Exploring quantum phases of matter with quantum processors

Michael Knap

Technical University of Munich

Co-funded by the European Union

Munich Quantum Valley

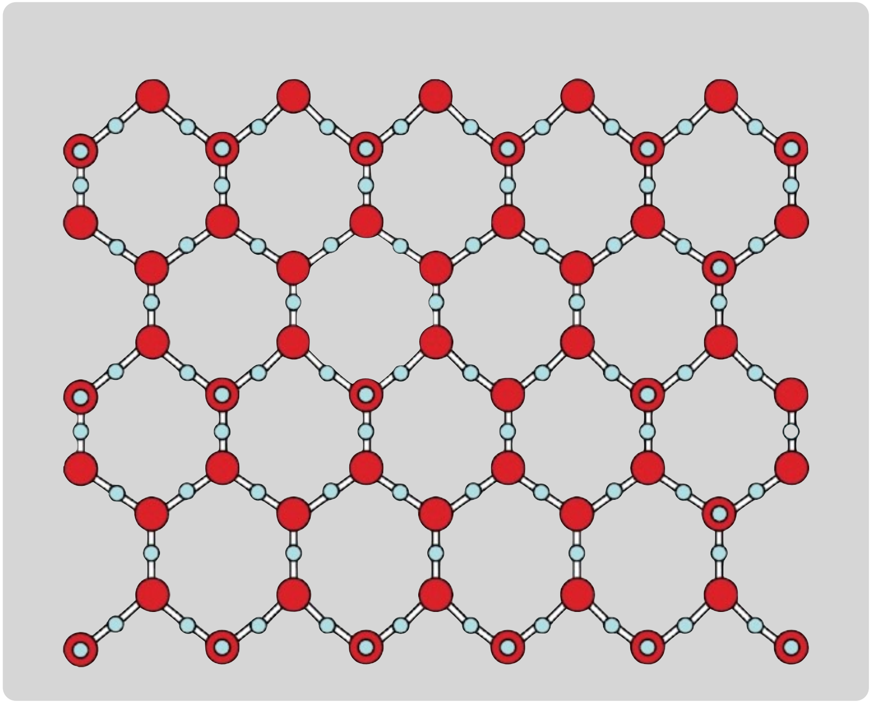
Y-J Liu J. Boesl B. Jobst W-T Xu A. Smith F. Pollmann K. Satzinger T. Cochran E. Rosenberg P. Roushan

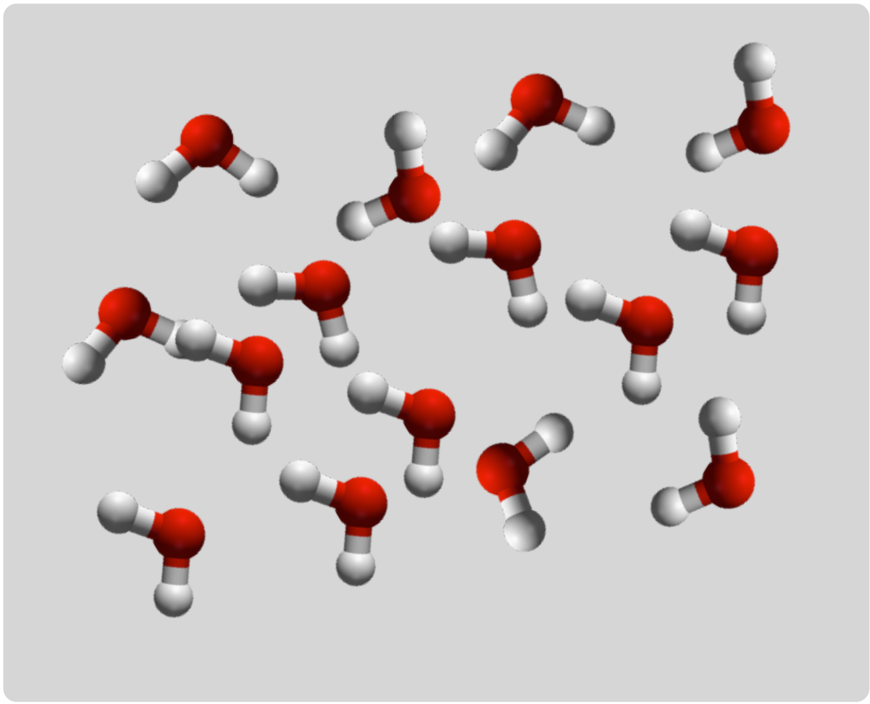
MCQST

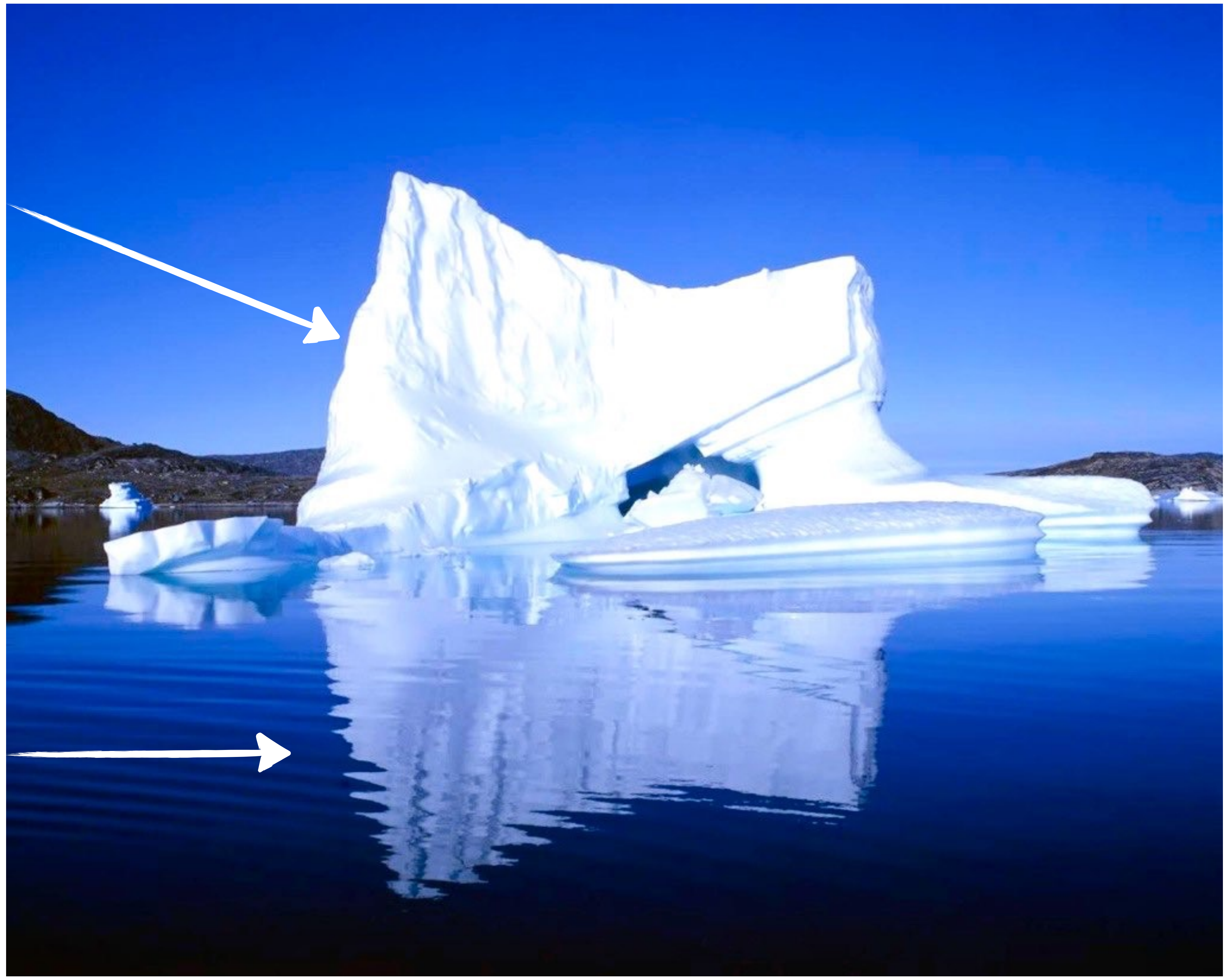
DFG

Warsaw, March 31 2025

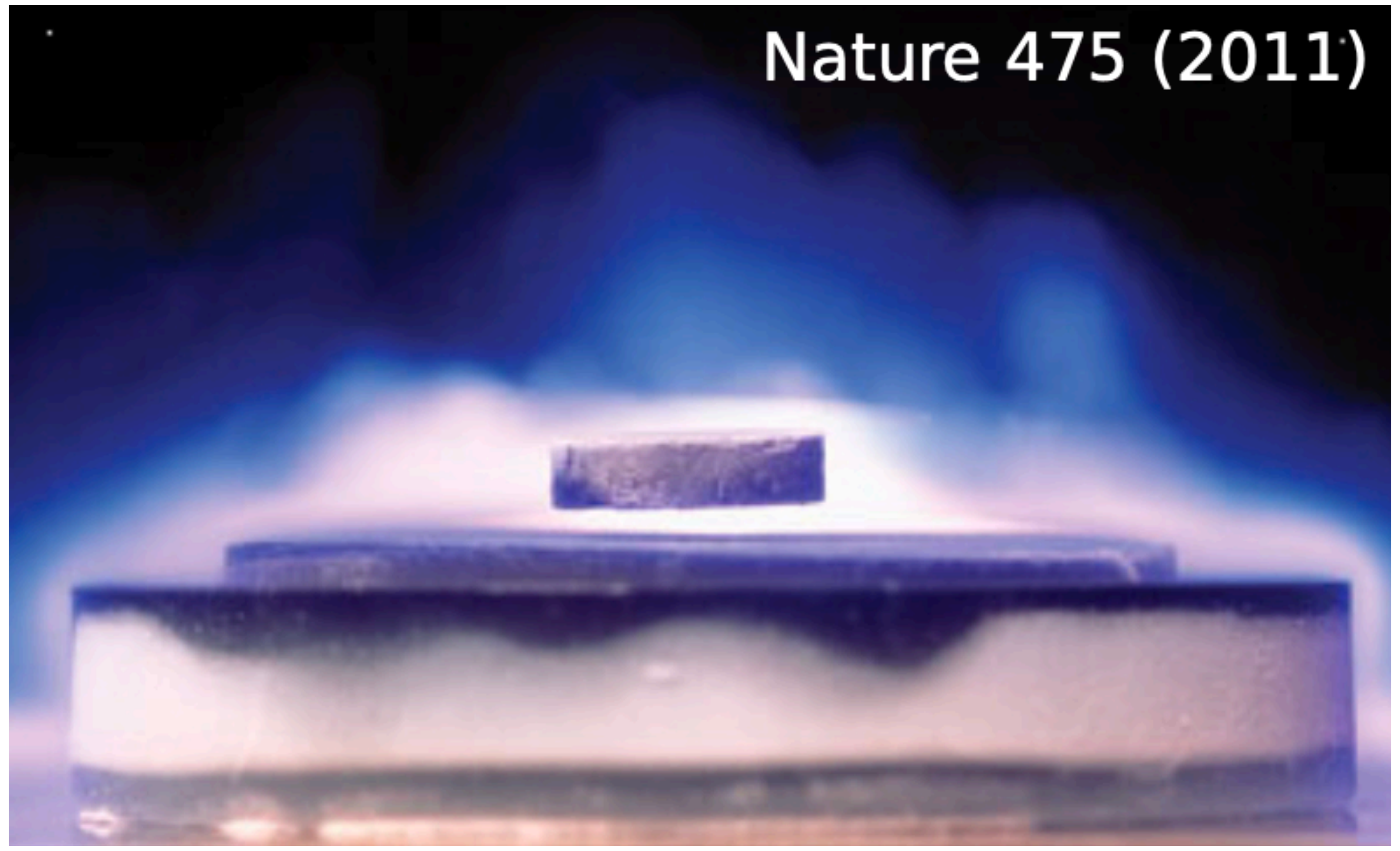
Matter occurs in different phases



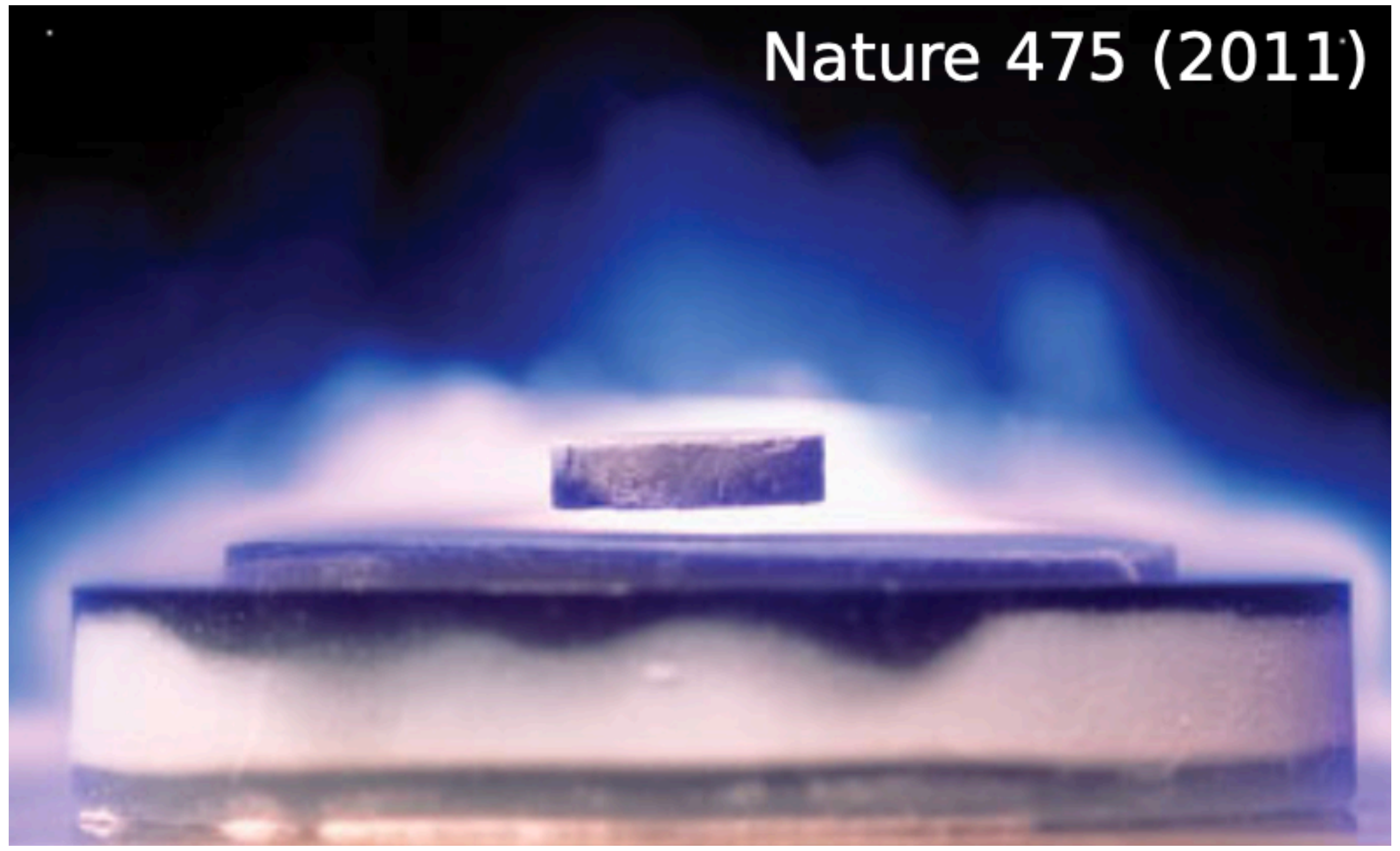




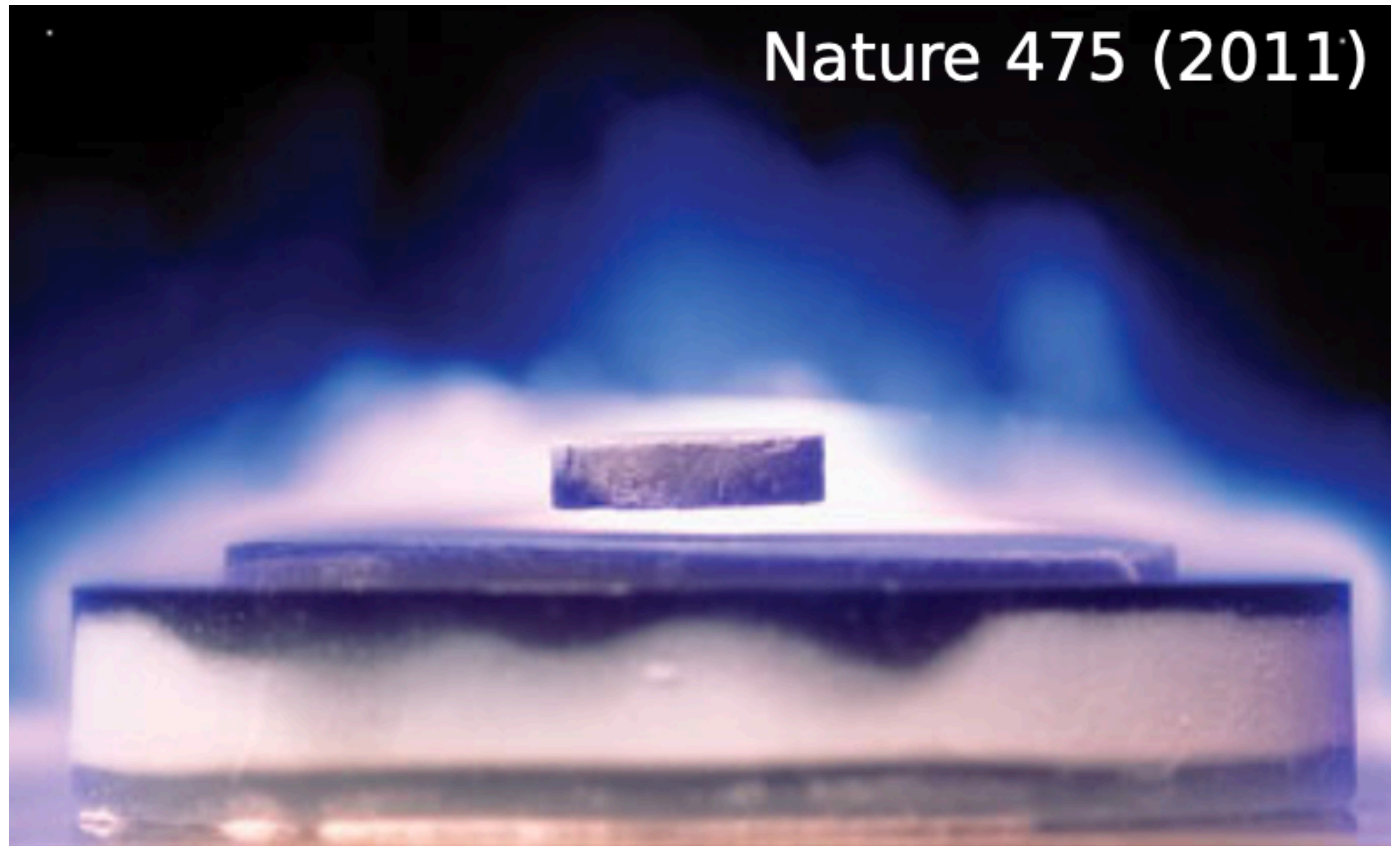
An exotic phase of matter …



An exotic phase of matter …



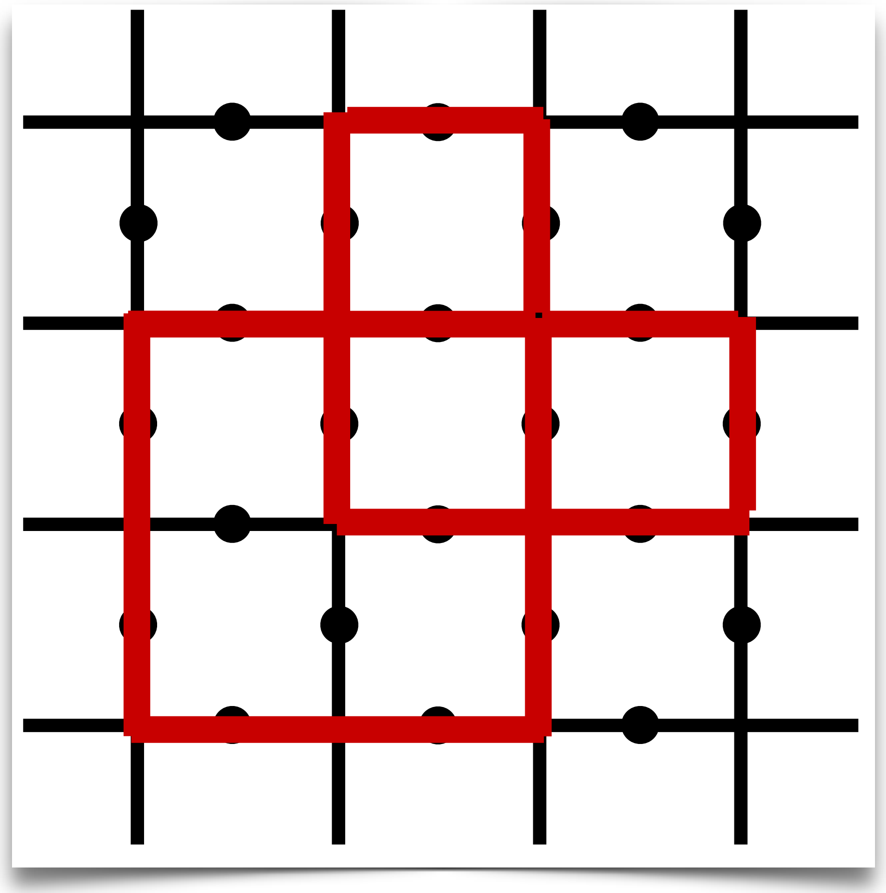
An exotic phase of matter …



Phases of matter can often be understood from local order parameters

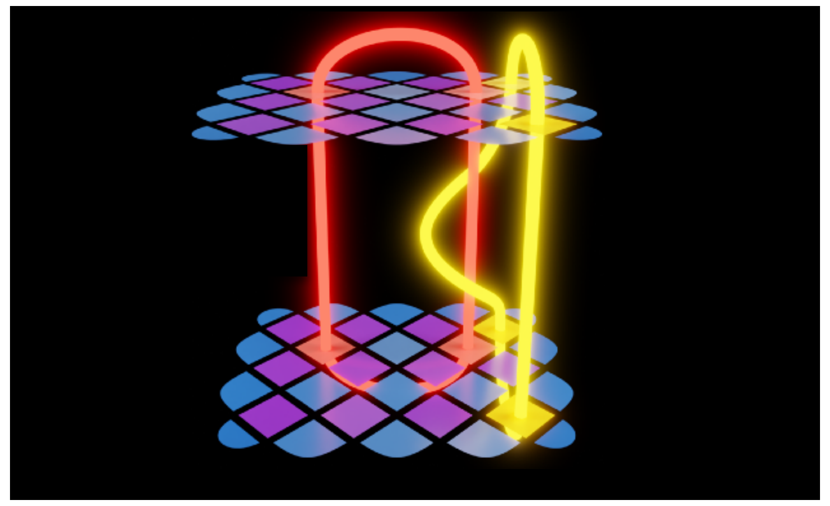
- Solids: translational symmetry breaking
- Superconductivity: electron pairing

An even more exotic phase of matter …



Phases with **topological order**: no local order parameters

- Fractional Quantum Hall Effect (FQHE), Quantum spin liquids, …

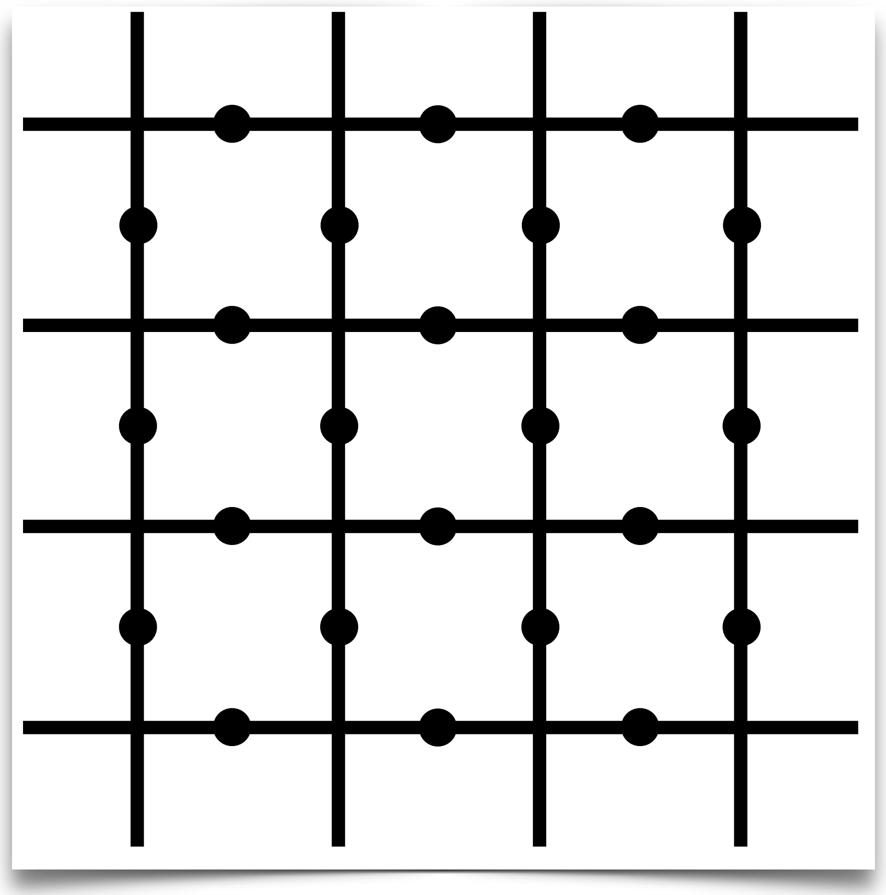


Fascinating features:

- Long-range entanglement
- Fractionalized excitations: anyons
- Foundation for error correction codes

Quantum Many-Body Problem

Simple models: collection of qubits (a.k.a. spin-1/2)



Dimension D of Hilbert space:

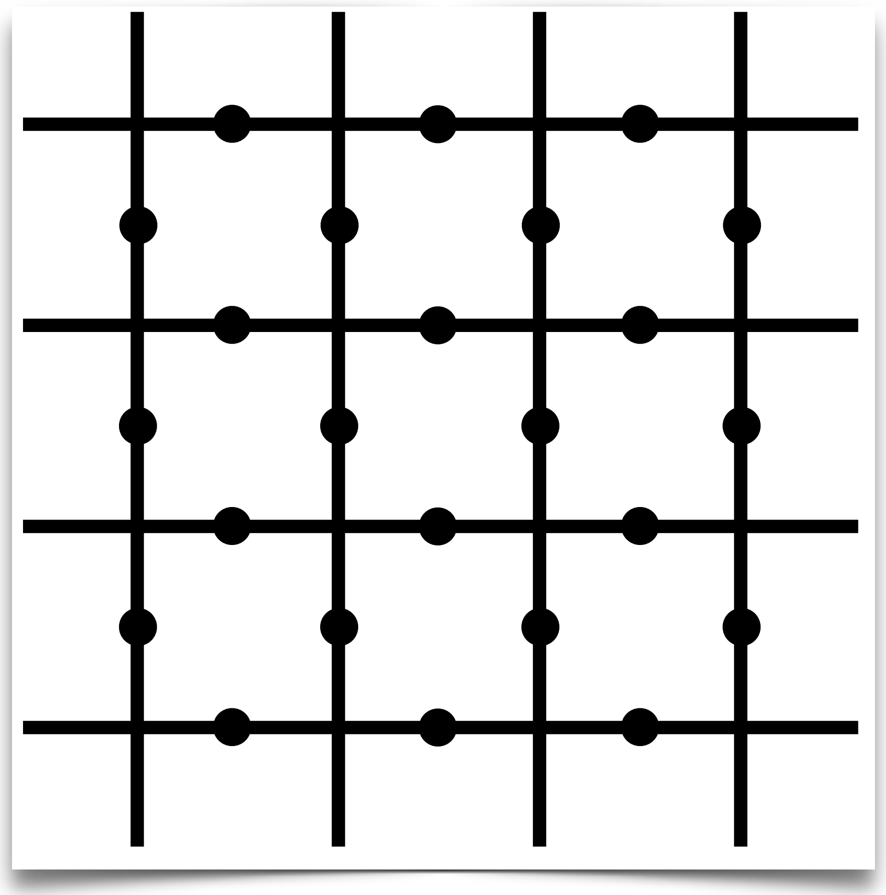
1 qubit: $D = 2$

2 qubits: $D = 4$

3 qubits:

Quantum Many-Body Problem

Simple models: collection of qubits (a.k.a. spin-1/2)



Dimension D of Hilbert space:

1 qubit: $D = 2$

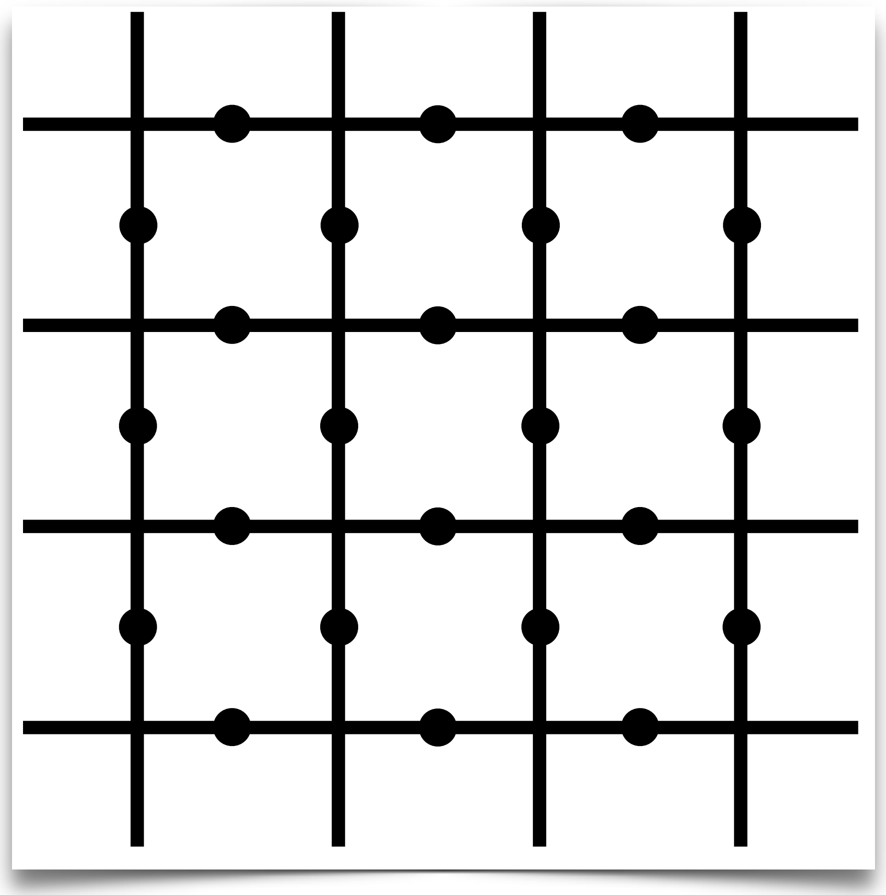
2 qubits: $D = 4$

3 qubits: $D = 8$

N qubits: $D = 2^N$

Quantum Many-Body Problem

Simple models: collection of qubits (a.k.a. spin-1/2)



Dimension D of Hilbert space:

1 qubit: $D = 2$

2 qubits: $D = 4$

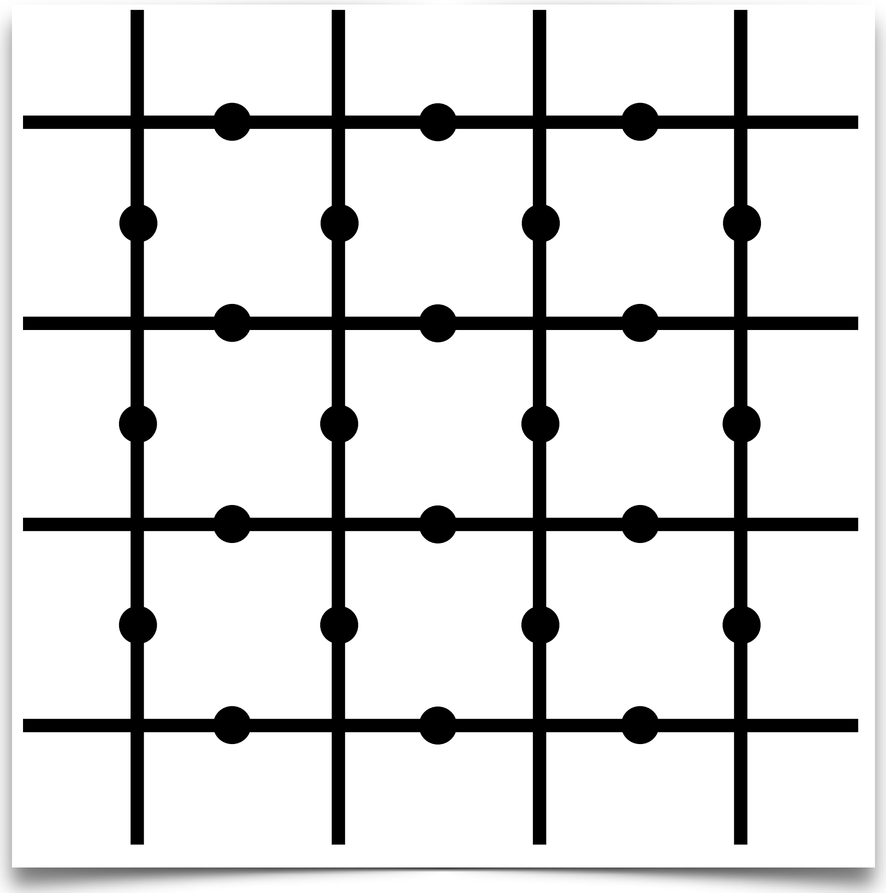
3 qubits: $D = 8$

N qubits: $D = 2^N$

100 qubits: Hamiltonian is a $10^{30} \times 10^{30}$ dimensional matrix

Quantum Many-Body Problem

Simple models: collection of qubits (a.k.a. spin-1/2)



Dimension D of Hilbert space:

1 qubit: $D = 2$

2 qubits: $D = 4$

3 qubits: $D = 8$

N qubits: $D = 2^N$

100 qubits: Hamiltonian is a $10^{30} \times 10^{30}$ dimensional matrix

Challenge:

Computational complexity grows exponentially with system size!

$$H|\psi\rangle = E|\psi\rangle$$

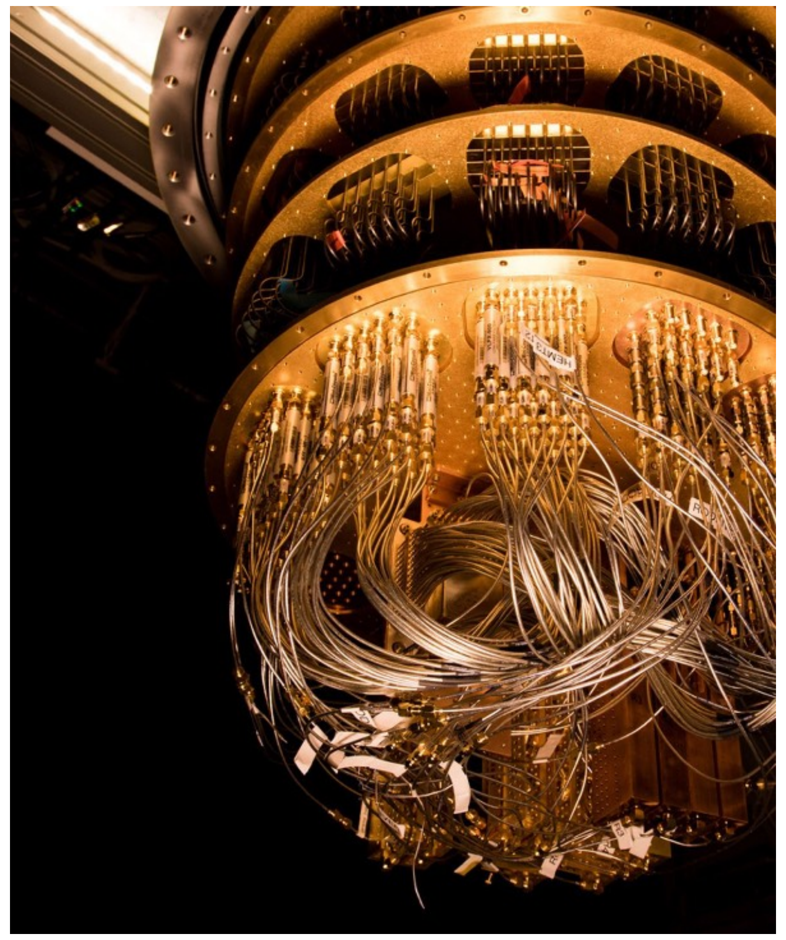
Complexity of Quantum-Many Body Problems

~40 qubits

Complexity of Quantum-Many Body Problems

~40 qubits

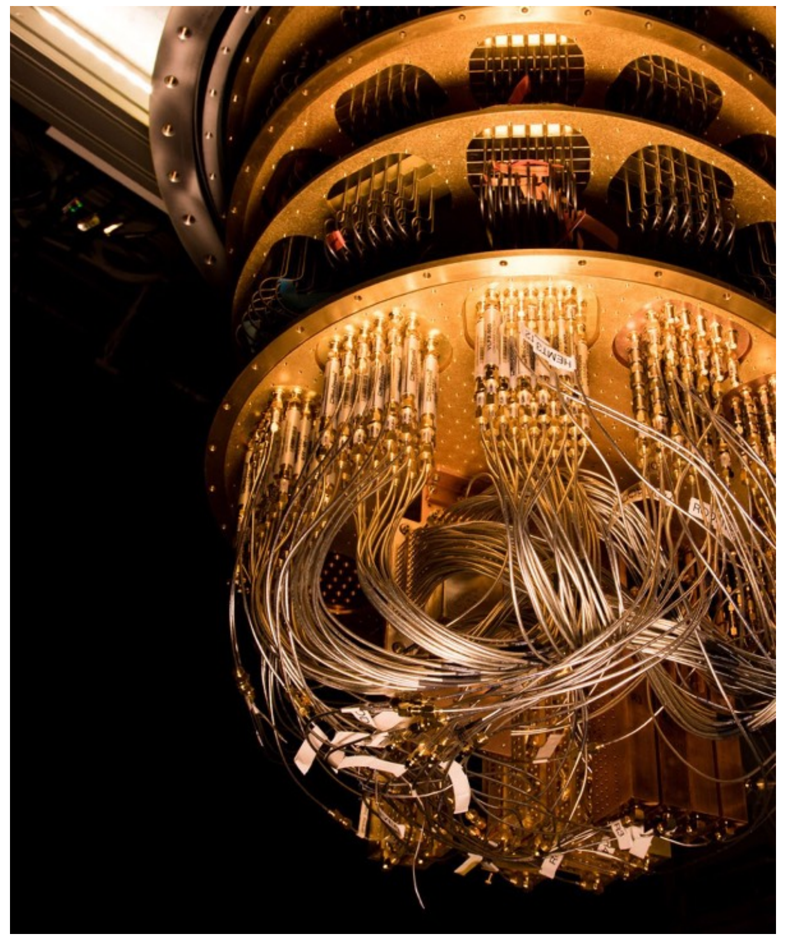
~50-100+ qubits



Complexity of Quantum-Many Body Problems

~40 qubits

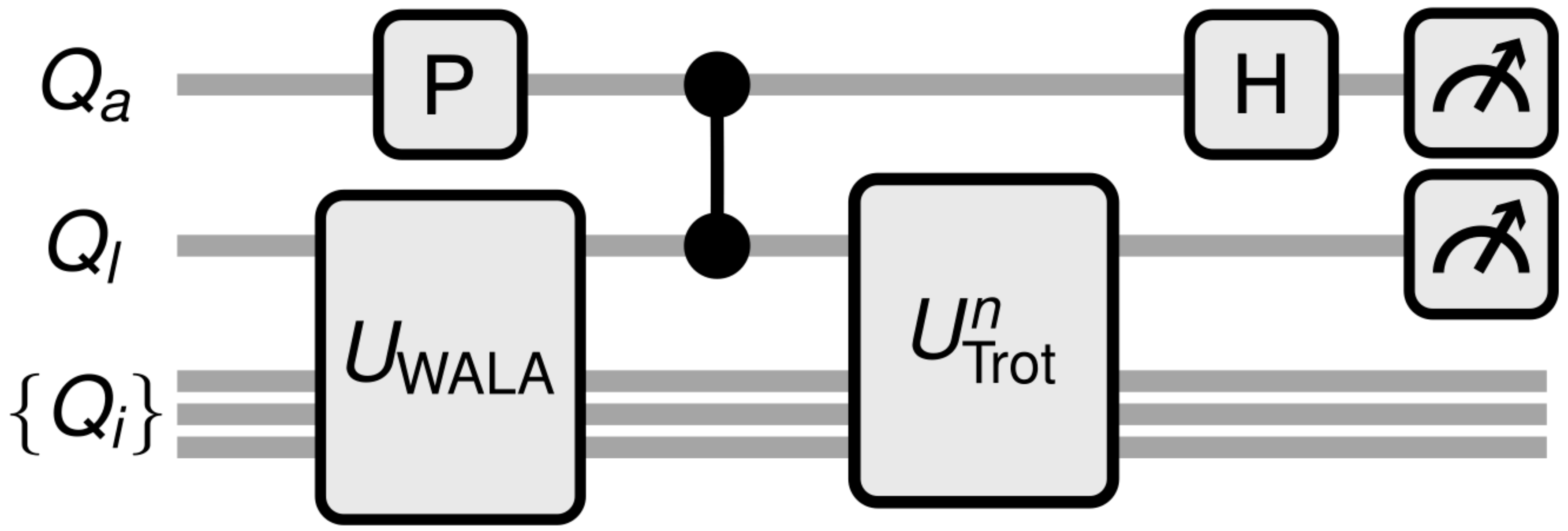
~50-100+ qubits

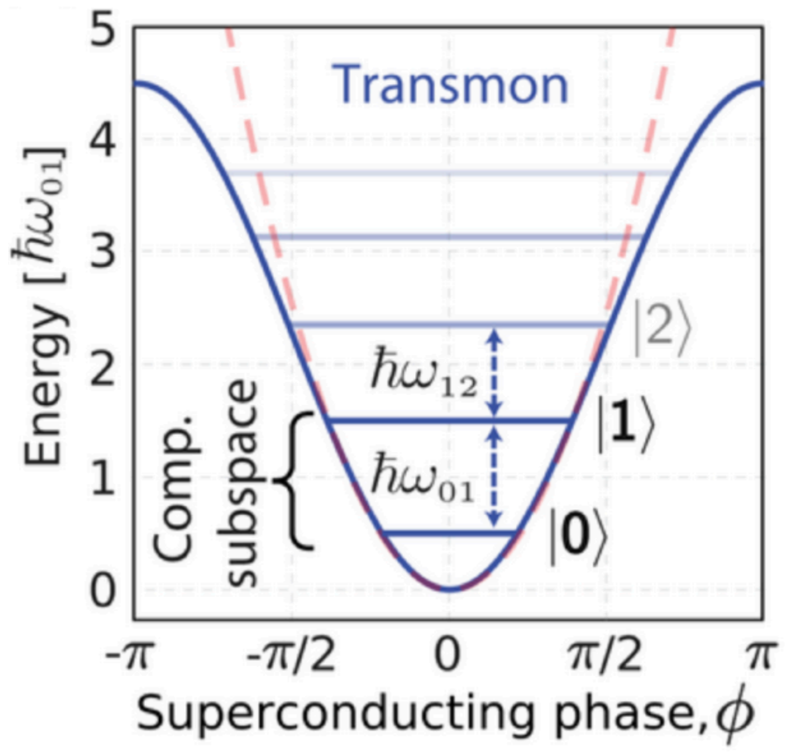


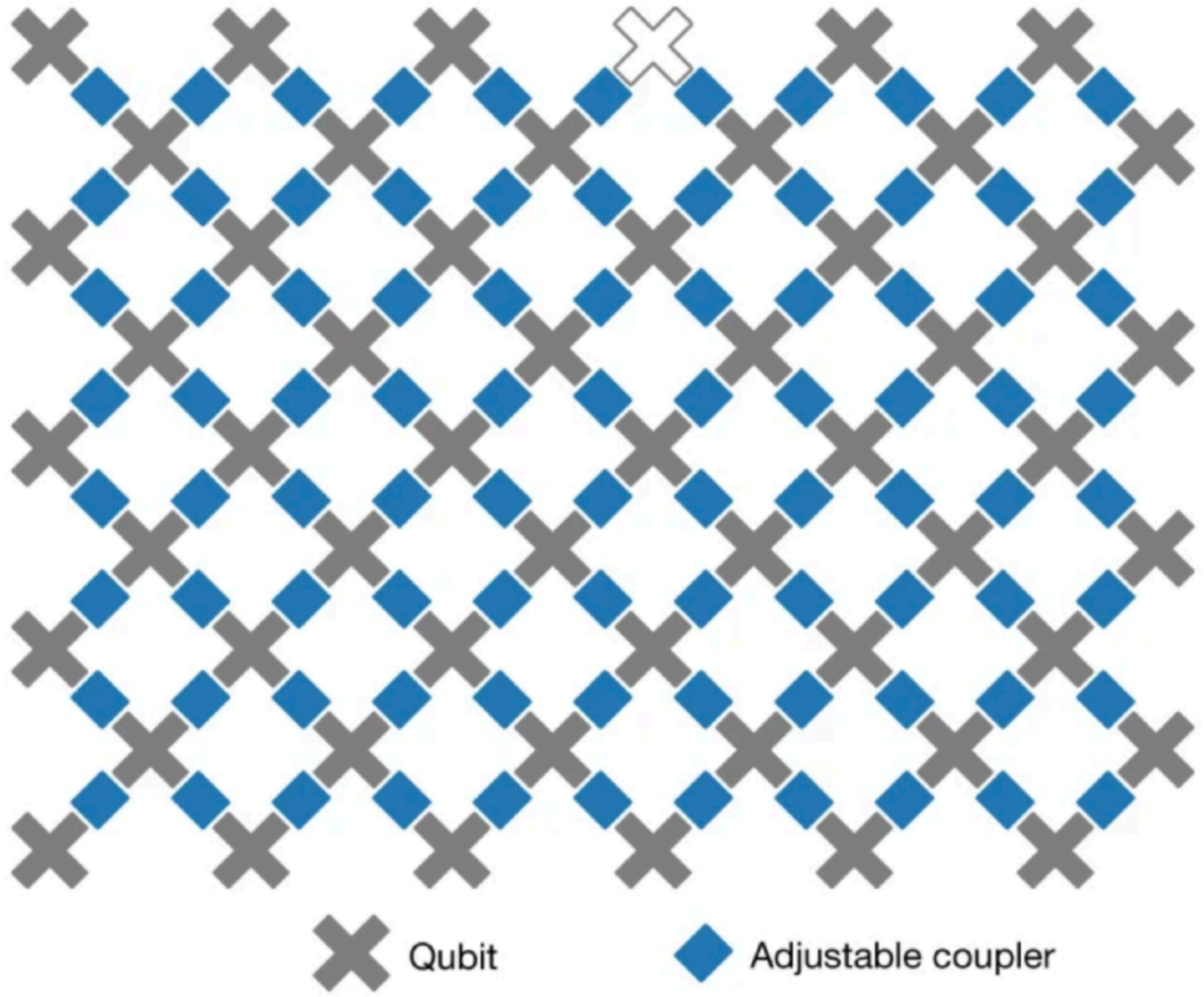
How to create entangled quantum states on quantum processors?

Quantum processors

Quantum algorithms to generate entangled quantum many-body states

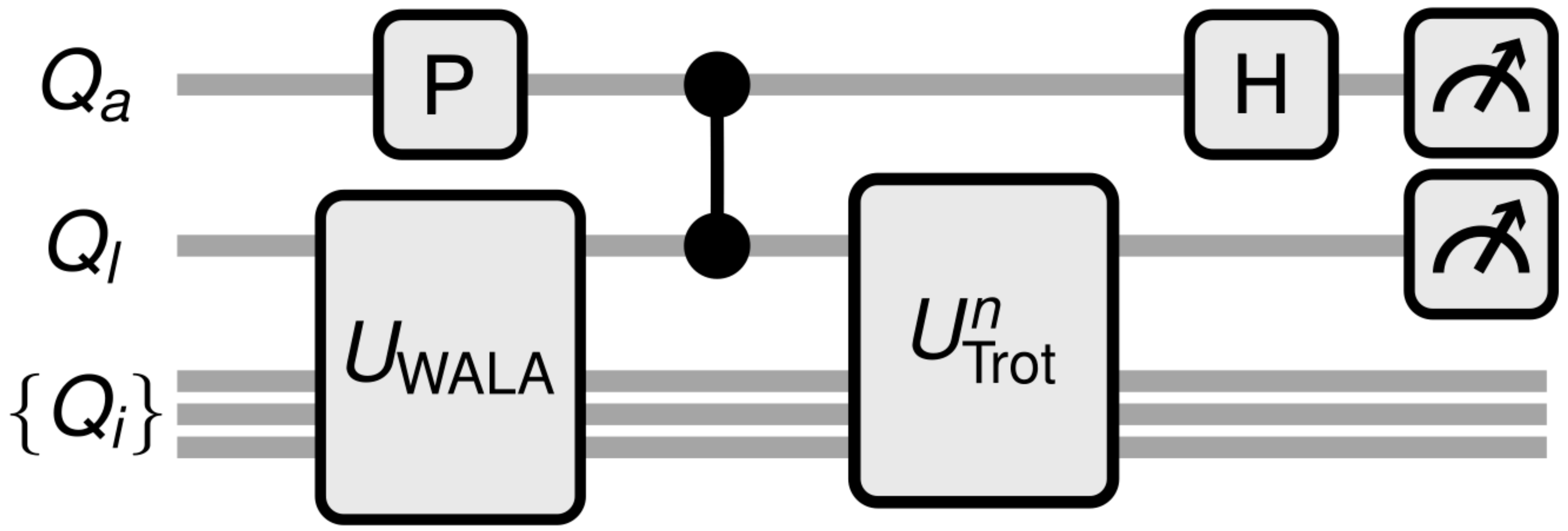


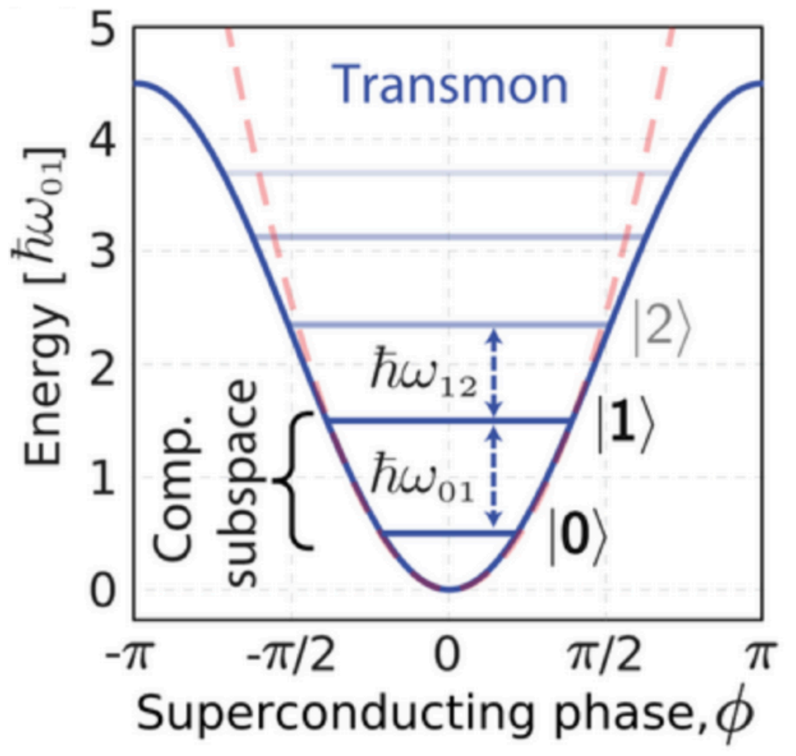


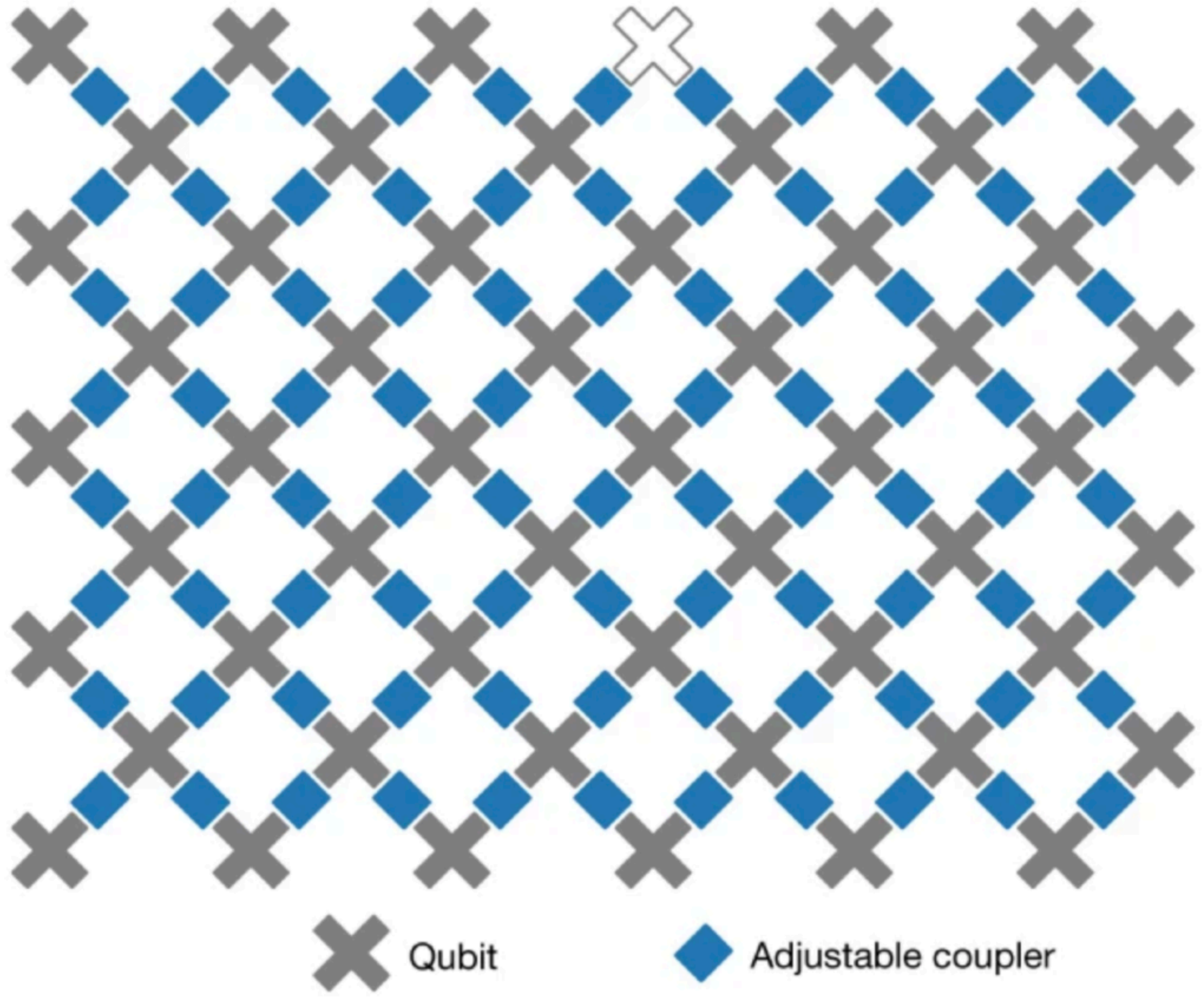


Quantum processors

Quantum algorithms to generate entangled quantum many-body states





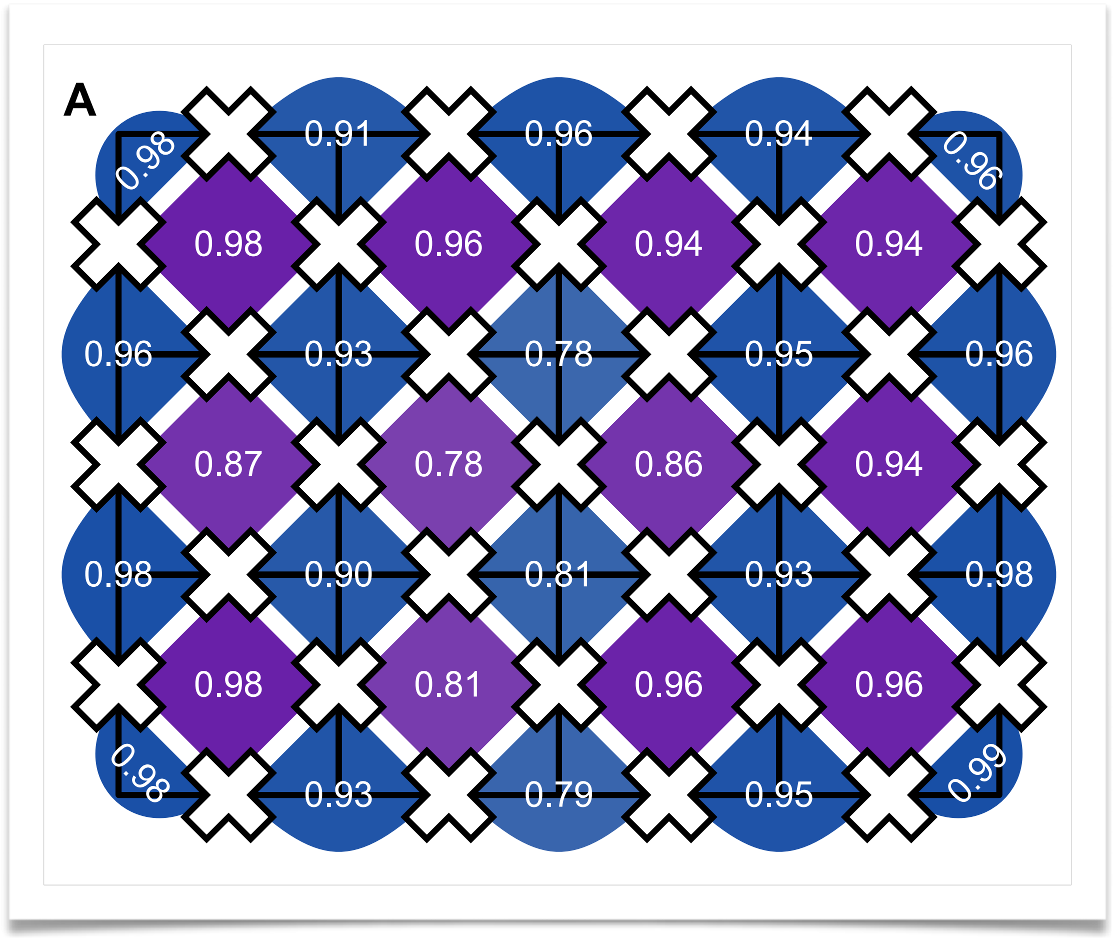


- Identify problems that are easy for quantum computers and hard for classical ones (and are physically interesting)

(I) Realizing topological order and fractionalization

[**Satzinger, Liu, Smith**, … MK, Pollmann, Roushan Science, 2021]

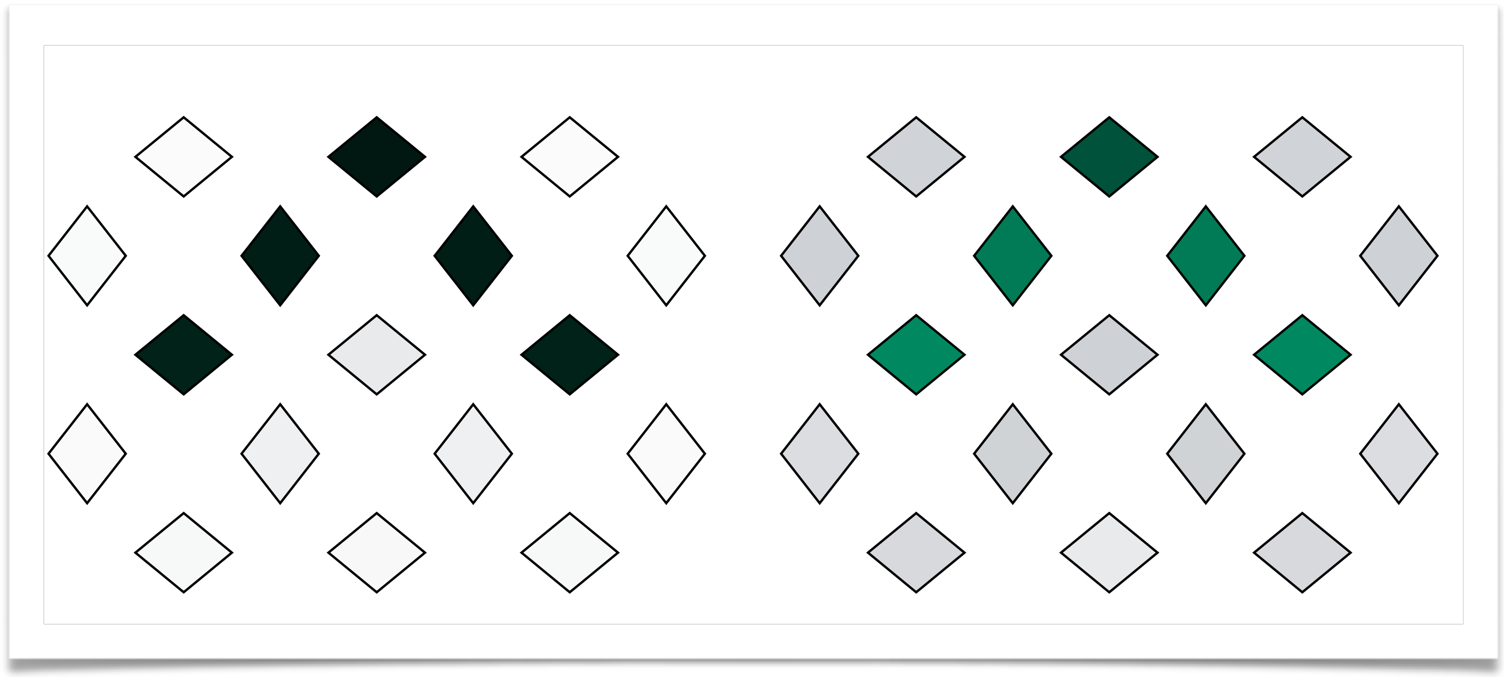
[**Boesl**, Liu, Xu, Pollmann, MK, arXiv:2501.18688]



(II) Visualizing dynamics of excitations

[**Cochran, Jobst, Rosenberg**, … Pollmann, MK, Roushan, arXiv:2409.17142]

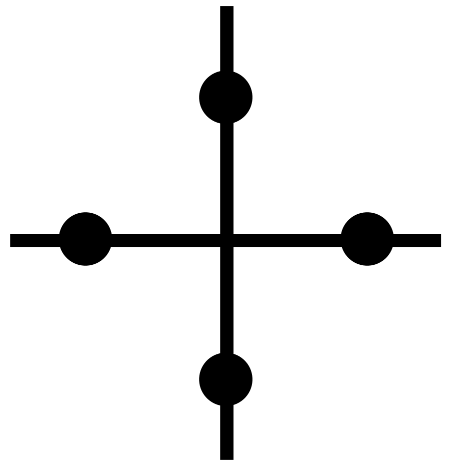
[**Xu**, MK, Pollmann, arXiv:2503.19027]



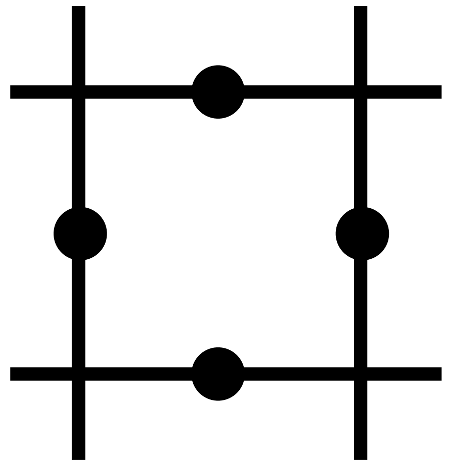
Topological order and fractionalized excitations

Toric code model [Kitaev, 2003]

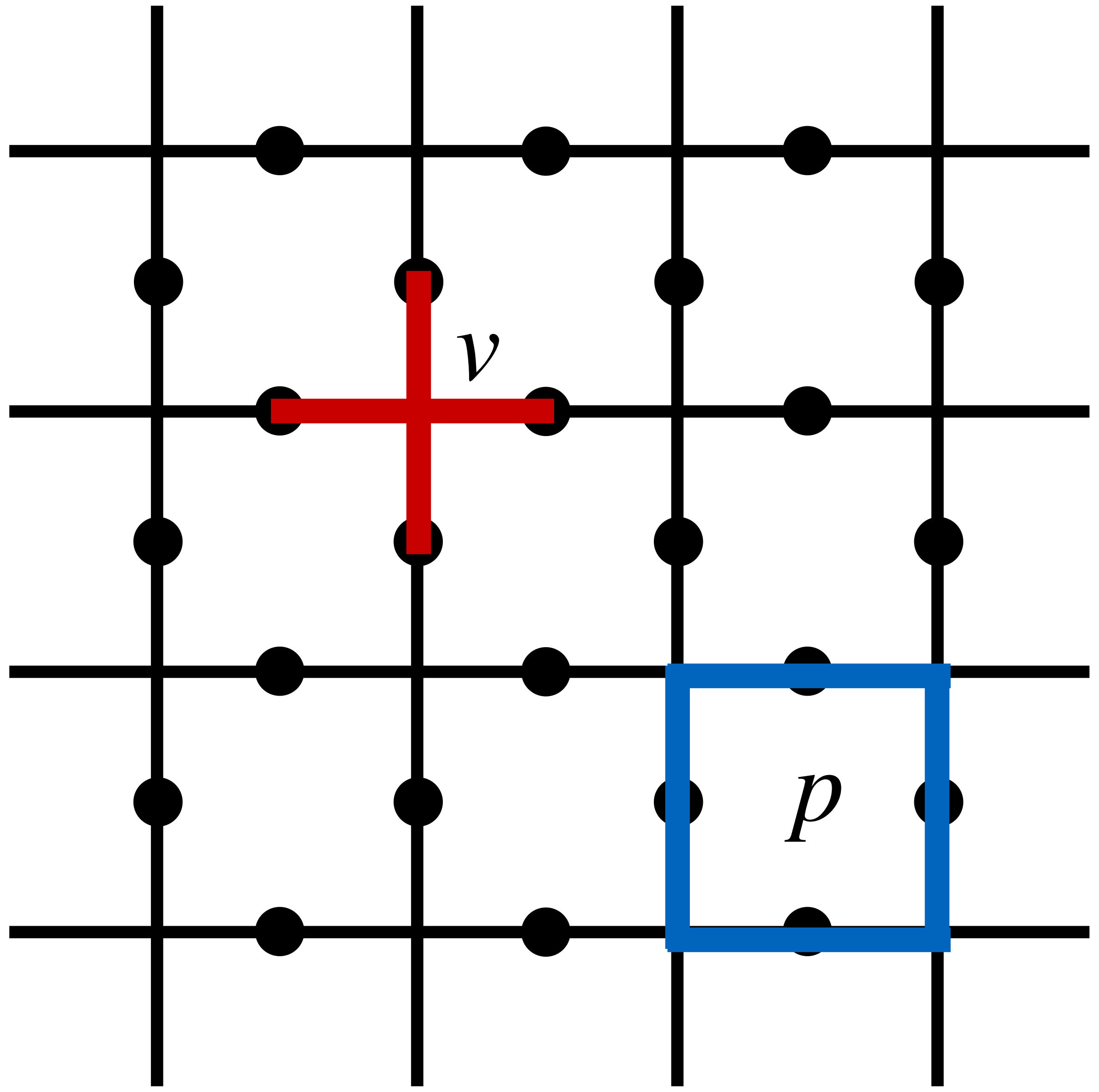
$$H = -J\sum_{v} A_v - J\sum_{p} B_p, \quad J > 0$$



$$A_v = \prod_{i \in v} Z_i$$



$$B_p = \prod_{i \in p} X_i$$



$[A_v, B_p] = 0 \rightarrow$ exactly solvable

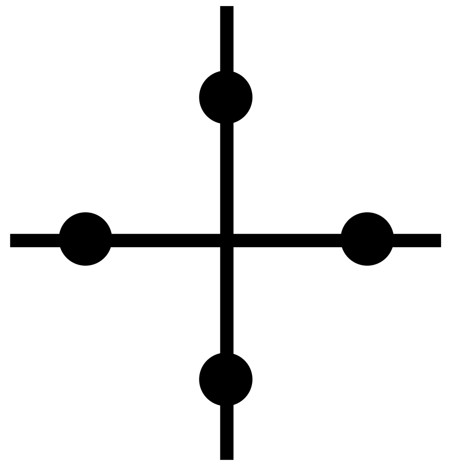
$A_v, B_p \rightarrow$ stabilizer operations of ground state $|\mathrm{TC}\rangle$

$$A_v|\mathrm{TC}\rangle = |\mathrm{TC}\rangle, \quad B_p|\mathrm{TC}\rangle = |\mathrm{TC}\rangle$$

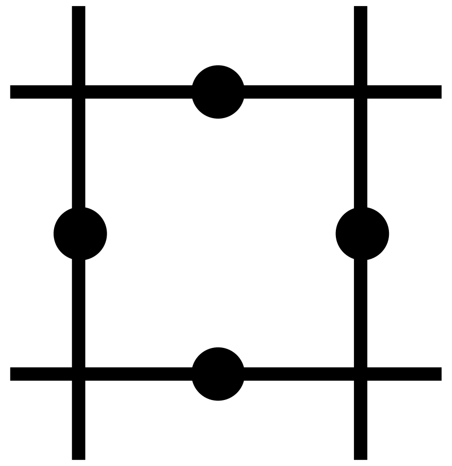
Topological order and fractionalized excitations

Toric code model [Kitaev, 2003]

$$H = -J \sum_{v} A_v - J \sum_{p} B_p, \quad J > 0$$

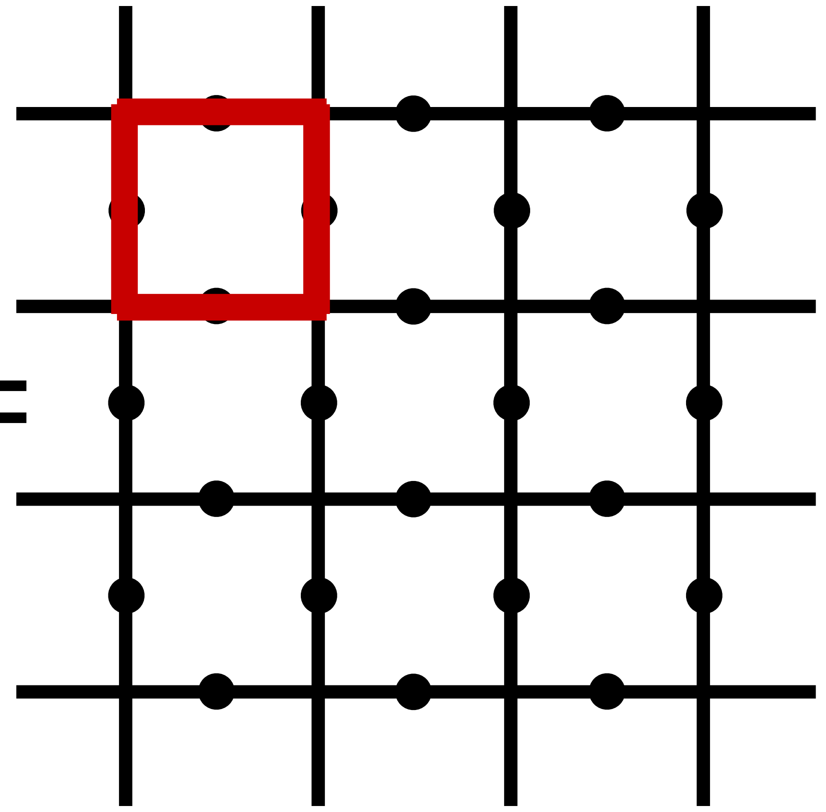


$$A_v = \prod_{i \in v} Z_i$$

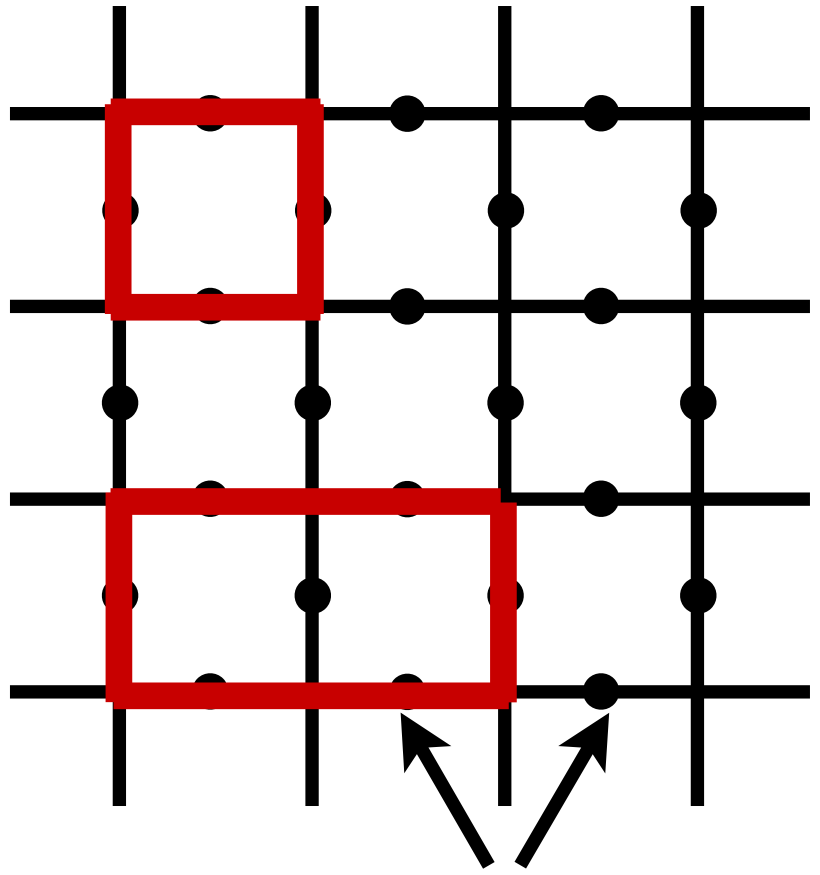


$$B_p = \prod_{i \in p} X_i$$

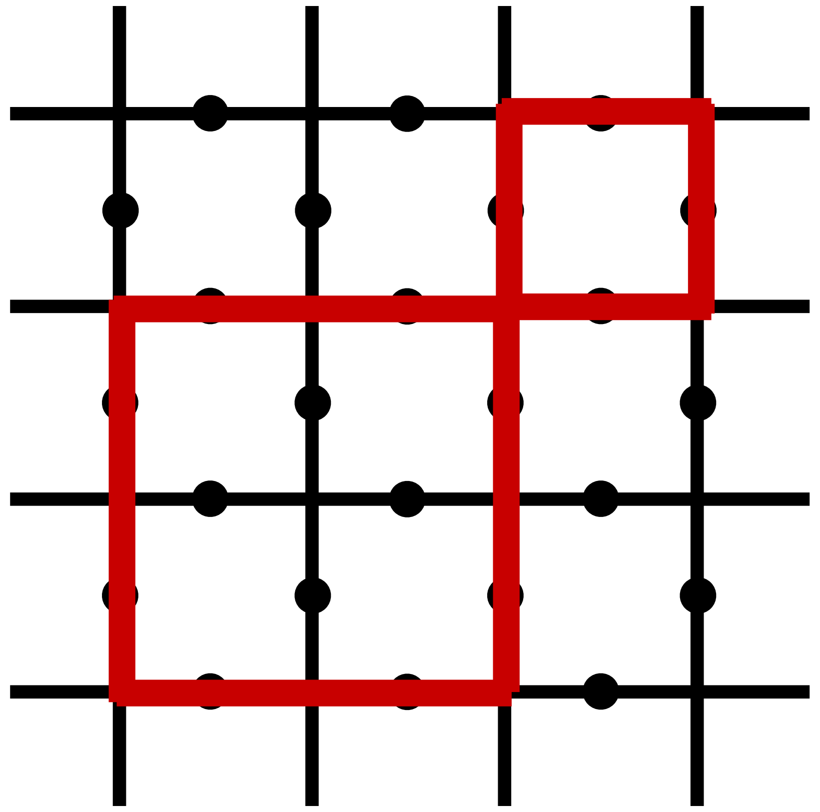
$|\mathrm{TC}\rangle =$



$+$



$+$



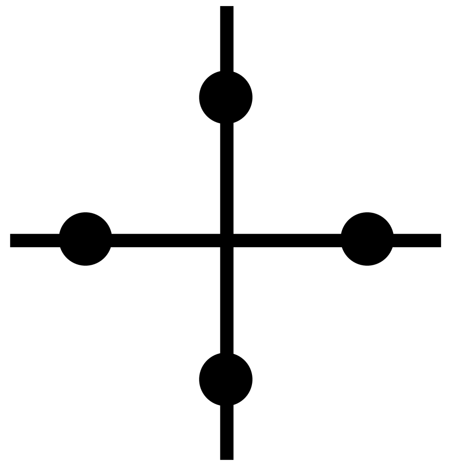
$+ \ldots \quad \propto \prod_{p} (1 + B_p) |0\rangle$

- Z_2 topological order

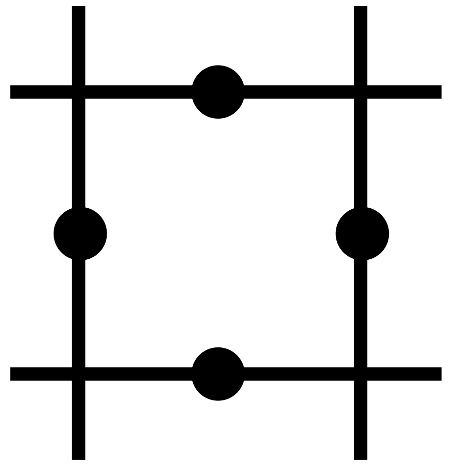
Topological order and fractionalized excitations

Toric code model [Kitaev, 2003]

$$H = -J\sum_v A_v - J\sum_p B_p, \quad J > 0$$

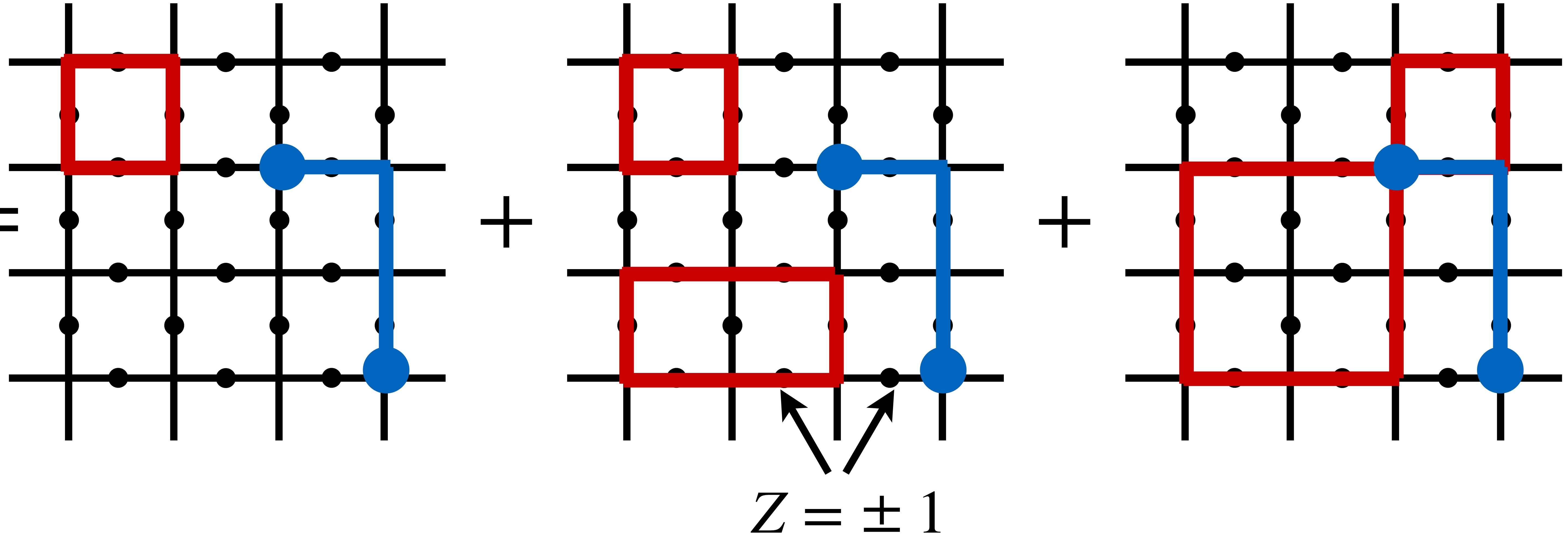


$$A_v = \prod_{i\in v} Z_i$$



$$B_p = \prod_{i \in p} X_i$$

$|\psi(X)\rangle =$

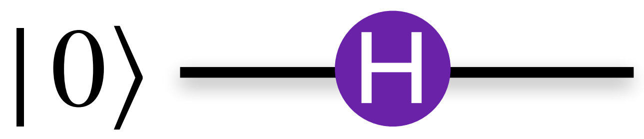


$+ \ldots \quad \propto \prod_e X_e |\mathrm{TC}\rangle$

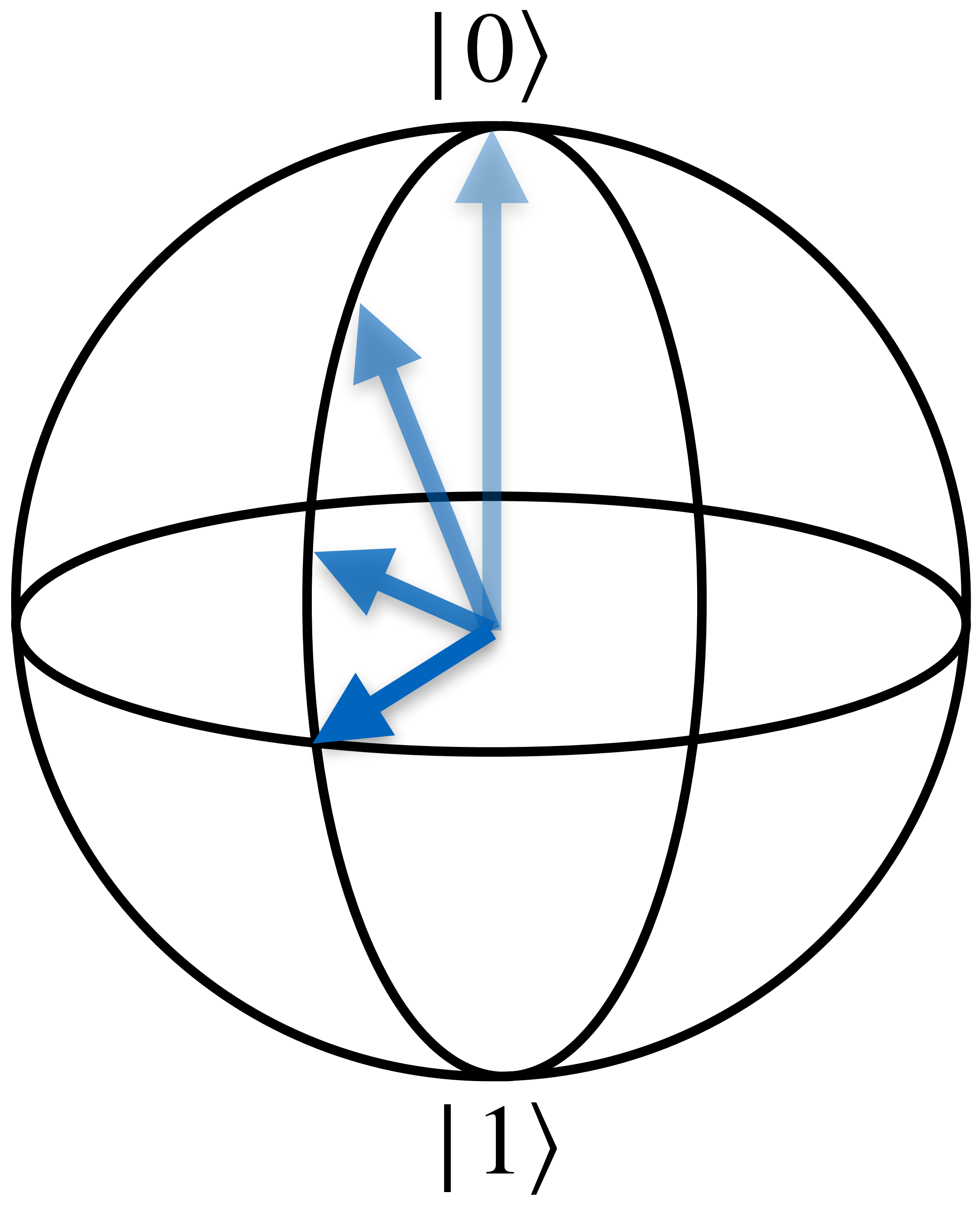
- Z_2 topological order
- fractionalized excitations

Elementary quantum operations

Single qubit operation: Hadamard gate

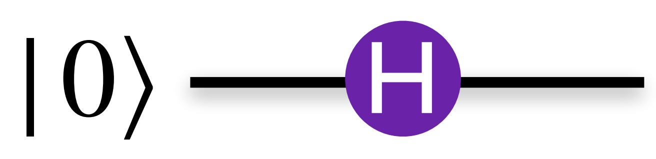


$$= \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$$

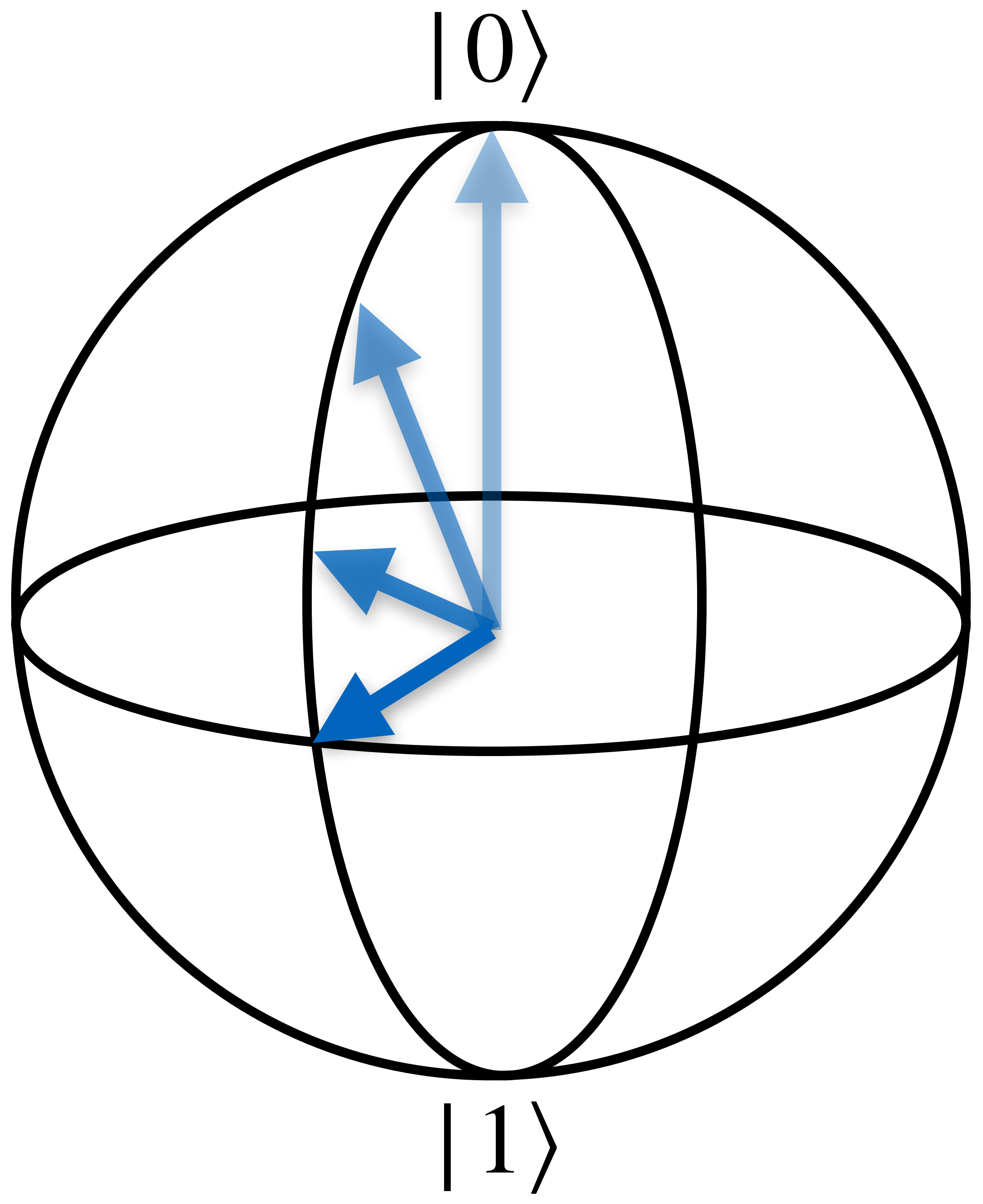


Elementary quantum operations

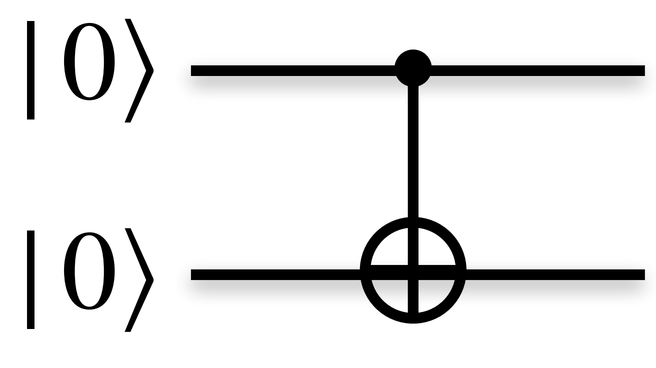
Single qubit operation: Hadamard gate



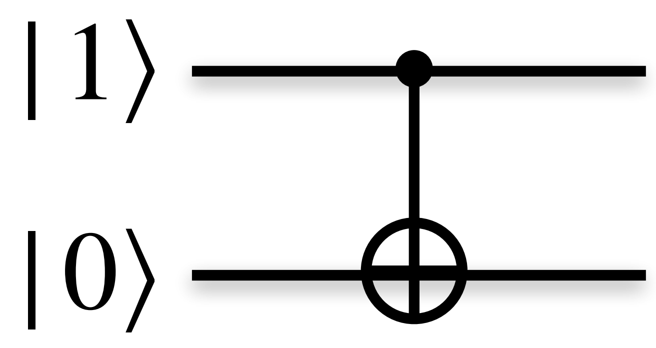
$$= \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$$



Two-qubit operation: CNOT



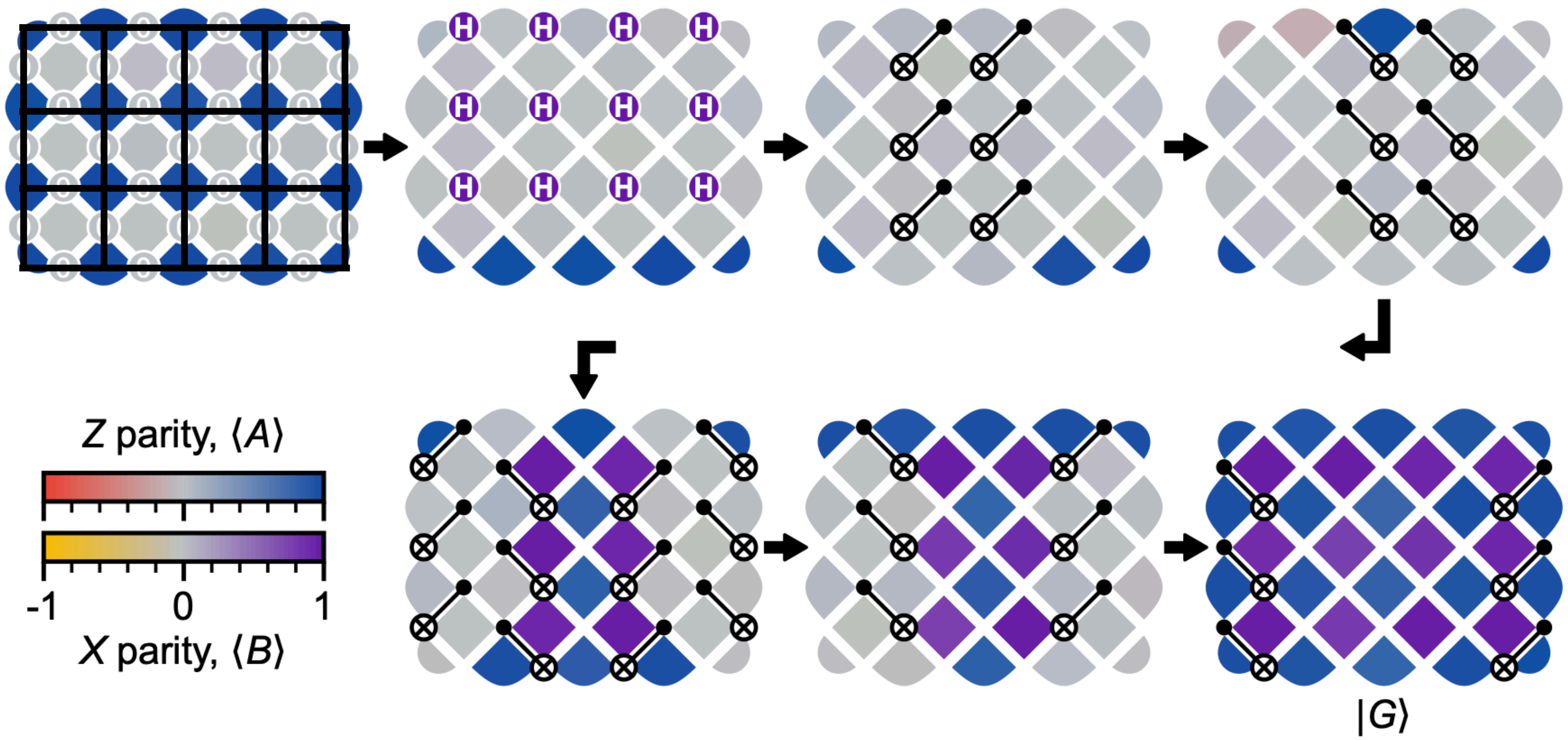
$$= \begin{matrix} |0\rangle \\ |0\rangle \end{matrix}$$



$$= \begin{matrix} |1\rangle \\ |1\rangle \end{matrix}$$

Realizing the toric code on a quantum processor

Toric code ground state: $|\psi_0\rangle \propto \prod_p (1 + B_p)|0\rangle$



- Toric code fixed point is generated sequentially
- Quantum circuit of linear depth in system size L_y


Yu-Jie Liu


Kevin Satzinger


Adam Smith

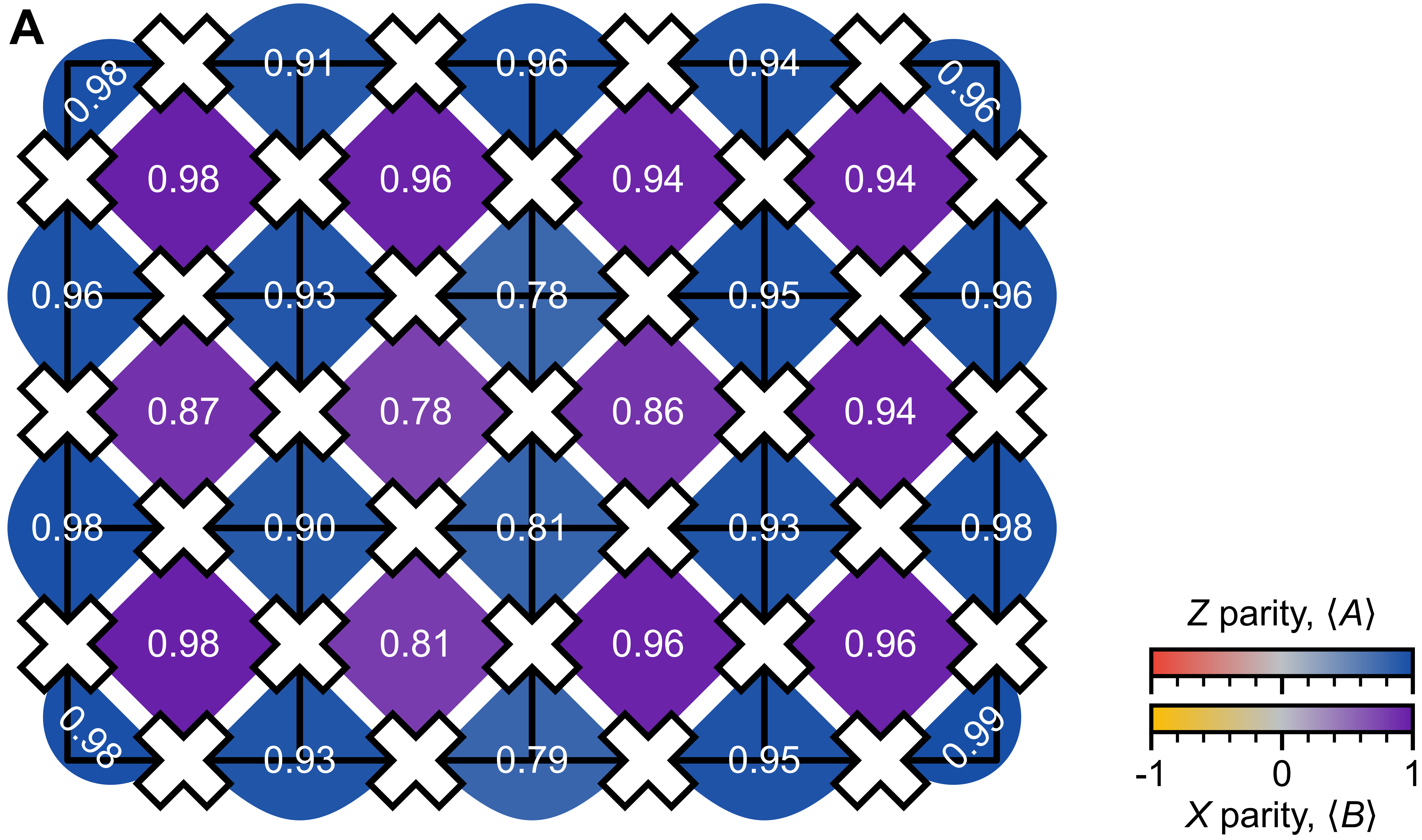

Pedram Roushan


Frank Pollmann

[**Satzinger, Liu, Smith**, … MK, Pollmann, Roushan, Science, 2021]

Realizing the toric code on a quantum processor

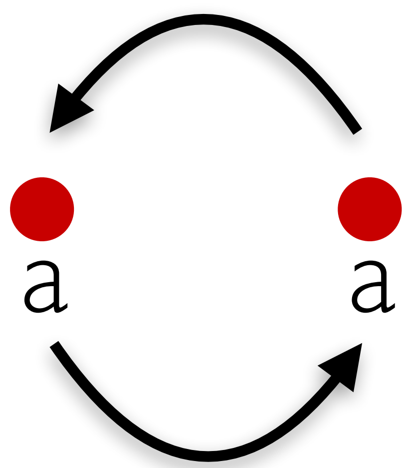
Toric code ground state: $|\psi_0\rangle \propto \prod_p (1 + B_p)|0\rangle$



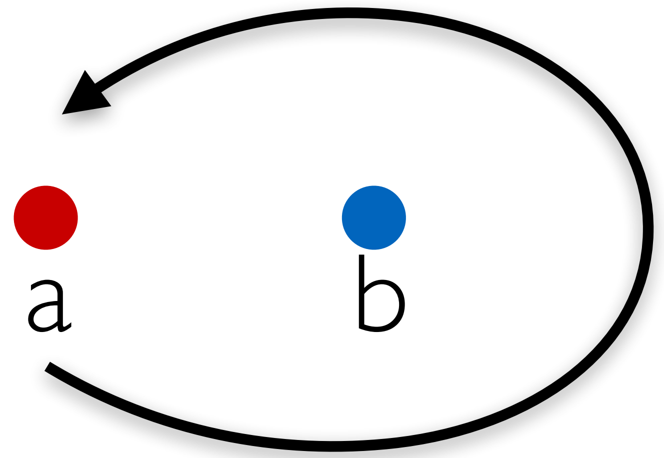
31 qubits, average stabilizer fidelity 0.92 ±0.06

[**Satzinger, Liu, Smith**, … MK, Pollmann, Roushan, Science, 2021]

Fractionalized excitations: Anyons



T_{aa}



S_{ab}

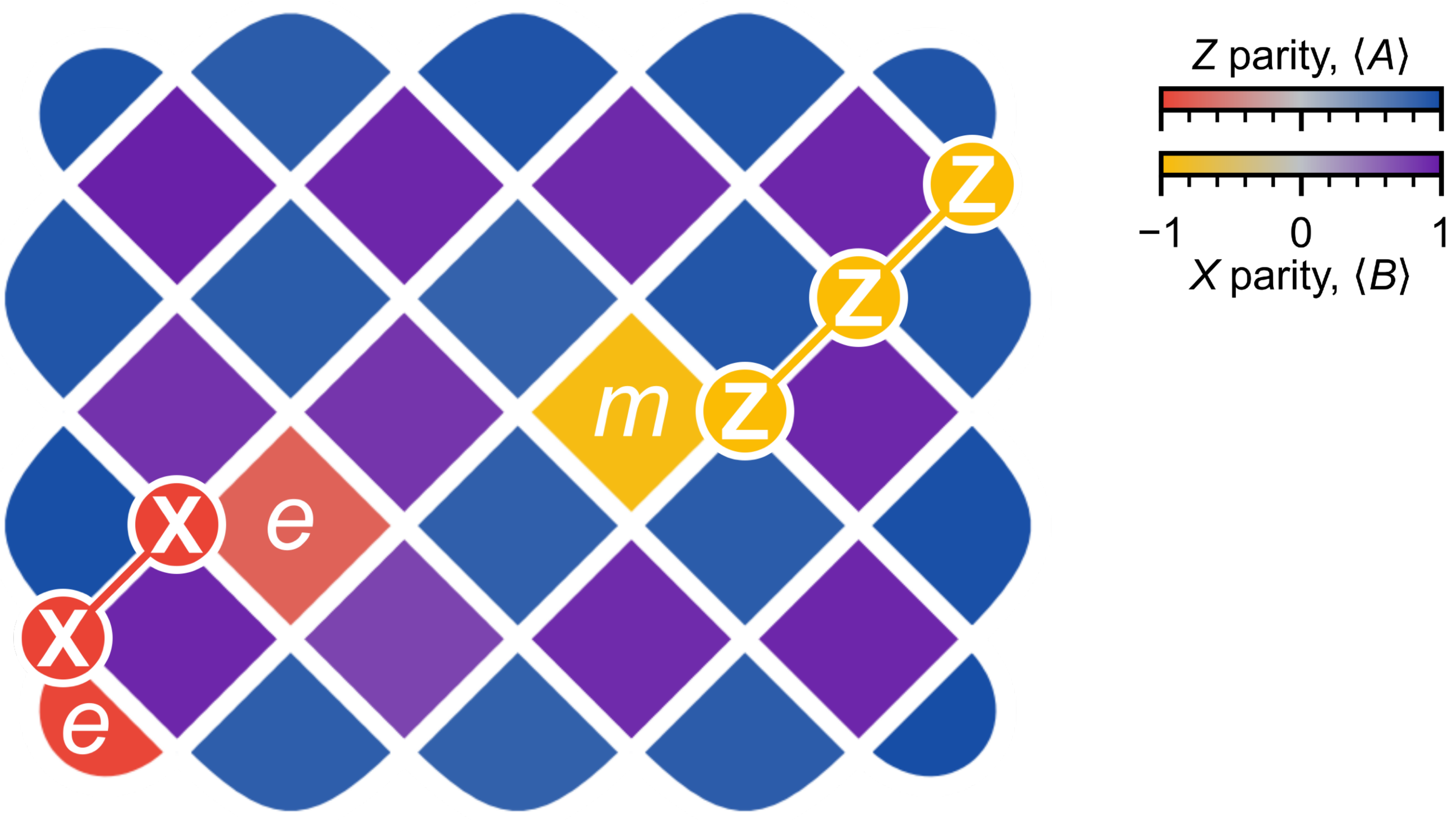
Exchange: T-matrix

Can take phases other than ±1

Mutual: S-matrix

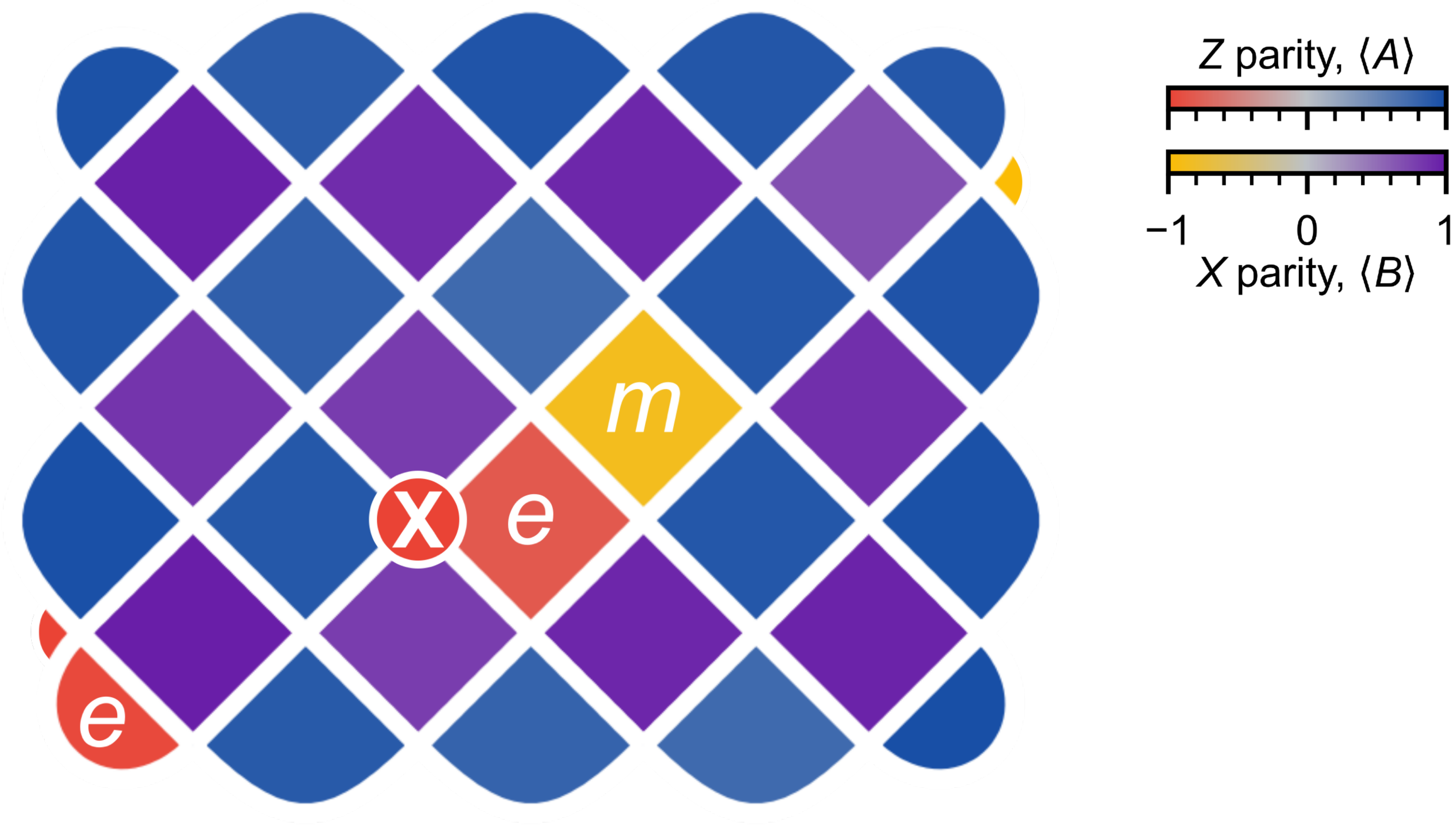
No analogue for fundamental fermions/bosons in 3D

Simulating anyonic statistics



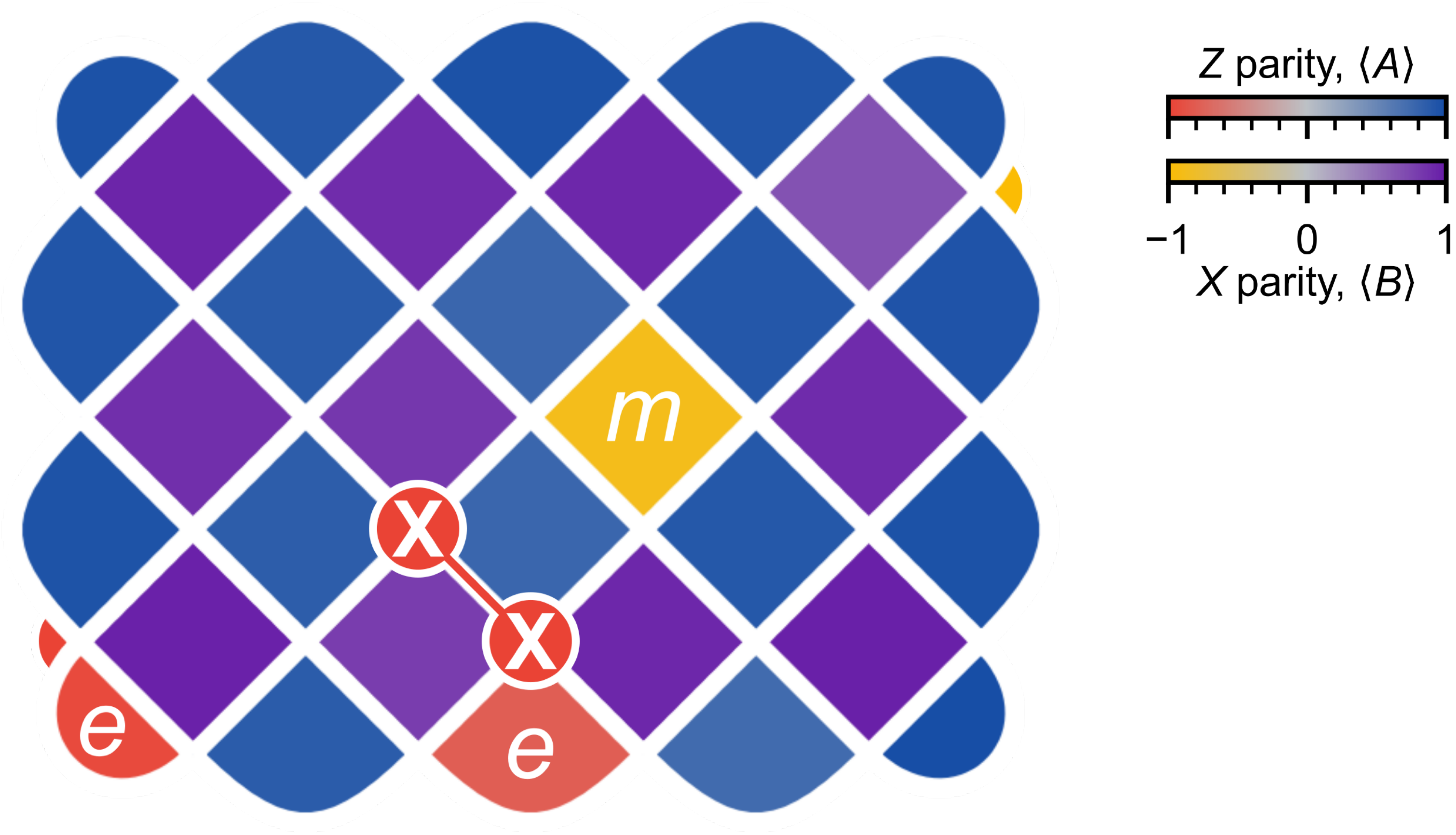
[**Satzinger, Liu, Smith**, … MK, Pollmann, Roushan, Science, 2021]

Simulating anyonic statistics



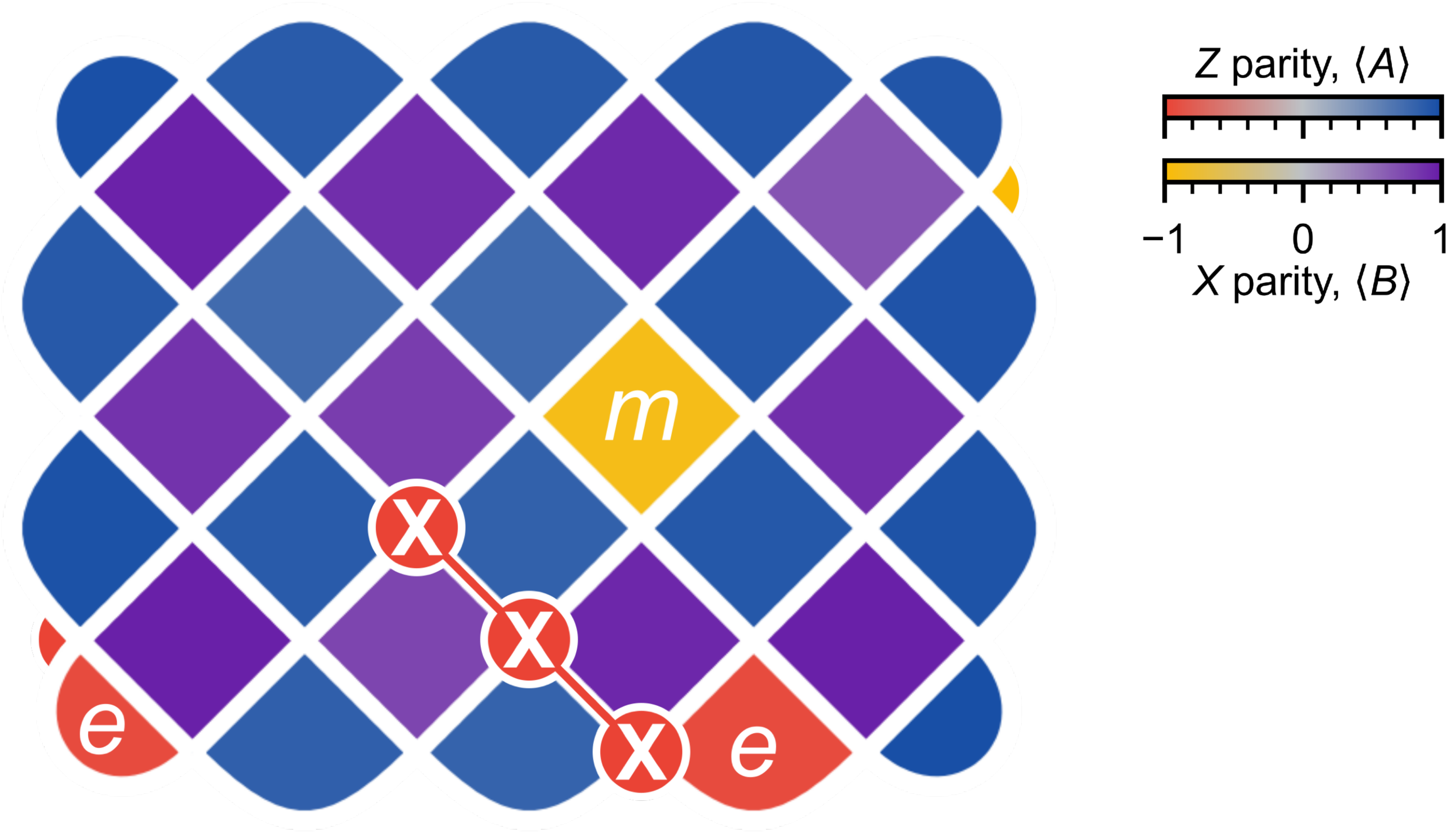
[**Satzinger, Liu, Smith**, … MK, Pollmann, Roushan, Science, 2021]

Simulating anyonic statistics



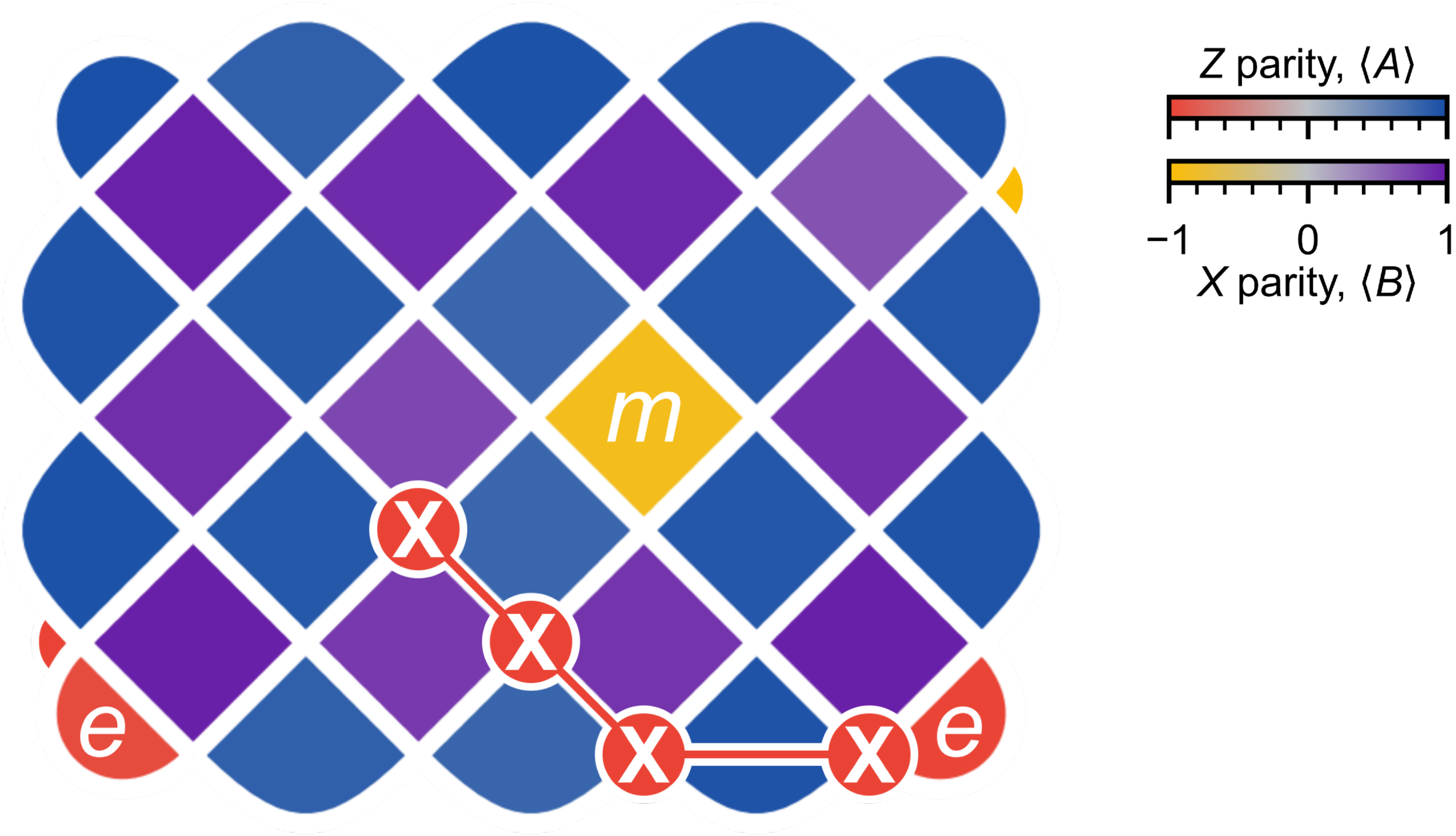
[**Satzinger, Liu, Smith**, … MK, Pollmann, Roushan, Science, 2021]

Simulating anyonic statistics



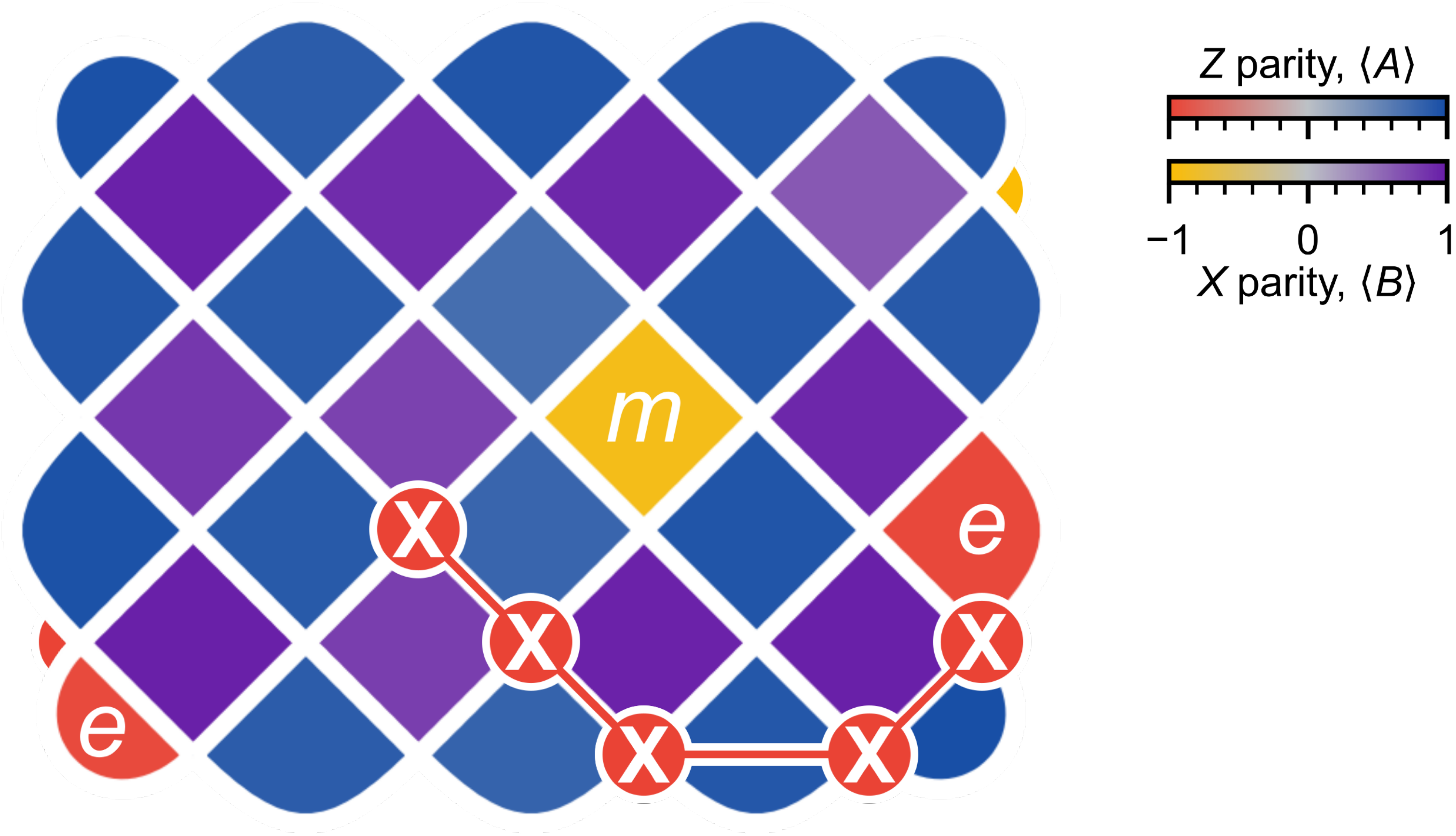
[**Satzinger, Liu, Smith**, … MK, Pollmann, Roushan, Science, 2021]

Simulating anyonic statistics



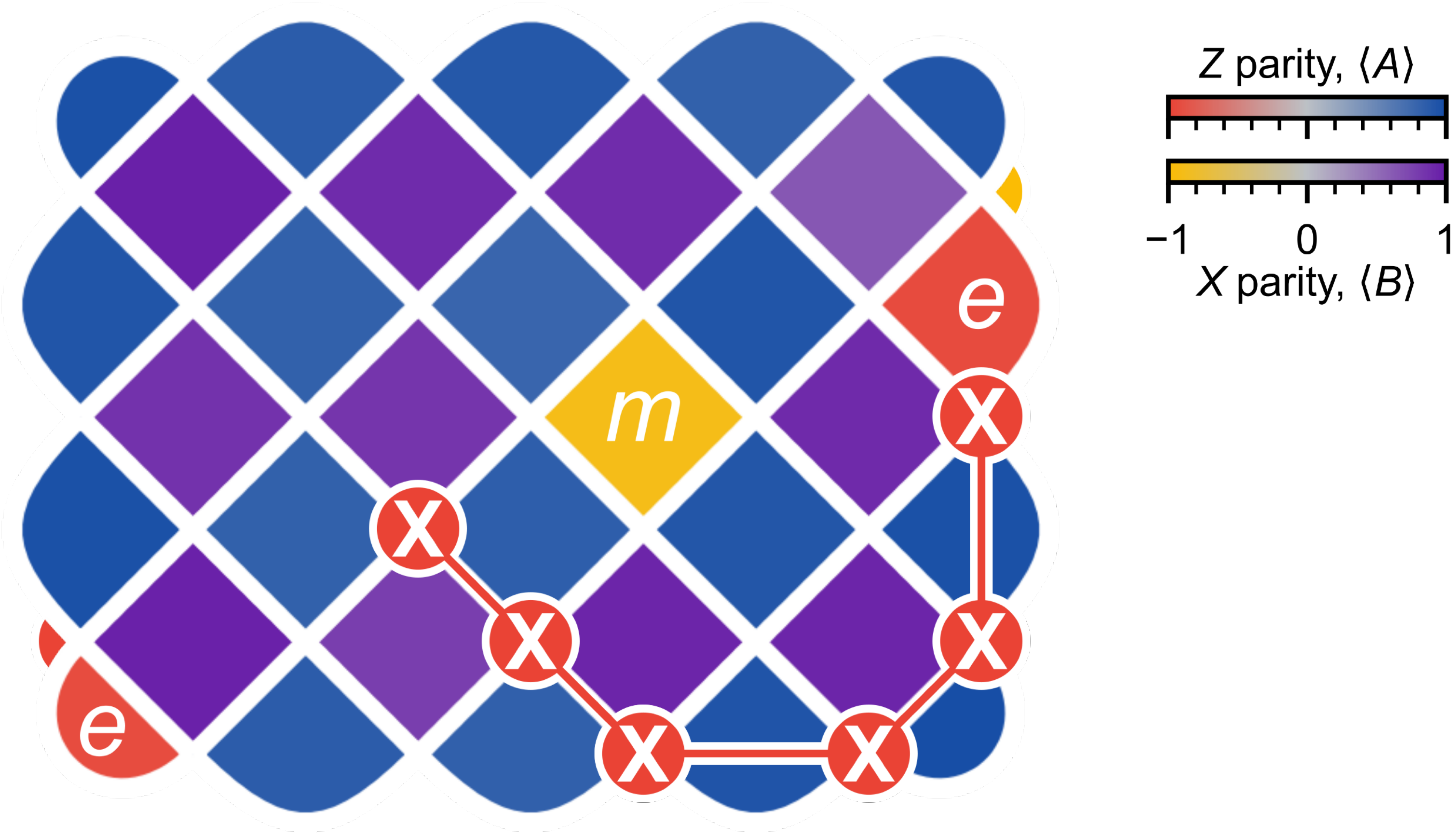
[**Satzinger, Liu, Smith**, … MK, Pollmann, Roushan, Science, 2021]

Simulating anyonic statistics



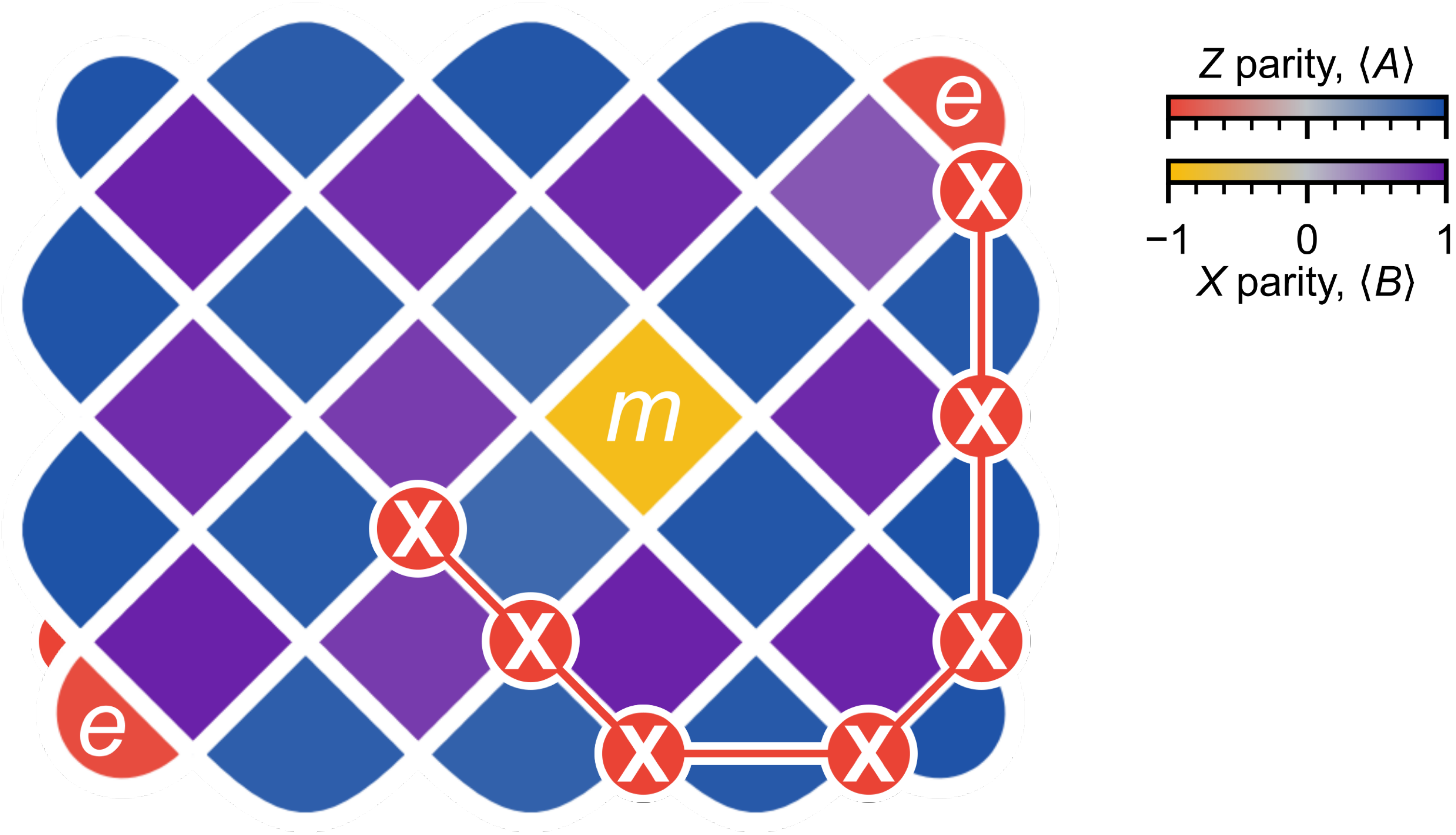
[**Satzinger, Liu, Smith**, … MK, Pollmann, Roushan, Science, 2021]

Simulating anyonic statistics



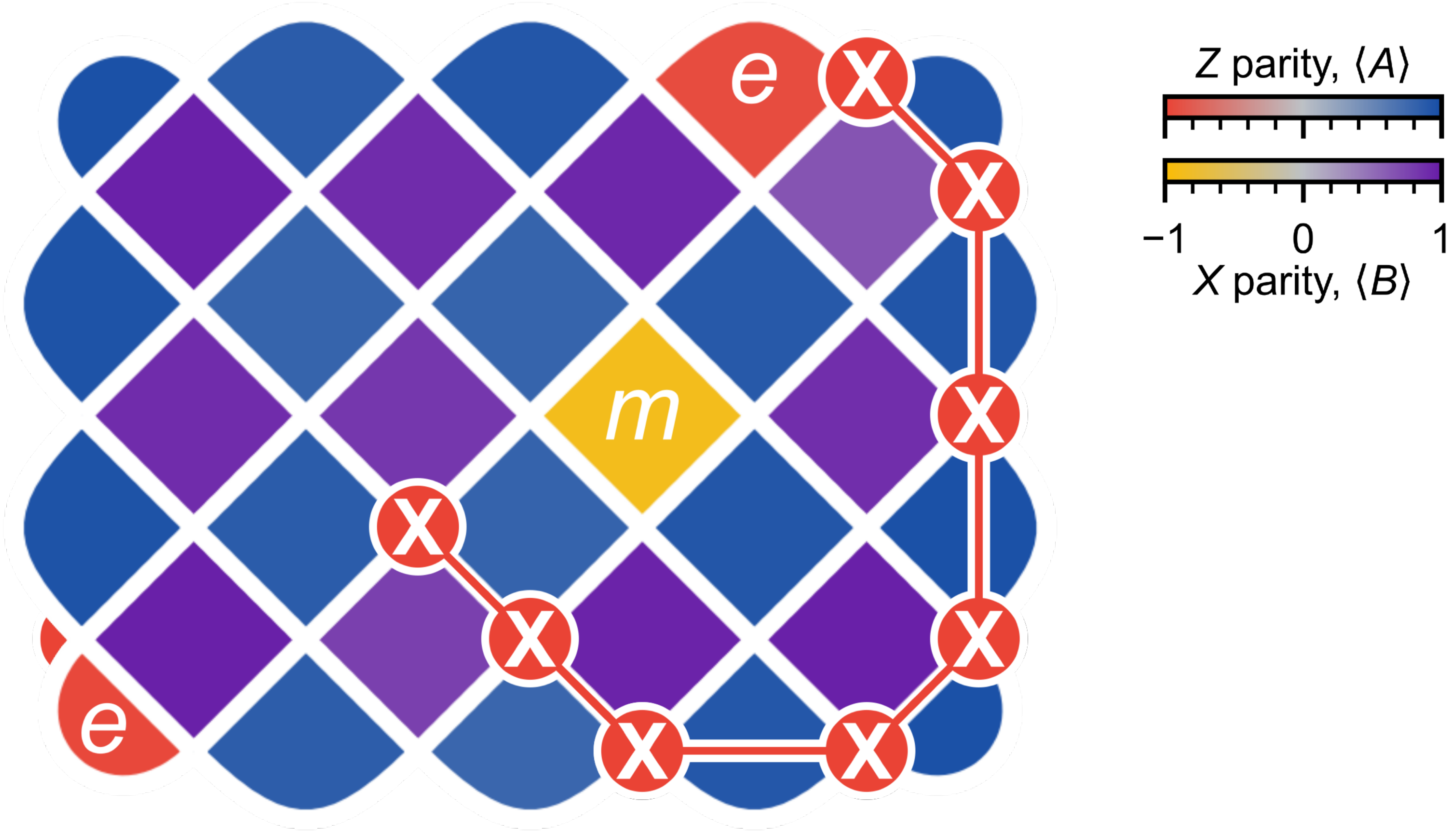
[**Satzinger, Liu, Smith**, … MK, Pollmann, Roushan, Science, 2021]

Simulating anyonic statistics



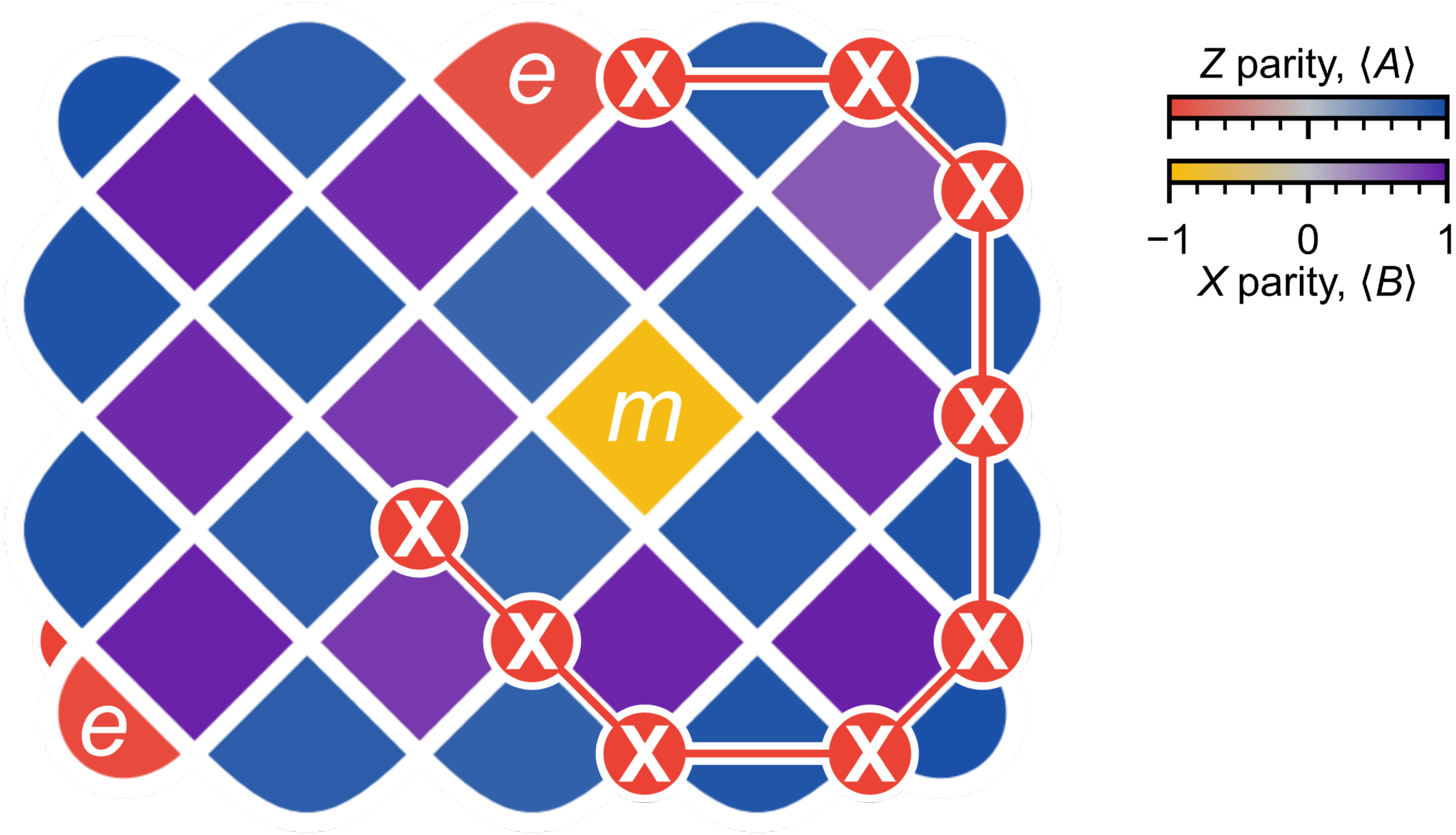
[**Satzinger, Liu, Smith**, … MK, Pollmann, Roushan, Science, 2021]

Simulating anyonic statistics



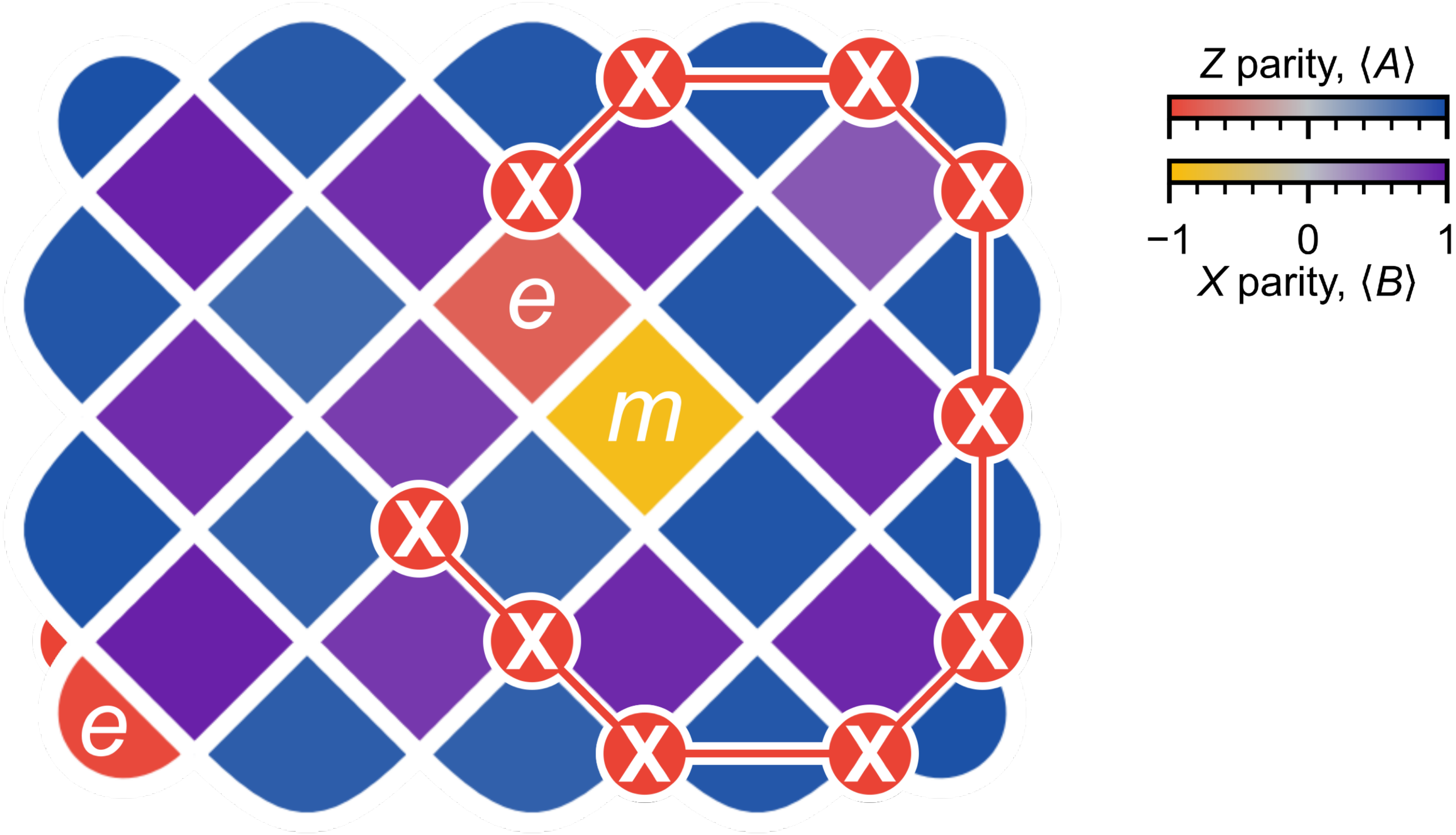
[**Satzinger, Liu, Smith**, … MK, Pollmann, Roushan, Science, 2021]

Simulating anyonic statistics



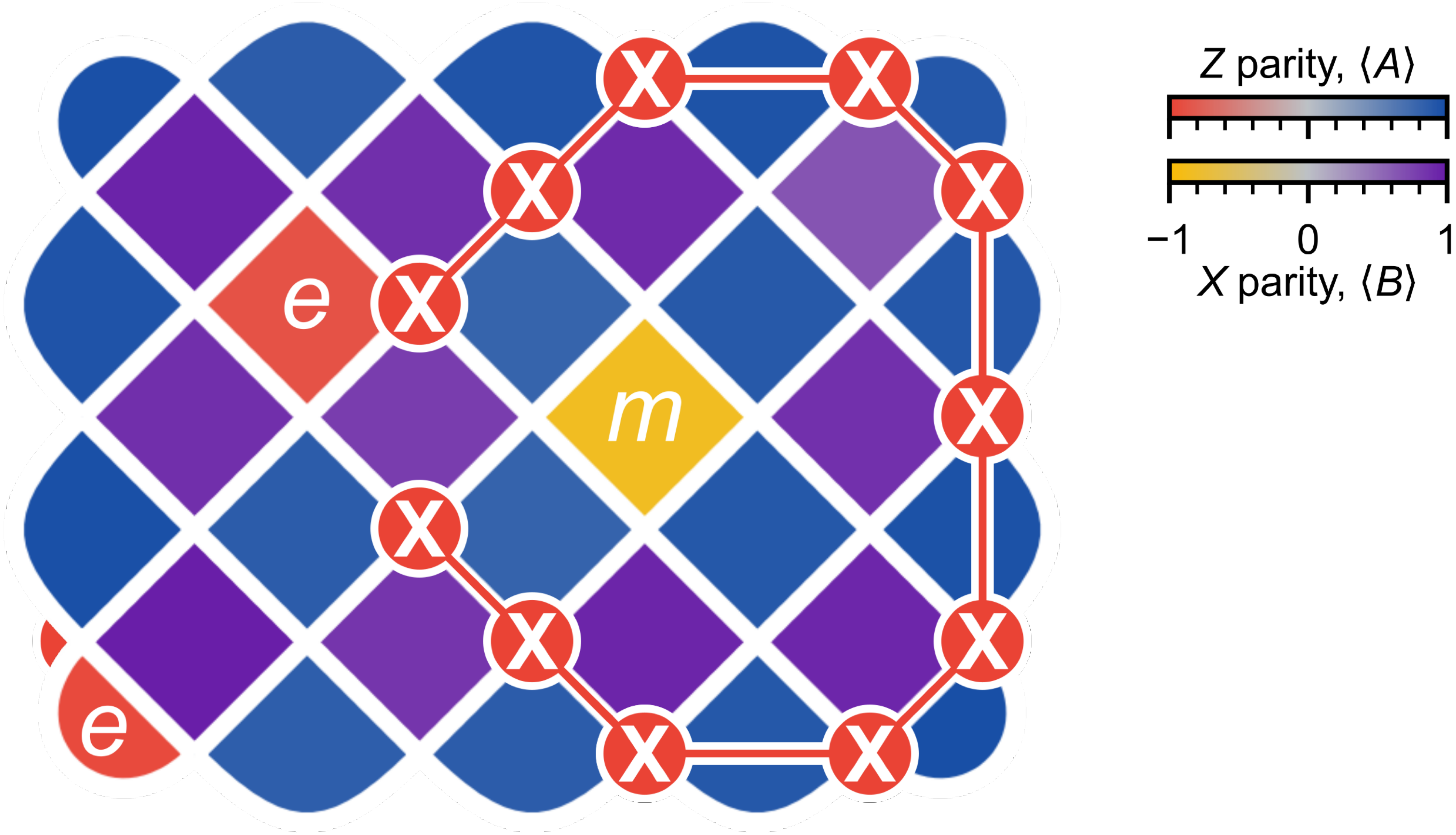
[**Satzinger, Liu, Smith**, … MK, Pollmann, Roushan, Science, 2021]

Simulating anyonic statistics



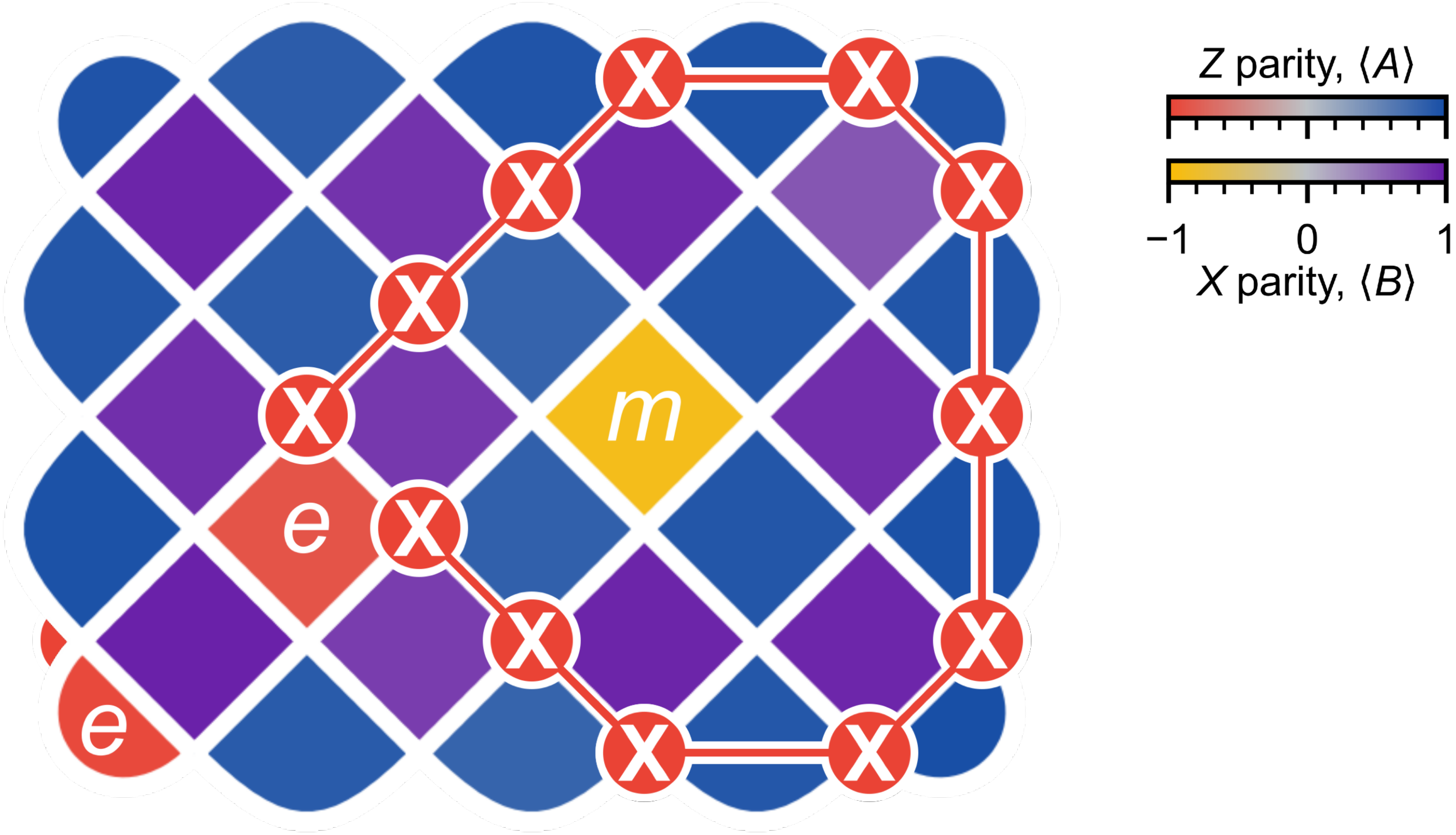
[**Satzinger, Liu, Smith**, … MK, Pollmann, Roushan, Science, 2021]

Simulating anyonic statistics



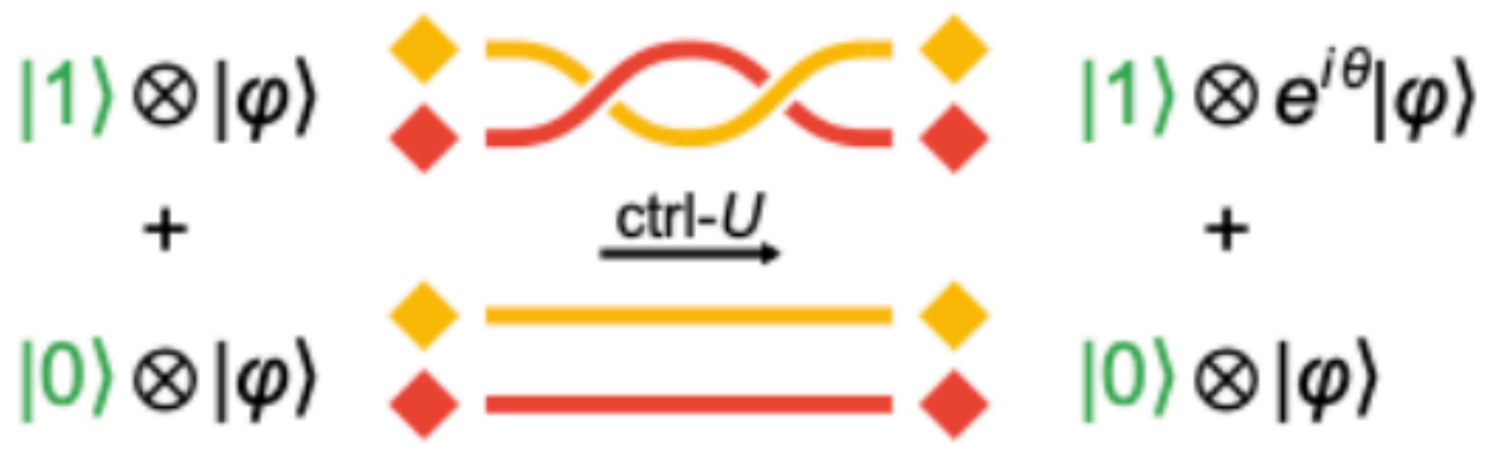
[**Satzinger, Liu, Smith**, … MK, Pollmann, Roushan, Science, 2021]

Simulating anyonic statistics



[**Satzinger, Liu, Smith**, … MK, Pollmann, Roushan, Science, 2021]

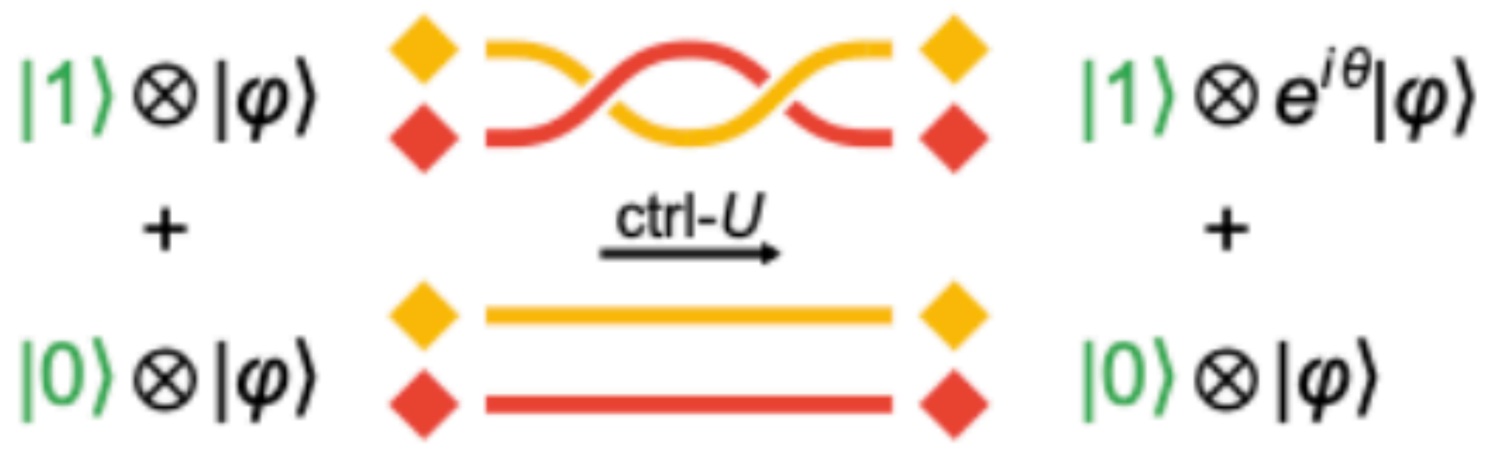
Simulating anyonic statistics



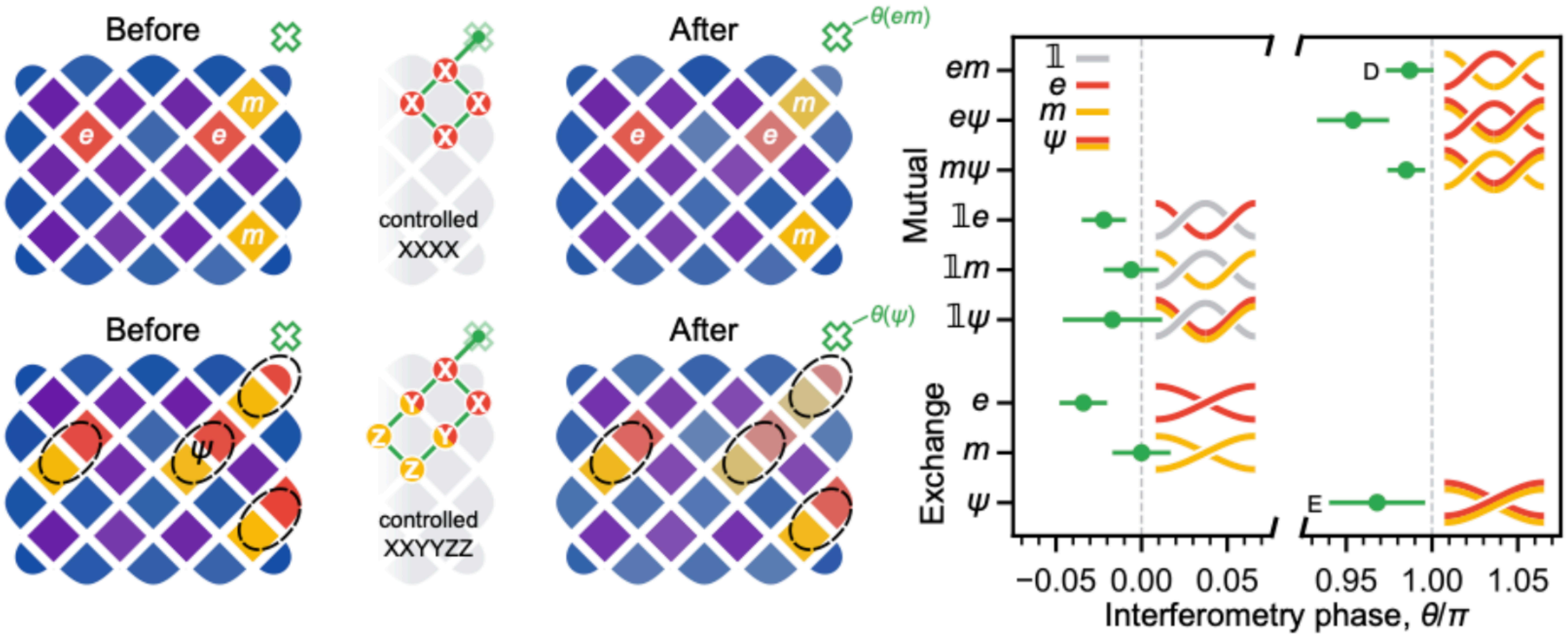
e.g. [Jiang et al. 2008]

[**Satzinger, Liu, Smith**, … MK, Pollmann, Roushan, Science, 2021]

Simulating anyonic statistics



e.g. [Jiang et al. 2008]

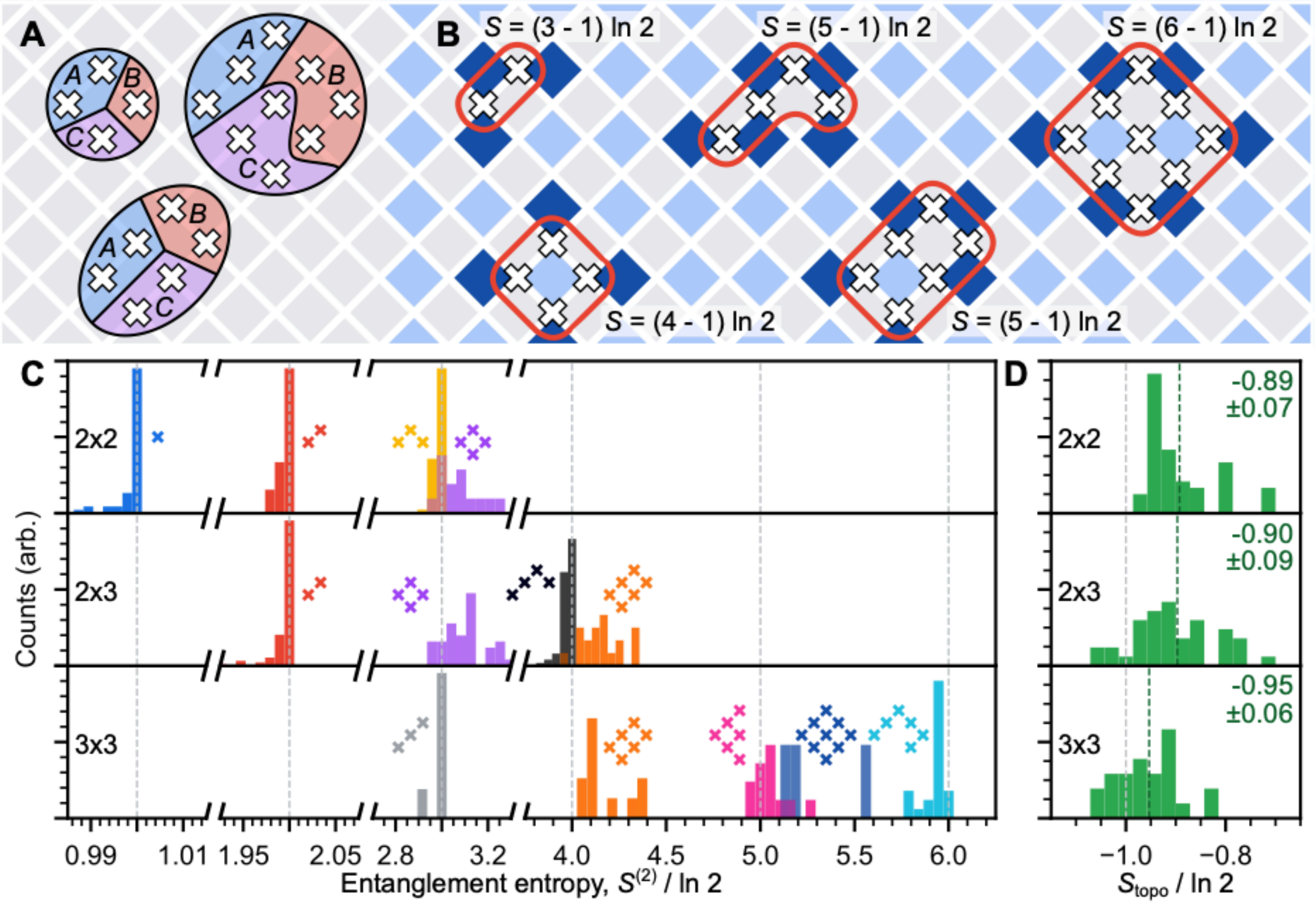


[**Satzinger, Liu, Smith**, … MK, Pollmann, Roushan, Science, 2021]

Signatures of topological order

- Topological entanglement

$$S = \alpha L - \gamma \qquad \gamma = \ln(2)$$

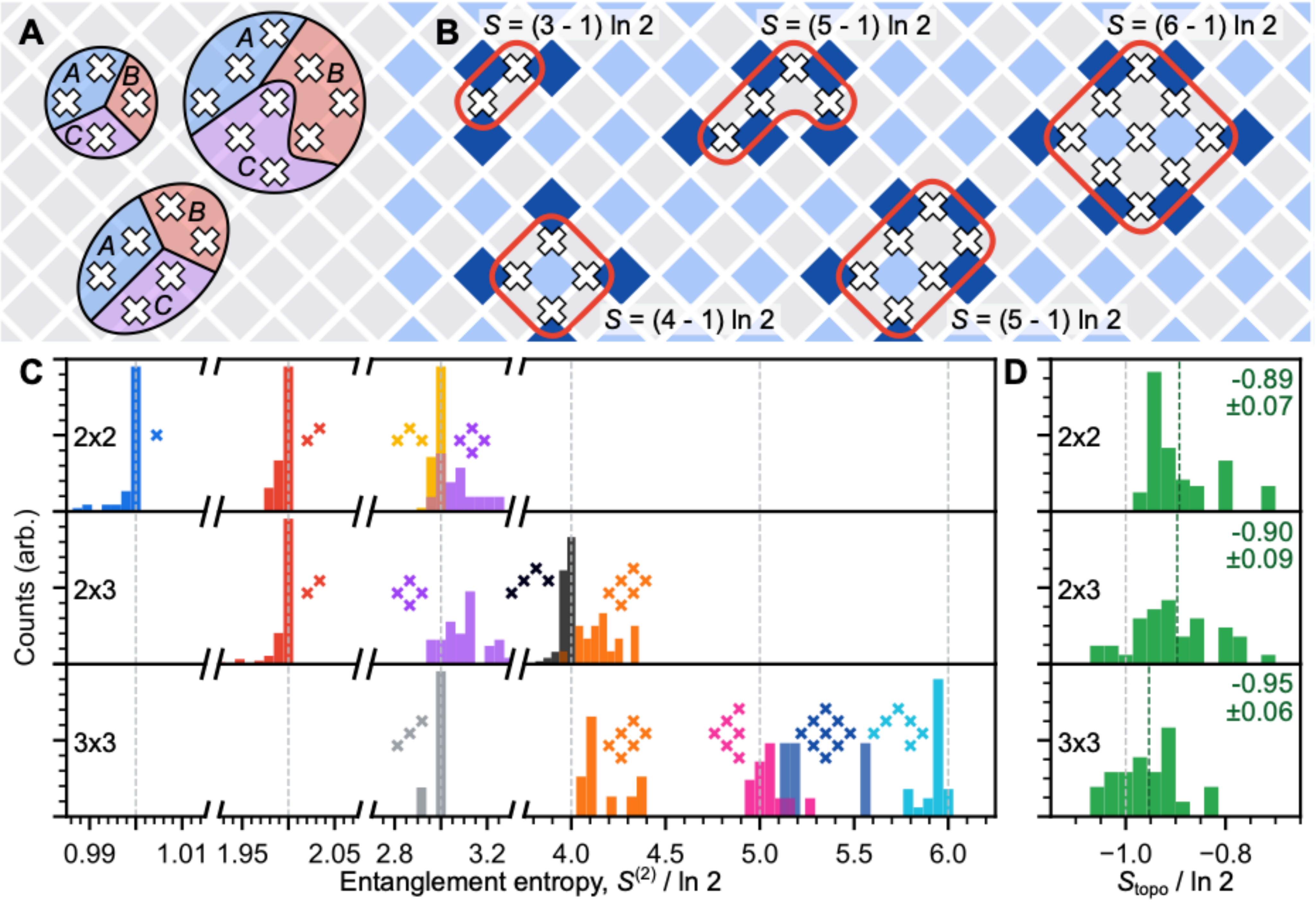


[**Satzinger, Liu, Smith**, … MK, Pollmann, Roushan, Science, 2021]

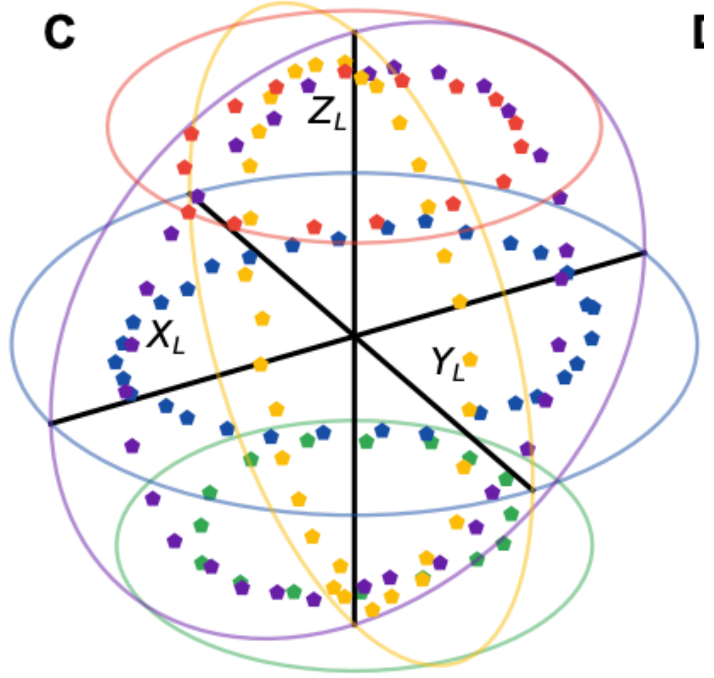
Signatures of topological order

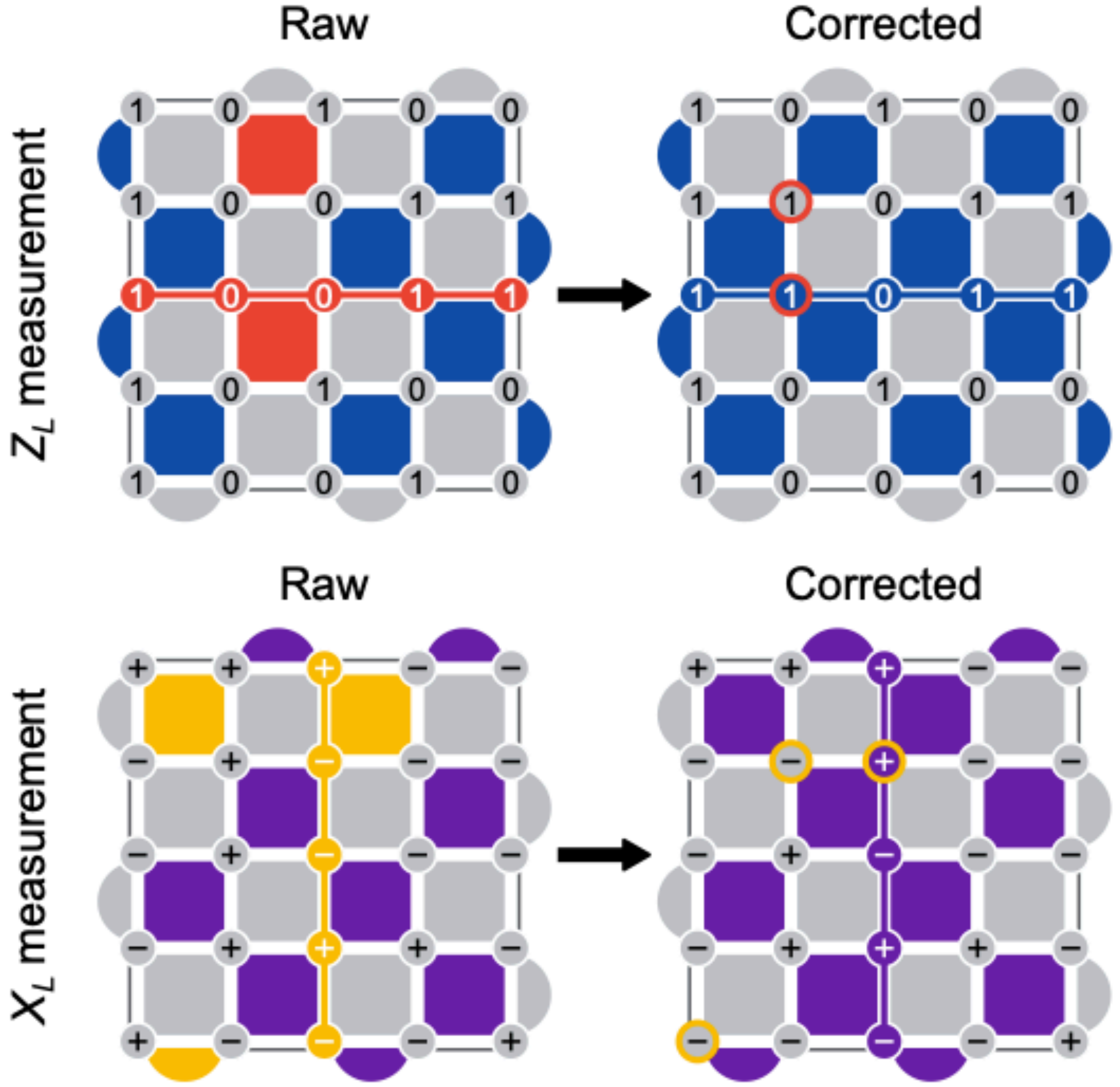
- Topological entanglement

$$S = \alpha L - \gamma \qquad \gamma = \ln(2)$$



- Distance-5 surface code



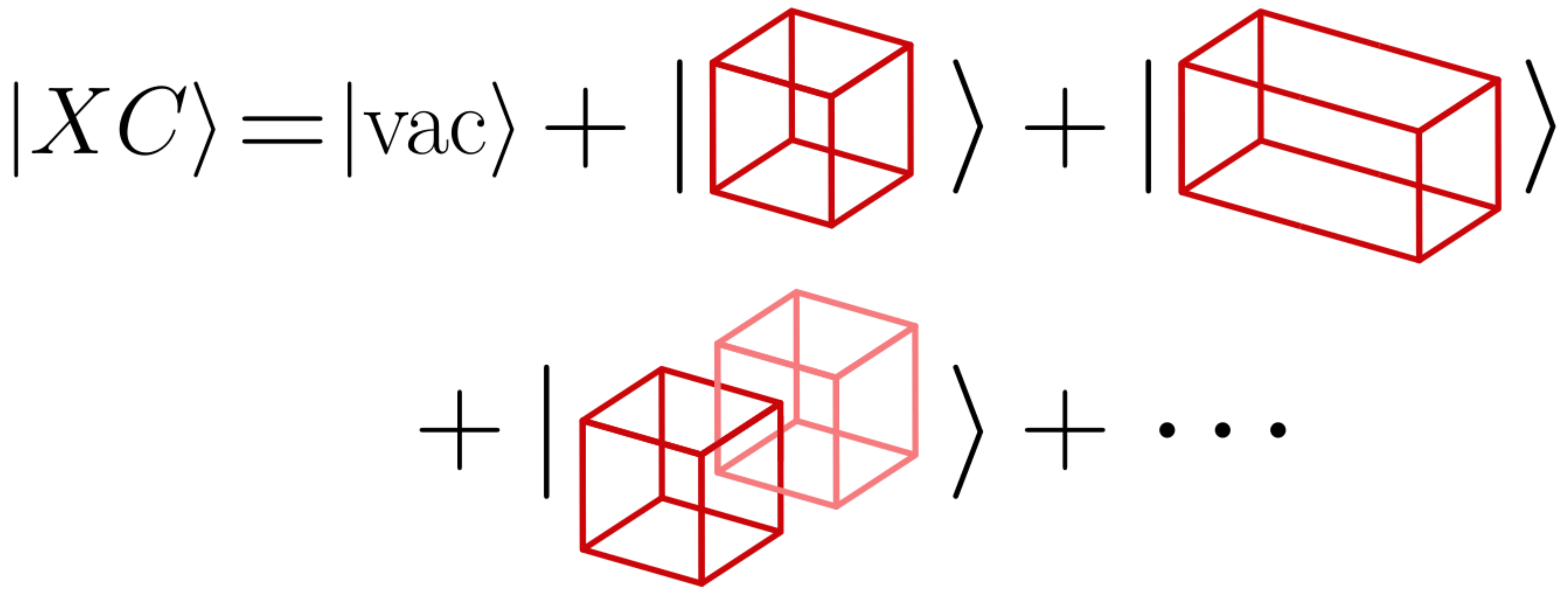


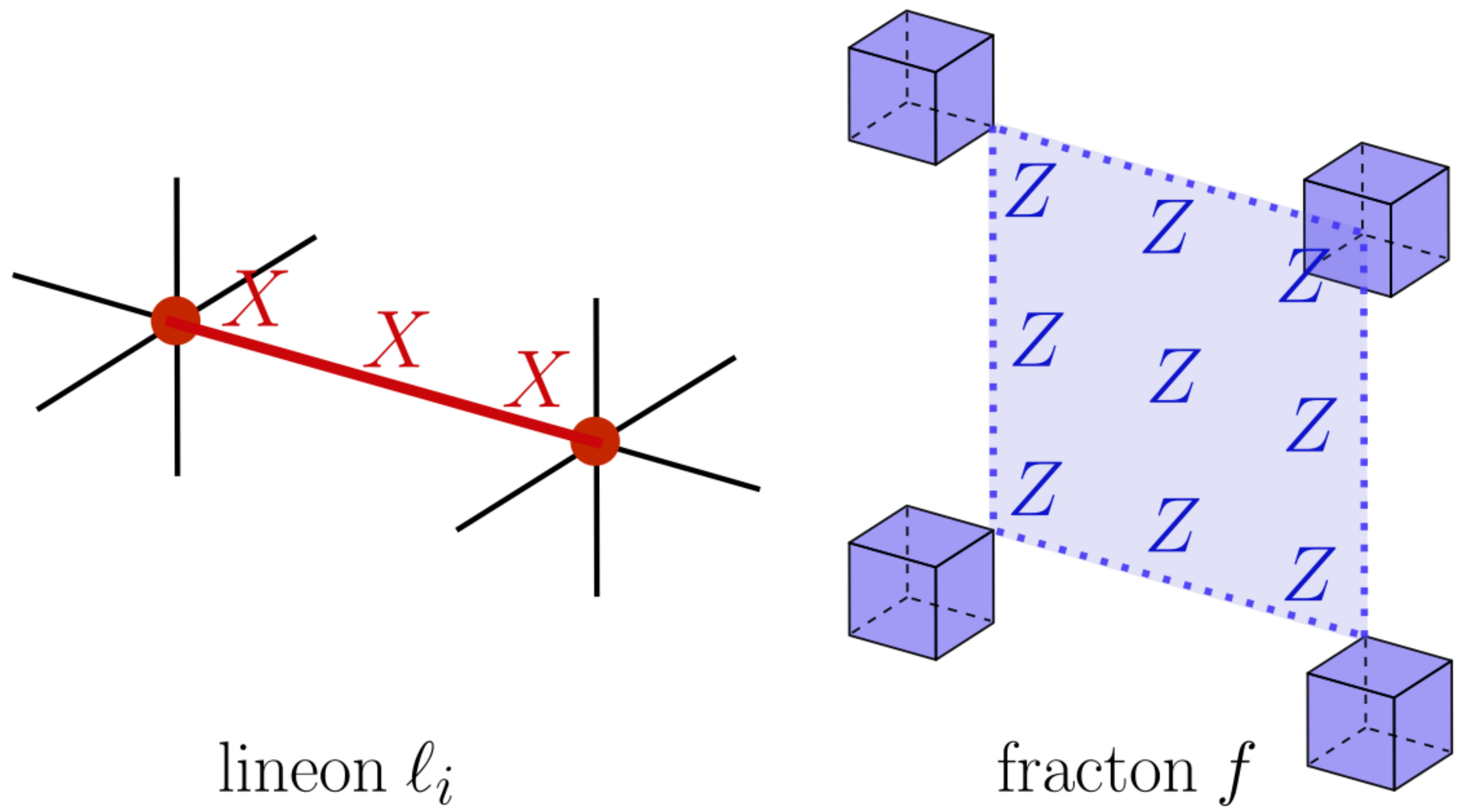
[**Satzinger, Liu, Smith**, … MK, Pollmann, Roushan, Science, 2021]

Preparation of other exotic states?

Fracton topological order

- Ground state of **X-Cube model** is condensate of cuboids
- GS = isometric TNS → linear-depth circuit






Julian Boesl


Yu-Jie Liu


Wen-Tao Xu

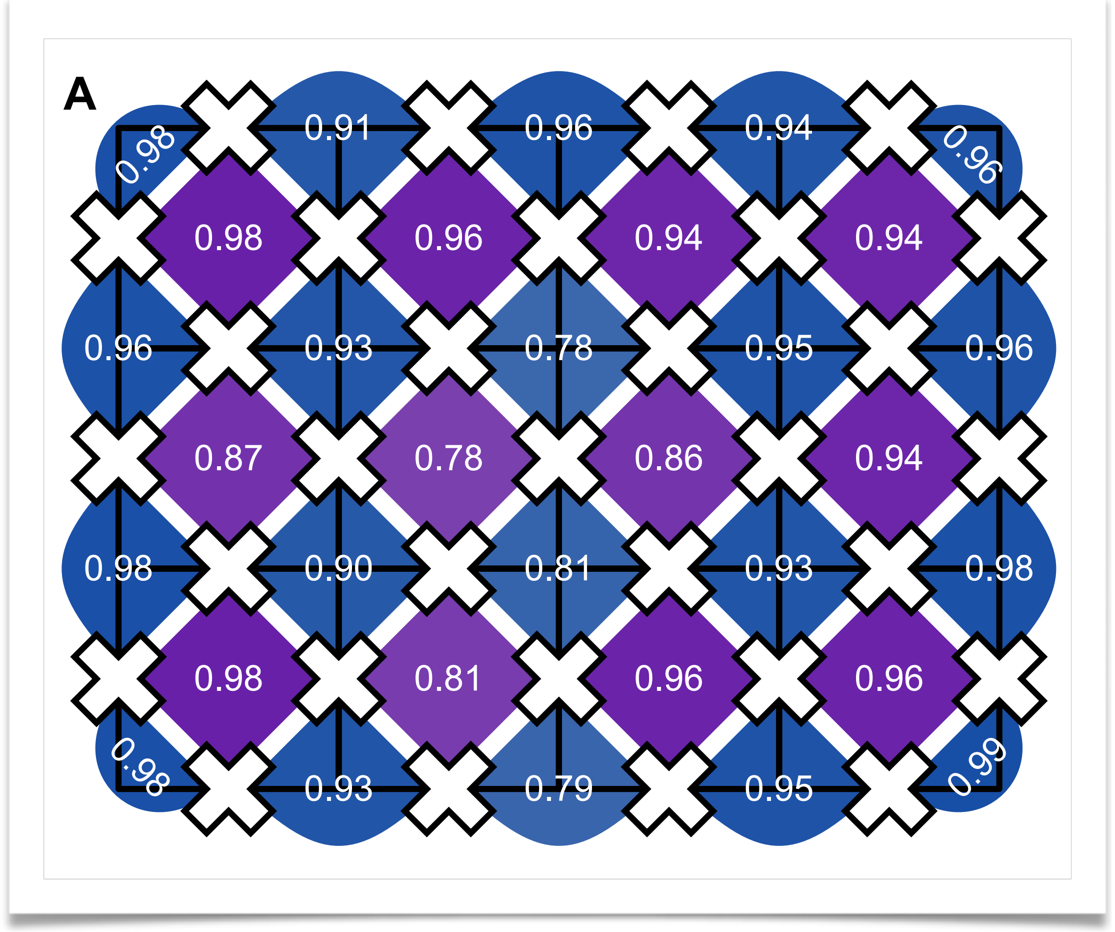

Frank Pollmann

[**Boesl**, Liu, Xu, Pollmann, MK, arXiv:2501.18688]

(I) Realizing topological order and fractionalization

[**Satzinger, Liu, Smith**, … MK, Pollmann, Roushan Science, 2021]

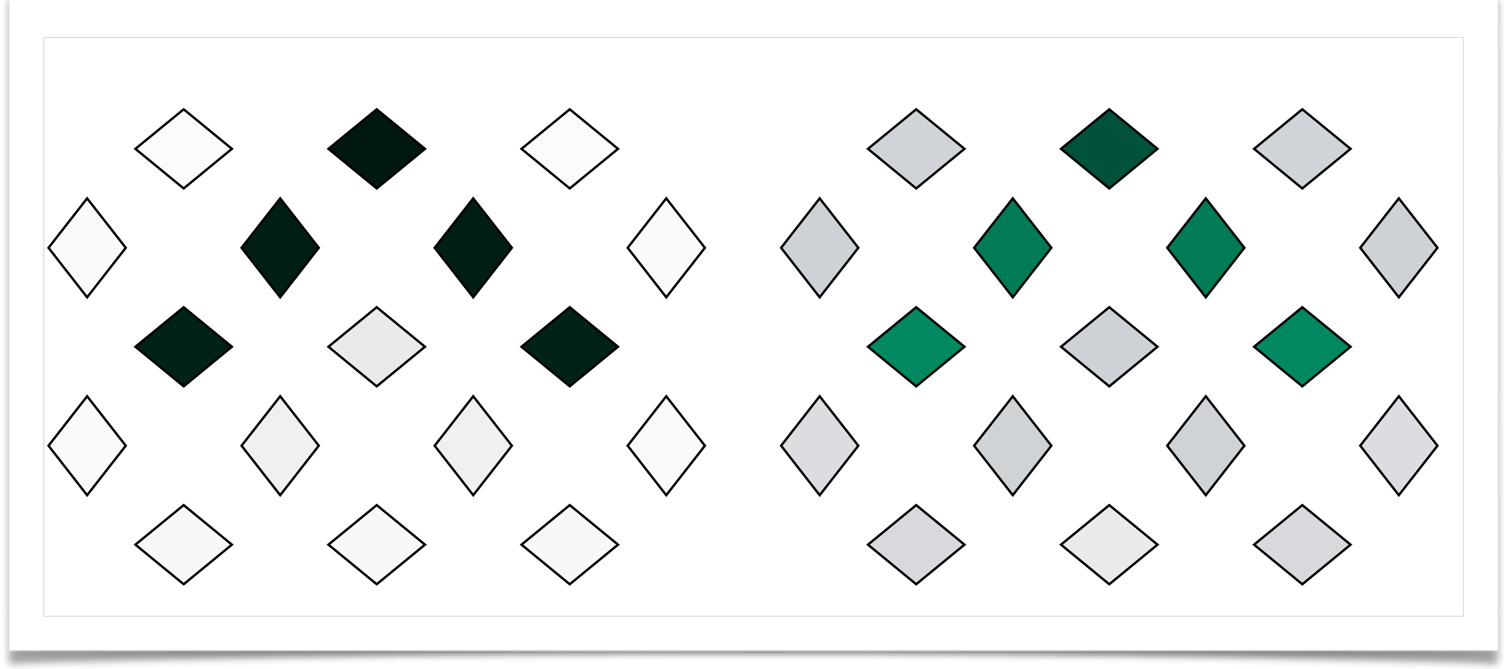
[**Boesl**, Liu, Xu, Pollmann, MK, arXiv:2501.18688]



(II) Visualizing dynamics of excitations

[**Cochran, Jobst, Rosenberg**, … Pollmann, MK, Roushan, arXiv:2409.17142]

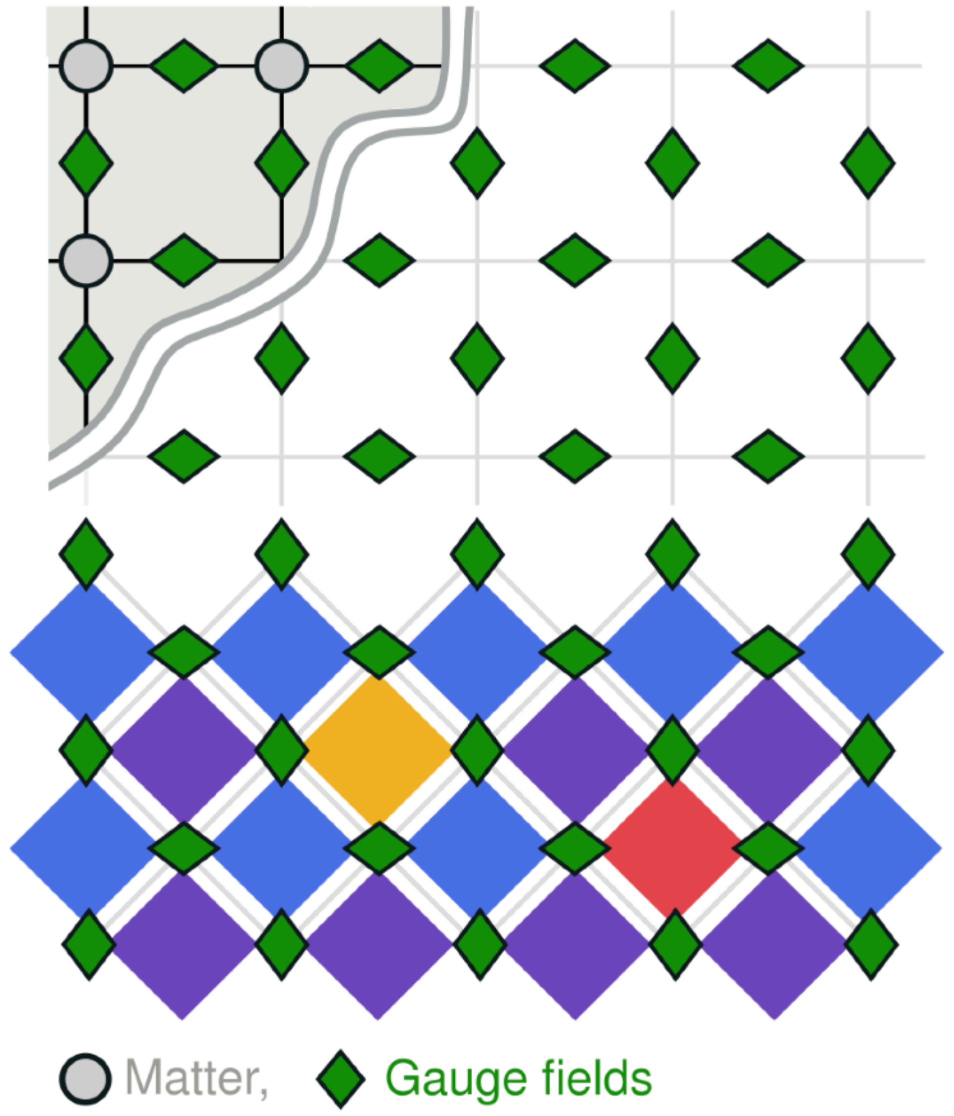
[**Xu**, MK, Pollmann, arXiv:2503.19027]

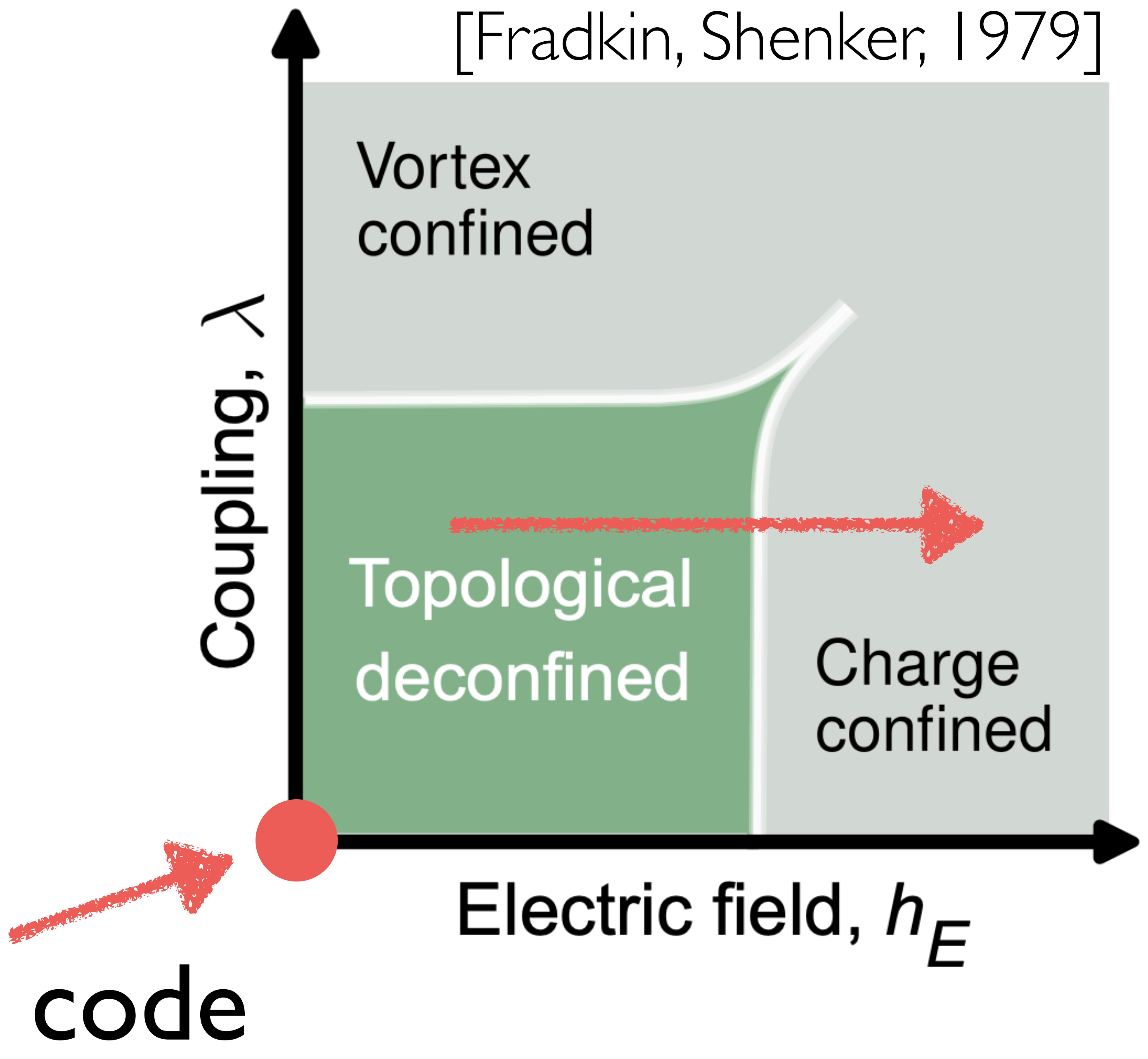


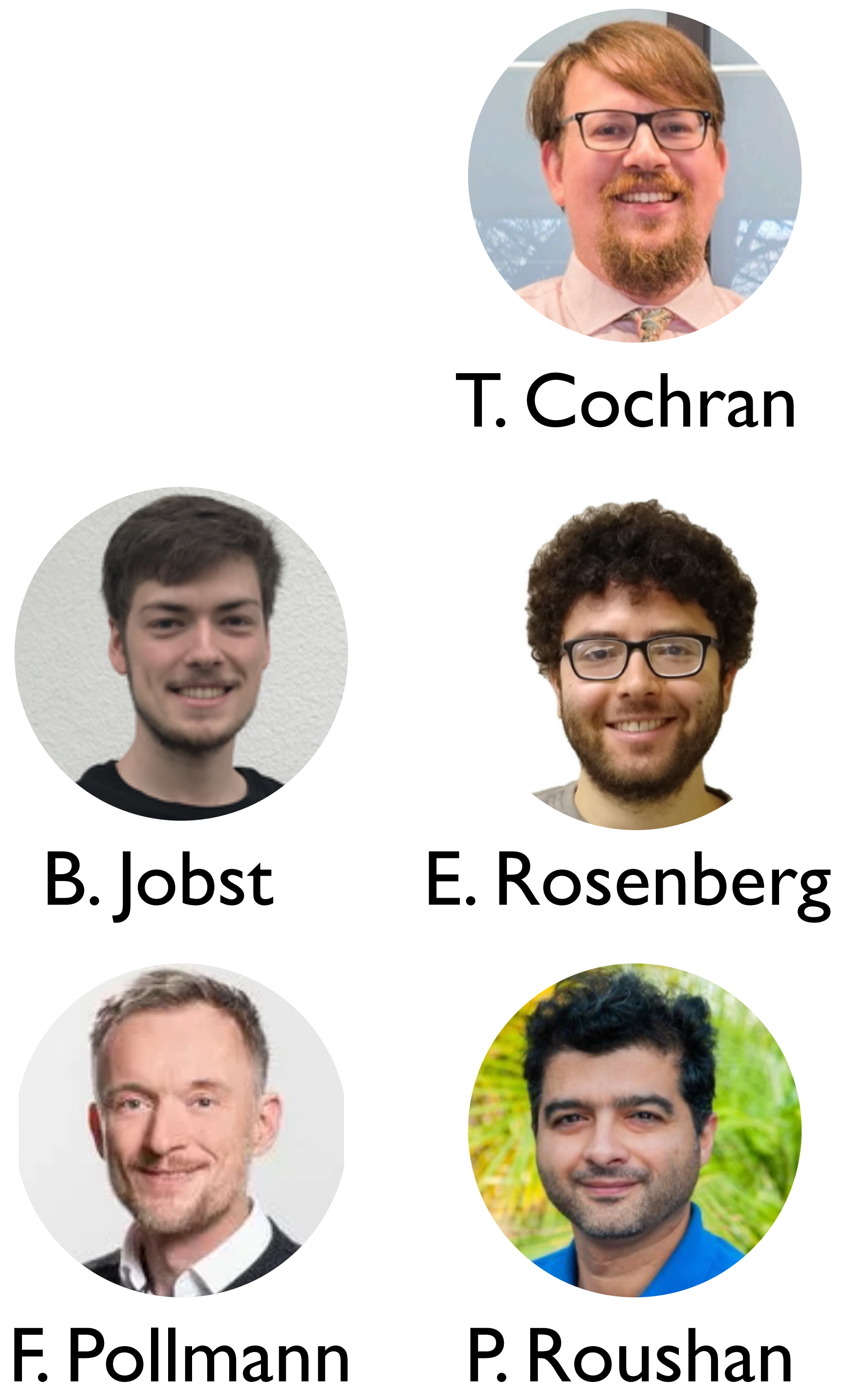
Confinement transition

- Toric code in a field is dual to (2+1)D $\mathbb{Z}_2$ lattice gauge theory

$$H = -J\sum_v A_v - J\sum_p B_p - h_E\sum_e Z_e - \lambda\sum_e X_e$$







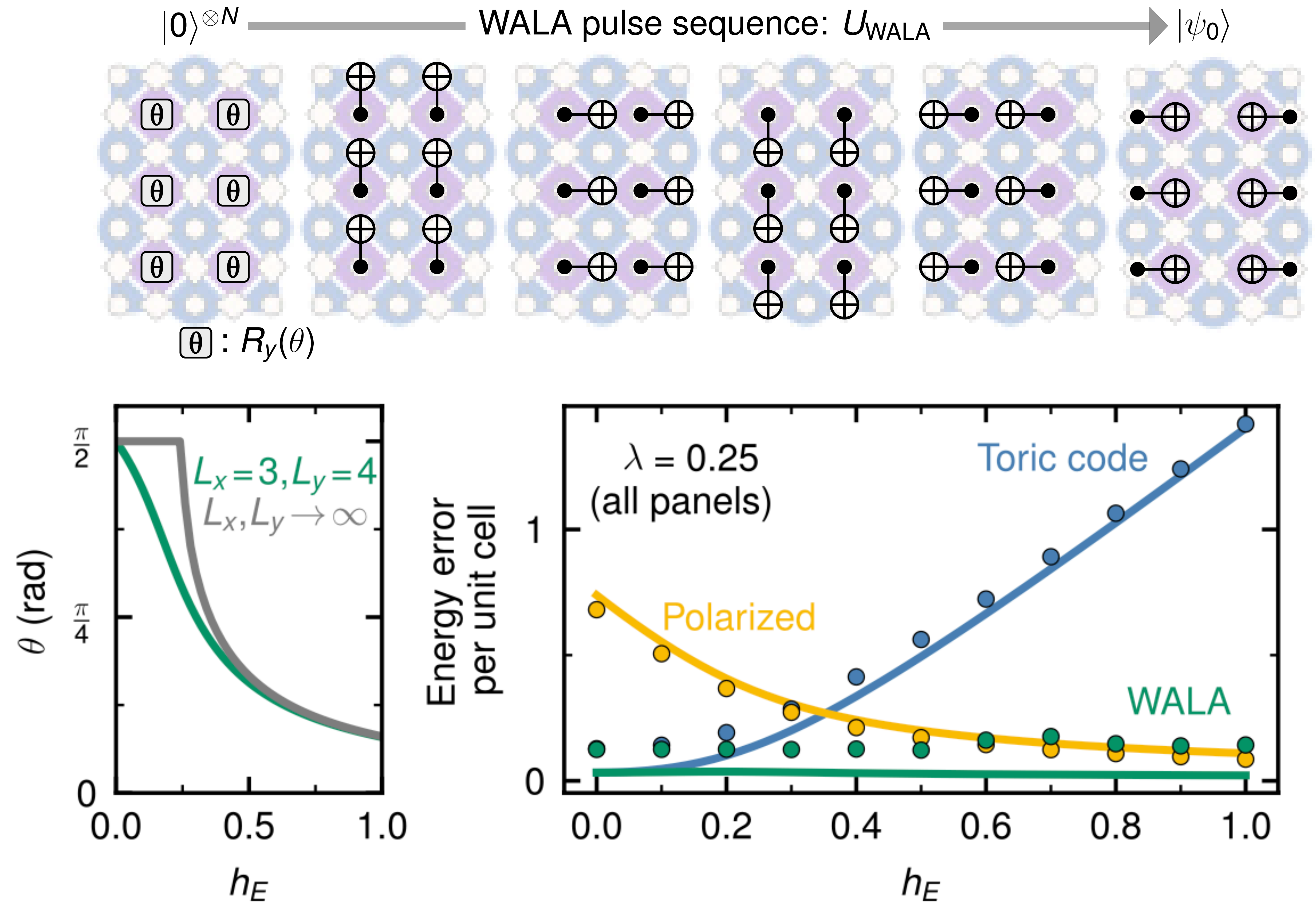
- Dynamical signatures of charge confinement?

[**Cochran, Jobst, Rosenberg**, … Pollmann, MK, Roushan, arXiv:2409.17142]

Preparing a low energy density state

Weight Adjustable Loop Ansatz (WALA):
[Dusuel and Vidal '15, Sun et. al '23]

$$|\psi\rangle = \prod_p \left(\cos(\theta/2)1 + \sin(\theta/2)B_p\right)|0\rangle$$

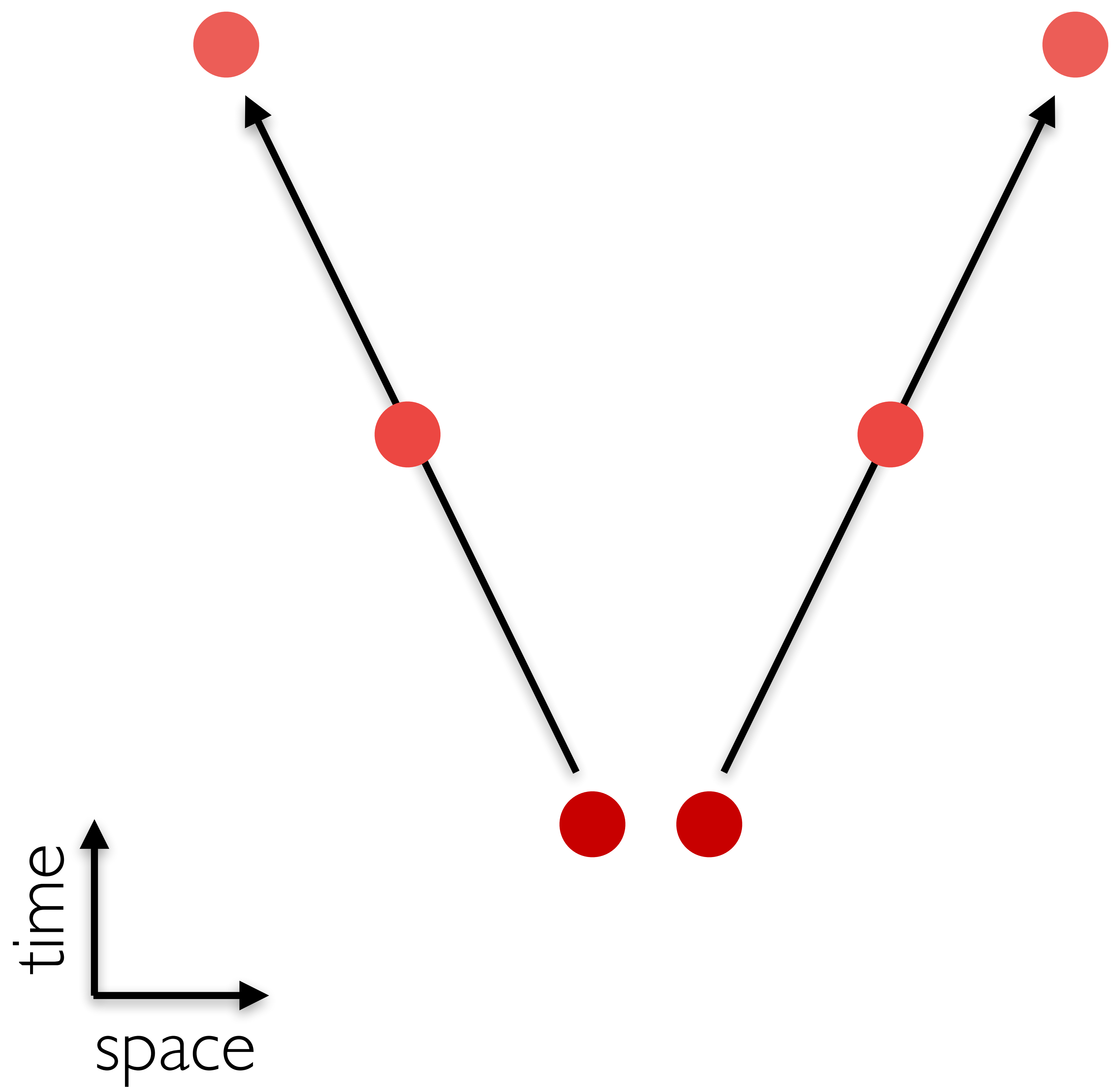


- Classically optimized WALA state is simple to prepare and has low energy density

[**Cochran, Jobst, Rosenberg**, … Pollmann, MK, Roushan, arXiv:2409.17142]

Dynamical signatures of confinement

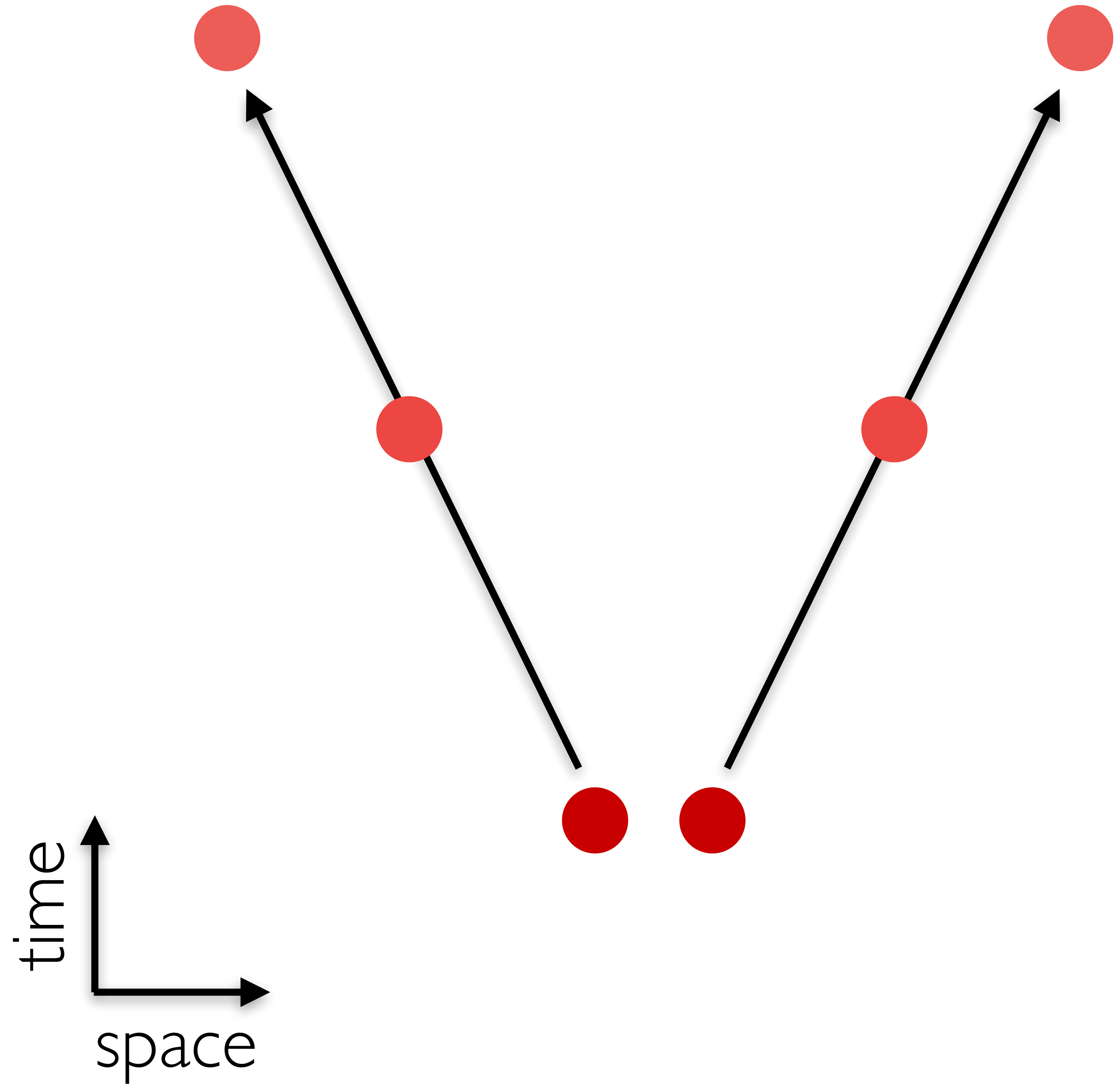
Deconfined charges



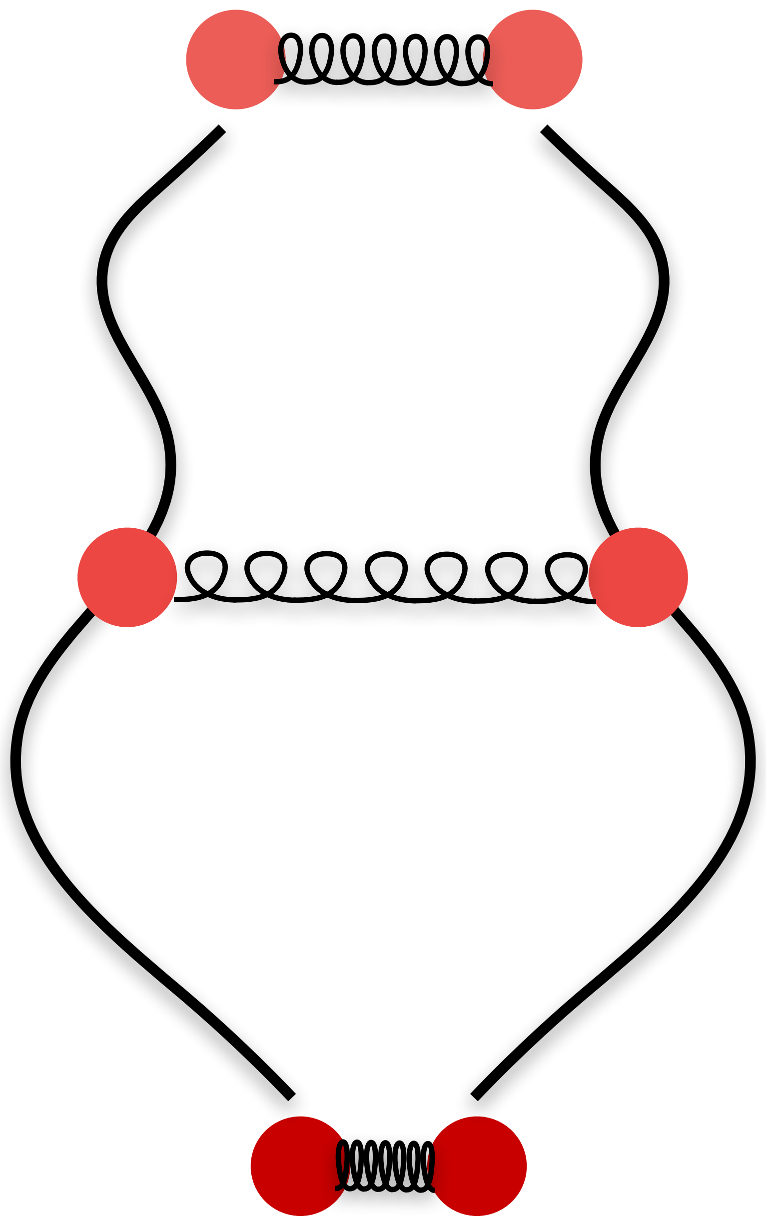
[**Cochran, Jobst, Rosenberg**, … Pollmann, MK, Roushan, arXiv:2409.17142]

Dynamical signatures of confinement

Deconfined charges

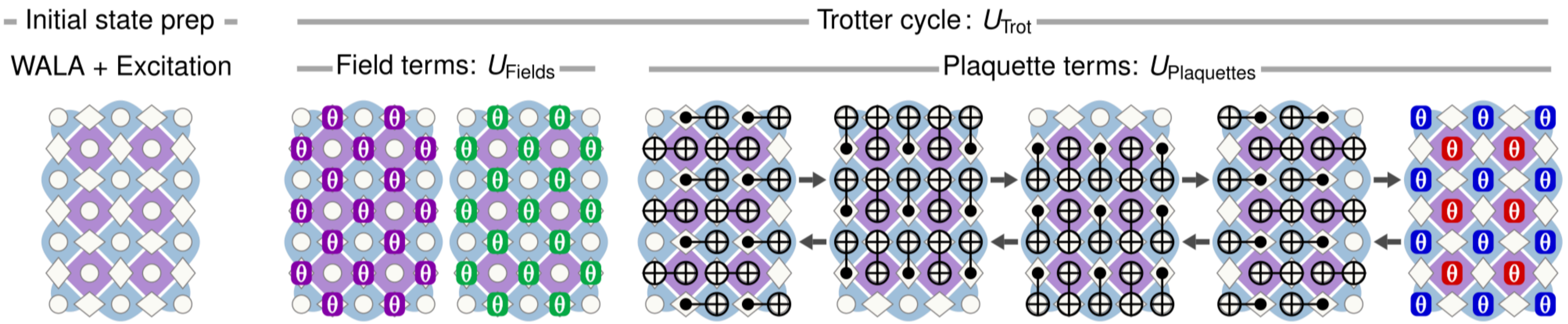


Confined charges

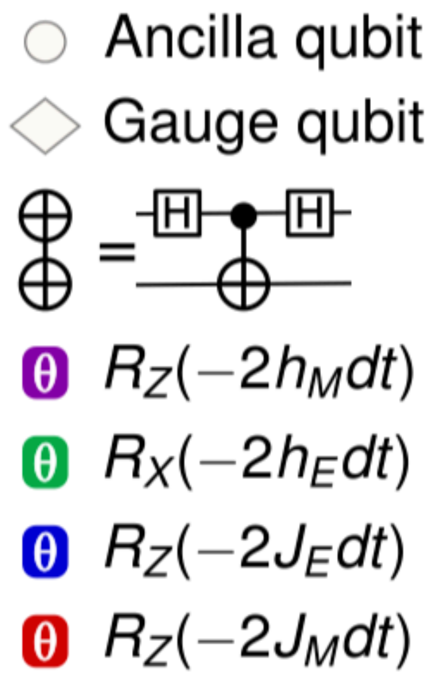


[**Cochran, Jobst, Rosenberg**, … Pollmann, MK, Roushan, arXiv:2409.17142]

Suzuki-Trotter evolution circuit

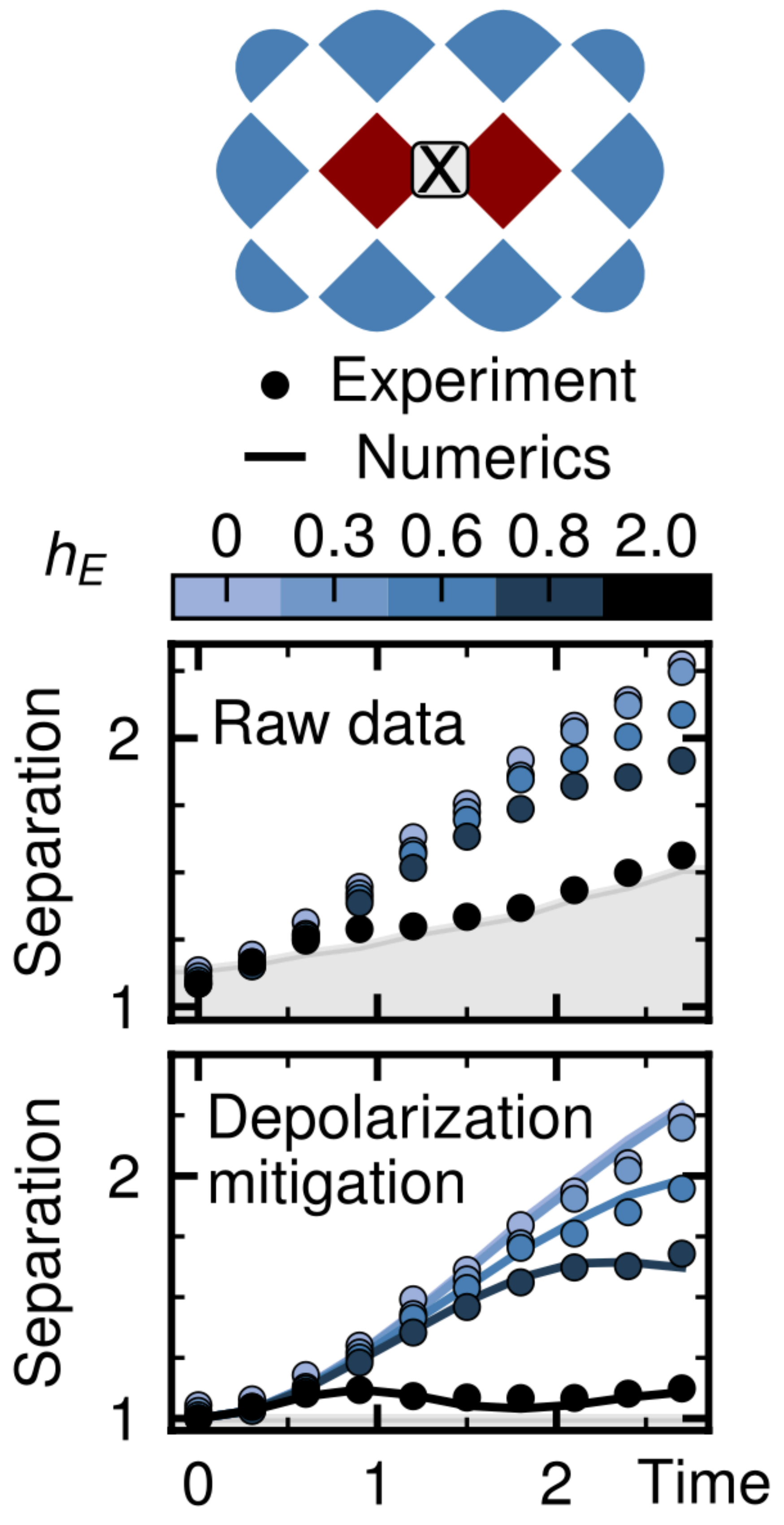


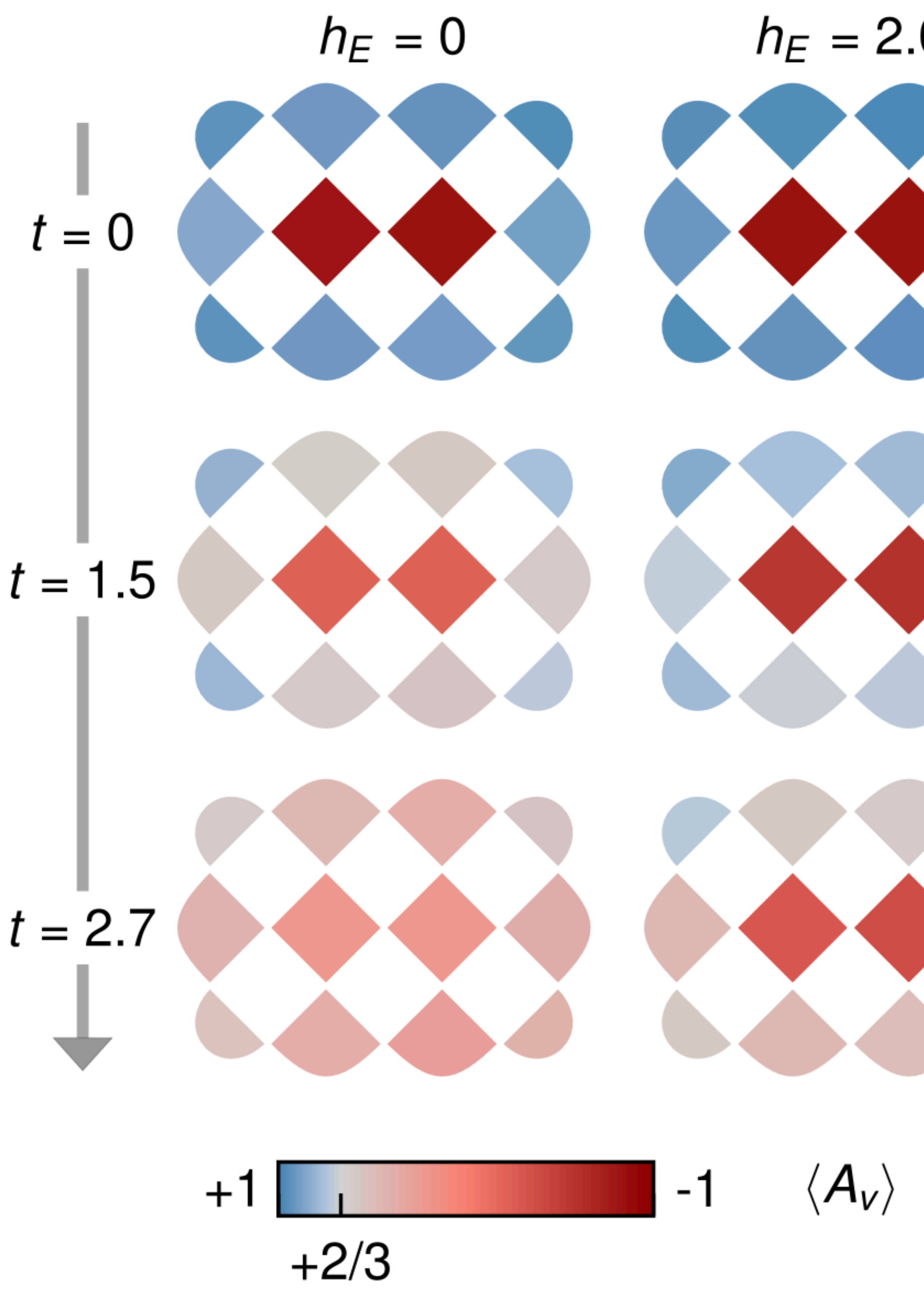
- Efficient Suzuki-Trotter using ancilla qubits (116 CZ gates per time step for the grid of 35 qubits)
- Post-select the measured data on the ancilla $|0\rangle$ state to mitigate decoherence

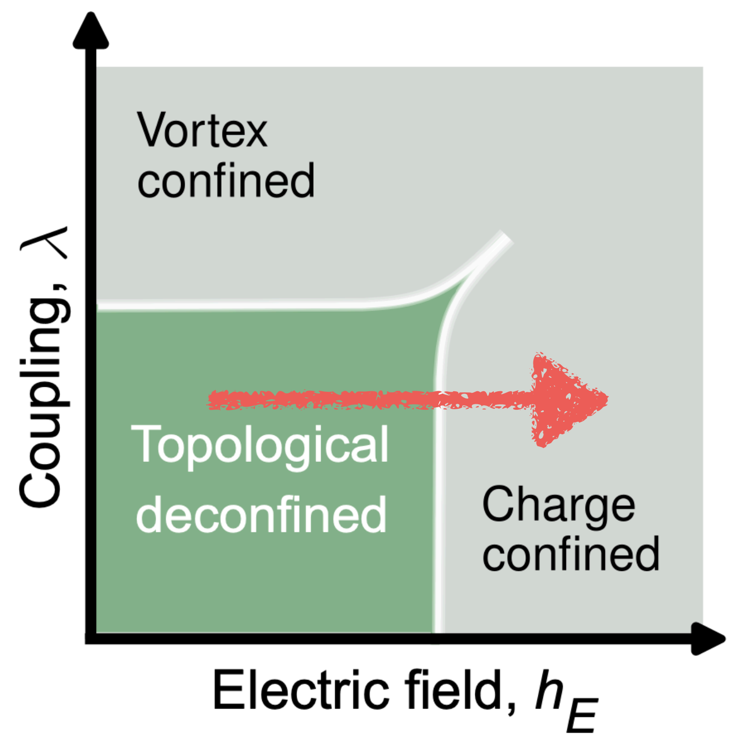


[**Cochran, Jobst, Rosenberg**, … Pollmann, MK, Roushan, arXiv:2409.17142]

Confinement dynamics of charges





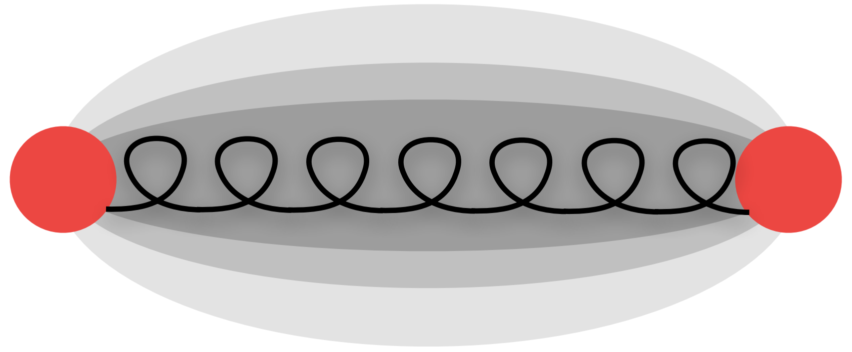


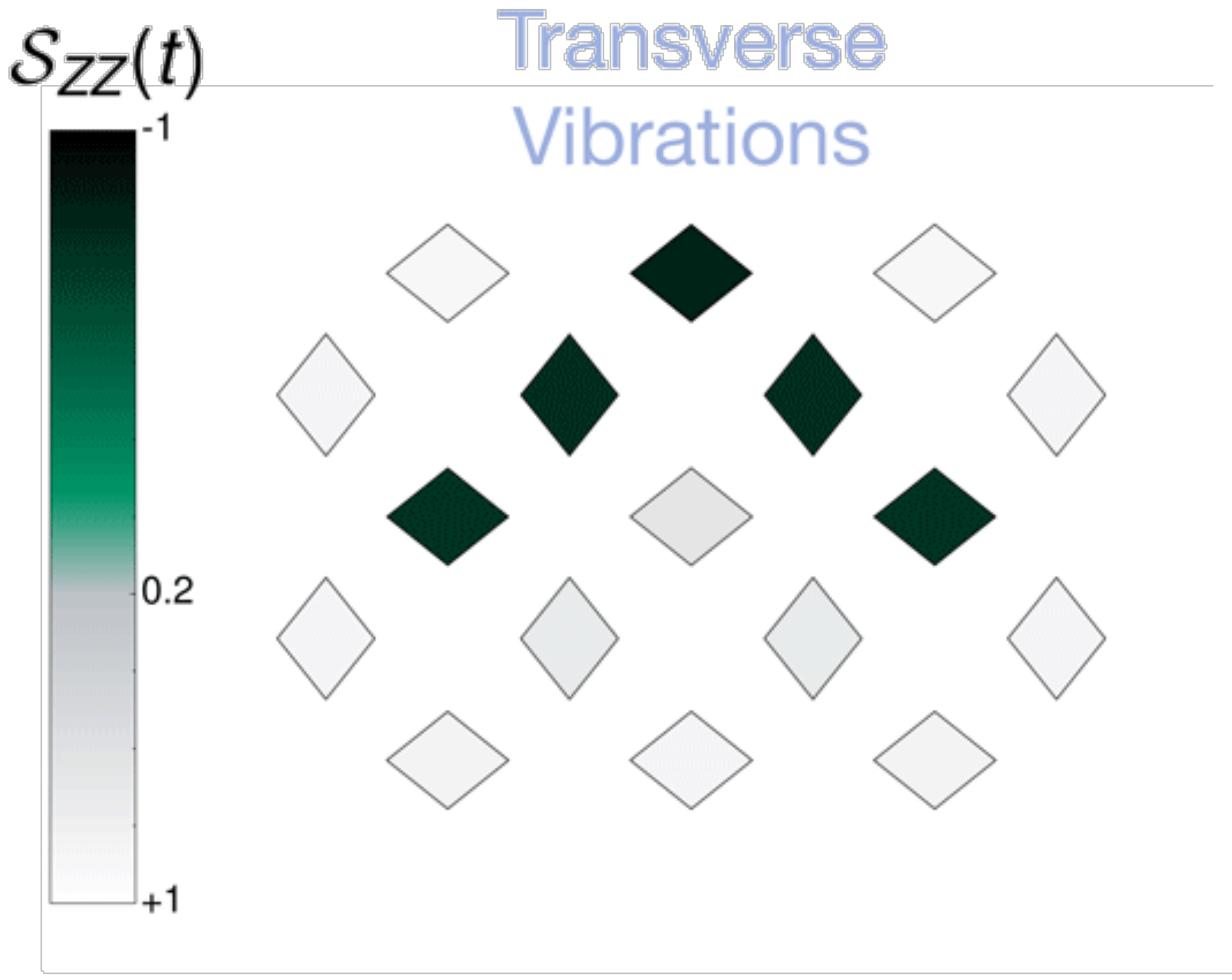
- Qualitatively distinct dynamical signatures as the electric field is tuned

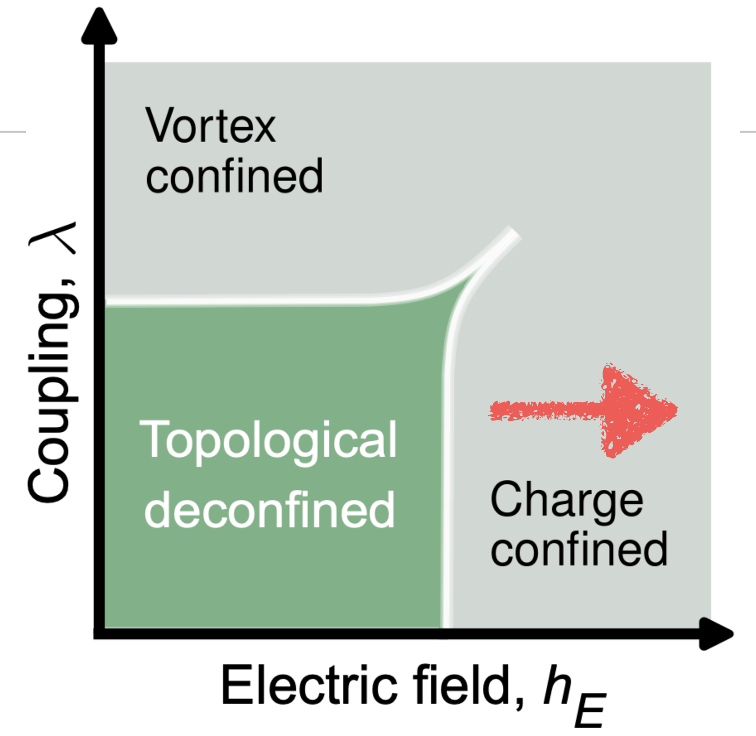
[**Cochran, Jobst, Rosenberg**, … Pollmann, MK, Roushan, arXiv:2409.17142]

Visualizing string dynamics

String vibrations



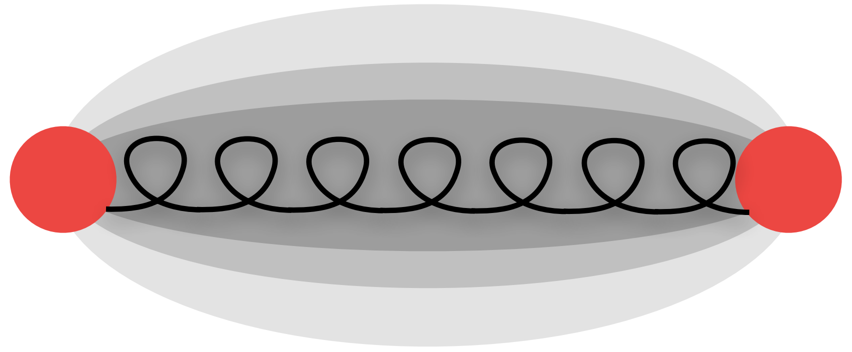


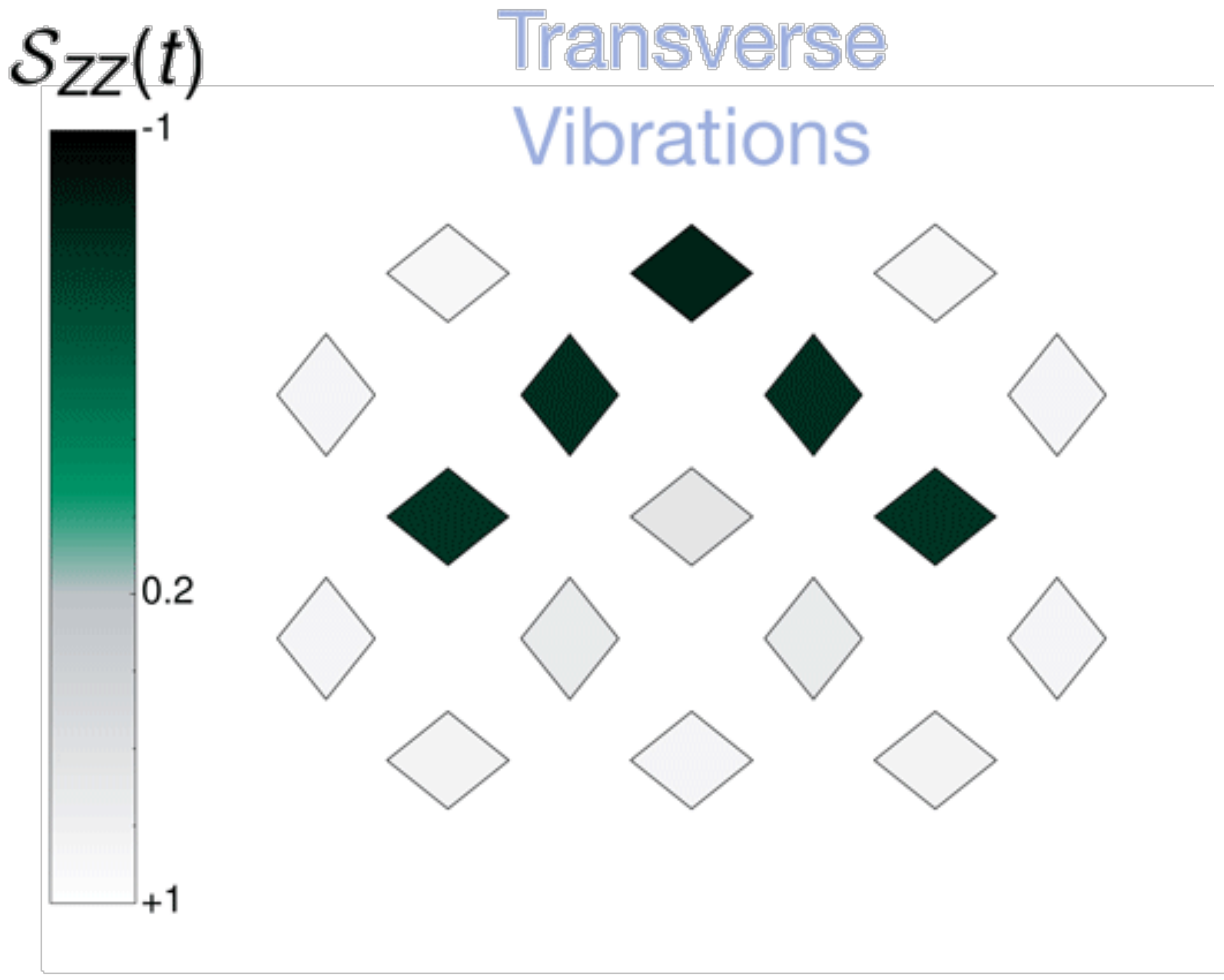


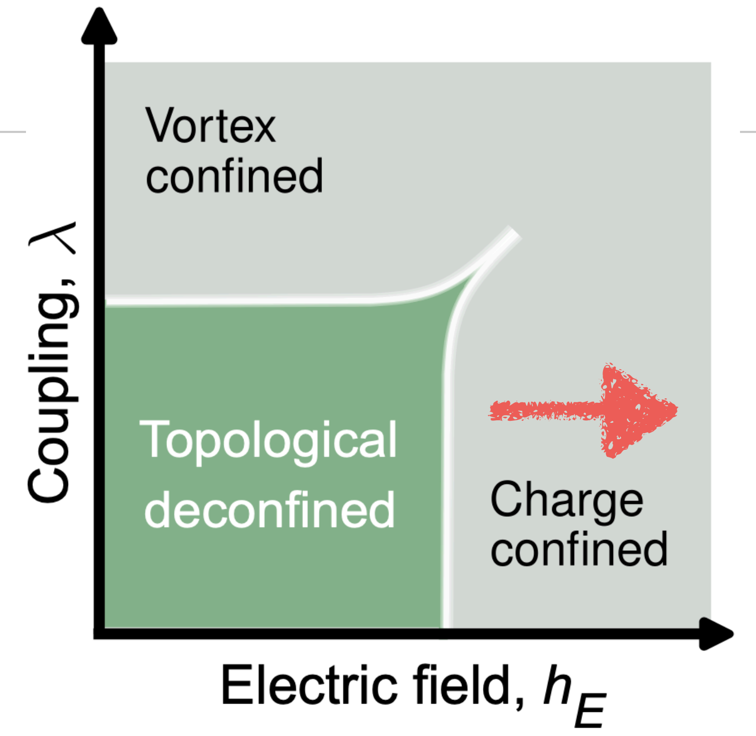
[**Cochran, Jobst, Rosenberg**, … Pollmann, MK, Roushan, arXiv:2409.17142]

Visualizing string dynamics

String vibrations



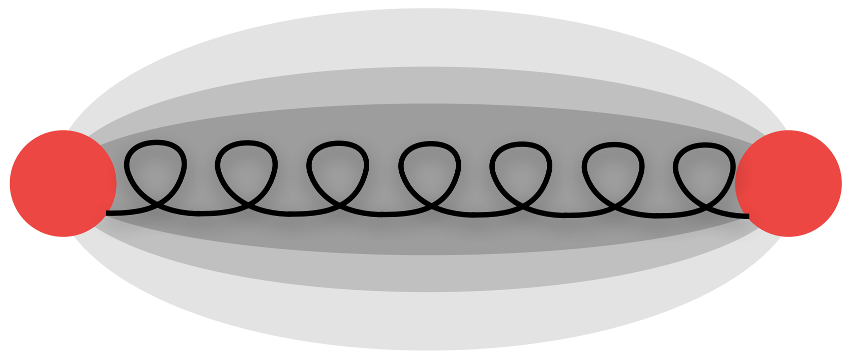


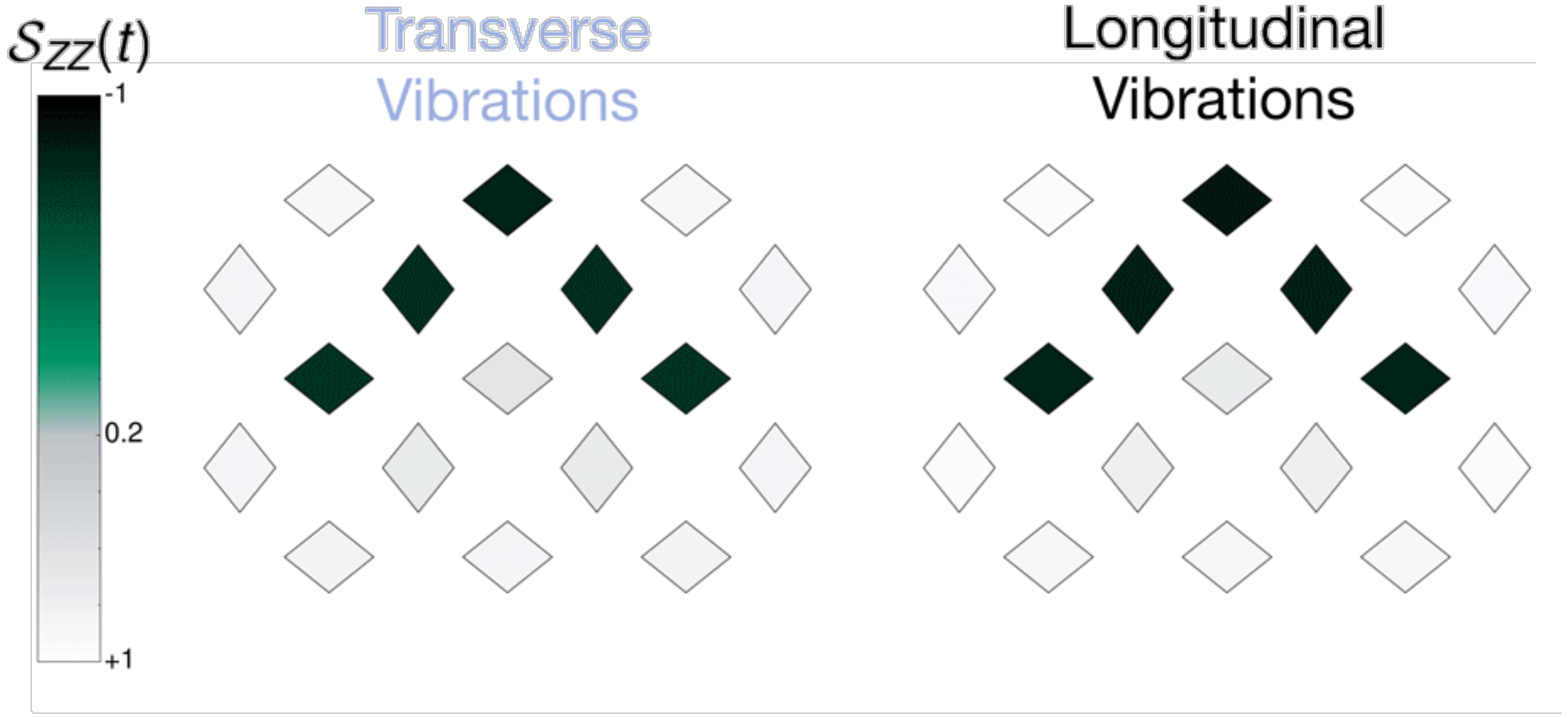


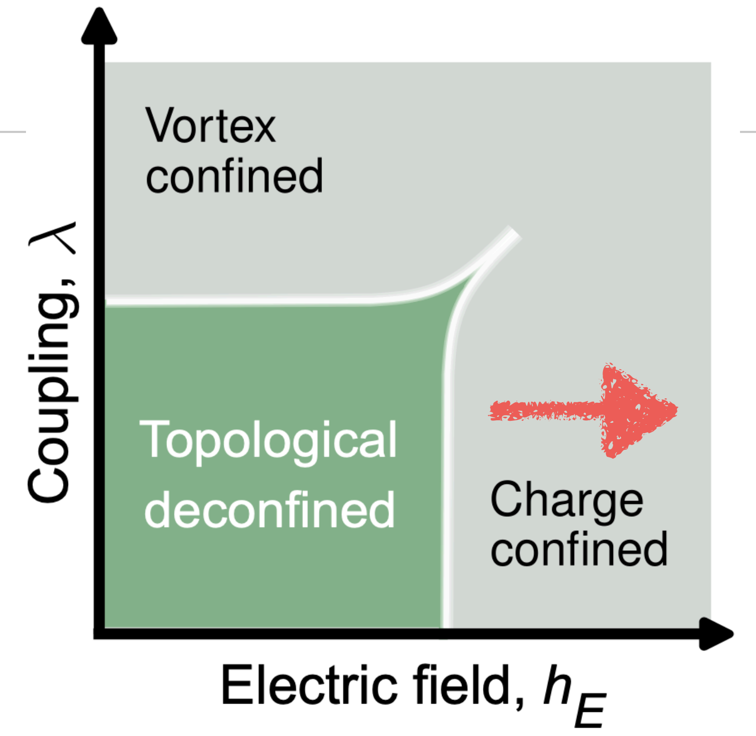
[**Cochran, Jobst, Rosenberg**, … Pollmann, MK, Roushan, arXiv:2409.17142]

Visualizing string dynamics

String vibrations



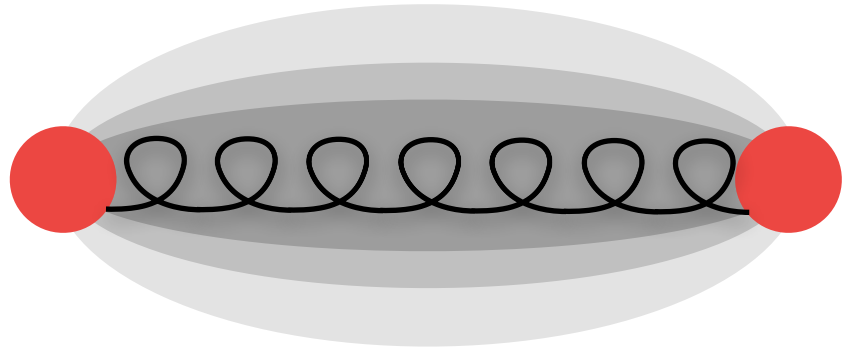


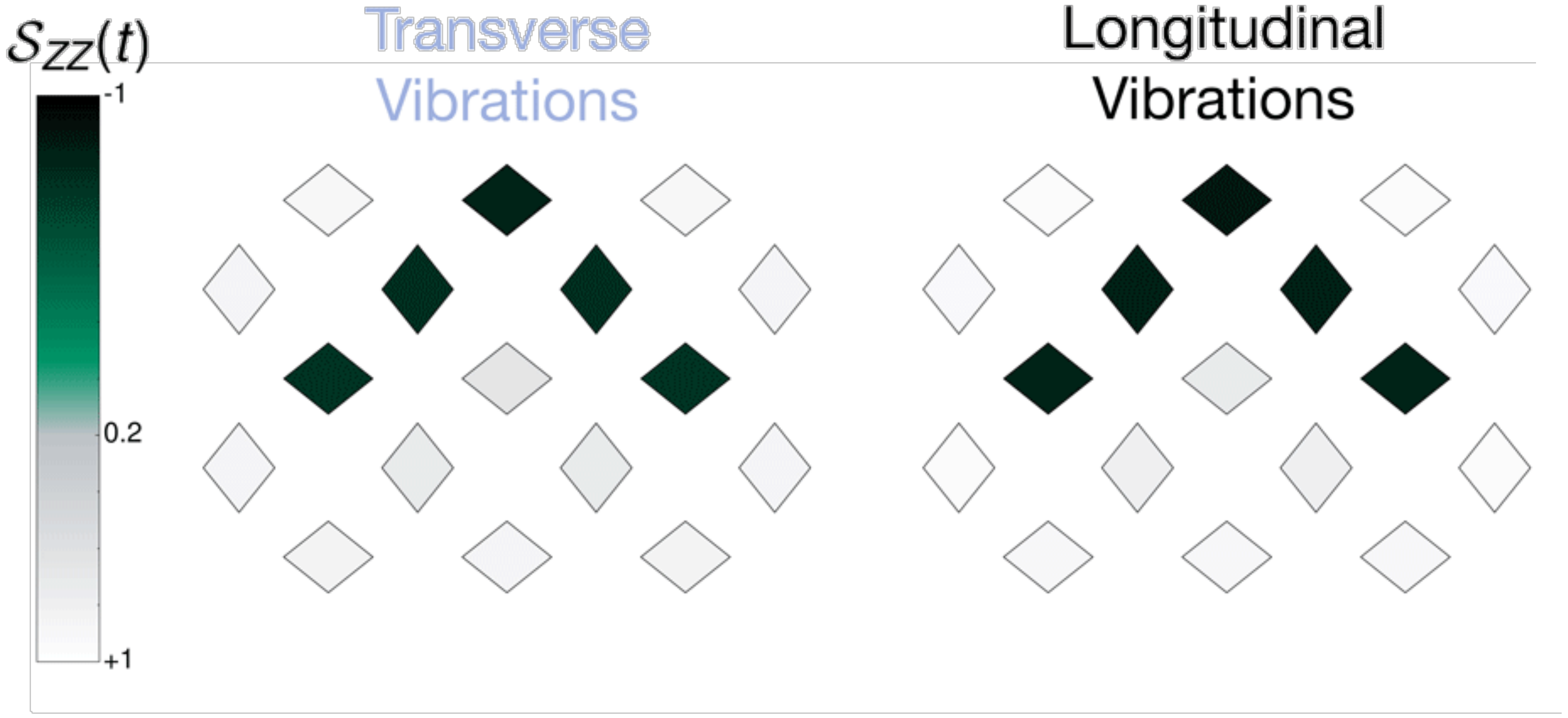


[**Cochran, Jobst, Rosenberg**, … Pollmann, MK, Roushan, arXiv:2409.17142]

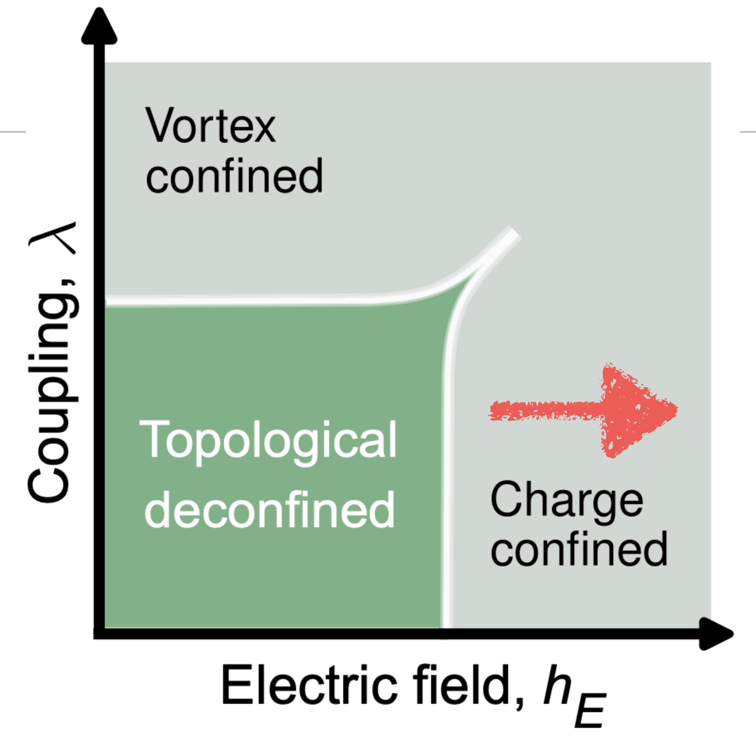
Visualizing string dynamics

String vibrations





- Equilibrium phase diagram (studied numerically for pure LGT $\lambda = 0$)

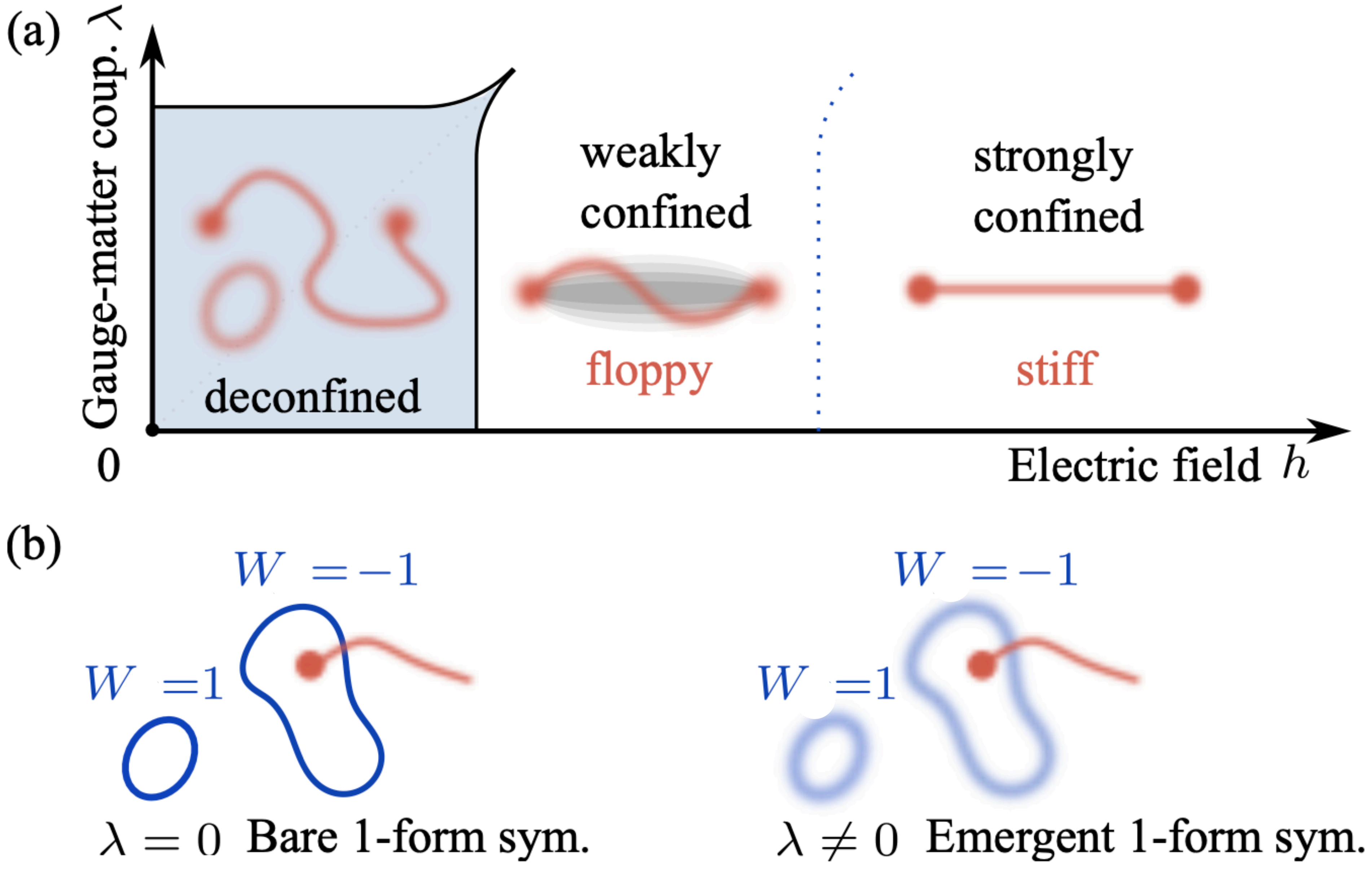


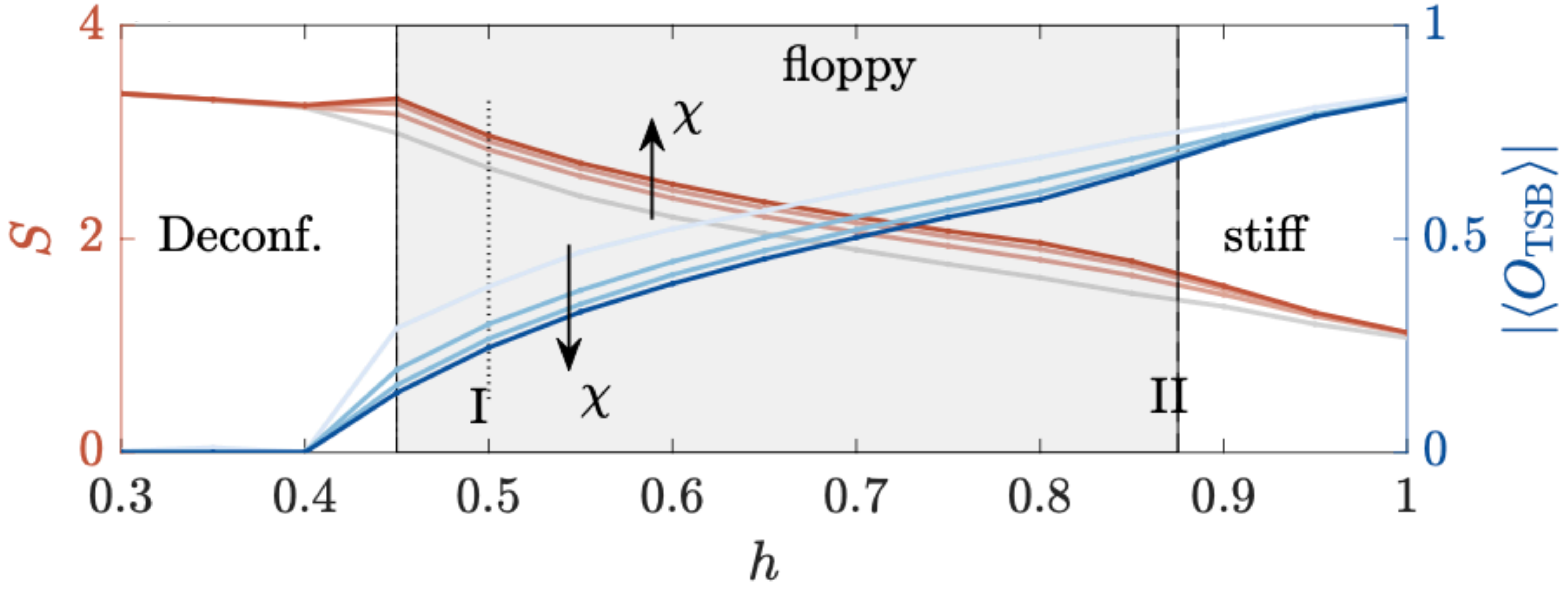
Equilibrium: [Lüscher, 1981; Hasenfratz^3, 1981]

[**Cochran, Jobst, Rosenberg**, … Pollmann, MK, Roushan, arXiv:2409.17142]

Roughening transition in a lattice gauge theory

Novel tensor network tools for finite gauge-matter coupling ($\lambda \neq 0$)



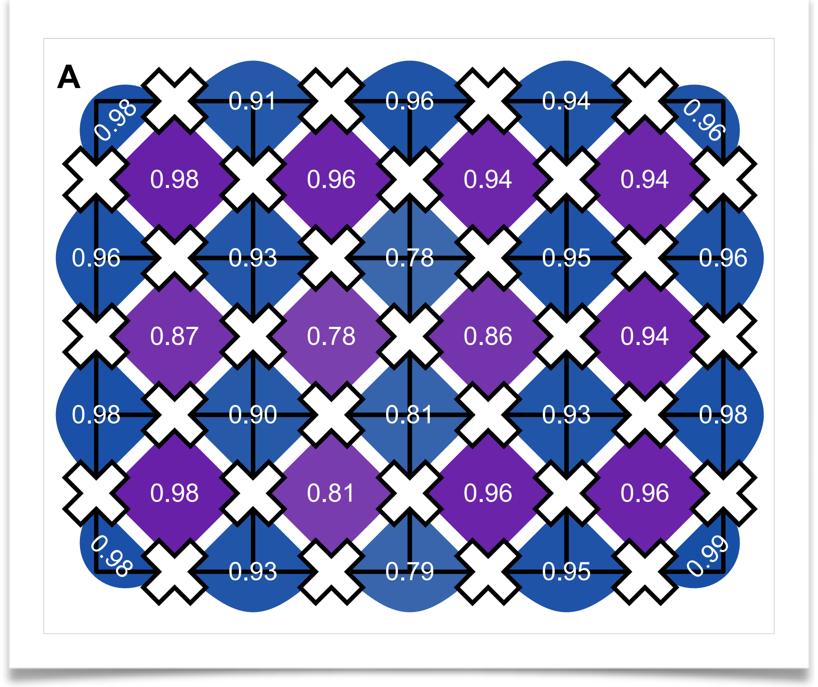



Wen-Tao Xu


Frank Pollmann

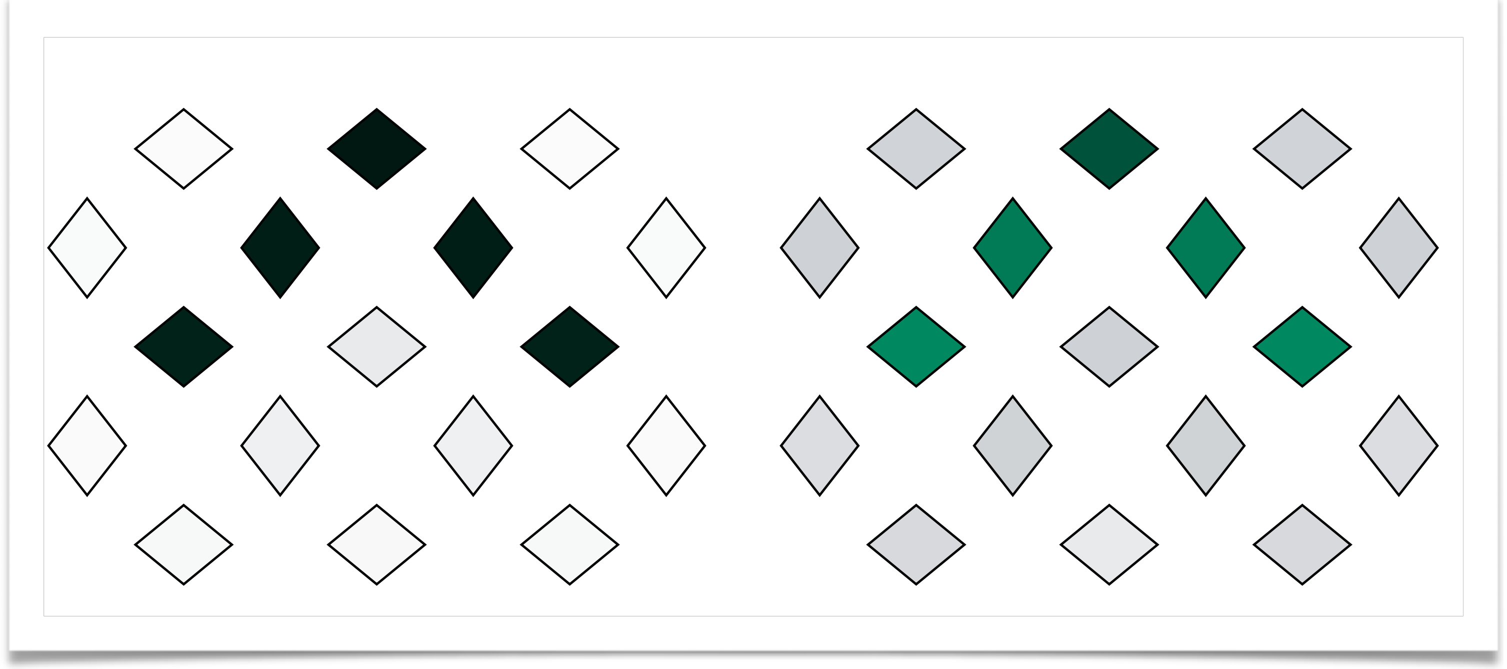
[**Xu**, MK, Pollmann, arXiv:2503.19027]

Exploring quantum phases of matter on quantum processors



(I) Realizing topological order and fractionalization

[**Satzinger, Liu, Smith**, … MK, Pollmann, Roushan Science, 2021]
[**Boesl**, Liu, Xu, Pollmann, MK, arXiv:2501.18688]



(II) Visualizing dynamics of excitations

[**Cochran, Jobst, Rosenberg**, … Pollmann, MK, Roushan, arXiv:2409.17142]
[**Xu**, MK, Pollmann, arXiv:2503.19027]


Y-J Liu


W-T Xu


J. Boesl


B. Jobst


A. Smith


F. Pollmann


K. Satzinger


T. Cochran


E. Rosenberg


P. Roushan