Załącznik 3. Autoreferat (English)

1 Name and Surname

Jan Chwedeńczuk

2 Scientific degrees

- 2007 r. PhD degree in Physics, University of Warsaw
  PhD thesis: *Two-body collisions in cold atomic gases*
  Advisor: prof. dr hab. Marek Trippenbach

- 2003 r. Master degree in Theoretical Physics, Quantum Optics (award for the best master thesis of the year, University of Warsaw, Faculty of Physics), University of Warsaw
  Thesis: *Parametric resonanse and the stability of the Bose-Einstein Condensate*
  Advisor: prof. dr hab. Marek Trippenbach

3 Employment in academic institutions.

- 2013 r. –
  Assistant professor, Institute of Theoretical Physics, Faculty of Physics, University of Warsaw

- 2010-2013 r.
  Scientific and technical specialist, Faculty of Physics, University of Warsaw; Postdoc under the TEAM project financed by the Foundation for Polish Science

- 2008-2010 r.
  Postdoc, Universitá degli studi di Trento, Italy

4 Scientific accomplishment.

a) Title of the scientific accomplishment

publication cycle: *Interferometry in cold quantum gases*

b) (Authors, publication titles, publication year, journal name),

[1] J. Chwedeńczuk, L. Pezzé, F. Piazza, A. Smerzi, *Rabi Interferometry and Sensitive Measurement of the Casimir-Polder Force with Ultra-Cold Gases*, Physical Review A **82**, 032104 (2010)

[2] Karol Gietka, Jan Chwedeńczuk, *Atom interferometer in a double-well potential*, Physical Review A **90**, 063601 (2014)

[3] P. Szańkowski, M. Trippenbach, J. Chwedeńczuk, *Parameter estimation in memory-assisted noisy quantum interferometry*, Physical Review A **90**, 063619 (2014)

[4] J. Chwedeńczuk, F. Piazza, A. Smerzi, *Phase Estimation With Interfering Bose-Condensed Atomic Clouds*, Physical Review A **82**, 051601(R) 2010

[5] J. Chwedeńczuk, F. Piazza, A. Smerzi, *Phase Estimation from Atom Position Measurements*, New Journal of Physics **13**, 065023 (2011)

[6] J. Chwedeńczuk, P. Hyllus, F. Piazza, A. Smerzi, *Sub shot-noise interferometry from measurements of the one-body density*, New Journal of Physics **14**, 093001 (2012)

[7] J. Chwedeńczuk, F. Piazza, A. Smerzi, *Multi-path Interferometer with Ultracold Atoms Trapped in an Optical Lattice*, Physical Review A **87**, 033607 (2013)

[8] T. Wasak, P. Szańkowski, R. Bücker, J. Chwedeńczuk, M. Trippenbach, *Bogoliubov theory for atom scattering into separate regions*, New Journal of Physics **16**, 013041 (2014)

[9] T. Wasak, P. Szańkowski, J. Chwedeńczuk, *Interferometry with independently prepared Bose-Einstein condensates*, Physical Review A **91**, 043619 (2015)

[10] T. Wasak, J, Chwedeńczuk, P. Ziń, M. Trippenbach, *Raman scattering of atoms from a quasi-condensate in a perturbative regime*, Physical Review A **86**, 043621 (2012)

[11] T. Wasak, P. Szańkowski, P. Ziń, M. Trippenbach, J. Chwedeńczuk, *Cauchy-Schwarz inequality and particle entanglement*, Physical Review A **90**, 033616 (2014)

c) Description of the scientific goal of the above mentioned works, obtained results and prospects of applications.

In what follows, references [1–11] refer to the cycle of publications being the basis of the present habilitation application, references [12–21] cover some other related papers of the applicant not belonging to the cycle, whereas all the remaining references are included in order to provide a proper background for the research described. Section 4.1 has and introductory character and is the starting point for the further discussion of the original research results. Original results are summarized in section 4.2 and are described in more detail in the subsequent sections 4.3-4.8.

4.1 Introduction and motivation of the research

Quantum metrology is a dynamically developing branch of physics. Its foundation can be embodied by a following question: to which degree can the quantum phenomena be useful for improving the measurement precision. It might seem that this is a merely technical issue, which means in this case that it is such that is of interest only for the experimental physicists and out of them only those who aim at improving the experimental precision. In other words, it is hard to regard this problem as such that could link to fundamentals of quantum mechanics. However it turns out to be the opposite — modern metrology and in particular its interferometric aspect — vastly contribute to our understanding of the essentials of theory of quanta. Using a simple example, we will now discuss the basic concepts of this field. This will allow to appreciate the importance of metrology and quantum interferometry in modern physics.

4.1.1 Basic metrological scheme

The basic scheme of quantum metrology is shown in Fig. 1. A quantum state described by the density matrix $\hat{\varrho}$ undergoes the evolution generated by the

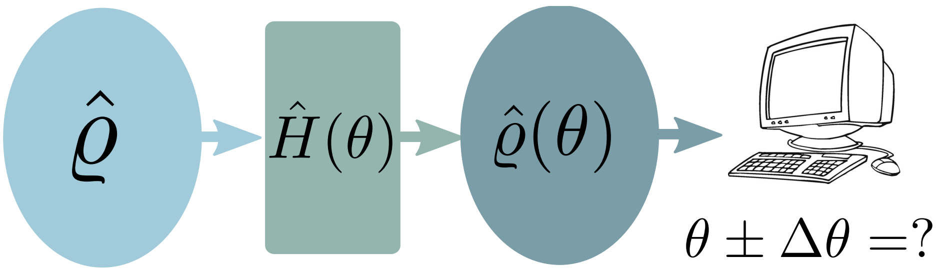

Figure 1: *Simplest metrological scheme, in which a quantum state $\hat{\varrho}$ evolves in a way, which depends on θ. As a result, we obtain a final state $\hat{\varrho}(\theta)$, on which a measurement is executed. The goal is to estimate the value of θ as precisely as possible (i.e., with as small as possible $\Delta\theta$).*

Hamiltonian $\hat{H}(\theta)$. The dependence of $\hat{H}$ on the parameter θ projects onto the form of the output state $\hat{\varrho}$. The goal faced by the quantum metrology is to estimate the value of θ with maximal possible precision, i.e., minimal possible estimation error $\Delta\theta$.

At this moment, two comments are in order. First of all, one might consider systems, where θ is its inherent property, rather then being imprinted onto the quantum state. A good example would be a system at thermal equilibrium, where θ is identified with its temperature. In such case, one should skip the first two stages of the scheme presented in Fig. 1 and imagine that from the beginning we have at hand the state $\hat{\varrho}(\theta)$, on which a measurement is performed. Second, in case of the scheme of Fig. 1, the parameter θ might be any physical quantity appearing in the Hamiltonian. Usually, θ is related to the coupling strength between some external field and the system described by the density matrix $\hat{\varrho}$.

The main foal of quantum metrology is to propose the detection scheme and a method of inferring the information from the gathered data, so that to obtain a minimal possible value of $\Delta\theta$. Moreover, with fixed $\hat{H}(\theta)$, an appropriate choice of the input state $\hat{\varrho}$ turns out to be crucial. As we will learn using the example of quantum interferometry, non-classical properties of this state are important for minimizing the estimation error.

4.1.2 Interferometry as a metrological problem

From now on we will concentrate on a particular aspect of metrology, namely the quantum interferometry. In this case, the parameter to be estimated, i.e., the θ, is the relative phase between two subsystems of the state described by $\hat{\varrho}$. The particular place, which interferometry occupies in the realm of

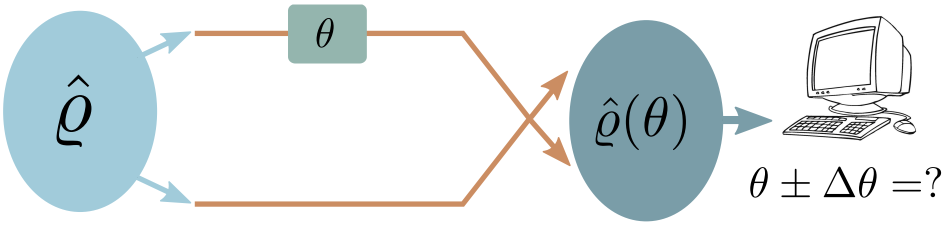

Figure 2: *Elementary interferometric scheme. A quantum state $\hat{\varrho}$ passes through two arms of an interferometer. On one of the arms, a phase θ is imprinted. The arms intersect to yield an interference signal, which provides the information about the value of the parameter. Similarly as for a generic metrological problem, the resulting output state $\hat{\varrho}(\theta)$ is a subject of measurement. Just as before, the goal is to estimate θ with small $\Delta\theta$.*

metrological inquiries is a consequence of a simplicity of the system shown in Fig. 2. Namely, it is a two-mode system (though extensions to multi-mode configurations are discussed in literatur [7]), which allows for many calculations to be executed analytically. However, the central concept of two-mode quantum linear interferometry (i.e., such that does not correlate the particles) can be formulated using the following theorem [22].

4.1.3 Interferometry and entanglement

Consider a collection of N particles in a two-mode quantum state $\hat{\varrho}$, which undergoes an interferometric transformation $\hat{U}(\theta)$. The theorem says that if the state does not contain genuinely quantum correlations, i.e., if it can be represented in the following form

$$\hat{\varrho} = \sum_i p_i \hat{\varrho}_i^1 \otimes \ldots \otimes \hat{\varrho}_i^N, \quad \text{where} \quad p_i \geq 0, \text{ and } \sum_i p_i = 1, \tag{1}$$

then the precision of the phase estimation cannot beat the shot-noise limit defined as $\Delta\theta = \frac{1}{\sqrt{N}}$. The second part of the theorem shows that nevertheless there exist states which cannot be represented in a form of Eq. (1) — those states we call "entangled" — which allow to surpass this limit, reaching up to the Heisenberg scaling $\Delta\theta \propto \frac{1}{N}$.

This is the central theorem of quantum interferometry. To summarize — it tells that quantum mechanics predicts the presence of states — strongly non-classical ones, by the way — which contain such correlations that can be treated as a resource for improving the precision of estimation. This theorem puts quantum interferometry in new light. From this perspective one can regard the interferometric problems as those which lead to the classification of entanglement. This classification is of particular interest, since it sets the entanglement in a wider context and assignes a practical aspect to it.

4.1.4 Current state of knowledge

Theoretical inquiries on interferometric systems and two-mode states focus on many aspects. A relevant issue is the impact of the environment on the performance and robustness of subtle quantum correlations [3, 23–25]. Important analysis was performed to generalize the above theorem to systems with fluctuating number of particles (with and without the atom-number coherences) [26]. The quest for novel interferometric schemes [2] and methods of obtaining entangled states [9, 27] is underway.

From the practical point of view, the most important advancements are encountered in the experimental branch of quantum interferometry. In the following, we briefly discuss the seminal experimental works.

4.1.5 Most important experimental achievements

The first paper we shall discuss was published in 2008 in a group run by Markus Oberthaler [28]. The experimentalist have prepared a Bose-Einstein condensate (BEC) of rubidium atoms, which was later split into the double-well potential, see Fig. 3(a). Next, after rising the barrier, the atom number fluctuations in each well was reduced below the shot-noise level, see Fig. 3(b). During this process, special attention was paid to sustain high inter-well coherence, witnessed by visible interference fringes obtained after the release of the gas from the trap, see Fig. 3(c). This phenomenon — reduction of the atom number fluctuations in each mode accompanied by high coherence — is called the spin-squeezing [27, 29]. One can show that the spin-squeezed states is in fact entangled and useful for quantum interferometry below the shot-noise level. The experiment [28] is a first example of a preparation of a many-body entangled state of this type.

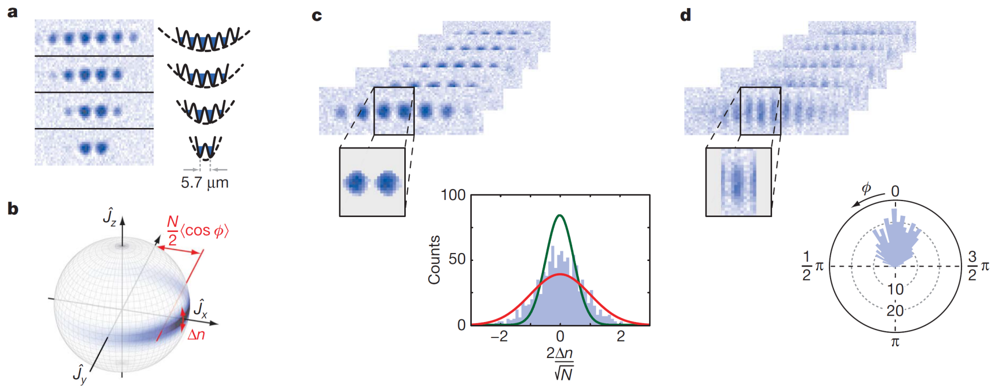

Figure 3: *Figure from [28] presenting the experimental scheme and most important results. (a) An optical lattice combined with the harmonic trap to give a double-well structure. (b) Measurement of the number of particles in each well and the histogram of outcomes, showing the comparison with the Poisson normal distribution. (c) Interference of a gas released from the two wells and the histogram of observed relative phases.*

In the years that followed, many papers appeared reporting on the generation of many-body spin-squeezed states in atomic systems [30–32]. Simultaneously, researchers began to look for alternative methods of generation on entangled states.

In [33] the effect of atom scattering from a mother cloud was employed to generate a quantum-correlated system. The experiment began from preparation of a spinor Bose-Einstein condensate with a total spin $F = 1$ in a state characterized by the projection number $M_F = 0$. Atom collisions lead to the spin-flips: a an initial pair with $M_F = 0$ each was transformed into a pair with $M_F = \pm 1$. Next, the clouds with different M_F where split using the external magnetic field with strong gradient (in analogy to the Sterna-Gerlach experiment). It has been proven that in each cloud with opposite M_F the number

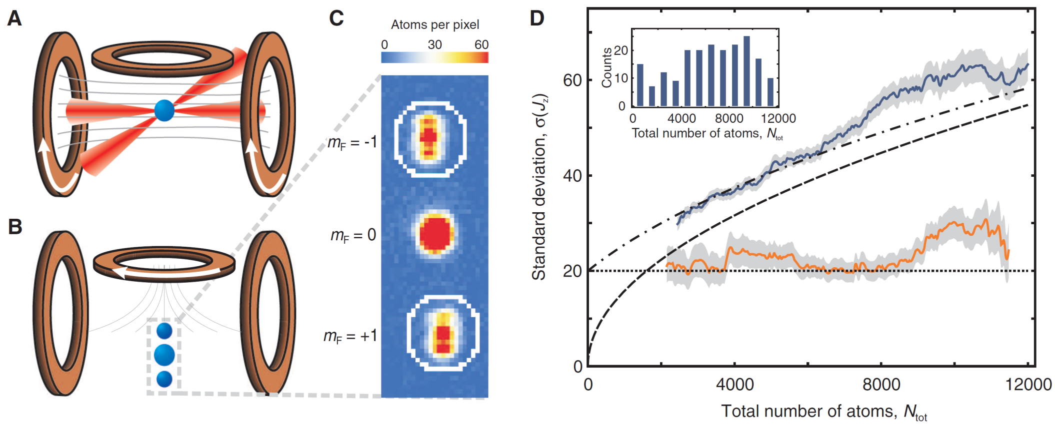

Figure 4: *Figure from [33] showing the experimental scheme and the most important results. (A) Bose-Einstein condensate of atoms with spin $F = 1$ and projection $M_F = 0$ in a magnetic trap. (B) Collisions between the atoms lead to generation of pairs of particles with opposite M_F. (C) Sterna-Gerlach experiment splits the clouds with different M_F. (D) Measurement of the atom number difference between the two clouds proofs the pair-scattering.*

of atoms in equal, which confirms the fact that the clouds were formed in the pair-scattering process. Next, this correlated state was used to estimate the rotation angle of the atomic analog of the non-symmetric beam-splitter. It was shown that the precision of the estimation is better then the shot-noise limit. In this way, the authors proofed that the state of scattered atomic pairs is non-classically correlated, in analogy to the state of photon pairs obtained in the parametric-down conversion process [34, 35]. It is important to stress, that a similar experimental scheme, basing on the scattering of pairs from a Bose-Einstein condensate, was employed in other experiments such as [36] obtained in the group of Jörg Schmiedmayer in Vienna or [37] by the group of Chris Westbrook from Palaiseau/Paris.

Finally, we mention a recent experimental work, also published by the group of Markus Oberthaler [38]. The main result of this work is based on the analysis of strongly correlated systems, which are not characterized by the spin-squeezing parameter. In the experiment, strong two-body interactions were employed to entangle the particles. Initially, the state was spin-

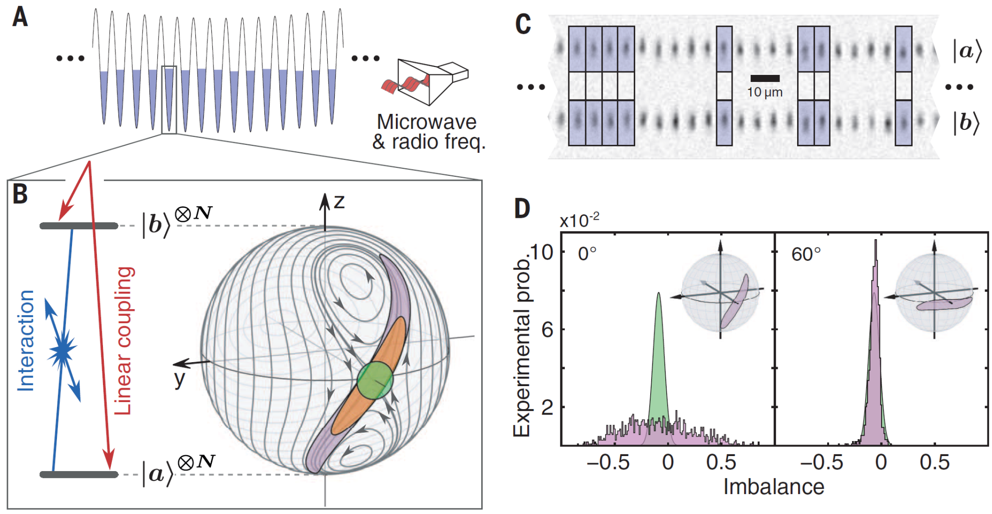

Figure 5: *Figure from [38] showing the experimental scheme and the most important results. (A) Bose-Einstein condensate split into the optical lattice. (B) In each site of the lattice, a correlated atomic state is generated by means of two-body collisions. (C) Measurement of the number of particles in two modes related to the internal degrees of freedom, performed in each lattice site. (D) Outcome of the measurement, showing strong correlation between the modes.*

squeezed, however at later stages of the evolution, the correlation increased leading to generation of a non-Gaussian entangled state. It was shown that a full knowledge about the correlations might be deduced from a sophisticated tool called the Fisher information [39, 40].

Modern interferometers operating on non-classical states of light or matter are rather proof-of-principle devices, demonstrating profound skills of the experimentalists. The effort, which is required to obtain a non-classical state and then to protect it from the destructive impact of the environment, is very high as compared to the revenue. In some sense, quantum interferometry is at a stage which resembles the situation of cars that run on electricity. The immense cost of the research and development and the price of the final product vastly surpass the cost of an energy-efficient car running on oil. Nevertheless, in a longer run, electric or hydrogen-driven cars seem to be the only possible path to follow for the car industry.

The similarity with interferometry is striking — in a long-time perspective, "nano-rulers" must be able to benefit from quantum entanglement. The reason for that lies in the potential applications of quantum interferometers. They are intended to probe some most etheric phenomena in the micro- and the nano-scale. For example, it is considered to use an atomic interferometer (which is such that operates on the matter-waves, in contrast to first interferometers, which used ligth) to estimate the value of the gravitational constant G (the "large" one, in contrast to the small g — which is the local acceleration). The prospective measurement would be based on the interaction of a massive object (a sphere made of steel would be a good example) with a non-classical two-mode state of matter. Such interaction would lead to the phase-imprint between the two modes, from which the coupling constant could be estimated. Since the number of particles N, which can be put in a highly susceptible quantum state is limited, it is crucial that the sensitivity of the phase estimation strongly scales with N.

To summarize, quantum interferometry is a rapidly developing branch of metrology, and it provides vital information about non-classical correlations between the particles. The advancement in the theoretical and experimental research proofs large interest, sparked by quantum interferometry, in the leading groups around the world.

4.2 Summary of the key results

Quantum interferometry, as argued in Section 4.1, connects fundamental problems of quantum mechanics with practical aspects. For this reason, research conducted in this field contributes to the better understanding of the basics of quantum physics and, in a broader perspective, might lead to creation of ultra-precise detection devices.

There are many alternative applications of atomic interferometers. The perspective of future benefits originating from the better understanding how the non-classical correlations emerge, how they are influenced by the environment and how the could be employed in realistic experimental setups, was the motivation for the research I conducted.

The main directions of my inquiries were: the quest for novel interferometric schemes, generation of non-classical correlations in many-body systems and the methods for their detection.

Within the search for novel interferometric schemes, I analyzed systems based on coherent oscillations of a quantum gas in a double-well potential [1–3]. I have demonstrated that basing on the observed frequency of oscillations, it is possible to precisely estimate the ground state energy difference between the two wells. The precision of the estimation of this parameter was expressed in terms both of elementary properties of the system as well as the particle entanglement. Besides, I have analyzed the impact of environment on the precision of estimation. The detailed results are presented in Section 4.3.

In the series of articles [4–6] I have concentrated on a system of two interfering atomic clouds with an unknown relative phase θ. I have demonstrated that such simple interferometric setup allows for precise determination of the phase basing on the least-squares method of fitting of the density curve to the acquired positions of atoms forming the pattern. I have also established a relation between the particle entanglement and the estimation precision. In the work [7] I have generalized these results to the case when the interference pattern emerges as a result of overlap of many wave-packets released from an optical lattice. The detailed results are discussed in Section 4.4.

The papers [8, 9] are devoted to the analysis of generation of non-classical correlations in atomic systems. The article [8], which was prepared in collaboration with Robert Bücker – an experimentalist from the Vienna group of prof. Jörg Schmiedmayer – was dealing with the process of emission of entangled atomic beams from a BEC. We have demonstrated that this system reveals strong entanglement, useful for precise quantum metrology. In [9] we have shown that due to the particle indistinguishability of bosons, two independently prepared BECs might be strongly and usefully entangled. The detailed discussion is contained in Section 4.5.

The article [10] presents an the analysis of the process of scattering of light on a BEC in the Raman scheme. The relation between the correlation functions of emitted particles and the properties of the mother cloud is presented. The details are contained in Section 4.6.

The work [11] contains a proof that the violation of the Cauchy-Schwarz inequality by the second order correlation function is a simple criterion for the particle entanglement. The derivation is based on the analogy between the coherent states of light and the classical states of matter. A relation between this inequality and other criteria for particle entanglement is also shown. For detailed discussion of these results, see Section 4.7.

4.3 Novel interferometric schemes

A relevant element of an interferometer, apart from the phase imprint, is the beam-splitter which is used to interfere the signal coming from the two arms of an interferometer (see the scheme of Fig. 2). To devise such an element for photonic interferometry is not a difficult task – one can find a beam-splitter for a moderate cost in the Internet. On the other hand, to build an atomic analog of a beam-splitter, is a challenging task. This means that to close the interferometric cycle, for instance in the Mach-Zehnder setup, is in case of atoms usually very difficult.

Usually, an atomic interferometer is generated by putting a cold gas in a double-well potential. In the Bose-Hubbard approximation, the Hamiltonian of such system takes an appealing form

$$\hat{H}_{\mathrm{bh}} = -E_J \hat{J}_x + \delta \hat{J}_z + U \hat{J}_z^2, \tag{2}$$

where the operators $\hat{J}_x = \frac{1}{2}(\hat{a}^\dagger \hat{b} + \hat{a}\hat{b}^\dagger)$ i $\hat{J}_z = \frac{1}{2}(\hat{a}^\dagger \hat{a} - \hat{b}^\dagger \hat{b})$ together with $\hat{J}_y = \frac{1}{2i}(\hat{a}^\dagger \hat{b} - \hat{a}\hat{b}^\dagger)$ form the closed algebra of the angular momentum (operators $\hat{a}/\hat{b}$ annihilate a boson in the left/right potential well). The physical interpretation of each term in this Hamiltonian is following: $-E_J \hat{J}_x$ drives the inter-well tunneling of particles, $\delta \hat{J}_z$, coming from the energy difference between the wells, imprints the relative phase, while $U \hat{J}_z^2$ is a result of the two-body collisions between the atoms. If one could “turn off” the interactions (for instance using the technique of the Feshbach resonances [41–43]) and equalize the energy in both wells (i.e. provide that $\delta = 0$), the above Hamiltonian would consist of the tunneling term only. By setting the time evolution t such that $t \cdot E_J/\hbar = \frac{\pi}{2}$, we obtain the evolution operator $\hat{U} = \exp\left[-i\frac{\pi}{2}\hat{J}_x\right]$, which is the atomic analog of the beam-splitter. It might seem therefore that the procedure described above allows for a quick generation of the symmetric signal-splitting device. However it turns out that it is difficult to reach this goal in the lab – this is because it is difficult to precisely fix $\delta = 0$; moreover, often the precise value of E_J is not know, which implies that the splitting is not symmetric.

These difficulties motivated our inquiries: the goal we set was to propose an interferometric scheme in a double-well potential not using the symmetric beam-splitter. To this end, we considered a Hamiltonian (2), assuming that the two-body collisions can be neglected, giving

$$\hat{H}_{\mathrm{bh}} = -E_J \hat{J}_x + \delta \hat{J}_z. \tag{3}$$

Since, as stated above, the operator $\hat{J}_z$ imprints the phase among the wells, the quantity $\delta t/\hbar$ can be identified with the phase θ. The complete evolution thus consists of *simultaneous* phase imprint and coherent oscillations

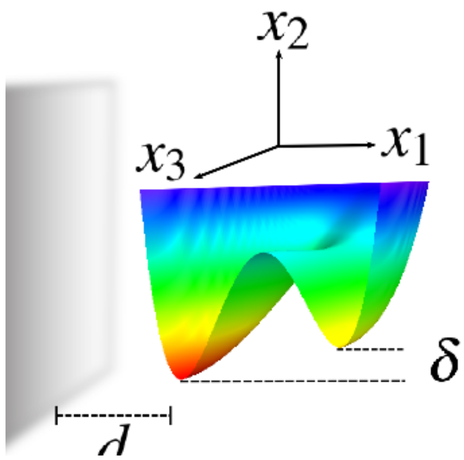

Figure 6: *Scheme of the setup, which consists of the quantum gas trapped in a double-well potential and a dielectric plate put at a distance d, which induces the Casimir-Polder force on the gas. The figure comes from the article [1].*

among interferometric arms. In this way, an alternative scenario to the Mach-Zehnder interferometer (MZI) is realized, in which the phase-imprint is “sandwiched” by the two beam-splitters and these two types of transformation do not happen at once.

In the paper [1] we have performed an analysis of the usefulness of such an system, assuming that the initial state of the gas is spin-squeezed (see Section 4.1). We have also assumed that the energy difference between the wells (i.e. $\delta \neq 0$) is a result of the action of the Casimir-Polder force acting from a dielectric surface put at a distance d from the potential wells [44–46], as schematically shown in Fig. 6. We have shown that such system is a very sensitive probe, allowing to distinguish between the thermal and the quantum branch of the Casimir-Polder interaction.

The above considerations were extended in [2].This work contains a precise analysis of the performance of an atomic interferometer in a double-well potential, depending on the type of quantum correlations present in the initial state. We have established under which conditions the sub shot-noise sensitivity can be reached using the spin- or the phase-squeezed states. Besides, we have verified how the phase sensitivity depends on the values of the parameters E_J and δ.

In the work [3] we have presented the results of our research on the impact of noise on the performance of an atomic interferometer realized in a double-well potential. We have assumed that each particle of the quantum gas is influenced by the environment in the same way. This is the only choice of indsitinguishable particles, which many-body wave-function must be symmetrized. The considered Hamiltonian was

$$\hat{H} = \delta \hat{J}_z + \boldsymbol{\omega}(t) \cdot \hat{\mathbf{J}}, \tag{4}$$

where δ, as previously, is the detuning between the wells, while the vector $\boldsymbol{\omega}(t)$ characterizes the impact of the environment To find the dynamics of the system in the presence of the noise modelled by $\boldsymbol{\omega}(t)$ requires first the finding of the evolution operator

$$\hat{\varrho}(t; \boldsymbol{\omega}_0) = \hat{U}(t; \boldsymbol{\omega}_0)\, \hat{\varrho}(0)\, \hat{U}^\dagger(t; \boldsymbol{\omega}_0). \tag{5}$$

for a single trajectory of the stochastic process $\boldsymbol{\omega}_0$, where

$$\hat{U}(t; \boldsymbol{\omega}_0) = \mathcal{T} \exp\left[-i \int_0^t dt' \left(\Omega \hat{J}_z + \boldsymbol{\omega}_0(t') \cdot \hat{\mathbf{J}}\right)\right], \tag{6}$$

while $\mathcal{T}$ stands for the time ordering. Next, the density matrix (5) must be averaged over to ensemble, which yields

$$\hat{\varrho}_S(t) = \sum_{\boldsymbol{\omega}_0} \hat{\varrho}(t; \boldsymbol{\omega}_0) = \int \mathcal{D}\boldsymbol{\omega}_0\, \mathcal{P}(\boldsymbol{\omega}_0)\, \hat{\varrho}(t; \boldsymbol{\omega}_0). \tag{7}$$

In our study we have considered the case, when $\boldsymbol{\omega}$ is a Gaussian process with the correlation time τ_c, i.e.

$$\overline{\boldsymbol{\omega}(t)} = 0 \qquad \text{and} \qquad \overline{\omega_i(t)\omega_j(t')} = \kappa(|t - t'|)\delta_{ij}, \tag{8}$$

where the indeces i and j stand for different cartesian coordinates, while $\kappa(|t - t'|)$ is the correlation function with the range of the order of τ_c. Having $\hat{\varrho}_S(t)$, we could determine the quantum Fisher information [1], under the assumption that the metrological parameter is δ. Our main result is that there is a method for preventing the destructive impact of the environment,

[1]The quantum Fisher information is a quantity, which with the interferometric sequence and the initial quantum state both fixed, provides the lower bound for $\Delta\theta$, optimized over all possible methods of estimation [40]

given that the whole interferometric sequence happens within τ_c. In other words, the correlation time of the noise coming from the environment sets a new time scale, within which the process of the loss of coherence is not yet dominating. If the sequence last for longer than τ_c, the negative impact of the environment limits the coherence of the system, and thus destroys the entanglement. We have also analyzed what is the impact of the environment on a particular estimation scheme – based on the measurement of the number of particles in each potential well. This work presents, for the first time, the analysis of the performance of a bosonic interferometer influenced by the environment, acting, according to the requirements of the indistinguishability, on each particle in the same manner.

4.4 Phase estimation from the interference pattern

The sequence described above, which connects the phase imprint with the coherent oscillations between the potential wells, is a simplification over the MZI. In the series of articles [4–7] we have considered a scheme, which is even simpler and consists of two elements only: pure phase imprint (in absence of tunneling between the wells) which is followed by the switch-off of the trapping potential. This leads to free expansion of both clouds, which in the far field overlap and form an interference pattern. From this pattern of fringes, the phase θ is to be estimated.

The questions which we posed were following: what is the ultimate phase sensitivity in such a scenario? which estimation scheme is optimal? which entangled states are useful for such type of interferometer? and: how much can be judged about the phase from the simple measurement of the density, without any knowledge about the correlations among the particles.

In the first paper belonging to this cycle we have analyzed how much information about the phase θ can be deduced knowing the full, i.e.the N-body correlation function of the interference pattern [4]. The model assumes that the quantum state is two-mode, which is

$$|\psi\rangle = \sum_{n=0}^{N} C_n |n, N-n\rangle, \tag{9}$$

where C_n is the probability amplitude for finding n particles in the mode a and $N-n$ in b. The information about the spatial properties is contained in the bosonic field operator, which after the phase-imprint reads

$$\hat{\Psi}(\mathbf{r}, t) = \psi_a(\mathbf{r}, t)\hat{a} + \psi_b(\mathbf{r}, t)e^{i\theta}\hat{b}. \tag{10}$$

The wave-packets $\psi_{a/b}(\mathbf{r}, t)$ are initially localized in the two potential wells, and subsequently freely evolve and overlap, giving the interference pattern. We have demonstrated, that if the quantum state (9) has all the coefficients C_n real and fulfilling the symmetry condition $C_n = C_{N-n}$, then the estimation from the N-th order correlation function

$$G_N(\mathbf{r}_1 \ldots \mathbf{r}_N; t) = \langle \hat{\Psi}^\dagger(\mathbf{r}_1, t) \ldots \hat{\Psi}^\dagger(\mathbf{r}_N, t)\hat{\Psi}(\mathbf{r}_N, t)\hat{\Psi}(\mathbf{r}_1, t)\rangle \tag{11}$$

is optimal, giving the value of $\Delta\theta$ bounded by the quantum Fisher information [40]. We have identified the entangled two-mode states of the form (9), which allow for breaking the shot-noise limit (SNL): these are the states with reduced fluctuations of the phase and the NOON-like states. Moreover, in [4] we have shown that the estimation from the measurement of the center-of-mass of the interference pattern also allows for breaking the SNL, for as long as the state (9) is NOON-like.

In [5] we have extended the inquiries of [4] to the case of estimation from lower-order correlation functions. We have demonstrated that the information about the phase increases as the order of correlation grows. Next, we have analyzed how sensitive to the drop of detection efficiency is the precision of estimation from the center-of-mass of the system described by the equations (9,10). As expected, NOON-like states, which are useful for this estimation scheme, impose serious requirements onto the detection techniques. In the work [5] we have also performed a full analysis of the estimation of the phase θ from the measurements of atom positions in the MZI scheme.

The most important results of this cycle are contained in [6]. We have assumed that the only information one has at hand in that about the one-body density of the interference pattern, i.e.,

$$\rho(\mathbf{r}, t) = \langle \hat{\Psi}^\dagger(\mathbf{r}, t)\hat{\Psi}(\mathbf{r}, t)\rangle. \tag{12}$$

This function of the variable $\mathbf{r}$ is fitted using the least-squares-fit method to the measured positions of single atoms, where the free fitting parameter is the unknown phase. We have shown that in this way, though no information about the correlations among the particles is known, one can beat the SNL. The usefully entangled states are those with reduced phase fluctuations. Our research was extended to the analysis of the impact of detection

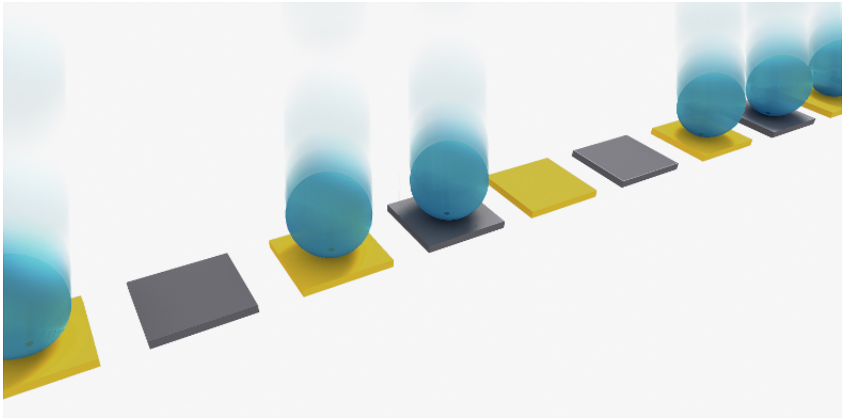

Figure 7: *Figure taken from [6] showing two basic sources of imperfection in the detection of positions of single atoms (represented by the blue spheres). The detectors (rectangular plates) might either not register an atom at all (second detector from the right) or it might happen that the signal is detected by one of the neighbouring detectors (fourth and fifth plate from the right).*

imperfections, schematically shown in Fig. 7. We have taken into account the possibility of both the limited spatial resolution as well as the unperfect detector efficiency. We have set the minimal constraints for the detection methods to have the possibility to beat the SNL.

In the work [7] we have extended the results of [6] into the case, when the quantum gas is trapped in an optical lattice with M sites. This means that one must generalize the state (9) into

$$|\psi\rangle = \sum_{n_1=0}^{N} \sum_{n_2=0}^{N-n_1} \cdots \sum_{n_{M-1}=0}^{N-n_1-\ldots-n_{M-2}} C_{n_1\ldots n_{M-1}} |n_1, n_2, \ldots, n_{M-1}, N-n_1-\ldots-n_{M-1}\rangle, \tag{13}$$

while the field operator between the phase imprint reads

$$\hat{\Psi}(x) = \sum_{k=1}^{M} \psi_k(x)\hat{a}_k. \tag{14}$$

Assuming that the phase θ grows linearly along the lattice, we have shown that in general the precision of the estimation will also grow linearly with M.

Moreover, basing on the results of [6] we have shown that also in the case of optical-lattice interferometry, the interference pattern form by the overlap of the gas released from the M sites allows for the phase estimation from the density beating the SNL. Also in this case, the $\Delta\theta$ scales with M.

To summarize, the cycle [4–7] presents an analysis of the phase estimation in the simplest interferometric scheme – from the pattern of interference fringes. We have analyzed how, depending on the order of correlation function and the entanglement present in the input state, the precision of the estimation $\Delta\theta$ scales with the number of particles N.

4.5 Generation of non-classical many-body states

As underlined in the Section 4.1, generation of non-classical many-body states of light or matter is the key part of quantum metrology. In the optical domain, usually the correlated photonic pairs are generated in the parametric down conversion process [34, 35]. For massive particles, the two-body collisions might generate strong correlations (see Section 4.1). In the works [9] and [8] we have analyzed some alternative methods of creating entangled states in ultra-cold gases.

In the work [8] we have presented a theoretical description of the emission of correlated atomic beams from a BEC. This effect has been observed in the experiment run by the group of Jörg Schmiedmayer in Vienna [47], from which the Fig. 8 is taken. This Figure shows the scheme for creating the twin beams: the BEC is shaken to provide the energy necessary to drive the system into the first excited state of the harmonic trap. The two-body collisions scatter pairs of atoms back into the ground state. The energy excess leads to the acquisition of the momenta $\pm\hbar k_0$ by the pair. This momentum is oriented along one line, which is a result of the geometry of the system. The increasing number of scattering events leads to the generation of two counter-propagating beams, shown in Fig. 8(b) in red.

In collaboration with dr. Robert Bücker, who was an author of [47], we have presented a theoretical model for this scattering process. Our work is based on the Bogoliubov approximation, which assumes that the particles scattered from the BEC wave-function $\psi(\mathbf{r})$ are a small part of the whole system. Therefore, the field operator $\hat{\Psi}(\mathbf{r})$ reads

$$\hat{\Psi}(\mathbf{r}) = \psi(\mathbf{r}) + \hat{\delta}(\mathbf{r}), \tag{15}$$

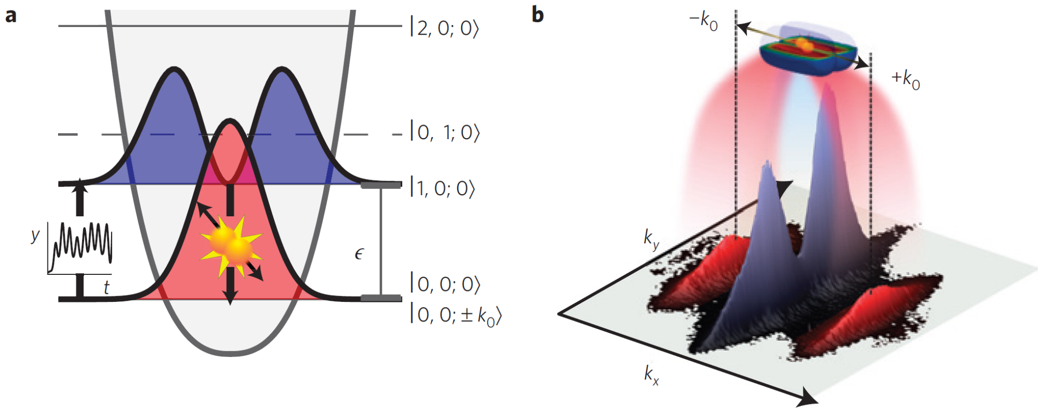

Figure 8: *Figure from [47] shows (a): the scheme of the BEC shaking, which excites the cloud into the first state of the trapping potential and (b): result of measurement of the BEC (blue) and twin-beam (red) system.*

where the operator $\hat{\delta}(\mathbf{r})$ is in some sense "small". The effective Hamiltonian describing the dynamics of scattering to the field $\hat{\delta}(\mathbf{r})$ is quadratic and reads

$$\hat{H}_{\mathrm{bog}} = \int d\mathbf{r}\, \hat{\delta}^{\dagger}(\mathbf{r}) \left(-\frac{\hbar^2 \nabla^2}{2m} + V(\mathbf{r}) + 2g|\psi(\mathbf{r})|^2 \right) \hat{\delta}(\mathbf{r}) \tag{16a}$$

$$+ \; g \int d\mathbf{r} \left(\hat{\delta}^{\dagger}(\mathbf{r}) \hat{\delta}^{\dagger}(\mathbf{r})\, \psi^2(\mathbf{r}) + \mathrm{h.c.} \right). \tag{16b}$$

Since the scattered pairs of particles have opposite momenta, oriented along the line, the Bogoliubov operator in the momentum space might be represented as

$$\hat{\delta}(\mathbf{k}, t) = \sum_i \varphi^{(i)}_{+k_0}(\mathbf{k}, t)\, \hat{a}^{(i)}_{+k_0}(t) + \sum_i \varphi^{(i)}_{-k_0}(\mathbf{k}, t)\, \hat{a}^{(i)}_{-k_0}(t), \tag{17}$$

where the sum runs over all possible scattering states, while $\varphi^{(i)}_{\pm k_0}(\mathbf{k}, t)$ describes the spatial properties of each mode. Thanks to the Eq. (17), the dynamics can be easily found using simple numerical methods. Having found $\hat{\delta}(\mathbf{k}, t)$ we have gained the full information about the system. This allowed for the evaluation of the correlations between the scattered atoms, as well as the estimation of the amount of metrologically useful entanglement in the system. For example, Fig. 9 taken from [8] shows the dependence of the quantum Fisher information (QFI) on time, juxtaposing this result with the

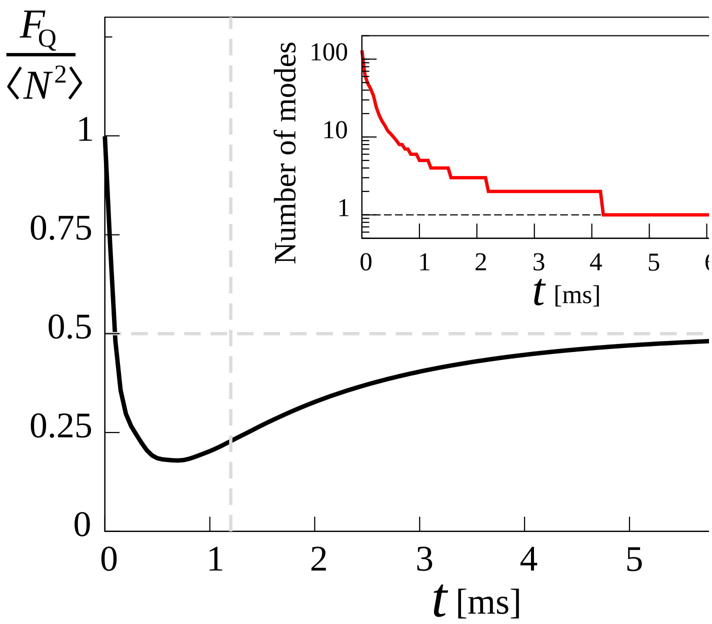

Figure 9: *Figure taken from [8] showing the quantum Fisher information as a funtion of the scattering time. The inset shows the number of active modes in each of the counter-propagating beams.*

number of significantly occupied modes in each beam. During the scattering, as the number of modes drops, the QFI grows, as expected from the multi-mode quantum interferometry. A simple theoretical model contained in [8] might in future be applied to describe the experiments where the particles are scattered in pairs into disjoined regions.

The study [9] presents another approach to the generation of entangled many-body states, much different from that presented in Section 4.1, as well as in the work [8]. The motivation to this inquiry was the Hong-Ou Mandle (HOM) effect [48], where two photons described by a pure state

$$|\psi\rangle = |1, 1\rangle \tag{18}$$

pass a beam-splitter giving at the output

$$|\psi\rangle = \frac{1}{\sqrt{2}} \left(|2, 0\rangle + |0, 2\rangle\right), \tag{19}$$

known as the NOON state. It is a non-classical state, with the superposition of two photons in one arm and none in the other + vice versa. The key observation is that in the state (18), the two photons might be taken from two independent sources. If so, where does the entanglement of (19) come from? The answer is related to the statistics of photons – these are bosons thus they interfere in a particular way. This interference cancels the coincidences, where in each arm one photon propagates. This destructive interference on the beam-splitter, is the essence of the HOM effect.

If two photons coming from independent sources might interfere in this way, is it also possible for a larger number of particles? It is known that the generalization of (18), i.e.,

$$|\psi\rangle = |N, N\rangle \tag{20}$$

passed through the beam splitter, gives an output which is close to the NOON state [49–52]. The question we posed was the following: can two independently prepared BECs (20), interfering on a beam splitter, give a non-classical state of matter, useful for quantum interferometry?

A realistic model of a BEC is such that takes into account atom number fluctuations from shot to shot. This means that the two-BEC state must be described by the density matrix

$$\hat{\varrho} = \left(\sum_{N_a=0}^{\infty} P_a(N_a)|N_a\rangle\langle N_a|\right) \otimes \left(\sum_{N_b=0}^{\infty} P_b(N_b)|N_b\rangle\langle N_b|\right), \tag{21}$$

where $P_{a/b}$ is the probability for having $N_{a/b}$ particles in the a/b BEC. For such an input state passing through the MZI we have calculated the QFI and have shown that it has a value larger than $\bar{N}$ [2] when the atom number fluctuations in each BEC are below the SNL. This is an important result, which indicates that it is sufficient to limit the fluctuations in two independently prepared BECs without any entangling procedure, to obtain a non-classical state of matter. Moreover, we have shown that the measurement of the number of atoms in each arm of the MZI allows for precision surpassing the SNL. The results of this work might in future allow for construction of a novel type of an interferometer, based on the many-body analog of the HOM effect.

4.6 Interaction of light and matter – Raman scattering

The publication [10] presents an analysis of scattering of atom-photon pairs in the Raman process resulting from the enlightening of a BEC with a laser

[2] where $\bar{N}$ is the mean number of particles in the system, thus $F_Q > \bar{N}$ means breaking the SNL [26]

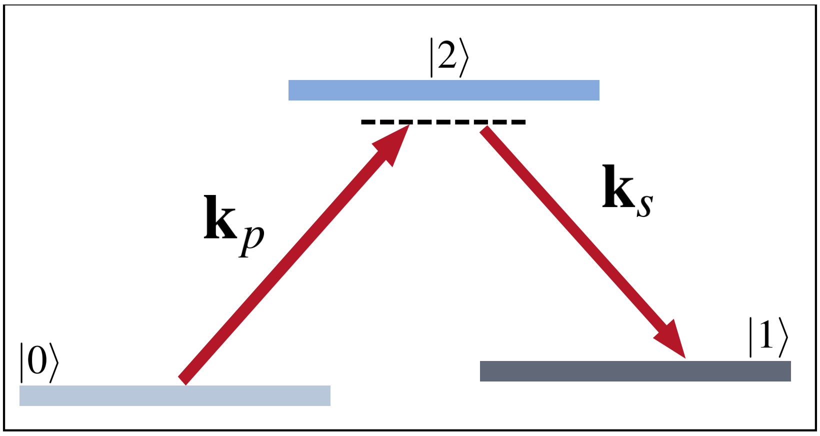

Figure 10: *A figure showing a single act of scattering of the incoming photon (having momentum* $\mathbf{k}_p$*), leading to an emission of a signal photon with momentum* $\mathbf{k}_s$ *together with the transition of an atom from* $|0\rangle$ *through* $|2\rangle$ *to* $|1\rangle$*.*

beam. The scheme of a single scattering event in shown in Fig. 10. A photon from the laser beam (called the pump) is absorbed by an atom, triggering an internat transition of an atom from $|0\rangle$ to $|2\rangle$. An immediate act of emission leads to an atomic transition $|2\rangle \to |1\rangle$. Since the momentum of the spontaneously scattered photon can be any, also the momentum of the atom after the interaction attains a random direction, though correlated with the photon. In such a way a pair atom-photon is emitted. The publication [10] presents an analysis of this effect, allowing for a possibility of a non-zero temperature of the mother cloud. If this is a case, a regular BEC is transformed into a quasi-BEC – an almost coherent system, where phase fluctuations limit the spatial coherence [53–55]. We have analyzed the impact of the magnitude of these phase fluctuations (i.e., the impact of finite temperature) on both the range of the atomic correlations and their amplitude. In our study we have taken into account also the finite expansion time of the atomic cloud (it is the time which it takes for the cloud to reach the detector, once released from the trap). The results of our study might be helpful in future preparations and analysis of the Raman scattering in atomic gases, in particular having in mind potential applications for quantum interferometry.

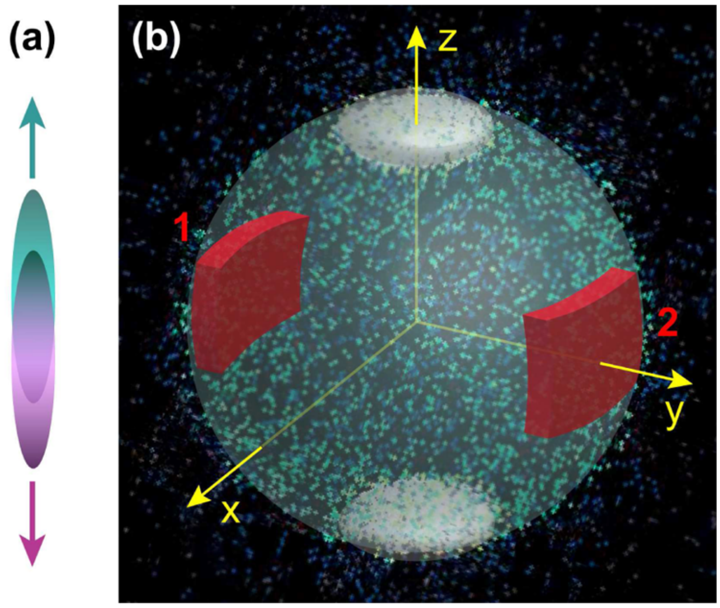

Figure 11: *Figure taken from [56]. (a): system of two cylindrically-shaped BECs colliding along the axis of symmetry; (b): an observed halo of scattered atoms, together with the two BECs (light areas at the bottom and at the top) and the two regions, in which the second order correlation function is measured (red cuts of the sphere).*

4.7 Entanglement criterion – Cauchy-Schwarz inequality

The study [11] presents a proof of a statement that the violation of the Cauchy-Schwarz inequality by the second-order correlation function in a system of bosons is a criterion for particle entanglement. The need for such a proof was a result of the work [56], in which the observed violation of the above inequality is treated as a criterion for non-classicality in a system of particles scattered during a collision of two BECs. As shown in Fig. 11, the two colliding BECs (Fig. 11(a)) emit pairs of particles, which, due to the energy and momentum conservation scatter into a sphere, which radius increases linearly in time (Fig. 11(b)). The second-order correlation function of the scattered atoms was measured, i.e.,

$$\mathcal{G}^{(2)}(x,x') = \left\langle \hat{\Psi}^\dagger(x)\hat{\Psi}^\dagger(x')\hat{\Psi}(x')\hat{\Psi}(x) \right\rangle . \tag{22}$$

Next, to increase the signal, this function was integrated over the volume of the two regions marked in red in Fig. 11(b). In this way, three object were obtained

$$\mathcal{G}^{(2)}_{aa} = \int_{V_a} dx \int_{V_a} dx' \mathcal{G}^{(2)}(x,x') \tag{23a}$$

$$\mathcal{G}^{(2)}_{ab} = \int_{V_a} dx \int_{V_b} dx' \mathcal{G}^{(2)}(x,x') \tag{23b}$$

$$\mathcal{G}^{(2)}_{bb} = \int_{V_b} dx \int_{V_b} dx' \mathcal{G}^{(2)}(x,x'). \tag{23c}$$

It was shown that these quantities satisfy

$$\mathcal{C} \equiv \frac{\mathcal{G}^{(2)}_{ab}}{\sqrt{\mathcal{G}^{(2)}_{aa}\mathcal{G}^{(2)}_{bb}}} > 1. \tag{24}$$

The authors, in analogy to the violation of a similar inequality in photonic systems, treated this result as a proof of non-classicality in the system of massive particles.

In order to specify what this statement means, one must note that a classical (separable) system of N bosons has a form

$$\hat{\rho} = \int \mathcal{D}\phi \, |\phi;N\rangle\langle\phi;N| \mathcal{P}(\phi), \tag{25}$$

where $|\phi;N\rangle = |\phi\rangle^{\otimes N}$ is a coherent state of N bosons, while $\mathcal{P}(\phi)$ is a probability distribution. This form stands in a clear analogy to a classical state of electromagnetic field according to the Glauber-Sudarshan P-representation [57, 58]. For photons, this form of the state is used to derive an analog of the inequality (24). This observation leads to an immediate conclusion that the violation of the (24) by massive particles is a criterion for particle entanglement in a system of massive bosons. The details of the proof are presented in [11]. This result can be generalized to systems, where the number of particles fluctuates from shot to shot, provided there is no coherence between different number states. Our proof provides an explanation for the non-classical phenomenon observed in [56] as well as in many other experiments with cold atoms, where the violation of the Cauchy-Schwarz inequality serves as a criterion for the quantumness of the state.

4.8 Summary

To summarize, the cycle of publications [1–11] presents a research on the non-classical properties of ultra-cold atoms. The contained analysis of types of correlations and methods of their detection is mostly devoted to the potential future application for quantum metrology.

Current state of knowledge, both in the experimental and the theoretical domain, to which the presented publications contribute, makes me believe that soon a fully operational atomic interferometer surpassing the SNL will be available.

Together with my colleagues we continue the research in the atomic interferometry domain. The paper analyzing the impact of hidden (parasite) modes on the efficiency of an interferometer, is due to appear. Soon, we plan to begin the collaboration with the group of prog. Helmut Ritsch from Innsbruck. Within this activity we will focus on a new aspect of atomic interferometry – a scheme based on generation of many-body entangled states in the optical cavities.

5 Other research accomplishments.

a) bibliometric data (valid on 10th of September 2015)

Number of published papers: 26 + 3 papers on arxiv
Citations: 121
Total impact factor: 81.22
Hirsch index: 8

b) research not contributing to the habilitation

In the years 2004-2013 I co-authored a cycle of works devoted to the collisions of two BECs [13, 15–20]. In this process, described in Section 4.7 and shown in Fig. 11, pairs of correlated atoms are scattered. The aim of these studies was to calculate the shape of the cloud of scattered atoms, the strength of correlations among the particles as well as the rate of scattering. In the work [17] we have managed to predict an outcome of a single scattering event, by drawing the positions of emitted atoms from the full probability distribution. In [18] the predictions of our model were compared with the outcomes of the experiments, reaching a very good agreement. In [13] we have presented an analysis of the scattering from the point of view of the usefulness of the system for quantum interferometry. We have studied what is the degree of the number squeezing between the opposite regions of the scattering halo and how this squeezing depends on the parameters of the collision – such as the shape of the condensates and their relative speed.

In [12] we have found a number of optimal measurements (such that saturate the bound set by the QFI) in various interferometric setups. In collaboration with prof. Augusto Smerzi from Florence, we have made an analytical calculations and managed to find a broad set of optimal measurements.

In [14] we have analyzed the dynamics of generation of non-classical states of matter in a process of dynamical squeezing of an initially coherent state [27, 29, 31, 32, 38]. We have found an efficient phase estimation method and have found the impact of the limited detection precision onto the performance of an interferometer.

In [21] we have presented an analysis of the properties of a field of photons emitted in the Raman scattering process. We have evaluated all the relevant correlation functions and the impact of the environment on the loss of coherence of the system.

c) awards

2003: Award for the best master thesis of the year at the Faculty of Physics, University of Warsaw
2006: Award for the best teacher of the year at the Faculty of Physics, University of Warsaw
2007: START scholarship founded by the Foundation for Polish Science
2008: Participation in the young researchers' meeting with the Nobel Prize

laureates, Lindau, Germany
2012: Scholarship for an outstanding young researcher, Ministry of Science and Higher Education, Poland

d) directing research projects

2012-2016: national project "SONATA" financed by the Nation Science Center, "Generation and application of non-classical atomic states for quantum interferometry"

e) participation in research projects

- KBN grant number N202 022 32/0701 2007-2010 "Many Body and Nonlinear Effects in Bose Einstein Condensates and waveguides" - Faculty of Physics, University of Warsaw
- KBN grant number 2P03B04325 2003-2006, "Nonlinear Optics of Atoms and Photons" - Faculty of Physics, University of Warsaw
- PhD grant N0413/P03/2005/29 2005-2006 "Two-body effects and elastic losses in a BEC" - Faculty of Physics, University of WarsawW
- Grant number N202 019 32/0698 2007-2008 "Parametric amplification of ultra-fast pulses of light, the role of non-axial configuration" - Faculty of Physics, University of Warsaw
- European grant number 06-EuroQUAM-FP-002 "Controlled Interactions in Quantum Gases of Metastable Atoms (CIGMA)" within the EUROCORES programme, 2006-2010
- Postdoc within the TEAM programme "Photonic Implementations of Quantum Enhanced Technologies", Foundation for Polish Science, contract number: TEAM/2009-3/1, 2010-2013
- NCN grant number 1678/B/H03/2011/40 "Analysis of non-linear and many-body effects in quantum atomic gases", Faculty of Physics, University of Warsaw
- Iuventus Plus project number IP2014 050073, 2015-2017, scientific supervisor of dr Tomasz Wasak

f) invited conference talks

- “Quantum Technologies Conference” Torun, Poland, 2011
- “Quantum Technologies Conference” Cracow, Poland, 2012
- “ProQuP workshop”, Paris, France, 2012
- “Quantum Information”, Florence, Italy, 2013
- “Quantum Optics”, Jachranka, Poland, 2013
- “Quantum Gases and Quantum Coherence”, Levico Terme, Italy, 2014
- “CAP-ProQuP Workshop”, Vienna, Austria, 2014

g) national and international collaboration

- Warsaw, Poland (2003-2007): collaboration with prof. Kazimierz Rzążewski from the Center of Theoretical Physics, Polish Accademy of Science (PAS), devoted to the analysis of the scattering of atoms from a BEC
- Oxford, Great Britain (2004): collaboration with the group of prof. Keith Burnett within the BEC2000 programme, devoted to the study of the Feshbach resonances in BECs
- Oxford, Great Britain (2005): collaboration with the group of prof. Keith Burnett within the QUDEDIS programme, devoted to the analysis of the scattering of atoms from a BEC
- Trento, Italy (2008-2013): collaboration with prof. Augusto Smerzi within the field of quantum metrology
- Florence, Italy (2013-): collaboration with prof. Augusto Smerzi within the field of the quantum Zeno effect and the Bell inequalities
- Munich, Germany (2012-2014): collaboration with dr. Francesco Piazza within the field of quantum metrology
- Innsbruck, Austria (2015-): collaboration with dr. Francesco Piazza within the field of quantum metrology in optical cavities

- Vienna, Austria (2013-2014): collaboration with dr. Robert Bücker devoted to the theoretical description of the scattering of atomic beams from a BEC
- Warsaw, Poland (2015-): collaboration with dr. Emilia Witkowska from the Institute of Physics, PAS devoted to the analysis of creation of squeezed states from a BEC
- Warsaw, Poland (2010-2014): collaboration with dr. Piotr Deuar from the Institute of Physics, PAS devoted to the analysis of the scattering of atoms from a BEC
- Cracow, Poland (2007-2009): collaboration with prof. Krzysztof Sacha from the Faculty of Physics, Jagiellonian University devoted to the analysis of quantum states of ultra-cold atoms in a double-well potential
- Torun, Poland (2007-2009): collaboration with dr. Michał Zawada from the Departmet of Physics, Nicholas Kopernikus University, devoted to the theoretical model of free expansion of an interacting BEC

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