Summary of Professional Accomplishments

1. Name : Paweł Piotr Caputa, born: 07.04.1983, Żywiec, Poland.
2. Diplomas and degrees:

   PhD in Theoretical Physics.
   Awarded: 24.10.2011.
   Thesis: "Non-planar corrections to the dilatation operator in AdS/CFT."
   Institution: The Niels Bohr Institute, Copenhagen University, Copenhagen, Denmark.
   Advisor: Prof. Charlotte Floe Kristjansen.

   Masters in Theoretical Physics.
   Awarded: 30.08.2008 (with distinction).
   Thesis: "Gluon scattering amplitudes: From weak to strong coupling and back."
   Institution: University of Amsterdam, Amsterdam, The Netherlands.
   Advisor: Prof. Jan de Boer.

3. Information on employment in research institutes and research visits:

   From 01.02.2020, Adiunkt (Assistant Professor) and
   "Quantum Information in Quantum Gravity" Group Leader.
   Faculty of Physics, University of Warsaw, Warsaw, Poland.
   Program Polish Returns 2019 founded by NAWA.
   Group website: http://qiqg.fuw.edu.pl

   01.10.2019 – 31.10.2019 Institute for Advanced Studies, Princeton, USA.
   Host: Prof. Juan Maldacena.

   01.09. – 30.10.2017 University of California, Santa Barbara, USA.
   Host: Prof. Don Marolf.

   01.01.2017 – 31.12.2019, Research Assistant Professor and Simons Foundation
   "It from qubit" fellow.
   Yukawa Institute for Theoretical Physics, Kyoto University, Japan.
   Host: Prof. Tadashi Takayanagi.

   01.11.2014 – 31.12.2016 Postdoctoral Researcher.
   Nordic Institute for Theoretical Physics (NORDITA), Stockholm, Sweden
   Host: Prof. Konstantin Zarembo.

   01.02.2014 – 31.10.2014 JSPS Postdoctoral Fellow (short-term).
   Yukawa Institute for Theoretical Physics, Kyoto University, Japan
   Host: Prof. Tadashi Takayanagi.

   01.03.2013 – 30.06.2013 Tata Institute for Fundamental Research, Mumbai, India
   Host: Prof. Gautam Mandal.

   01.02.2012 – 31.01.2014 Postdoctoral Researcher.
   Mandelstam Institute for Theoretical Physics, University of the Witwatersrand,
   Johannesburg, South Africa.
   Host: Prof. Robert de Mello Koch.

   9.2010 – 4.2011 Brown University, Providence, USA.
   Host: Prof. Anastasia Volovich.

4. Description of the achievements, set out in art. 219 para 1 point 2 of the Act.

4.1 Title: The scientific achievement presented below consists of a series of publications focused on ***"Dynamics of entanglement and complexity in the AdS/CFT correspondence."***

4.2 Publications in chronological order (in my field authors are listed in alphabetical order please see the attached statements from my collaborators about my contributions):

[A1] **P. Caputa**, G. Mandal, R. Sinha, "Dynamical entanglement entropy with angular momentum and U(1) charge", *JHEP 1311, 052 (2013).*

[A2] **P. Caputa**, M Nozaki, T. Takayanagi, "Entanglement of Local Operators in large N CFTs", *PTEP. (2014) 093 B 06.*

[A3] **P. Caputa**, A. Stikonas, J. Simon, T. Takayanagi, "Quantum Entanglement of Localized Excited States at Finite Temperature", *JHEP 1501, 102 (2015).*

[A4] **P. Caputa**, A. Stikonas, J. Simon, T. Takayanagi, K. Watanabe, "Scrambling time from local perturbations of the eternal BTZ black hole", *JHEP 1508, 011 (2015).*

[A5] **P. Caputa**, T. Numasawa, A. Veliz-Osorio, "Out-of-time-ordered correlators and purity in rational conformal field theories", *PTEP (2016) no.11, 113 B 06.*

[A6] **P. Caputa**, N. Kundu, M. Miyaji, T. Takayanagi, K. Watanabe, "Anti-de-Sitter space from Optimization of Path-Integrals in CFTs", *Phys. Rev. Lett. 119 (2017) no.7, 071602.*

[A7] **P. Caputa**, N. Kundu, M. Miyaji, T. Takayanagi, K. Watanabe, "Liouville Action as Path-Integral Complexity: From Continuous Tensor Networks to AdS/CFT ", *JHEP 1711 (2017) 097.*

[A8] **P. Caputa**, M. Miyaji, T. Takayanagi and K. Umemoto, "Holographic Entanglement of Purification from Conformal Field Theories ", *Phys. Rev. Lett. 122, 111601 (2019).*

[A9] H. A. Camargo, **P. Caputa**, D. Das, M. P. Heller, R. Jefferson, "Complexity as a novel probe of quantum quenches: universal scalings and purifications", *Phys. Rev. Lett. 122 (2019) no.8, 081601.*

[A10] **P. Caputa**, J. M. Magan, "Quantum Computation as Gravity", *Phys. Rev. Lett. 122, (2019).*

4.3 Detailed description of the achievement:

The main goal of the research that I summarize in this habilitation thesis was to **explore basic mechanisms behind holography and shed light on how strongly interacting systems encode gravity**. For that, I have been developing two closely related research programs in the context of the AdS/CFT correspondence: the first focused on exploring the structure of quantum entanglement and its dynamics in quantum field theories and the second on developing measures of complexity of quantum states in conformal field theories.

In the first five projects described below, I studied dynamics of entanglement in conformal field theories in various dimensions. My main motivation was to test and develop probes of entanglement in these many-body quantum systems and study differences in spreading of quantum entanglement and scrambling between holographic and non-holographic CFTs.

In [A1], we developed "charged entanglement entropy" in 2D CFTs and studied its evolution after global quantum quench in the presence of conserved charges. In particular, we analyzed the time evolution of entanglement entropy in the charged thermofield-double (TFD) state that, holographically, corresponds to a rotating eternal black hole in anti-de Sitter spacetime. We computed and matched entanglement entropies on both sides of the holographic correspondence and found that they saturate to a value that depends on the charge (according to the Generalized Gibbs ensemble hypothesis).

In [A2,A3,A4] we analyzed the evolution of Renyi and entanglement entropies in a class of CFT states excited by a local operator. First, in [A2], we set up the problem in CFTs in various dimensions and derived universal results for the evolution of Renyi entropies with the focus on differences between non-holographic (rational or integrable) models and large-N or large-c CFTs that play important roles in holography. We also developed the holographic dual of these excited states in terms of a massive particle propagating in AdS spacetime and matched our CFT results with holographic computations using the HRT prescription (explained below).

Then, in [A3], we generalized our analysis to finite temperature and local operator excitations on top of thermal states. The results obtained for large-c holographic CFTs were matched with the HRT prescription used in the black hole geometry with a back-reaction from a massive particle dual to the local operator. Finally, in article [A4], we used our setup to derive the scrambling time from the evolution of the mutual information in perturbed thermofield-double state. In these three works, we not only developed and tested quantum information tools for probing CFTs (including those with holographic dual) but also formulated criteria that distinguish holographic CFTs from integrable ones. The main one being the logarithmic growth of entanglement entropy with time in a quantum state excited by a local primary operator in holographic CFTs. On the contrary, in rational CFTs the entropy just increases by the logarithm of operator's quantum dimension.

In the meantime, the new ideas about constraining holographic CFTs matured in the concept of the Out-of-Time-Ordered Correlators (OTOC) as probes of quantum chaos. In [A5], we studied these 4-point functions and derived universal result for their late time behaviour in all rational CFTs in two dimensions. It turns out that in these integrable models, OTOCs approach to a certain constant (the so-called "monodromy constant") fully specified in terms of the modular S-matrix of the rational CFT.

In the second line of research, I focused on the problem of extracting holographic geometries from CFT and on quantifying complexity of quantum states prepared by Feynman's path integrals. In [A6], we introduced the path integral optimization and path integral complexity. We showed how using the path integral optimization procedure, after minimizing the path integral complexity (Liouville action in 2D), one can extract a hyperbolic geometry that can be interpreted as a slice of a holographic dual spacetime.

In [A7], we further developed the path integral complexity in two- and higher-dimensional CFTs, explored its quantum computational properties, studied optimal metrics and analyzed it from the perspective holographic complexity proposals. Then, article [A8] was a very important application of our optimization to the first computation of entanglement of purification (EoP) in 2D CFTs. EoP was proposed to be holographically dual to the area of the entanglement wedge cross-section in AdS and we managed to verify this in our example.

In [A9], exploring possible candidates for complexity measures in QFTs, we applied geometric approach to complexity in free Gaussian field theory (exactly solvable) to explore the dynamics of a quantum quench. We analyzed the so-called slow and fast quenches and found universal scalings of complexity with the quench rate. Our work established geometric complexity as a novel and useful probe of quantum quenches, in addition to correlation functions or entanglement entropy used before.

Finally, in [A10], based on intuitions from path integral complexity, we initiated geometric approach to complexity in conformal field theories. In 2D CFTs, we introduced the notion of Virasoro quantum circuits and showed that by an appropriate choice of cost functions, Nielsen's complexity action becomes the Polyakov action of 2D gravity (geometric action on the co-adjoint orbits of the Virasoro group). Our idea provided a natural definition of circuit complexity in interacting, holographic CFTs and has been further extended and fruitfully developed since then.

The plan of this presentation is as follows. I will start with a brief introduction to the two main problems addressed in my works. Then, I will elaborate on the main results in each paper, their impact and implications. Finally, I will summarize and list several new directions that emerged from these works that I'm following with my group at present.

Introduction.

The AdS/CFT, proposed by Maldacena [1] (see also [2,3]), is a correspondence that relates certain quantum field theories which describe critical many-body systems (conformal field theories or CFT for short) with theories of quantum gravity on negatively curved Anti-de Sitter geometry (AdS for short) in one higher dimension. For this reason, by analogy with familiar holography, AdS/CFT is usually referred to as an example of a "holographic duality".

There are several reasons why AdS/CFT is important and attracts a lot of attention. Firstly, one of our biggest and most difficult open problems in theoretical high-energy physics is to unify two pillars of the 20th century physics, quantum mechanics and general relativity, into one theory of quantum gravity. On this front, the AdS/CFT correspondence provides us with a precise toy model and first working definition of a theory of quantum gravity in AdS spacetime by the dual, strongly interacting CFT (still, very non-trivial to analyze but a concrete model of quantum gravity).

Secondly, in its most rigorous version, AdS/CFT relates supersymmetric conformal field theories in d spacetime dimensions to theories of quantum gravity (string theory) in d+1 dimensional AdS spacetime. However, there is a wide belief and evidence that holography holds more generally (in models without supersymmetry and away from conformality) and can teach us important lessons about the notoriously difficult to study physical regime of strong interactions (present in e.g. quark-gluon plasma [4] or strange metals [5]).

Still, a conventional, "particle physicists"-like approach to the duality seems to hide the principles of how it works. Indeed, even though the AdS/CFT correspondence is already more than 20 years old, we are still lacking the basic principle behind it and the answer to the key problem:

How states of strongly coupled quantum field theories encode spacetime geometry?

This question drives some of the most recent research trends in theoretical physics and brings together scientists across several disciplines. From these efforts, we managed to discover a few important clues in the above puzzle. One of them, is the hypothesis that:

***Spacetime geometry is intimately related to the structure of quantum information and quantum entanglement in states of holographic quantum field theories*.**

Pursuing this interesting idea allowed to discover several strong evidences that quantum information and spacetime geometry are intimately related.

Most of the recent developments started with the proposal of Ryu and Takayanagi [6] to identify an area $\mathscr{A}$ of a co-dimension 2 (fixed time) surface in Anti-de Sitter spacetime with the CFT Von Neumann entropy S_A (entanglement entropy) of a boundary sub-region A to which the surface is attached:

$$S_A = \frac{\mathscr{A}}{4G_N}\,, \tag{1}$$

where G_N is the Newton constant. This expression, the "RT formula" for short, can be thought of as a generalization of the famous Bekenstein-Hawking relation for black hole entropy in terms of the area of its horizon. See [7,8] for more technical details of the RT proposal.

In the special case of 2+1 dimensions, the RT proposal asserts that a geodesic on the hyperbolic plane computes the entanglement entropy of an interval in (1+1)-dimensional, strongly interacting conformal field theory at the boundary. This holographic proposal has been verified by explicit comparison with CFT [9] and its general derivation in AdS/CFT was given by Maldacena and Lewkowycz [10] (see [11] for a pedagogical review).

For more general, time-dependent spacetimes, RT formula was generalized by Hubeny, Rangamani and Takayanagi [10] (HRT) to covariant prescription where entanglement entropy at Lorentzian time t is computed by the area of extremal surfaces homologous to the region A at the boundary. Similarly to its static predecessor, it was checked by various nontrivial computations and its derivation was also outlined in [13]. In most of the results in the first five works on dynamics of entanglement, it was in fact mainly the HRT prescription that we tested and used to extract new interesting results for holographic CFTs.

In order to test RT and HRT proposals, one needs to compute entanglement entropy in conformal field theories. This is a non-trivial task since, as many of the quantum information concepts, they are not observables in the QFT sense. Moreover, one also has to face the problem of dealing with infinite dimensional Hilbert spaces of QFTs. The later problem can be circumvented by introducing a UV cutoff and extracting interesting information from "bare quantities" with explicit cutoff dependence. The former issue turns out to be solvable in a very elegant way in 2d CFTs [9].

Namely, using the replica trick, entanglement entropy can be computed as a limit of $n \to 1$, of Renyi entropies $S_A^{(n)}$:

$$S_A = \lim_{n\to 1} S_A^{(n)} = \lim_{n\to 1} \frac{1}{1-n} Tr\left(\rho_A^n\right). \tag{2}$$

On the other hand, the trace of the n-th power of the reduced density matrix can be defined, in the path-integral formalism, as a partition function on the n-sheeted Riemann surface with cuts corresponding to the interval A. The cuts are glued together in an appropriate, consecutive way so that when we perform the integration around their end-points we move from the (n-1)-th to the n-th sheet (and from the n-th to the 1st). The trace of the density matrix is also normalised so we need to divide this partition function by the n-th power of the partition function on a single copy and

$$Tr\left(\rho_A^n\right) = a_n \frac{Z_n}{(Z_1)^n}, \tag{3}$$

where a_n is a model-dependent (non-universal) constant. In a series of elegant works, Cardy and Calabrese [7,14-18] showed that ratio (3) can be equivalently described by correlation function of twist operators at the end-points of the entangling region A. Twist operators are CFT primaries with conformal dimension given in terms of the CFT central charge c and the replica number n as

$$h = \bar{h} = \frac{c}{24}(n - 1/n). \tag{4}$$

and since two-point functions are universally fixed in 2D CFTs, this result allowed to derive numerous single interval entropies including those for finite size or finite-temperature CFTs. These were in fact the first formulas reproduced by the holographic RT proposal. Moreover, it was argued that in large-c holographic CFTs with sparse spectrum of low energy operators [19], higher-point correlators of twist operators should be dominated by vacuum conformal blocks [20,21]. This then allowed for comparing entropies for multi-interval regions with large-c holographic computations [22]. In works [A1-A4], we have explored this computable setup of 2D CFTs and matched entropies with HRT results. In addition, in [A2], we used the replica trick itself to compute the growth of entropies in rational CFTs where we employed known correlators to extract new results for dynamics of Renyi entropies.

As I already stressed, holographic CFTs are believed to be special among those describing generic critical points of many-body systems. Interestingly, many of their universal properties are determined by the fact that their states are dual to AdS geometries that contain black holes. Indeed, it is a very well known fact from the early days of the AdS/CFT that thermal states of CFTs describe black hole geometry in the bulk. However, it was the quantum information "revolution" in AdS/CFT that followed and explored this fact so extensively that new tools had to be developed and gave rise to new surprising lessons about black holes.

Firstly, already in the early days of holography, Maldacena proposed a duality between the thermofield double state of two CFTs (a purification of the thermal state) and eternal black hole [23], i.e., maximal extension of the black hole geometry that consists of two asymptotic black hole spacetimes connected by a wormhole. In the new light of quantum information, this duality made us realise that it is in fact pure quantum entanglement in CFT states that corresponds to connectedness of the bulk regions [24]. This was further promoted to a conjecture that bits of entanglement in CFTs always give rise to wormholes in the holographic bulk (dubbed as ER=EPR conjecture) [25].

More quantitatively, universal properties of black holes in AdS, such as high-energy scattering near their horizons, gave rise to new CFT probes that helped to make a sharp distinction between models with a holographic dual and more familiar rational CFTs. In [26-28], the so-called thermal Out-of-Time-Ordered-Correlators (OTOC) of generic operators W and V were proposed as measures of quantum chaos in CFTs

$$C_\beta(t) = \frac{\langle W(t)VW(t)V\rangle_\beta}{\langle VV\rangle_\beta \langle W(t)W(t)\rangle_\beta}. \tag{5}$$

It was also argued that, in thermal states of holographic models dual to black holes of Einstein's gravity, this correlators should universally decay with a maximal Lyapunov coefficient $\lambda = 2\pi/\beta$. In [A5] we analysed these measures from a more general perspective of 2D CFTs and derived a universal result for their late time behaviour in terms of the modular S-matrix of a CFT.

The second line of research that I have been actively developing in focuses on the question of how CFT states may encode the holographic geometry of their AdS dual. The most promising development in this direction, in my opinion, has been the so-called AdS/Tensor Networks correspondence (AdS/TN for short) that I will now briefly describe.

Tensor Networks that started the AdS/TN were developed by Vidal [29]. With original goal of simulating many-body quantum systems he proposed an efficient algorithm to approximate quantum states of critical Hamiltonians (corresponding to CFTs in the continuum limit) by a Tensor Network called MERA. This network is built from a basic set of objects called tensors and is optimized (e.g. min. of energy; variational ansatz) taking into account the structure entanglement in the critical quantum state (the so-called "entanglement renormalisation").

After the optimization, the quantum state is represented by a geometric (discrete) network of tensors (MERA). Surprisingly, this optimized network resembles a discrete slice of the Anti de-Sitter spacetime that is holographically "dual" to the approximated quantum state (see Figure 1, right).

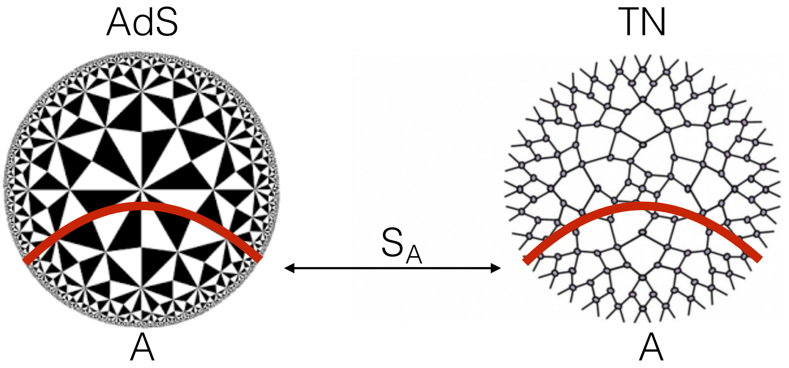

Figure 1. **Tensor Networks and holography.**
The correspondence between Tensor Networks and AdS space might be the key insight in understanding mechanisms behind holography and how quantum states encode holographic geometries. In both approaches, the entanglement entropy S_A of a subset A of the physical degrees of freedom is computed by a "geodesic".

Then, the crucial observation came from the work of Swingle [30], who pointed that MERA itself may represent a time-slice of Anti-de Sitter spacetime and could be thought of as a "discrete version of holography" (see also [31-36] for important developments following this work and [37,38] for recent reviews). One strong supporting argument was that a discrete Ryu-Takayanagi formula, a shortest piecewise path in the effective network geometry, indeed sets the upper bound on entanglement entropy of a subset of the boundary degrees of freedom. This interesting idea and incorporating it to genuine, strongly-interacting CFTs was the main motivation for my works on path-integral optimization [A6-A7] that I will describe below.

Finally, the last concept that plays a important role in my works is complexity. In holography, it became important also in connection with the AdS/TN developments when Hartman and Maldacena [39] analyzed gravity dual of the global quench using time evolved thermofield double state and its dual eternal black hole. As in generic global quenches [17], entanglement entropy saturates after some time of order the interval size and this was elegantly reproduced from the HRT prescription (see also [40,41] for earlier studies of holographic thermalisation). However, Susskind [42] noticed that even after the saturation time, the Einstein-Rosen bridge in dual geometry continues to grow with time, hence knowing only the entanglement entropy may not be sufficient to fully determine holographic geometry. His proposed that this growth should be related to the dual state's complexity (measured by the size of the tensor network representing the TFD state at time t), and with collaborators [43,44], suggested how one could estimate it using gravity constructions like volume of a maximal time-slice or gravity action on the Wheeler-DeWitt patch. Both of them seemed plausible from the gravity perspective, however the concept of "complexity" in dual CFTs was completely unexplored.

As I will review below, we introduced the idea of path integral complexity as a continuous measure of the size of path-integral tensor network that prepares CFT states. This became one of the leading candidates for CFT complexity. Later on, the geometric approach of Nielsen to circuit complexity [45,46] was adapted to exactly-solvable free boson setup [47,48] (coupled harmonic oscillators). In this construction, after deciding on a set of quantum gates and the cost of applying them, we are interested in quantum circuits that prepare a target state (usually a vacuum of some interesting Hamiltonian or other entangled state) starting from some simple reference state. The set of unitary operations with gates that we have at our disposal can be represented geometrically as a "manifold" on which different paths represent different quantum circuits. Then we must decide on some physical way of associating the cost with each circuit, i.e., non-unique metric on the geometry of unitaries, that allows to estimate state's complexity as the minimal length of a geodesic between the reference and target state.

Each of the above steps is rather non-trivial and comes with an ambiguity that requires physical reasoning, especially in continuous quantum field theories. Nevertheless for free Gaussian systems there are some "natural" choices for this construction (e.g., position and momentum Gaussian gates or operations on covariance matrix) and one is able to make progress in computing states complexity [47,48]. I will describe below how one can e.g. employ such complexity to extract interesting and universal "scaling information" about quantum quenches.

Moreover, there is another natural choice of gates in QFTs in terms of the symmetry algebra of the theory [49]. In 2D CFTs this is particularly constraining since the symmetry sector consists of two copies of the infinite-dimensional Virasoro algebra. I will also describe below how in [A10] we generalized Nielsen's approach to 2D CFTs and discussed possible choices of fixing these ambiguities that hint on further connections with gravity.

After this brief introduction, I will now discuss each of the ten works in more detail.

Details of my works.

1. "Dynamical entanglement entropy with angular momentum and U(1) charge."

In this work, I started studying dynamics of quantum information in the setup of quantum quenches in CFTs and holography. In particular, in [A1], we were interested in thermalization of the quantum system in the presence of additional global or local conserved charges. A general paradigm is that in the presence of conserved charges, the finial equilibrium state is represented by charged density matrix according to the Generalized Gibbs Ensemble (see e.g. [50]). Various examples of this phenomenon were found before in lattice models using correlation functions (see a relevant review [51]), but our work was the first one in 2D CFTs to confirm it with quantum information tools such as entanglement entropy.

In order to make progress, we first had to introduce a concept of charged entanglement entropies in 2D CFTs. We started, with a thermal state with temperature and angular momentum as well as its purification, the charged thermofield double (TFD) state, and analyzed unitary time evolution focusing on dynamics of entanglement entropy. On the gravity side, such states correspond to a rotating BTZ black hole spacetime and its two-sided extension, that solves Einstein's equations with negative cosmological constant. In this setup, using conformal maps, we computed entanglement entropy of a single interval in 2D CFTs and time-dependent entanglement entropy between two intervals in each copy of a CFT in the charged TFD state. Correspondingly, we derived holographic entanglement entropies using HRT prescription and found perfect match between both results. Interestingly, the speed of entanglement production as well as the plateau to which the entropy saturated in the global quench protocol dependent on the chemical potential (see Fig 2.).

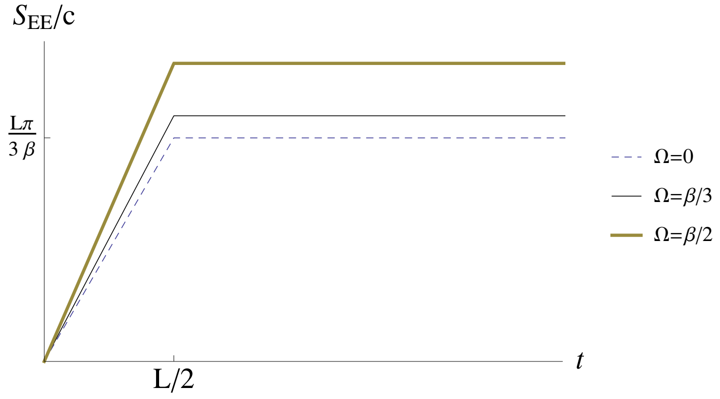

Figure 2. **Entanglement entropy with conserved charge in large-c 2D CFTs and holography.** Figure shows evolution of entanglement entropy for interval of size L computed in charged TFD state with chemical potential Ω. In the presence of the conserved charge, entanglement entropy after the quantum quench can grow faster and saturate to the thermal value according to the Generalized Gibbs Ensemble prediction.

In addition, to explore charged entropies, we studied: holographic entanglement entropy with the U(1) charge described in gravity by a U(1) Chern-Simons gauge field, a QFT setup of a charged massive scalar field using covariance matrix approach [52,53] and the first law of entanglement entropy [54] in the presence of conserved charges. All these original computations provided non-trivial checks of this interesting tool, sensitive to symmetries of the system. Moreover, at the time, the motivation and need for charged entanglement entropies seemed purely theoretical but with time this idea gained more momentum and a related holographic work appeared in [55], extremal limit of the entropy was further developed in [56] and entanglement of charged operator excitations was investigated in [57]. More recently, the idea of "symmetry resolved" entanglement became very popular in condensed matter community [58] and computations generalizing our initial results are finding more use in this context [59].

From a broader perspective, with this work I became interested in quantum information in quantum field theories and AdS/CFT. It helped me to formulate a program of testing field theories from the perspective of quantum information and entanglement spreading that I have been developing for the last years. In particular, I started specializing in quantum quenches in field theories and quantum information and computation in AdS/CFT.

2. "Entanglement of Local Operators in large N CFTs."

Another important setup for studying the dynamics of quantum information in quantum field theories and holography consists of states excited by insertion of local operators. This is a special case of the so-called local quantum quench [60] where one is usually interested in dynamics of quantum information measured by entanglement entropy at time t. More precisely, focusing on CFTs, at some initial time we start with an arbitrary quantum state and then insert a local primary operator at some spatial distance from the interval A of which reduced matrix we are interested in (see Fig. 3). Since the insertion breaks translation invariance, unitary Hamiltonian evolution leads to a non-trivial change of entanglement structure between A and its complement. On general grounds one expects that, as time progresses, energy from the local excitation spreads through the system and changes the entropy only after it reaches the interval A. Then, depending on the properties of the Hamiltonian as well as the entanglement structure of the initial state, one finds different characteristic behaviours in the growth of the entropy. This setup was first considered in [61-63].

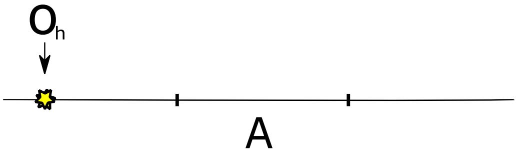

Figure 3. **Setup for computing Renyi entropies of states excited by local operators.** Local CFT operator O_h is inserted in some distance from the interval A. We then perform time evolution and compute the change in Renyi entropies of A with time t.

In [A2], we developed and explored this setting focusing on differences between evolution of Renyi and entanglement entropies in holographic and non-holographic CFTs in various dimensions. Technically, in CFTs one can compute n-th Renyi entropies by the replica trick using partition functions on n-sheeted geometries or as correlator of twist operators [14], but now in the presence of 2n primary operators. Depending on a theory, these correlators can be computed directly (e.g., in free theories) or extracted in the large-c limit using conformal blocks. On the gravity side, one can construct time-dependent geometries corresponding to the excited, non-equilibrium, CFT states and compute entanglement entropies using the HRT prescription.

Before our work, it was found that in two-dimensional rational CFTs (RCFTs) entropy increases by a constant amount equal to the logarithm of quantum dimension of the local operator [63]. On the other hand, using conformal blocks and bootstrap arguments, we found that for strongly interacting holographic models, the entanglement production is faster, and the entropy grows logarithmically with time. Moreover, we constructed the dynamical gravity dual geometry that represents the excited state by taking the back-reaction from a massive particle on the AdS spacetime (see [64] for a similar dual of a geometric local quench). Then, using the HRT prescription we found that our result perfectly matches the large-c CFT analysis. In addition, we performed a novel holographic computation of the 2n-point correlators on the replicated geometry using geodesic approximation in the hyperbolic black hole background with temperature related to the replica index n. This computation also confirmed the logarithmic growth of the entropy in the holographic regime.

Last but not least, we managed to analyze this setup in free N=4 SYM theory with SU(N) gauge group and found a very different behaviour of Renyi entropies (with $n \geq 2$) and entanglement entropy. Namely, exciting the state with a singlet operator $\mathrm{Tr}(Z^J)$, with an SU(N) matrix Z, was seen by Renyi entropies as changing entanglement of the state by number J of EPR pairs, whereas entanglement entropy increased by $\sim \log(N)$ implying that it is sensitive to the internal degrees of freedom of the excitation.

Our results were among the first works that started probing dynamics of locally excited CFT states from the perspective of quantum information and were later extended to various modified setups with different families of excitations as well as computations with other quantum information tools. More universal results was derived in [65] and importing new developments from CFT bootstrap [66], it was argued that, for heavy operator (operator with large conformal dimension) in holographic CFTs one may also find saturation of entanglement according to the Eigenstate Thermalization Hypothesis (ETH) [67].

3. "Quantum Entanglement of Localized Excited States at Finite Temperature."

In this work [A3], we generalized our setup to locally and globally excited states at finite temperature in 2d CFTs (see also parallel work [65]), and we derived various new results for entanglement evolution in thermal states excited by local primary operators. Using conformal transformations, we could again compute Renyi and entanglement entropies as well as mutual information in large c CFTs. In particular, for locally excited states, we found initial logarithmic growth of the entropy, but now, after times of order the inverse temperature, we observed universal saturation to the logarithm of the value of thermal entropy.

Then, we analyzed single interval entanglement entropy in eigenstates of the CFT Hamiltonian prepared path integral on a disc with a primary operator inserted at the origin. Interestingly, for heavy operators, we derived a very general result in large-c CFTs with sparse spectrum where the vacuum block gives a dominant contribution to 4-point correlators. In this case, entanglement entropy, computed by 4-point correlators with two heavy excitations and two light twist operators (in the limit of $n \to 1$) can be derived using the Heavy-Light conformal block [68] and is universally expressed in a "finite temperature-like" result:

$$S_L = \frac{c}{6}\log\left(\frac{\beta_h}{\pi}\sinh\left(\frac{\pi L}{\beta_h}\right)\right), \qquad T_h = \beta_h^{-1} = \sqrt{\frac{24h}{c} - 1}, \tag{6}$$

where β_h is the inverse effective temperature of the excitation with dimension h. This important result fits well with the expectation that holographic CFTs should generically correspond to chaotic Hamiltonians where the ETH hypothesis [67] is satisfied. Usually, the ETH is formulated in terms of correlators and other local observables, but our result also emphasizes the fact that it can be equally well studied and explored using quantum information measures.

Finally, from the perspective of holography, locally excited thermal states are extremely interesting since the dual picture corresponds to a massive particle in the AdS-black hole background that is sent from the boundary at time t=0 towards the black hole (e.g. after times of order β it also gets to the horizon region). Indeed, we have explicitly constructed such geometry by taking the back-reaction from the point particle on the BTZ metric. This metric allowed us to compute the length of extremal geodesic at time t and verify the result from the large-c CFT analysis. Our gravity solution can be modelled in various limits by shock-wave metrics but it is more general and valid for insertion of operators at arbitrary positions and time. Indeed, our work was further generalized to interesting AdS3/CFT2 setups [69-71] and is still being used in recent developments in lower-dimensional holographic duality with the SYK model as well as the "islands" computations [72,73].

4. "Scrambling time from local perturbations of the eternal BTZ black hole."

This work [A4], was the final application of the techniques that we developed in [A2,A3] to the computation of the evolution of mutual information after local excitation. More precisely, Shenker and Stanford [74] argued that, due to their chaotic nature, black holes scramble information in the fastest possible manner. Using holography, they showed that when we perturb an eternal black hole by a local excitation then the mutual information between two intervals A and B on each side will vanish after the so-called scrambling time proportional to the logarithm of the black hole entropy (i.e., an example of the butterfly effect). Their analysis was done using holography and supported by a random/chaotic qubit model, so it was very important to verify to what extend their claim is true in holographic CFTs. Fortunately, the tools that we developed studying local operator excitations, both in CFTs and in gravity, turned out to be tailor-made for performing this computation.

In 2D CFT, we considered the time evolution of the thermofield double state (maximally entangled state of two CFTs) perturbed by a local primary operator at some time t_ω in the past in one of the CFTs. The thermofield double state is holographically dual to the maximally extended BTZ black hole (with two asymptotic boundaries) and the excitation corresponds to a massive point particle sent from the boundary (left on Fig. 4). The dual geometry of the excited state is obtained by taking the back-reaction from the particle i.e., solving Einstein's equations with a point-like source on particle's trajectory (similar solutions were found in [75] and used for quenches in [63]).

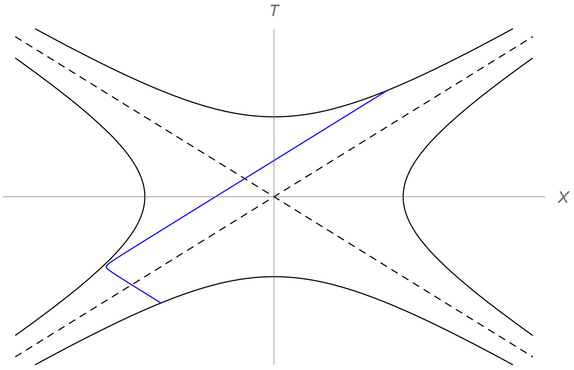

Figure 4. **Gravity dual metric for evaluation of the scrambling time.** Dual metric to the TDF state locally excited by a primary operator at time t_ω in the past on the left copy of a CFT. The metric is obtained by taking the back-reaction on the Kruskal background from the massive point particle following the blue trajectory. In 3D this can be done by appropriate boosted maps.

The main object of our interest was then the mutual information between two spatial intervals A and B on the left and right boundary respectively, computed in this locally excited state. In CFT this computation involves a 6-point correlator of 4 (light) twist operators and 2 primary operators. In the large-c limit and assuming the vacuum conformal block dominance we evaluated the correlator and computed the evolution of the mutual information as a function of t_ω. Indeed we found that it vanishes for the value of the "scrambling time" given generally by

$$t^*_\omega = f(L,\beta) + \frac{\beta}{2\pi}\log\left(\frac{\pi S}{4E}\right), \tag{7}$$

where function f contains geometric details of the intervals A and B (L is the interval length), β is the inverse temperature of the state, E is the energy of the excitation and finally S represents the entropy density of the original system (proportional to c/β).

On the gravity side, we managed to solve Einstein's equations analytically and explicitly found geodesic lengths needed to compute entanglement entropies and mutual information holographically using the HRT prescription. Our results showed perfect agreement with the large-c CFT computation of the scrambling time. This was a very important and an elaborate check of not only the holographic HRT prescription but also the arguments behind the chaotic nature and scrambling of black holes as seen from the perspective of dual CFT. Our works were also a part of important developments that lead to the discovery of new probes of holographic CFTs and quantum chaos as I will now describe.

5. "Out-of-Time-Ordered Correlators and purity in rational conformal field theories."

Studies of black holes, including the scrambling time and the butterfly effect as well as evolution of mutual information led to a paradigm that black holes are the fastest objects in nature in processing quantum information [76]. To be more precise, holographic CFTs that describe them should exhibit strong quantum chaotic properties. This gave rise to a new proposal for a diagnose of quantum chaos in QFTs in terms of the Out-of-Time-Ordered-Correlators (OTOC) [24-26]. As in every QFT, Lorentzian correlators can be obtained from Euclidean ones by analytic continuation which takes into account the ordering of operators in Lorentzian time. The ordering that is relevant for the OTOC (5) can be interpreted as computing the overlap between thermal states with operators inserted at initial time t=0 and some later time t and the state with opposite order of the insertions. This ordering is more natural from the perspective of the thermofield double state with two pairs of operators at different times on each side. Gravitationally, such four-point functions compute high-energy scattering of excitations near the black hole horizon and, based on universal results for this process, it was conjectured that OTOCs in CFT states dual to black holes decay with the maximal possible Lyapunov exponent equal $\lambda=2\pi/\beta$ [28] (see also [77] for recent discussion).

In [A5], with the aim of better understanding these putative probes of quantum chaos, we begun exploring the difference in their evolution in chaotic versus integrable models and what kind of information about 2D CFTs they are sensitive to. Interestingly, by carefully analyzing the analytic continuation from Euclidean to Lorentzian correlators, we realized that OTOCs at late time require performing a monodromy transformation on the conformal blocks that four-point correlators are expanded in. This allowed us to give a very elegant proof for the late time value of OTOCs in arbitrary RCFTs (non-chaotic) in 2D (see also [78,79] for more results). It turns out that in RCFTs, at late time, OTOCs for two operators O_i and O_j approach to a constant value given in terms of the modular S-matrix of the model (the so-called monodromy constant):

$$C^{\beta}(t)_{ij} \to \frac{1}{d_i d_j}\frac{S^*_{ij}}{S_{00}}\,, \tag{8}$$

where d_i's are the quantum dimensions of the two pairs of operators used in the OTOC and S^*_{ij} is the complex conjugate of the modular S-matrix.

We confirmed this powerful result using known correlators in the SU(N) WZW model, the compact boson theories and other RCFTs. Interestingly, from the perspective of 3D Chern-Simons theory that is closely related to 2D conformal blocks [80] this constant describes expectation value of the Wilson loop on the Hopf link computing Jone's polynomials [80,81]. This also suggests that even in RCFTs OTOCs are sensitive to a very fine and complex information about the model. These ideas are still being developed at present in the context of operator complexity in 2D CFTs.

In addition, we compared differences in evolution of purity (second Renyi entropy) studied in [A2] and OTOCs. We found that while in RCFTs, in the large-N limit, purity can grow logarithmically with time similarly to holographic and chaotic models, OTOCs still approach to our monodromy constant confirming that model remains integrable. This interesting observation highlights the fact that, in order to make a sharp distinctions between holographic and non-holographic theories only one tool/probe may not be sufficient.

6. "Anti-de-Sitter space from Optimization of Path-Integrals in CFTs."

In [A6], we initiated the program of extracting holographic geometries from CFT states using path integrals. The main idea for our construction came from tensor networks [82,83], where one starts from a particular representation of a quantum state (wave function) and then via a corse-graining procedure (TNR [83]) an optimal tensor network (geometry) is produced. Naturally, for holographic, strongly-interacting CFTs, such procedure must be implemented directly in the continuum limit and using universal tools (see e.g. cMERA [84]).

Our starting point was the representation of a quantum state in QFT by Feynman's path integral on Euclidean plane with prescribed boundary condition. In fact, in QFT the wave function is a functional of this boundary condition. Then, we proposed to replace the flat metric on the Euclidean plane by a more general, curved one keeping the boundary condition unchanged. This is the step from the left to the middle in Fig 5. Next, in order to mimic the optimization procedure, we defined the "path integral complexity" action as logarithm of the ratio of two wave functions (curved and flat). This action is a functional of the background metric (e.g., in 2D CFTs it is given by the famous Liouville action) and we select the optimal metric as the one that minimizes the path integral complexity. We tested this procedure in numerous examples in CFTs and found that our optimal metrics are always hyperbolic. This established the path integral optimization procedure as a continuous counterpart of the AdS/TN observation valid beyond free theories.

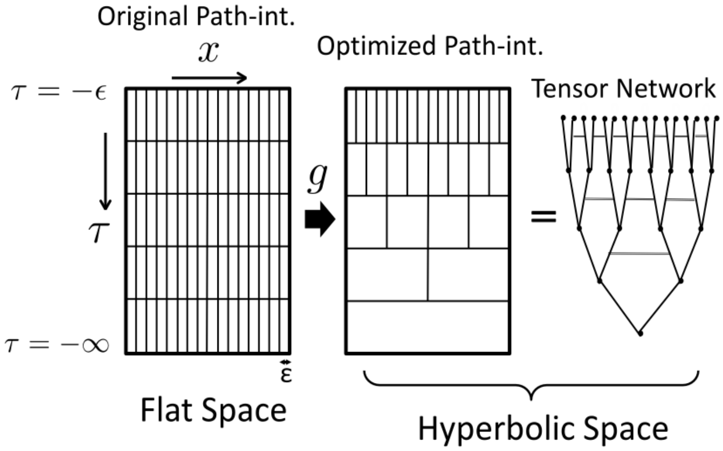

Figure 5. **Path Integral Optimization for the vacuum state.**
Original path integral on flat space is replaced with one on curved geometry chosen by minimisation of path integral complexity functional. Optimal metrics are hyperbolic geometries realising continuous version of the AdS/TN observations.

More specifically, in [A6], we started by analyzing two-dimensional conformal field theories where metrics on the 2D Euclidean plane can be always written in the Weyl-flat form

$$ds^2 = e^{2\phi(\tau,x)}(d\tau^2 + dx^2). \quad (9)$$

By definition, the CFT action is invariant under the Weyl rescaling but the path-integral measure is anomalous and transforms with the exponent of the Liouville action [85] that becomes our path integral complexity in 2D:

$$S_L[\phi] = \frac{c}{24\pi}\int dx \int d\tau \left((\partial_\tau \phi)^2 + (\partial_x \phi)^2 + e^{2\phi}\right) \quad (10)$$

This path integral complexity is universally defined for all 2D CFTs with central charge c (in front of the action) and the optimization procedure for the vacuum state corresponds to solving the Liouville equation on the half plane with the boundary condition that the UV cutoff is reproduced at the boundary of τ. The correct solution with the appropriate boundary condition is given by the metric on the hyperbolic plane that has constant negative curvature.

We applied this method to thermofield double states as well as primary and vacuum in CFT on the circle, and found that optimal metrics for path integrals preparing these states are always hyperbolic. Moreover, we interpreted them as slices of dual bulk geometries in AdS. This was followed by many interesting developments [86-92] and recently we proposed the "gravity dual" prescription of the path integral optimization in holographic CFTs using Hartle-Hawking wave functions [93,94].

7. "Liouville Action as Path-Integral Complexity: From Continuous Tensor Networks to AdS/CFT."

In this work we further explored Liouville action as path integral complexity and generalized it to higher-dimensional CFTs. Firstly, we interpreted the minimal on-shell value of the Liouville action as a relative measure of complexity (see [95] for review) of continuous path integral tensor networks. This comes from the fact Liouville action naturally depends on two metrics, the reference one (that we took flat in eq. (10)) and the second one related by the Weyl factor (9). Moreover, a shift of the Weyl factor accompanied by rescaling of the reference metric is a symmetry of the 2D geometry. However, the Liouville action is not invariant under this procedure and one has to subtract the volume of the reference metric to define the "improved" relative complexity action $I[g_1, g_2]$. Interestingly, this action satisfies the so-called cocycle properties in terms of three metrics:

$$I[g_1, g_2] = -I[g_2, g_1], \qquad I[g_1, g_2] + I[g_2, g_3] = I[g_1, g_3]. \tag{11}$$

The first property makes it clear that, unlike quantum computational measures based on distance, our relative path integral complexity action can be negative. This property is natural from our TN motivated definition that counts the relative number of tensors between two continuous tensor networks: g_1 and g_2.

In the second part of [A7], we proposed a generalization of the relative path integral complexity action to higher dimensions. Guiding principles behind it were: the co-cycle properties above, solutions of the optimization given by constant curvature slices of AdS geometries in higher dimensions, match with holographic results such as entanglement entropy for spherical regions and reduction to the Liouville action for d=2. The formula for that fulfils all these constraints is given by:

$$I[\phi,\hat{g}] = N\int d^dx\sqrt{\hat{g}}e^{d\phi}\left(e^{-2\phi}\partial_a\phi\partial^a\phi + \frac{\hat{R}}{(d-1)(d-2)}e^{-2\phi} + \mu\right) \tag{12}$$

and computes relative complexity between background metric $\hat{g}$ and its Weyl rescaled form (9). In higher dimensions this is only a subset of metrics, nevertheless, we analyzed various solutions to the path integral optimization with this action with our boundary condition and checked that they all have constant negative curvatures (hyperbolic). The overall factor related to the Newton constant N~(d-1)/16πG was fixed by match with entanglement entropy results and the coefficient of the potential, related to the UV divergence, can be absorbed by constant shift of the Liouville field. Finally, the limit of d=2 should be taken on the level of the difference of improved actions $I[\phi, \hat{g}] - I[0,\hat{g}]$ and yields $S_L[\phi, \hat{g}] - S_L[0,\hat{g}]$.

Furthermore, we evaluated $I[\phi, \hat{g}]$ on-shell on our examples and found a universal spatial volume scaling of the leading UV divergence. In addition, in [86] Czech argued that various terms of our complexity action can be naturally interpreted from the perspective of MERA-like tensor networks where the volume term counts the number of unitary and the kinetic term the isometric tensors respectively. Our result became the first definition of CFT complexity applicable to holographic CFTs and was followed by many interesting works that tested and extended it further in various directions. Despite all these appealing properties, the higher-dimensional action (12) remained a conjecture, but recently [92,93] we found a very strong evidence for $I[\phi, \hat{g}]$ reproducing it as a "UV limit" of the gravity action that computes semi-classical Hartle-Hawking wave functions.

8. “Holographic Entanglement of Purification from Conformal Field Theories.”

This work was the final application of the path integral optimization to computation of entanglement of purification (EoP) in AdS/CFT. In [96] authors proposed a new correspondence between the minimal area of the entanglement wedge cross-section E_w (see also [97,98] for other proposals relating the area of the cross-section to CFT computations) in AdS and quantum information measure of entanglement for mixed states called entanglement of purification E_P [99]:

$$E_W(\rho_{AB}) = \frac{\mathscr{A}(\Sigma_{AB}^{min})}{4G_N} \quad \leftrightarrow \quad E_P(\rho_{AB}) = min_{|\Psi_{A\bar{A}B\bar{B}}\rangle}[S_{A\bar{A}}] \,. \tag{13}$$

In the above formula the density matrix ρ_{AB} of two regions A and B is computed from a general wave function $\Psi_{A\bar{A}B\bar{B}}$. Entanglement of purification E_P is then computed as entanglement entropy $S_{A\bar{A}}$ of the union of A and its complement $\bar{A}$ minimized over all possible purifications that give rise to ρ_{AB}. On the other hand, on the gravity side, the density matrix is described by the entanglement wedge with cross section Σ_{AB} with area $\mathscr{A}(\Sigma_{AB})$.

The above correspondence was argued based on various non-trivial quantum information theoretic properties of EoP that were found to be satisfied by the area of the entanglement wedge cross-section in gravity as well [96]. However, prior to our work, there has not been an explicit CFT computation that could support this interesting relation. The main difficulty in computing EoP in CFTs comes from its mathematically elegant by formal definition with minimization over “all possible” purifications that becomes an ill-defined problem in continuous quantum field theories. On the contrary, the gravity dual proposal with the minimal wedge cross-section is well defined and straightforward to evaluate, providing a sharp prediction for the correct CFT counterpart.

Our new idea in this work was to focus on a class of “geometric purifications” corresponding to Weyl rescalings of the background metric in CFT that, by the path integral optimization construction, have minimal path integral complexity. In this class, the evaluation of the EoP boils down to the computation of entanglement entropy in a 2D CFT on hyperbolic geometries. Using standard CFT techniques on hyperbolic space, we evaluated EoP in these purifications and verified that it perfectly matches the gravity result. This was the first computation of EoP in quantum field theory and our result was recently reproduced from numerical computation in free Gaussian system [100,101]. This work was also one of the first important applications of the path integral optimization and established it as a powerful tool for probing many-body systems.

9. “Complexity as a novel probe of quantum quenches: universal scalings and purifications.”

Similarly to the developments in entanglement entropy in many-body systems [102,103] it was natural to start with testing potential measures of complexity in exactly solvable, free Gaussian systems [47]. Even in these simple settings there are numerous approaches that one can consider. However, as discussed in the introduction, Nielsen’s geometric definition of circuit complexity based on the geodesic length in the manifold of unitaries stands out as a potentially useful tool in QFTs [45,46]. Indeed, initial results [47,48] that applied it to compute complexity of the vacuum states showed reasonable behaviour and universal scaling with the volume of the system. In [A9] we decided to employ this new tool in the context of quantum quenches and test whether it was sensitive to the interesting non-equilibrium physics, previously only probed with correlation functions [104] and entanglement measures [105].

The setup that we considered in [A9] is given by the harmonic chain with time dependent mass as follows:

$$H(t)=\frac{1}{2}\sum_{n=1}^{N}\left(\Pi_n^2+(\phi_{n+1}-\phi_n)^2+m^2(t)\phi_n^2\right). \tag{14}$$

The mass profile is chosen such that at early times ($t\to-\infty$) it is constant, then it passes through a massless “critical point” at $t=0$ with velocity δt (the so-called “quench rate”) and returns to its initial value at late times ($t\to\infty$). One generic choice of the profile described above that we adapted is given in terms of hyperbolic functions (see eq. (5) in [A9]). The interesting question then is how physical quantities evolve with time during this process and how they depend on the quench rate δt? Often, depending on the quench rate, one can describe the physics of such non-equilibrium evolution using scaling arguments, with the most famous example of the Kibble-Zurek scaling [106,107] for “slow quenches” as well as recently found universal scalings in the fast quenches [108].

In our work we were interested in circuit complexity between the reference state corresponding to the ground state of the early-time Hamiltonian and a target state described by the wave function of the Hamiltonian at time t. As in previous works, we defined circuits in terms of position and momentum gates or, equivalently, we described them in terms of transformations of the covariance matrix for this Gaussian system. It turned out that the algebra of “sufficient set” of our unitary gates corresponds to Nielsen’s geometry given by the hyperbolic space $\mathbb{H}_2$ (one for each momentum mode separately) and our task was to find minimal geodesics in this geometry at time t. Having solved this problem, we adapted the cost function given by the L_2 norm, summed (and regulated) the answers from each mode and studied its dependence on the quench rate.

In this proof of concept work, we analyzed complexity from the “universality” point of view and indeed verified that it is sensitive to these universal scalings i.e., shows particular scalings depending on the quench rate and hence can be used as a novel probe of quantum quenches. For example, we verified that at the critical point $t=0$, it scales linearly with the quench rate (see Fig. 6). Similar results were confirmed in [109,110] and complexity is now frequently used as an interesting new probe of non-equilibrium physics [111].

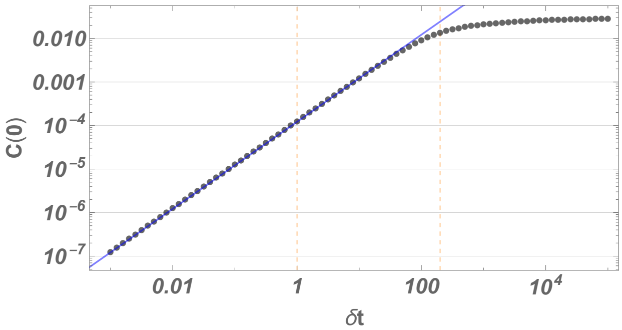

Figure 6. **Scaling of circuit complexity with the quench rate.**
Figure shows log-log plot of complexity of the (1+1)D model (14) at the critical point t=0 as a function of the quench rate δt. The straight line reveals the linear scaling in the fast regime.

10. "Quantum Computation as Gravity."

Last but not least, I will finish with a summary of [A10] where, building on the intuition from the path integral complexity as well as Nielsen's geometric complexity we proposed how to formulate and compute circuit complexity in conformal field theories. We started from CFTs in 2 dimensions which enjoy a large, universal "symmetry sector" governed by two copies of the infinite dimensional Virasoro algebra generated by the modes of the chiral and anti-chiral components of the energy momentum tensor T and $\bar{T}$ respectively. In this setup, we defined "Virasoro circuits" built from the energy momentum tensor. More precisely, our symmetry circuits in 2D CFTs have the form [A10]:

$$U(\tau) = P\exp\left(\int_0^\tau d\tau'(Q(\tau') + \bar{Q}(\tau'))\right), \qquad Q(\tau) = \int_0^{2\pi} \frac{d\sigma}{2\pi}\epsilon(\tau,\sigma)T(\sigma), \tag{15}$$

where P stands for the path ordering, T is the energy momentum tensor of 2D CFT expanded in terms of Virasoro generators L_n and similarly for ($\bar{T}$). These circuits take us from some given reference state (e.g. vacuum or other eigenstate of the Hamiltonian) to an arbitrary Virasoro descendants. Nielsen's geometric approach is then naturally implemented by defining circuit complexity by the length of the minimal geodesic in the Virasoro group (infinite dimensional geometry). At first it appears that this setup is too simple but actually in holographic large-c CFTs points on these "orbits" can be represented in terms of geometries (Banados metrics [112]) that are fixed by specifying the expectation values of T and $\bar{T}$. Of course, the universal symmetry sector can be naturally extended to include (non-universal) primary operators in gates Q instead of T.

This is not the only connection to gravity that we observed in our setup. Namely, in the Nielsen's approach, geodesics length clearly depends on the cost function (metric on the space of unitaries). In [49] it was argued that, in QFTs, it is natural to define them in terms connected or disconnected higher point correlation functions of the "instantaneous gates" Q. Interestingly, we showed that in the large-c limit various norms are equivalent up to 1/c corrections and the action that emerges from the geometric complexity is given by the famous Polyakov action of 2D gravity [85] or more mathematically, the geometric action on the coadjoint orbits of the Virasoro group [113]. This explains the title and our main claim that, in CFTs, there exist cost functions that naturally lead to gravity actions governing circuit's complexity. It turns out that this feature is universal in many generalizations of our analysis including extended symmetries in 2D [114] and global symmetry sector of higher dimensional CFTs [115].

Moreover, we explored different possibilities of defining complexity functional in our 2D CFT setup, including connected correlation functions (e.g., the Fubini-Study metric for the 2-norm) as well as more mathematical approaches based on the Euler-Arnold equations on the Virasoro group. Based on large-c scalings as well as time evolution, we argued that some cost functions are less natural (e.g. 1/c suppressed) from the perspective of holography but nevertheless very interesting on their own right in this completely unexplored setting. Since our work only initiated this new exciting direction in CFTs, the final rules of the game for holographic CFTs and exact gravity dual of our construction still remain to be determined and this is being very actively developed at present (see e.g. [116]).

Conclusions

Summarizing, ideas and tools from quantum information and computation turned out to be extremely fruitful in the study of holographic correspondence. At present, we are still in the explorative phase and this "unreasonable effectiveness" of quantum information in gravity remains mysterious. My works on this subject, discussed in this habilitation, were focused on aspects of dynamics of quantum information as well as on extracting geometry and complexity of states in conformal field theories. In particular, I have developed analytical tools to study quantum quenches in CFTs and their gravity dual geometries. My works [A1-A5] on charged entropies, local operator quenches and back-reacted geometries from point particles in AdS are among the important developments in my field. On the other hand, path integral optimization in CFTs [A6-A8], that I have developed with my collaborators, not only sheds new light on the possible mechanism behind AdS/CFT (extracting geometry from CFT states) but also remains one of the leading approaches to field theory complexity. Even though complexity in QFTs is still at its infancy, new applications such as [A9] and new ideas based on symmetry and universality as [A10] are pushing these developments forward. I am certain that further studies of black holes in AdS as well as in observational/realistic spacetimes will bring more fascinating surprises in the future and will allow us to understand the reason behind the holographic nature of gravity.

Bibliography

[1] J.M. Maldacena, Adv.Theor.Math.Phys. 2 (1998) 231-252
[2] S.S. Gubser, I.R. Klebanov and A.M. Polyakov, Phys.Lett. B428 (1998) 105-114
[3] E. Witten, Adv.Theor.Math.Phys. 2 (1998) 253-291
[4] E. Shuryak, Rev.Mod.Phys. 89 (2017) 035001
[5] S.A. Hartnoll, A. Lucas and S. Sachdev, "Holographic quantum matter", The MIT Press (March 9, 2018)
[6] S.Ryu and T.Takayanagi, Phys. Rev. Lett. 96, 181602 (2006)
[7] T. Nishioka, S. Ryu, T. Takayanagi, J. Phys. A 42 (2009), 504008
[8] T. Nishioka, Rev. Mod. Phys. 90 (2018) no.3, 035007
[9] P. Calabrese and J. Cardy, J.Stat.Mech. 0406 (2004) P06002
[10] A. Lewkowycz, J. Maldacena, JHEP 08 (2013), 090
[11] M. Rangamani and T. Takayanagi, Lect.Notes Phys. 931 (2017) pp.1-246
[12] V.E. Hubeny, M. Rangamani, T. Takayanagi, JHEP 07 (2007), 062
[13] X. Dong, A. Lewkowycz and M. Rangamani, JHEP 11 (2016), 028
[14] P. Calabrese and J.L. Cardy, J. Stat. Mech. 0504 (2005), P04010

[15] P. Calabrese and J.L. Cardy, Int. J. Quant. Inf. 4 (2006) 429
[16] P. Calabrese and J.L. Cardy, Phys. Rev. Lett. 96 (2006), 136801
[17] P. Calabrese and J.L. Cardy, J. Stat. Mech. 0706 (2007), P06008
[18] P. Calabrese and J.L. Cardy, *J. Phys. A* 42 (2009) 504005
[19] T. Hartman, C. A. Keller, B. Stoica, JHEP 09 (2014), 118
[20] M. Headrick, Phys. Rev. D, 82 (2010), 126010
[21] T. Hartman, arXiv:1303.6955 [hep-th]
[22] T. Faulkner, arXiv:1303.7221 [hep-th]
[23] J.M. Maldacena, JHEP 04 (2003), 021
[24] M. Van Raamsdonk, Gen. Rel. Grav. 42 (2010), 2323-2329
[25] J. Maldacena, L. Susskind, Fortsch. Phys. 61 (2013), 781-811
[26] S.H. Shenker, D. Stanford, JHEP 12 (2014), 046
[27] A. Kitaev, Hidden correlations in the Hawking radiation and thermal noise, talk given at Fundamental Physics Symposium, Stanford U.S.A
[28] J. Maldacena, S.H. Shenker, D. Stanford, JHEP 08 (2016), 106
[29] G. Vidal, Phys. Rev. Lett. 101 (2008) 110501
[30] B. Swingle, Phys. Rev. D86 (2012) 065007
[31] M. Nozaki, S. Ryu, T. Takayanagi, JHEP 1210 (2012) 193
[32] F. Pastawski, B. Yoshida, D. Harlow, J. Preskill, JHEP 1506 (2015) 149
[33] P. Hayden, S. Nezami, X-L. Qi, N. Thomas, M. Walter, Z. Yang, JHEP 1611 (2016) 009
[34] G. Evenbly, arXiv:1704.04229 [quant-ph]
[35] B. Czech, L. Lamprou, S. McCandlish, J. Sully, JHEP 1607 (2016) 100
[36] J. Haegeman, et al., Phys. Rev. Lett. 110 (2013) 100402, [1102.5524]
[37] R. Orús "Tensor networks for complex quantum systems", Nature Reviews Physics.
[38] A. Jahn, J. Eisert, arXiv:2102.02619 [quant-ph]
[39] T. Hartman and J. Maldacena, JHEP, 05 (2013), 014
[40] V. Balasubramanian, A.Bernamonti, J. deBoer, N. Copland, B. Craps, E. Keski-Vakkuri, B. Muller, A. Schafer, M. Shigemori, W. Staessens, Phys. Rev. D 84 (2011), 026010
[41] H. Liu, S. J. Suh, Phys. Rev. Lett. 112 (2014), 011601
[42] L. Susskind, Fortsch. Phys. 64 (2016) 24–43
[43] L. Susskind and D. Stanford, Phys.Rev. D90 (2014) no.12, 126007
[44] A. R. Brown, et al., Phys. Rev. Lett. 116 no. 19, (2016) 191301.
[45] M.A. Nielsen, arXiv:quant-ph/0502070
[46] M. A. Nielsen et al., Science 311, 1133 (2006)
[47] R. Jefferson, R.C.Myers, JHEP 1710 (2017) 107
[48] S. Chapman, M.P. Heller, H. Marrochio, F. Pastawski, Phys. Rev. Lett. 120 (2018) no.12, 121602
[49] J. M. Magan, JHEP 09 (2018), 043
[50] A. Polkovnikov, K. Sengupta, A. Silva, M.Vengalattore, Rev. Mod. Phys. 83 (2011), 863
[51] L. Vidmar, M. Rigol, J. Stat. Mech. (2016) 064007
[52] I. Peschel 2003 J. Phys. A: Math. Gen. **36** L205
[53] H.Casini, M. Huerta, J. Phys. A 42 (2009), 504007
[54] J. Bhattacharya et al., Phys.Rev.Lett. 110 (2013) no.9, 091602
[55] A.Belin, L.Y.Hung, A.Maloney, S.Matsuura, R.C. Myers, T.Sierens, JHEP 12 (2013), 059
[56] P. Caputa, V. Jejjala, H. Soltanpanahi, Phys.Rev.D.89 (2014) 4, 046006
[57] P. Caputa, M. Nozaki, T. Numasawa, Phys. Rev. D 93 (2016) no.10, 105032
[58] M. Goldstein, E. Sela, Phys. Rev. Lett. 120 (2018) no.20, 200602
[59] S. Murciano, G. DiGiulio, P. Calabrese, JHEP 08 (2020), 073
[60] P. Calabrese and J. Cardy, J.Stat.Mech. 1606 (2016) no.6, 064003
[61] M. Nozaki, T. Numasawa, T. Takayanagi, Phys. Rev. Lett. 112 (2014), 111602
[62] M. Nozaki, JHEP 10 (2014), 147
[63] S. He, T. Numasawa, T.Takayanagi, K.Watanabe, Phys. Rev. D 90 (2014) no.4, 041701
[64] M. Nozaki, T. Numasawa, T. Takayanagi, JHEP 05 (2013), 080
[65] C.T. Asplund, A. Bernamonti, F. Galli and T. Hartman, JHEP 02 (2015), 171
[66] Y. Kusuki, M. Miyaji, JHEP 08 (2019), 063
[67] M. Srednicki, Phys Rev E 50 (1994) 888
[68] A.L. Fitzpatrick, J.Kaplan, M.T. Walters, JHEP 08 (2014), 145
[69] J. R. David, S. Khetrapal, S.P. Kumar, JHEP 10 (2017), 156

[70] I.Y. Aref'eva, M.A. Khramtsov, M.D. Tikhanovskaya, JHEP 09 (2017), 115
[71] A. Sivaramakrishnan, Annals Phys. 381 (2017), 41-67
[72] T.J. Hollowood, S.P. Kumar, JHEP 08 (2020), 094
[73] A. Miyata, T. Ugajin, arXiv:2104.00183 [hep-th]
[74] S.H. Shenker, D. Stanford, JHEP 03 (2014), 067
[75] G.T. Horowitz, N. Itzhaki, JHEP 02 (1999), 010
[76] N. Lashkari, D. Stanford, M. Hastings, T. Osborne, P. Hayden, JHEP 04 (2013), 022
[77] C. Murthy, M. Srednicki, Phys. Rev. Lett. 123 (2019) no.23, 230606
[78] D.A. Roberts, D. Stanford, Phys. Rev. Lett. 115 (2015) no.13, 131603
[79] E. Perlmutter, JHEP 10 (2016), 069
[80] E. Witten, Commun. Math. Phys. 121 (1989), 351-399
[81] V.F.R. Jones, Bull. Am. Math. Soc. 12 (1985), 103-111
[82] G. Evenbly, G. Vidal, Phys.Rev.Lett. 115 (2015) 180405
[83] G. Evenbly, G. Vidal Phys. Rev. Lett. 115, 200401 (2015)
[84] J.Haegeman, T.J. Osborne, H.Verschelde, F.Verstraete, Phys. Rev. Lett. 110 (2013) no.10, 100402
[85] A.M. Polyakov, Mod. Phys. Lett. A 2 (1987), 893
[86] B. Czech, Phys.Rev.Lett. 120 (2018) no.3, 031601.
[87] A. Milsted and G. Vidal, arXiv:1805.12524 [cond-mat.str-el]
[88] A. Milsted and G. Vidal, arXiv:1807.02501 [cond-mat.str-el]
[89] A.Tilloy and J.I. Cirac , arXiv:1808.00976 [cond-mat.str-el]
[90] H.A.Camargo, M.P. Heller, R.Jefferson, J.Knaute, Phys.Rev.Lett. 123(2019)no.1, 1601
[91] A.Bhattacharya, et al., JHEP 1807 (2018) 086
[92] R.A. Janik, JHEP, 1901, 225 (2019)
[93] J.Boruch, P. Caputa, T. Takayanagi, Phys. Rev. D 103 (2021) no.4, 046017
[94] J. Boruch, P. Caputa, D.Ge, T. Takayanagi, arXiv:2104.00010 [hep-th].
[95] J. Watrous, in Encyclopedia of complexity, pp. 7174–7201. Springer, 2009.
[96] T. Takayanagi, K. Umemoto, Nature Phys. 14 (2018) no.6, 573-577
[97] S. Dutta, T. Faulkner, JHEP 03 (2021), 178
[98] K. Tamaoka, Phys. Rev. Lett. 122 (2019) no.14, 141601
[99] B M. Terhal et al., Journal of Mathematical Physics 2002, 43:9, 4286-4298
[100] A. Bhattacharyya, T. Takayanagi, K. Umemoto, JHEP 04 (2018), 132
[101] H.A. Camargo, L. Hackl, M.P. Heller, A. Jahn, T. Takayanagi, B. Windt, Phys. Rev. Res. 3 (2021) no.1, 013248
[102] L. Bombelli, R.K. Koul, J. Lee, R.D. Sorkin, Phys. Rev. D 34 (1986), 373-383
[103] M. Srednicki, Phys. Rev. Lett. 71 (1993), 666-669
[104] D.Das, S.R. Das, D.A. Galante, R.C. Myers, K. Sengupta, JHEP 11 (2017), 157
[105] P. Caputa, S.R. Das, M. Nozaki, A. Tomiya, Phys. Lett. B 772 (2017), 53-57
[106] T W B Kibble 1976 J. Phys. A: Math. Gen. 9 1387
[107] W. Zurek *Nature*. 317 (6037): 505–508
[108] S.R. Das, D.A. Galante, R.C. Myers, JHEP 02 (2015), 167
[109] D. W. F. Alves, G. Camilo, JHEP 06 (2018), 029
[110] S. Liu, JHEP 07 (2019), 104
[111] G. Di Giulio, E. Tonni, JHEP 05 (2021), 022
[112] M.Banados, AIP Conf. Proc. 484 (1999) no.1, 147-169
[113] A. Alekseev and S. L. Shatashvili, Commun. Math. Phys. 128, 197 (1990).
[114] J. Erdmenger, M. Gerbershagen, A.L. Weigel, JHEP 11 (2020), 003
[115] N. Chagnet, S. Chapman, J. de Boer, C. Zukowski, arXiv:2103.06920 [hep-th].
[116] M. Flory, M.P. Heller, Phys. Rev. Res. 2 (2020) no.4, 043438

OTHER ACCOMPLISHMENTS:

Grants:

1. **NCN Sonata Bis 9 (2020/06/01-2025/05/31), Warsaw, Poland**
Principal Investigator
Title: ”Quantum Information in Quantum Field Theories and Holography: Dynamics and Complexity.”
Grant number: **UMO-2019/34/E/ST2/00123.**
Amount: 2 185 000 PLN.

2. **NAWA Polish Returns 2019 (01.02.2020 - 31.01.2024), Warsaw, Poland**
Principal Investigator
Title: ” Holographic Geometry and Quantum Information”.
Grant Number: **PPN/PPO/2019/1/00010/U/0001.**
Amount: 2 010 000 PLN.

3. **KAKENHI Starting Grant from JSPS (2017-2019), Kyoto, Japan**
Principal Investigator,
Title: ”From Einstein equations to Tensor Networks”,
Grant Number: **17H06787**.
3 Million Yen (~24k Euro) for 2 years 2017-2019.

Fellowships and Awards:

1**. Simons ”It from Qubit” Fellowship (2017/01/01-2019/12/30)**
Prestigious fellowship from the Simons ”It from Qubit” collaboration.
Around 180 k USD for 3 years at the Yukawa Institute in Kyoto.
Principal Investigator: Tadashi Takayangi.

2**. JSPS Short Term Scholarship (9 months in 2014)**
Japanese Scholarship for Promotion of Science for 2014.
4.5 Million Yen (~36k Euro) for 9 months at the Yukawa Institute in Kyoto, Japan.

3. **Claude Leon Foundation Postdoctoral Fellowship (2013)**
Scholarship for Postdoctoral Researchers for 2014/15, WITS, South Africa.
Around 34k EUR for 2 years (declined for the JSPS fellowship).

4**. PhD fellowship at the Niels Bohr Institute (11/2008-10/2011)**
Project Title ”Integrable theories of particles and strings”.
Around 80k EUR for 3 years.

5. **HSP Huygens Scholarship (01/2008-08/2008)**
A scholarship for students to perform their Master’s research in The Netherlands.
Awarded by the Dutch Minister of Education, Culture and Science.
Around 1400 EUR/month.

6. **Shell Theoretical Physics Award (2008)**
Annual prize of 2000 EUR for the best Master students in The Netherlands.

7. **Scholarship for excellent results of studies (2004-2007)**
Wroclaw University, Poland

Selected talks and seminars:

1. "Holographic Path-Integral Optimization", University of Crete, Greece, 2021.
2. "Path Integral Optimization from CFT to AdS", University of Barcelona, Spain, 2021.
3. "Path Integral Optimization from CFT to AdS", Wurzburg University, Germany, 2020.
4. Invited talk (overview of my area): ”Complexity of Energy-Momentum Circuits in AdS/CFT. Workshop: “Complexity from Quantum Information to Black Holes”. Amsterdam University 2020.
5. CERN Theory Colloquium, CERN, Switzerland, 2019
   Title: “Quantum Information for Quantum Field Theories: From Black Holes to Complexity”.
6. Workshop at Simons Center for Geometry and Physics, Stony Brook, USA, 2019
   Title: “Sphere Partition Functions and cut-off AdS”.
7. Quantum Fields and Strings Seminar at Perimeter Institute, Canada, 2018
   Title: “Path Integral Optimization and Complexity in 2d CFTs”.
8. Quantum Information in Quantum Gravity 4, Florence, 2018
   Title: “From Liouville to Nielsen”.
9. The Relativistic Quantum Information North 17, Kyoto 2017
   “Out of Time Ordered Correlators and Quantum Chaos”.
10. Theory Seminar at UC Santa Barbara, USA, 2017
    Title: “Path Integral Complexity”.
11. Seminar: “Rencontres Theoriciennes” at ENS Paris, 2016
    Tite: “Entanglement of local operators”.
12. Holography Program at Galileo-Galilei Institute, Florence, 2015
    Title: “Quantum Entanglement of local excitations”.
13. Japanese Strings, YITP, Kyoto, 2014
    Title: “Entanglement of local operators in large N CFTs”.
14. String Theory Seminar at TIFR, Mumbai, 2013
    Title: “On correlators with giant gravitons”.

Teaching and Supervision:

Master’s students:

1. Mario Benites, Stockholm University, 2015
   Title: ”Covariant Prescription for Holographic Entanglement Entropy”

Defended with the highest grade (Now a PhD student at Florida State University, USA)

2. Jan Boruch, University of Warsaw. Expected to graduate in June 2021.
   Research topic: “Entanglement wedge cross-section in shock wave geometries”.
   Planning to apply for a PhD in my group.
3. Michal Baczyk, ETH Zurich, Switzerland. Expected to graduate in May 2021.
   Research topic: “Petz map in free quantum field theories”.
   Planning to apply for a PhD in my group.

PhD students:

1. Dimitrios Patramanis, University of Warsaw. 01.11.2020 - present.
   Research topic: “Quantum Information in Quantum Gravity”.

Mentoring (co-supervising) PhD students at YITP, Kyoto, Japan:

1. Masamichi Miyaji, 2017-2019 (From 2020 a Postdoc at UC Berkeley, USA).
2. Kento Watanabe, 2014-2017 (Now Postdoc at UC, USA).
3. Tokiro Numasawa, 2014-2015 (Now Simons Postdoc at MIT, USA).
4. Masahiro Nozaki, 2014 (Now Postdoc at UC Berkeley, USA).

Mentoring (co-supervising) PhD students at WITS in Johannesburg, South Africa:

1. Gareth Kemp, 2012-2013 (Now lecturer at U. of Johannesburg).
2. Badr A.E. Mohammed, 2012-2013 (Now professor at SUST, Sudan).

Invited Lectures:

1. “Complexity in Quantum Field Theories”
   Three lectures at ”International PhD School in Theoretical Physics”, Pretoria, South Africa, 10/2018.
2. “Introduction to Entanglement in CFTs”
   Two introductory lectures for the Theory Group at OKC and NORDITA, Stockholm, Sweden, 04/2015.
3. “Introduction to Integrability”
   Three lectures on Integrability at ”International PhD school in Theoretical Physics”, Johannesburg, South Africa, 09/2013.
4. “Introduction to String Theory for Mathematicians”
   Two introductory lectures (4 h) on String Theory for the Topology Group.
   Copenhagen, Denmark, 2010.

Organized Conferences:

1. ”Quantum Information and String Theory 2019” 27.05-28.06, 2019 YITP Kyoto.
2. ”Holography, Entanglement and Higher Spin Gravity II” 14-16.03.2018 YITP Kyoto.
3. ”Holography and Quantum Dynamics” 11.11.2017 YITP Kyoto.

4. ”Fourth Joburg Workshop on String Theory”, University of Witwatersrand, Johannesburg, South Africa, Sep. 20, 2013.

Commissions of trust and service for the scientific community:

1. I frequently review articles for top journals in my field: Physical Review Letters, Physical Review B and D, Entropy Journal, Journal of High Energy Physics (JHEP), Journal of Statistical Physics (JSTAT), European Physical Journal C (EPJC), Journal of Physics A Mathematical and Theoretical.
2. My group (with AEI Golm, Germany) has been organizing a joint “GQFI-WST virtual seminar” on quantum information in quantum field theories and quantum gravity. This has been a great success in my domain (especially during the pandemic) and recorded talks are available on YouTube.
3. I have been a member of the committee selecting top postdoctoral candidates from all over the world (around 460 this year) to the European postdoc system (run by KU Leuven in Belgium). The committee evaluates candidates and then, in the panel discussion, selects top candidates that will be offered positions within European research groups focused on String Theory, AdS/CFT correspondence and quantum gravity.
4. JSPS Alumni board member, Stockholm, Sweden, since 2014.
5. Together with dr Jakub Kryś, we are starting the JSPS Alumni Club Poland. The club will provide an interaction platform for previous and prospective participants of the JSPS programs and new opportunities for students as well as academic exchanges between Poland and Japan.

Paweł Caputa

(Applicant’s signature)