects the periodic nature of the estimated parameter. It has been proven that analogously to the lossless case [31], the search for the optimal measurement and estimator may be restricted, without loss of optimality, to the so called covariant measurements [32], i.e. sets of measurements operators  $\Pi_{\tilde{\varphi}}$ , such that  $\int \frac{d\tilde{\varphi}}{2\pi} \Pi_{\tilde{\varphi}} = \mathbb{1}$ ,  $\Pi_{\tilde{\varphi}} = U_{\tilde{\varphi}} \Pi_0 U_{\tilde{\varphi}}^\dagger$ , where  $\Pi_0$  is fixed positive semi-definite operator. Moreover, the optimal operator  $\Pi_0$  has been derived and reads explicitly:

$$\Pi_0^{\text{opt}} = \bigoplus_{n'=0}^n |e_{n'}\rangle\langle e_{n'}|, \quad |e_{n'}\rangle = \sum_{k=0}^{n'} |k, n' - k\rangle, \quad (11)$$

where the direct sum is carried out over sectors representing different total number of surviving photons. This allowed the authors to derive a formula for the minimal achievable estimation cost:

$$\bar{C}_{\min} = 2 - \lambda_{\max}, \quad (12)$$

where  $\lambda_{\max}$  is the largest eigenvalue of an  $(n+1) \times (n+1)$  matrix  $A$ , whose non-zero elements read:

$$A_{k-1,k} = A_{k,k-1} = \sum_{l_1, l_2=0}^{k, n-k} \beta_k^{l_1, l_2} \beta_{k-1}^{l_1, l_2}. \quad (13)$$

and

$$\beta_k^{l_1, l_2} = \sqrt{B_{l_1}^k(\eta_1) B_{l_2}^{n-k}(\eta_2)}, \quad B_l^n(\eta) = \binom{n}{l} (1-\eta)^l \eta^{n-l}, \quad (14)$$

with  $\eta_1, \eta_2$  being the effective light transmission coefficients in the two arms of the interferometer respectively. Making use of the above tools, the optimal states and the optimal precision have been numerically calculated for up to  $n = 100$  photons.

The most significant result of the paper, however, was the derivation of *analytical* bounds on precision based on the algebraic properties of the matrix  $A$ , the first of this kind in the literature. In the case of one-arm losses ( $\eta_1 = \eta, \eta_2 = 1$ ) and equal losses in both arms ( $\eta_1 = \eta_2 = \eta$ ) the derived analytical bounds read respectively:

$$\Delta\varphi \geq \sqrt{\frac{1-\eta}{4\eta n}}, \quad \Delta\varphi \geq \sqrt{\frac{1-\eta}{\eta n}}. \quad (15)$$

Hence, the results of [4] provided for the first time an analytical argument for the claim that the presence of decoherence implies the  $1/\sqrt{n}$  asymptotic scaling of precision, and that the quantum enhancement amounts to a constant factor improvement rather than a better scaling exponent. The results obtained are illustrated in Fig 2. It should be pointed out that the paper was cited 19 times in less than a 3 year period.

The Bayesian approach to metrology has been developed further in [3], where the problem of Bayesian phase estimation with the cost function  $C(\varphi, \tilde{\varphi}) = 4 \sin^2[(\varphi - \tilde{\varphi})/2]$  and *arbitrary* a priori knowledge has been addressed. Making use of the tools typically applied to the problem of separability of quantum states—positive partial transposition method—it has been proven that the optimal estimation precision is given by the following expression:

$$\Delta\varphi = 2\sqrt{\left(\frac{1}{2} - \|R\|_1\right)}, \quad (16)$$

where  $\|\cdot\|_1$  denotes the trace norm of a matrix, while matrix  $R$  reads explicitly

$$R = \frac{1}{2} \int_{-\pi}^{\pi} d\varphi p(\varphi) e^{i\varphi} \rho_\varphi, \quad (17)$$

where  $\rho_\varphi$  is the final state on which the measurement is being performed in order to estimate the parameter  $\varphi$ .

Moreover, it has been shown that the optimal estimation strategy may be realized by performing a standard von-Neumann measurement in the eigenbasis of  $U = V_R U_R^\dagger$  operator,

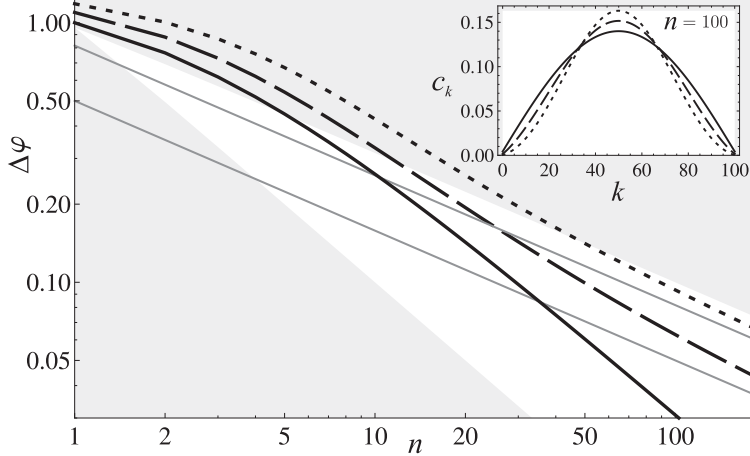


Figure 2: Log-log plot of the minimal estimation uncertainty as a function of the number of photons used for equal losses in both arms:  $\eta = 1$  (solid line),  $\eta = 0.8$  (dashed line),  $\eta = 0.6$  (dotted line). White region in the middle corresponds to  $1/n < \Delta\varphi < 1/\sqrt{n}$ . Gray lines depict analytical asymptotic bounds for  $\eta = 0.8$ ,  $\eta = 0.6$ . The inset provides an insight into the structure of the optimal states for  $n = 100$  and for the three levels of loss respectively.

where  $V_R, U_R$  are unitary matrices entering the singular value decomposition of the  $R$  matrix  $R = U_R \Lambda_R V_R^\dagger$ , whereas the phases corresponding to the eigenvalues of  $U$  are in fact the optimal estimator values. Additionally, an iterative numerical method has been proposed that allows for an efficient derivation of the optimal states, measurements and estimators.

The above tools have been employed in order to study the unexplored intermediate estimation regime interpolating between the results obtained using the QFI and Bayesian methods making use of the symmetry arising due to the assumption of a flat a priori distribution. Precision gains relative to the a priori knowledge have been studied and the regime of sufficiently narrow a priori distributions has been identified such that the optimal estimation strategies were analogous to the ones obtained in the QFI approach.

#### 4.6 Efficient methods for deriving bounds on estimation precision in single parameter estimation for general decoherence models.

The above presented numerical and analytical results, leading to an observation of the key impact of decoherence on the asymptotic scaling of precision, were restricted to a single class of models dealing with loss as the only decoherence source. The main goal of [1] was to formulate a general approach to the problem of finding the bounds on the optimal estimation precision achievable in quantum metrology in presence of decoherence, without restricting the considerations to a concrete decoherence model. Two efficient methods for finding the bounds have been proposed: one based on the mathematical idea of the classical simulation (CS) of a channel [33] and the other based on the ideas of channel extension (CE) [34]. Both methods are applicable for models in which decoherence acts independently on each particle (see Fig. 3a). Importantly, thanks to the CS method it is possible to derive bounds

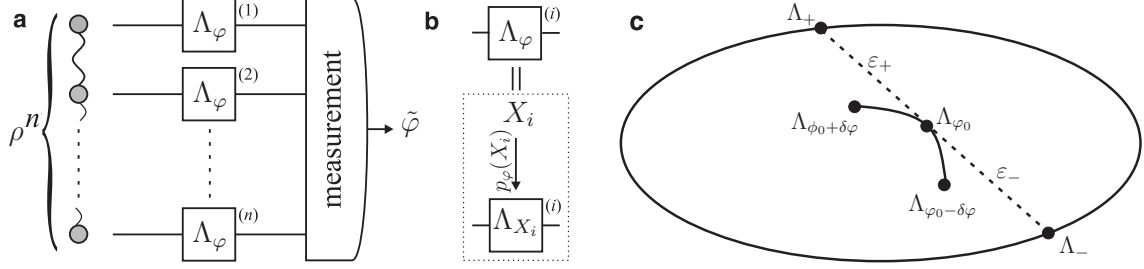


Figure 3: **(a)** General scheme of quantum metrology with uncorrelated noise.  $n$ -particle quantum state is being sent into  $n$  parallel  $\varphi$ -dependent channels. Based on the results of the measurement performed on the output state the value of parameter is estimated using an estimator  $\tilde{\varphi}$ . **(b)** Idea of the classical simulation of a quantum channel. A channel  $\Lambda_\varphi$  is treated as a mixture of other channels  $\Lambda_X$ , where the dependence on  $\varphi$  is moved to the mixing probabilities  $p_\varphi(X)$ . **(c)** A geometric construction that allows to find “distances” of a channel from the boundary of the set of quantum channels and in this way derive the bound on precision via Eq. (18).

on estimation precision using solely the geometry of quantum channels. In [1] it has been proven that when the  $\varphi$ -estimation problem is regarded a channel estimation problem, were the channel moves along a certain trajectory in the set of quantum channels (see Fig. 3c) the CS method allows to derive a simple bound on the achievable precision in the form:

$$\Delta\varphi \geq \sqrt{\frac{\varepsilon_+\varepsilon_-}{n}}, \quad (18)$$

where  $\varepsilon_\pm$  are “distances” of the channel  $\Lambda_\varphi$  along the tangent to its trajectory to the boundary of the set of quantum channels in the sense that the channels at the boundary are given by  $\Lambda_\pm = \Lambda_\varphi \pm \varepsilon_\pm \partial_\varphi \Lambda_\varphi$ . It implies that for an arbitrary decoherence process for which  $\varepsilon_\pm > 0$  (which is a generic situation) the above construction allows to immediately conclude that the asymptotic scaling of precision must necessarily have a  $1/\sqrt{n}$  character, and reaching the Heisenberg scaling is not possible.

Additionally, in [1] the CE method has been developed and applied it to a quantum channel written in the Kraus form  $\Lambda_\varphi(\rho) = \sum_i K_i(\varphi)\rho K_i^\dagger(\varphi)$  making use of the fact that the estimation uncertainty is always lower bounded by

$$\Delta\varphi \geq \frac{1}{2\sqrt{n \min_h \|\alpha_K\|}}, \quad (19)$$

where  $\alpha_K$  reads explicitly

$$\alpha_K = \sum_i \left[ \left( \partial_\varphi K_i(\varphi) - i \sum_j h_{ij} K_j(\varphi) \right)^\dagger \left( \partial_\varphi K_i(\varphi) - i \sum_{j'} h_{ij'} K_{j'}(\varphi) \right) \right], \quad (20)$$

$\|\cdot\|$  is the operator norm, while minimization over the  $h$  matrix is subject to an additional constraint

$$\sum_{ij} h_{ij} K_i^\dagger(\varphi) K_j(\varphi) = i \sum_i \partial_\varphi K_i^\dagger(\varphi) K_i(\varphi). \quad (21)$$

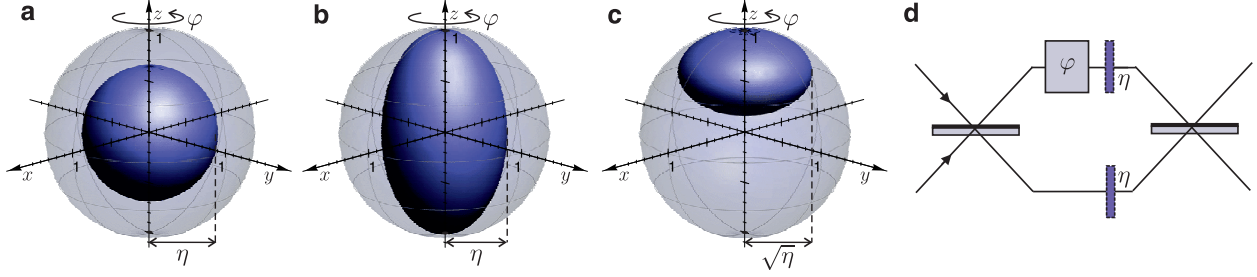


Figure 4: Graphical representation of popular decoherence models. Qubit decoherence processes are illustrated as transformations of the Bloch ball (a) Depolarization (b) Dephasing (c) Spontaneous emission (d) Lossy interferometer, where  $\eta$  is the transmission coefficient.

Model	Depolarization	Dephasing	Spontaneous emission	Loss
$\Delta\varphi_{\text{CS}}$	$\sqrt{\frac{(1-\eta)(1+3\eta)}{4\eta^2n}}$	$\sqrt{\frac{1-\eta^2}{\eta^2n}}$	-	-
$\Delta\varphi_{\text{CE}}$	$\sqrt{\frac{(1-\eta)(1+2\eta)}{2\eta^2n}}$	$\sqrt{\frac{1-\eta^2}{\eta^2n}}$	$\sqrt{\frac{1-\eta}{4\eta n}}$	$\sqrt{\frac{1-\eta}{\eta n}}$

Table 1: Bounds on precision scaling as  $1/\sqrt{n}$  derived for different decoherence models using the methods of the classical simulation (CS) and the channel extension (CE), where  $0 \leq \eta \leq 1$  is the decoherence parameter of a given model with  $\eta = 1$  corresponding to no decoherence at all while  $\eta = 0$  to the full loss of coherence.

It has been demonstrated that the above optimization problem can be solved with the help of the so called semi-definite programming, for which efficient numerical procedures exist, which in particular guarantee that the the optimum found is the global one. Additionally, it has been proven that the CS method is a special case of the more general CE method and as such the CE method yields the bounds which are equivalent or tighter than the ones obtained with the CS method. The only drawback of the CE is that it does not lead to a nice intuitive geometric picture analogous to the one which is provided by the CS method.

Making us of the above methods, bounds on estimation precision have been found for different decoherence models relevant in quantum metrology such as: depolarization, dephasing, spontaneous emission and loss (see Fig. 4). It has been shown, in particular, that the CS method provides bounds equivalent to the ones obtained with the CE method in the case of dephasing, but cannot be applied to pure loss and spontaneous emission decoherence models as these channels lie on the border of the set of quantum channels and the tangent to the trajectory points away from the set:  $\varepsilon_{\pm} = 0$ . Comparison of the bounds derived using the CS and CE methods is presented in Table 1.

The methods have also been applied to one of the most spectacular implementation of quantum enhancement ideas—the gravitational wave detector GEO600, were squeezed vacuum states have been used in order to beat the classical precision bounds [35]. The results have been welcomed by the experimentalists as they have in particular shown that in practice the technique based on interfering a coherent state of light with a squeezed vacuum state allows for almost optimal performance in the regime of large number of photons, and

there is no need to consider more sophisticated quantum states of light as inputs to the interferometer.

It should be pointed out that after being published in Nature Communications the paper has already been cited 8 times (no self-citations) over the period of approximately 7 months (according to Web of Science) which reflects the importance of the results and gives hope that the paper will contribute significantly to the future development of the whole field of quantum metrology.

## 4.7 Multi-parameter models in quantum metrology

Apart from single parameter estimation problems, which were the focus of all of the previously discussed papers, the cycle of publications contains two positions [2, 10] which deal with multi-parameter estimation.

The motivation for the research done in [2] was the observation that in theoretical models dealing with non-classical optical interferometry an additional phase reference beam is often neglected in the analysis. The reference beam is indispensable to give a physical meaning to phases of coherent/squeezed states used in optical interferometry and is commonly used in implementation of homodyne measurements. It has been demonstrated, that the naive use of the QFI in optical interferometry with coherent/squeezed states may lead to apparently paradoxical conclusions that the optimal estimation strategy depends on the way the estimated phase delay is distributed among the two arms.

This apparent paradox, arise due to implicit assumption that all phase delays as defined with respect to an external reference beam. A proper approach to this problem has been developed, which involves the use of a two-parameter estimation theory, and in particular the calculation of the whole quantum Fisher matrix—two phases corresponding to relative phase delays between each of the arm the reference beam need to be treated as two parameters entering the estimation problem. This has allowed for a rigorous calculation of the precision of estimation of the relative phase delay between the two arms of the interferometer as a function of the strength of the reference beam, and two extreme cases were analyzed in detail: strong reference beam, which can be treated as an ideal phase reference, and the case when the reference beam is lacking, and one needs to average input states of light over their phases which in this case have no physical significance. In the case of loss, precision increase with the increasing power of the reference beam has also been demonstrated.

Problem of optimal alignment of Cartesian reference frames by sending  $n$  qubit states has been analyzed in [10]. This problem had already been solved in the Bayesian approach [36], but the solution to the problem in the QFI approach, applicable when the two reference frames are only slightly rotated with respect to each other, had not been known. Mathematically, this problem is equivalent to the estimation of an  $SU(2)$  group element in the vicinity of the identity element, by measuring an  $n$ -qubit state on which the  $n$ -fold tensor representation of the group acts. As such this is an example of a three-parameter estimation problem. Analyzing the Fisher matrix in this case, it has been proven that the search for the optimal states may be restricted to the fully symmetric subspace—which is not true in the Bayesian approach—while the optimal states may be elegantly depicted using the so called Majorana representation. In Majorana representation an arbitrary symmetric  $n$ -qubit state may be represented as  $n$  points on the Bloch sphere. The symmetric state is obtained

by writing an  $n$  qubit state as a product of  $n$  respective single qubit states corresponding to each of the points on the Bloch sphere after which a full symmetrization is performed. Thanks to this tool, an intuitive insight into the structure of the optimal states has been obtained and in particular it has been shown that points on the Bloch sphere corresponding to the vertices of the platonic solids result in states that are optimal from the point of view of the problem of alignment of reference frames.

## 4.8 Conclusions

The cycle of publications presented constitutes as substantial contribution to the dynamically developing field of quantum metrology. It should be stressed that apart from numerous results on optimality of various estimation strategies obtained within concrete theoretical models, universal mathematical tools have been developed that allow for an efficient analysis of a huge class of realistic quantum models. Therefore, the impact of the above works goes beyond the spectrum of the particular models considered such as optical interferometry or atomic spectroscopy, since the tools developed may easily be applied to different metrological tasks such as e.g. magnetometry, as well as apparently non related topics such as robustness of quantum algorithms against noise.

Moreover, thanks to the use of various advanced mathematical tools such as, Bayesian estimation, Fisher information, covariant measurements and the geometry of quantum channels in the context of realistic quantum optical models, the cycle of publication closes the gap between sometimes a bit detached theoretically-mathematical and experimental aspects of quantum metrology.

## 5 Other research accomplishments

### a) bibliometric data (as for 16 May 2013)

Total number of papers: 23 + 3 preprints

Citations with auto-citations excluded: 258

Total impact factor: 125

Hirsch index: 8

### b) research not contributing to the habilitation

The remaining research not covered in the cycle of publication deals with a wide variety of topics in quantum information theory. The most recent papers on quantum metrology (preprints) are a natural continuation of the research contained in the cycle of publications and are described in Sec. 5.1. The remaining sections are not related directly with quantum metrology and deal with quantum cryptography and communication (Sec. 5.2), entanglement theory and the role of correlations in quantum information theory (Sec. 5.3) as well as the search for optimal approximate cloning operations (Sec. 5.4).

## 5.1 Recent research in quantum metrology not included in the cycle of publications.

Methods presented in [1] have been further developed in [11] in order to obtain tighter bounds in the regime of finite  $n$  without the need to go to the asymptotic limit  $n \rightarrow \infty$ . A detailed analysis of relations between seemingly unrelated methods used for the derivation of the bounds in quantum metrology has been performed covering methods such as: classical simulation (CS), quantum simulation (QS), right logarithmic derivative (RLD) and the channel extension (CE) method. The methods have been ordered in a hierarchy with respect to the tightness of the bounds they provide and the scope of models to which they may be applied. It has been proven that the RLD method provides in general tighter bounds than the CS methods, whereas the CS and the QS methods are a special cases of the CE method. As a result, it has been shown that the CE method is the most universal tools for derivation of the bounds among all the methods discussed in the literature.

Additionally, the problem of frequency rather than phase estimation in the presence of decoherence has been addressed in the framework of continuous quantum evolution described with GKSL equations (Gorini-Kossakowski-Sudarshan-Lindblad). The issue of the optimal choice of the evolution time has been investigated and expressions for the optimal frequency estimation precision under fixed resources—number of atoms  $n$  and the total time of an experiment  $T$ —have been derived. Application of the CS, QS, RLD and CE methods have also been presented to the problem of estimation of the decoherence parameter itself contrasting this problem with the unitary parameter estimation such as phase or frequency. In the case of estimation of the decoherence parameter it has been proven that all the methods considered result in equivalent bounds, hence the simplest and the most intuitive CS method is sufficient for obtaining the optimal bounds for precision.

An attempt to link quantum metrology with an extremely useful many-body physics concept of *matrix product states* (MPS), used for an efficient state characterization, has been the main objective of [12]. In face of the results of [1] stating that the asymptotic scaling of precision has typically the  $1/\sqrt{n}$  character, it is possible to use an intuitive argument that in order to achieve the optimal precision in the regime of large  $n$ , it is not necessary to employ strong entanglement among all the particles involved in the experiment, but it is sufficient to consider states where substantial degree of entanglement is only present inside clusters containing only finite number of particles. Reasoning along these lines, the class of MPS appears to be an ideal choice for an efficient description of states where only short-range correlations are involved. Indeed, it has been proven in [12] that in lossy interferometry it is possible to approximate faithfully the optimal states by considering MPS of relatively low number of free parameters (low MPS bond dimensions) both when the QFI itself is being maximized as well as when a concrete measurement scheme, e.g. Ramsey interferometry, is considered. This description not only facilitates the numerical search for the optimal states, but at the same time provides a much more intuitive description of the structure of the optimal states than is possible with the help of the standard Hilbert space formalism.



## 5.2 Quantum cryptography and communication

In [14], a study of security of four state quantum key distribution protocols, such as BB84 [37], SARG04 [38], has been performed, where heralded single-photon pulses were generated in parametric down conversion processes. A regime of experimental parameters has been identified, where multichannel detection allows for an enhancement of the secure communication distance and the results have been compared with the ones obtained in the implementations based on standard weak coherent pulses.

In entanglement based quantum cryptography the notion of private states [39], which allow for a secure secret key distillation, is a key concept. The main goal of the [15] paper was to experimentally generate a four photon private state, in which the amount of distillable secret key would exceed significantly its distillable entanglement—demonstrating experimentally a theoretical fact that secret key and entanglement distillation tasks are not equivalent. Results of the experiment have been analyzed thoroughly and the tomographic reconstruction of the prepared state have been performed using two independent methods: maximum likelihood estimation method, as well as the method based on the Bayesian approach and gaussian approximation of probability distributions on the set of density matrices. The reconstruction was accompanied by a credible assessment of reconstruction uncertainties of the quantities of interest, i.e. distillable secret key and distillable entanglement, proving that indeed the content of the secret key was larger than that of the distillable entanglement.

In [17] security of the so called quantum secret sharing protocols has been analyzed under the assumption of local eavesdropping attacks. The goal of the protocols considered was to securely distribute a message from a sender to two parties in such a way that the two parties could decode the message only in cooperation with each other. This problem may be solved by performing two independent quantum key distribution protocols from the sender to each of the receiving parties. It has been shown in [17], however, that under local eavesdropping attacks a more secure strategy is to make use of entangled states and send them to the receiving parties. In this case security is guaranteed for appreciably higher noise levels than in the two independent quantum key distribution approach.

Analysis of communication capacity through quantum channels, such as optical fibers, has been addressed in [20]. Motivated by the depolarization processes occurring in optical fibers, a general model has been formulated based on the mathematical theory of diffusion on the  $SU(2)$  group. Using the model, information capacity as a function of correlation strength of the noise acting on the consecutive photons has been investigated. Analytical formulas for the evolution of general multi-photon state in such channels have been derived, and explicit optimization of communication protocols have been performed in the case of the simplest non-trivial case of three-photon states. States most robust to the noise have been identified and the corresponding communication capacities calculated.

## 5.3 Entanglement and correlations

[13] paper was an extension of the previously published paper [15] demonstrating experimental details of the process of preparation of four photon states relevant from the point of view of entanglement theory, such as: private states, Smolin states. Results of a detailed tomographic reconstruction of the four photon density matrices corresponding to these states

have also been presented.

In [16], the strength of polarization correlations between two light beams that can be achieved in the semiclassical regime using statistical mixtures of coherent states and binary on/off detectors has been analyzed. It has been proven that correlation visibility is bounded by 33%, which is in striking contrast with 100% visibility, theoretically achievable by employing entangled states. The results have been illustrated with an experiment performed on a pair of laser beams undergoing correlated depolarization.

The problem of decorrelation of quantum states, i.e. preservation of as much local information in the states of the subsystems as possible while removing completely correlations between them, has been studied in [18, 19]. In [19] the problem has been stated and solved for the case of two-qubit and two-mode gaussian states decorrelation. States for which decorrelation is possible have been characterized and it has been proven that the states arising as a result of the action of the optimal cloning machines are not decorrelable. In [18] mathematical techniques have been described that allow to find the optimal decorrelating transformations. It has been proven that decorrelation is not possible in the case of continuous sets of states of  $d$ -dimensional quantum systems. A construction of optimal transformation decorrelating gaussian states of light has also been found.

[21, 24] papers investigated the impact of correlations in multi-qubit systems on the quality of estimation of local properties such as direction of the local Bloch vector. In [24] a complete analysis of this problem in the case of permutationally invariant states has been presented. Within the class of states with the same length of the local Bloch vectors (degree of mixedness of subsystems), optimal correlated states have been identified that allow for the most precise estimation of the direction of the local Bloch vectors. It has also been demonstrated that depending on the type, correlations may facilitate or hinder the process of estimation of local properties of the subsystems. In particular, it has been shown that the states resulting from the action of the optimal cloning of qubits contain the type of correlations that are the worst from the point of view of the estimation precision of the local Bloch vector among all the states with the same degree of mixedness of the subsystems. Inspired by [24], the [21] paper contained a proposal for making use of the observed effects of correlations on the possibility of estimating local quantities in order to encode the information on the spacial direction in an  $n$ -qubit state in a way that its decoding would be only possible when performing collective measurements on majority of the qubits, whereas would be virtually impossible when performing local measurements with the assistance of classical communication. An explicit protocol has been presented and its robustness against the loss of a small fraction of qubits has been demonstrated.

Properties of the so called multi-partite concurrences used in the characterization of entanglement have been studied in [23] from the point of view of their behavior under local operations and classical communication (LOCC). Only the quantities monotonically non-increasing under LOCC may be employed in quantitative characterization of entanglement. Sufficient and necessary conditions for monotonicity of the multi-partite concurrences have been derived.

[25] paper was one of the first attempts to link the theory of quantum chaos with the theory of quantum entanglement. Entanglement production in a system of coupled kicked tops has been studied as a function of a dynamics parameter (tops kicking strength) that is responsible for the transition from the regular to chaotic dynamics. Average final entangle-

ment has been investigated, where averages have been performed over two different ensembles: ensemble of products of spin coherent states and products of random states. It has been shown, that any conclusion drawn regarding the entanglement production depending on the chaoticity of the subsystems is strongly dependent on the choice of one of these ensembles. Analytical bounds on asymptotic generation of entanglement have been derived as a function of entanglement of eigenvectors of the evolution operator.

## 5.4 Cloning of quantum states

Faithful cloning of unknown quantum states is forbidden by the laws of quantum mechanics. It is, nevertheless, possible to find optimal approximate cloning transformations, which provide the best possible cloning fidelity allowed within the quantum mechanics framework. These topics are important for the analysis of security of quantum key distribution protocols, since it is often the case that the optimal attacks on the protocols amount to performing the optimal cloning transformation.

In [26] the problem of optimal cloning of spin-coherent states has been solved. Optimal cloning fidelity has been found numerically as a function of quantum system dimension, and the extrapolated asymptotic cloning fidelity has been estimated to be  $\approx 0.68$ . Because of the fact that the spin-coherent states become isomorphic to the standard coherent states in the asymptotic limit of large system dimension this allowed to conjecture that most probably gaussian transformations cloning coherent states and achieving  $2/3$  fidelity are not optimal. This hypothesis was indeed confirmed in [40].

Cloning of entangled qubit states using local operations and classical communication has been investigated in [22]. A threshold for the strength of entanglement has been found above which classical communication helps in increasing the cloning fidelity as compared with a strategy based on independent cloning of each of the qubits. Moreover, it has been shown that bound entanglement is not a useful resource for increasing the cloning fidelity.

### c) prizes

- 2012, III-rd degree individual prize of the Rector of the University of Warsaw for the research on the realization of quantum technologies.
- 2011, Polish Ministry of Science and Higher Education Scholarship for young scientist for the years: 2011-2014
- 2006, Foundation for Polish Science prize START for young scientists
- 1998, Polish Physics Olympiad Laureate; Honorable mention on the International Physics Olympiad (Iceland)

### d) directing research projects

- 2013-2016, Directing research activities at the University of Warsaw as part of the SIQS consortium (*Simulators and Interfaces with Quantum Systems*) a project financed under the 7-th EU framework programme

- 2012-2013, Directing research activities at the University of Warsaw under the European project QUASAR (*Quantum States: Analysis and Realizations*) finance by the Polish NCBiR under the ERA-NET CHIST-ERA programme
- 2006-2007, Polish research grant, 1 P03B 129 30: *Optimal communication in the presence of correlated noise - usefulness of multipartite entanglement*

#### e) participation in research projects

- 2010-2013, integrating project QESSENCE (*Quantum Interfaces, Sensors, and Communication based on Entanglement*), 7-th EU framework programme; Workpackage leader: *Multiparameter Estimation and Non-Linear Metrology*
- 2010-2013, *Photonic implementations of quantum enhanced technologies*, Foundation for Polish Science TEAM project, Post-Doc
- 2008-2011, CORNER (*Correlated Noise Errors in Quantum Information Processing*) EU FET project.
- 2007-2008, QAP (*Qubit Applications*), EU integrated project
- 2004-2005, QUPRODIS (*Quantum Properties of Distributed Systems*), 5-th EU framework programme
- 2004-2007, Polish research grant, 1 P03B 042 26, *Chaos in open system*
- 2004-2006, National solicited grant PBZ-MIN-008/P03/2003, *Quantum computing and engineering*

#### f) invited conference talks

- 19-22.02.2013, Toronto, Canada, Workshop on Mathematical Methods of Quantum Tomography, *All you need is squeezing! Optimal schemes for realistic quantum metrology*
- 20-24.06.2012, Toruń, Poland, Symposium on Mathematical Physics, *Quantum enhanced metrology and the geometry of quantum channels*
- 21-25.05.2012, Torino, Italy, Advances in Foundations of Quantum Mechanics and Quantum Information with atoms and photons, *Almost all decoherence models lead to shot noise scaling in quantum enhanced metrology*
- 18-19.05.2012, Sopot, Poland, KCIK Symposium, *Quantum enhanced metrology and the geometry of quantum channels*
- 17-21.01.2012, Moffett Field, California, USA, First NASA Quantum Future Technologies Conference, *The illusion of the Heisenberg limit*
- 17.09.2011, Toruń, Poland, International symposium „Quantum metrology with photons and atoms”, *Quantum enhanced metrology*

- 05.09.2011, Lublin, Poland, XLI Meeting of Polish physicists, *Fundamental bounds in quantum metrology*
- 22-26.08.2011, Bad Honnef, Germany, Wilhelm and Else Heraeus Physics School: Modern Statistical Methods in Quantum Information Processing, *Quantum enhanced metrology*
- 11-15.07.2011, Sarajevo, Bosnia and Herzegovina, 20th International Laser Physics Workshop, *Experimental extraction of secure correlations from a noisy private state*
- 27.06-01.07.2011, Międzyzdroje, Poland, II Polish Optical Conference, *Fundamental bounds in optical metrology*
- 9-12.09.2009, Gdańsk, Poland, NATO Advanced Research Workshop, Quantum Cryptography and Computing, *Entanglement enhances security in secret sharing.*
- 8-12.06.2009, Zakopane, Poland, Quantum Optics VII, *Quantum enhanced phase estimation in the presence of loss.*

#### **g) national and international collaboration**

- Germany, Hannover (2012-2013). Collaboration with Roman Schnabel (leader of the LIGO group) on the application of fundamental theoretical bounds to the GEO600 gravitational wave detector
- UK, Nottingham (2011-2013). Collaboration with Madalin Guta on mathematical aspects of quantum metrology and quantum estimation theory which resulted in [1].
- Switzerland, Zurich (2012-2013). Collaboration with Martin Fraas on quantum metrology in the context of calibration of atomic clocks.
- Poland, Gdańsk, (2011-2013). Collaboration with Paweł Horodecki on the theory of private states, quantum states reconstruction and the issue of additivity in quantum metrology, which until now resulted in [13, 15].
- UK, Oxford (2008-2010). Collaboration with I. A. Walmsley group from the University of Oxford on optimal phase estimation in the presence of loss which resulted in [5, 7–9].
- Italy, Pavia (2005-2008). Collaboration with G.M.D’Ariano group from the University of Pavia on quantum cloning and optimal decorrelation of quantum states which resulted in [18, 19].
- Spain, Barcelona (2004-2008). Collaboration with Maciej Lewenstein group from ICFO-Institut de Ciències Fòniques on local quantum information processing, which resulted in [17, 22].
- Germany, Dresden (2003-2005). Collaboration with Andreas Buchleitner group from the Max-Planck-Institut für Physik Komplexer Systeme on multi-partite entanglement measures which resulted in [23].

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