At 89 I’m just about old enough for a Nobel

The Nobel Prize in Physics 2020
Black Holes

Jerzy Lewandowski
Uniwersytet Warszawski

Imperial College, London, 1-10 July 1965

Kip Thorne's memories…

in elevator, IMPAN

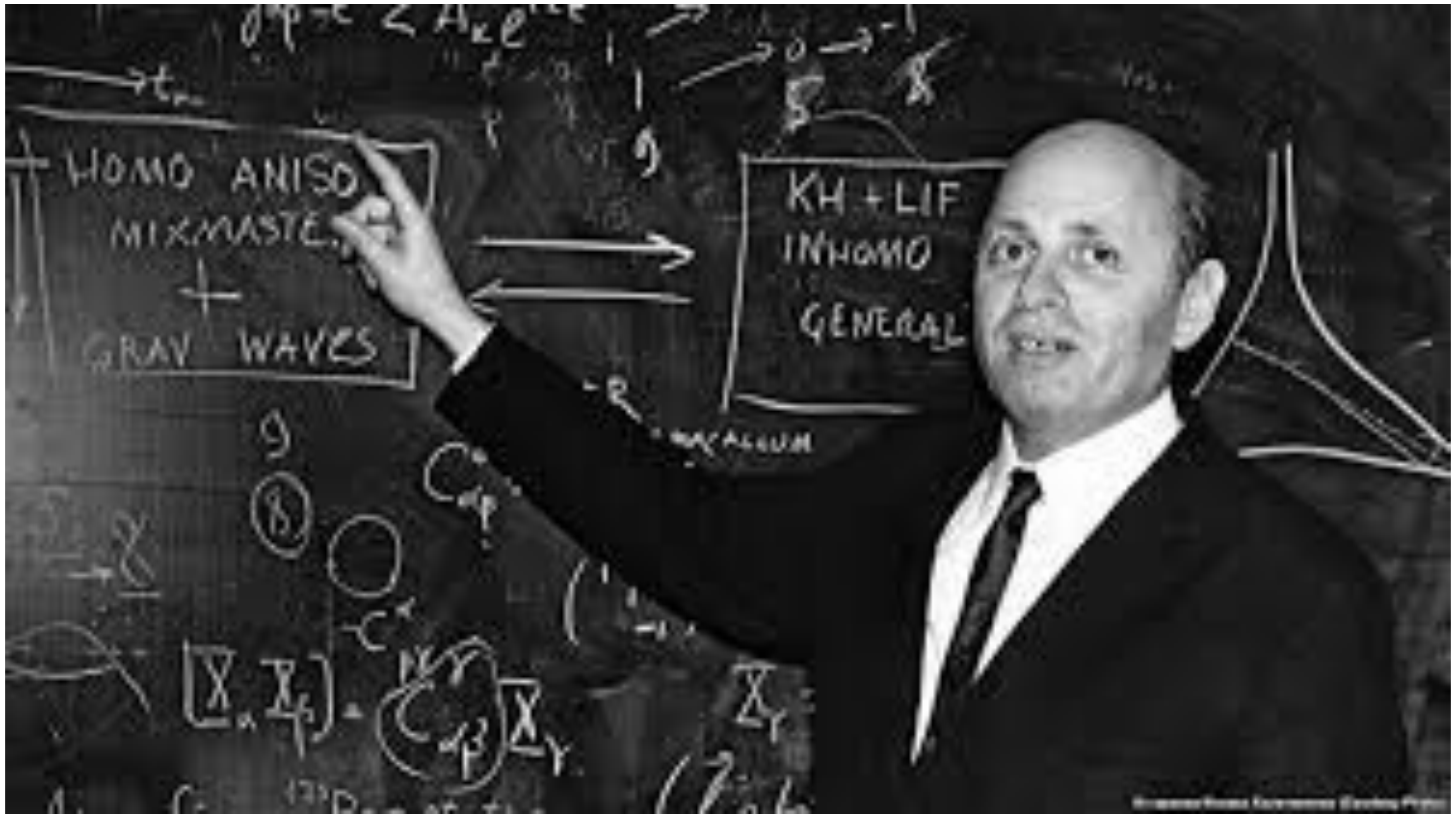
BLACK HOLES
AND
TIME WARPS

Einstein's Outrageous Legacy

KIP S. THORNE
THE FEYNMAN PROFESSOR OF THEORETICAL PHYSICS
CALIFORNIA INSTITUTE OF TECHNOLOGY

The Khalatnikov-Lifshitz results the gravitational collapse

It was a warm summer day in 1965. The world's leading general relativity researchers had gathered for the 4th International Conference on GR. The lecture room was filled to over flowing as Isaak Khalatnikov rose to speak. He and Evgeny Lifshitz in Moscow had proved (or so they thought) that when a real star with random internal deformations implodes it can not create a singularity at the center.



Isaak Khalatnikov

Evgeny Lifshitz

The Khalatnikov-Lifshitz results the gravitational collapse

Khalatnikov spoke dragging a microphone with him. Using the standard equation-intensive methods known well to all the theoretical physicists he demonstrated that random perturbations must grow as a star implodes.

-This means **- he asserted -** ***that if the implosion is to form a singularity, it must be one with completely random deformations in its spacetime curvature*****.**

The Khalatnikov-Lifshitz results the gravitational collapse

He then described how he and Lifshitz had searched, among all types of the singularities permitted by the laws of GR, for one with completely random deformations. He exhibited mathematically one type of singularity after another, he catalogued the types of singularity almost *ad nauseam*. Among them non had completely random deformations.

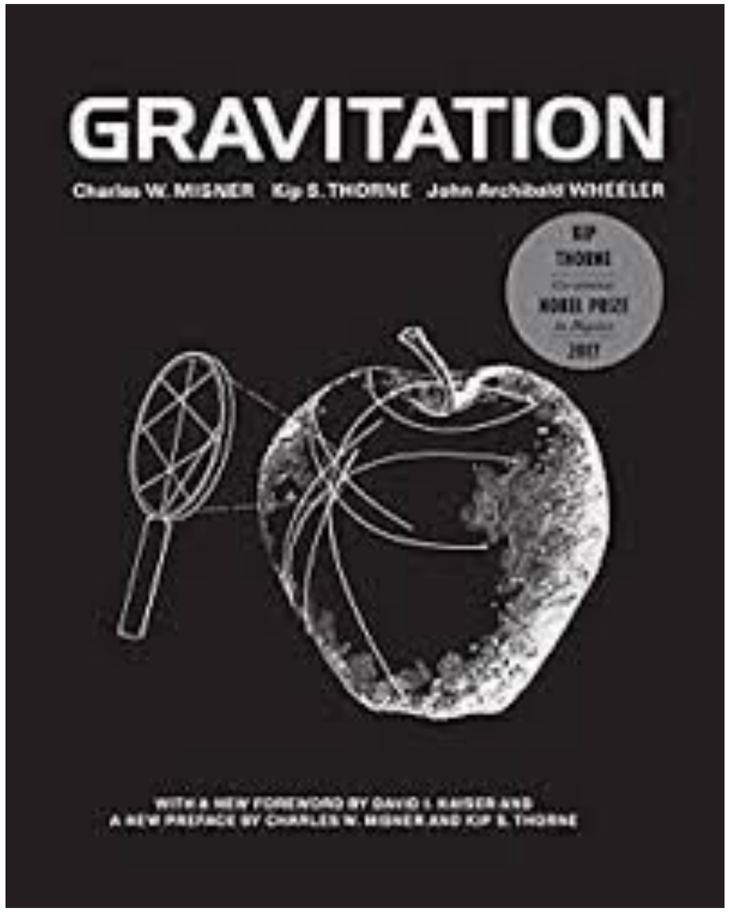
The Khalatnikov-Lifshitz results the gravitational collapse

-*Therefore* - he concluded bringing his 40-minut lecture to a close - *an imploding star with random perturbations can not produce a singularity. The perturbations must save the star from destruction.*

Misner's objection

As the applause ended, Charles Misner, one of Wheeler's most brilliant former students, leaped up and objected strenuously.

Charles Misner



Misner’s objection

Excitedly, and in rapid-fire English, Misner described the theorem that Penrose had proved a few months earlier. If Penrose’s theorem was right then Khalatnikov and Lifshitz were wrong.

Who is Penrose???

The Soviet delegation was confused and incensed. Misner's English was too fast to follow, and Penrose's theorem relied on topological arguments that were alien to relativity experts, the Soviets regarded it as suspect. By contrast the Khalatnikov-Lifshitz analysis was based on tried-and-true methods. *Penrose* - they asserted - *was probably wrong*.

Epilogue of this episode 4 years later:
a one more type of singularity found

One more type of singularity found

4 years later, in September 1969, Kip Thorn was visiting Zel’dovich in Moscow. Lifshitz came to him with a manuscript that he, Khalatnikov and Vladimir Belinsky had written. They had found one more singularity permitted by the laws of General Relativity - Penrose was right.

Kip Thorne

Vladimir Belinsky

Yakov Zeldovich

What was Penrose's result?

Outline of the history of the black hole theory

Sir Isaac Newton 1687 ***Philosophiæ Naturalis Principia Mathematica***

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Roger Penrose, Brandon Carter, Steven Hawking, Werner Israel, David Robinson, Gary Bunting, Paweł Mazur the black hole theory

Can black holes exist in physical reality?

Do black holes exists in the physical reality?

Can black holes exist in physical reality?

Annals of Mathematics
Vol. 40, No. 4, October, 1939

ON A STATIONARY SYSTEM WITH SPHERICAL SYMMETRY CONSISTING OF MANY GRAVITATING MASSES

By Albert Einstein

(Received May 10, 1939)

horizons

The essential result of this investigation is a clear understanding as to why the "Schwarzschild singularities" do not exist in physical reality. Although the theory given here treats only clusters whose particles move along circular paths it does not seem to be subject to reasonable doubt that more general cases will have analogous results. The "Schwarzschild singularity" does not appear for the reason that matter cannot be concentrated arbitrarily. And this is due to the fact that otherwise the constituting particles would reach the velocity of light.

Do black holes exists in the physical reality?

GRAVITATION
AND COSMOLOGY:
PRINCIPLES AND APPLICATIONS
OF THE GENERAL THEORY
OF RELATIVITY

STEVEN WEINBERG
Massachusetts Institute of Technology

8 The Schwarzschild Singularity*

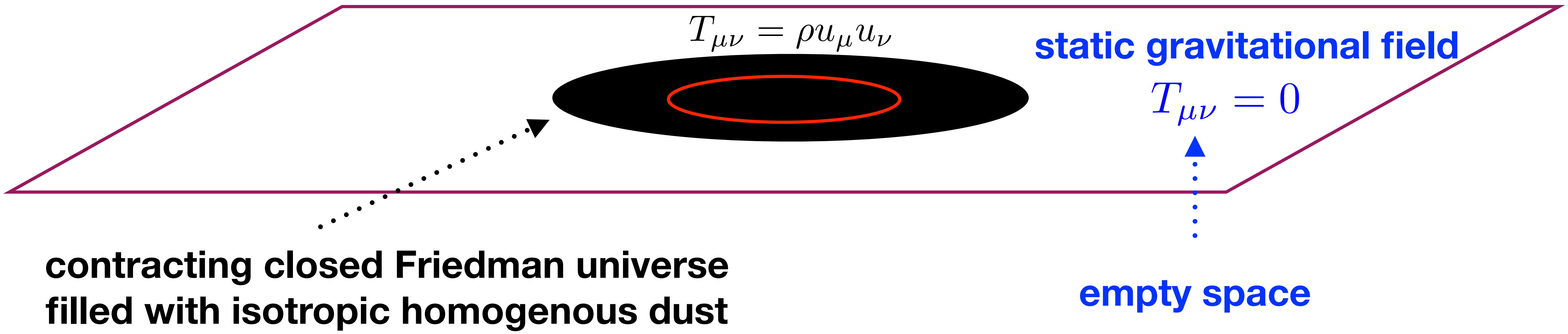
The reader will probably have noticed that the Schwarzschild solution (8.2.12) becomes singular at $r = 2MG$. This radius corresponds to $\rho = MG/2$ and $R = MG$, so we see that this singularity also occurs when the metric is expressed in its isotropic form (8.2.14) or in its harmonic form (8.2.15). The radius $2GM$ at which the singularity occurs in standard coordinates is called the *Schwarzschild radius* of the mass M.

It should immediately be stressed that there is no Schwarzschild singularity in the gravitational field of any known object in the universe.

Is black hole a likely final stage of the gravitational collapse of a compact object?

The Oppenheimer - Snyder model

A model of the implosion of a spherical star consisting of isotropic dust necessarily ending in singularity.



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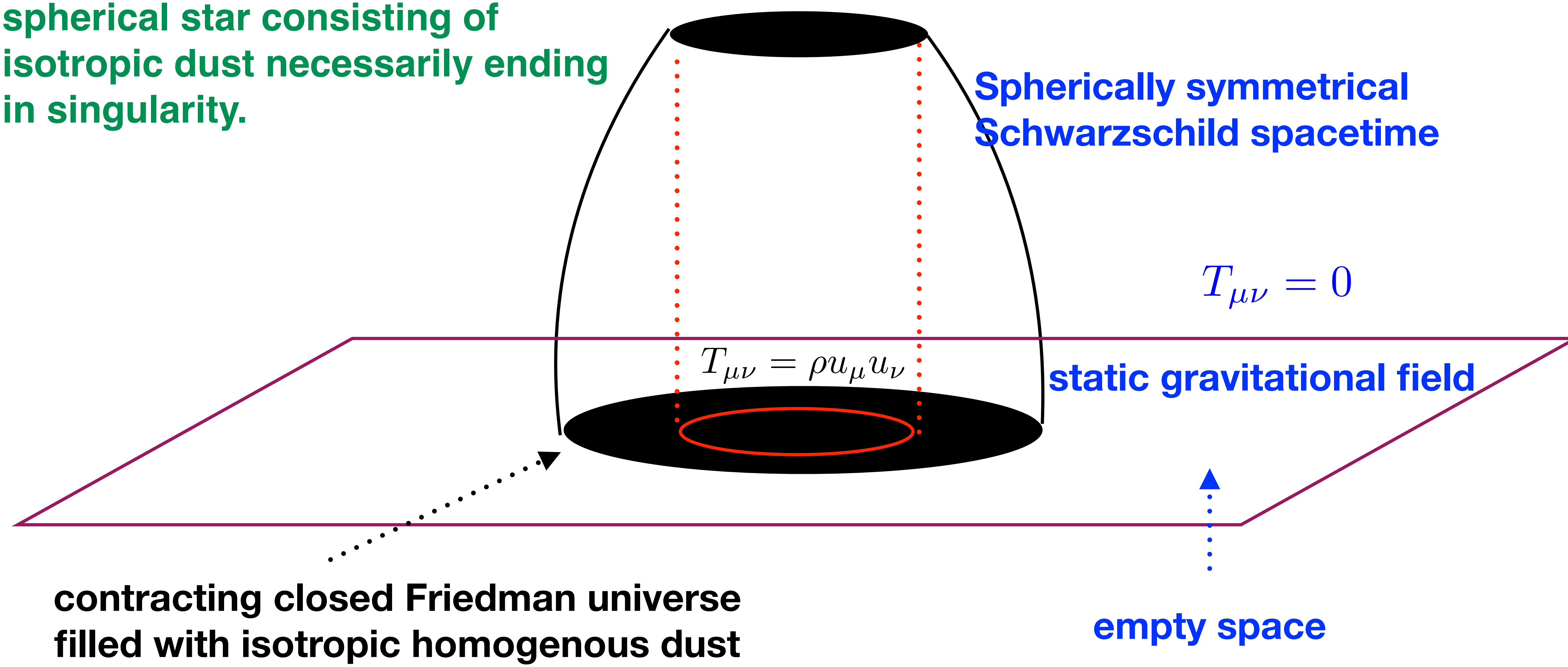

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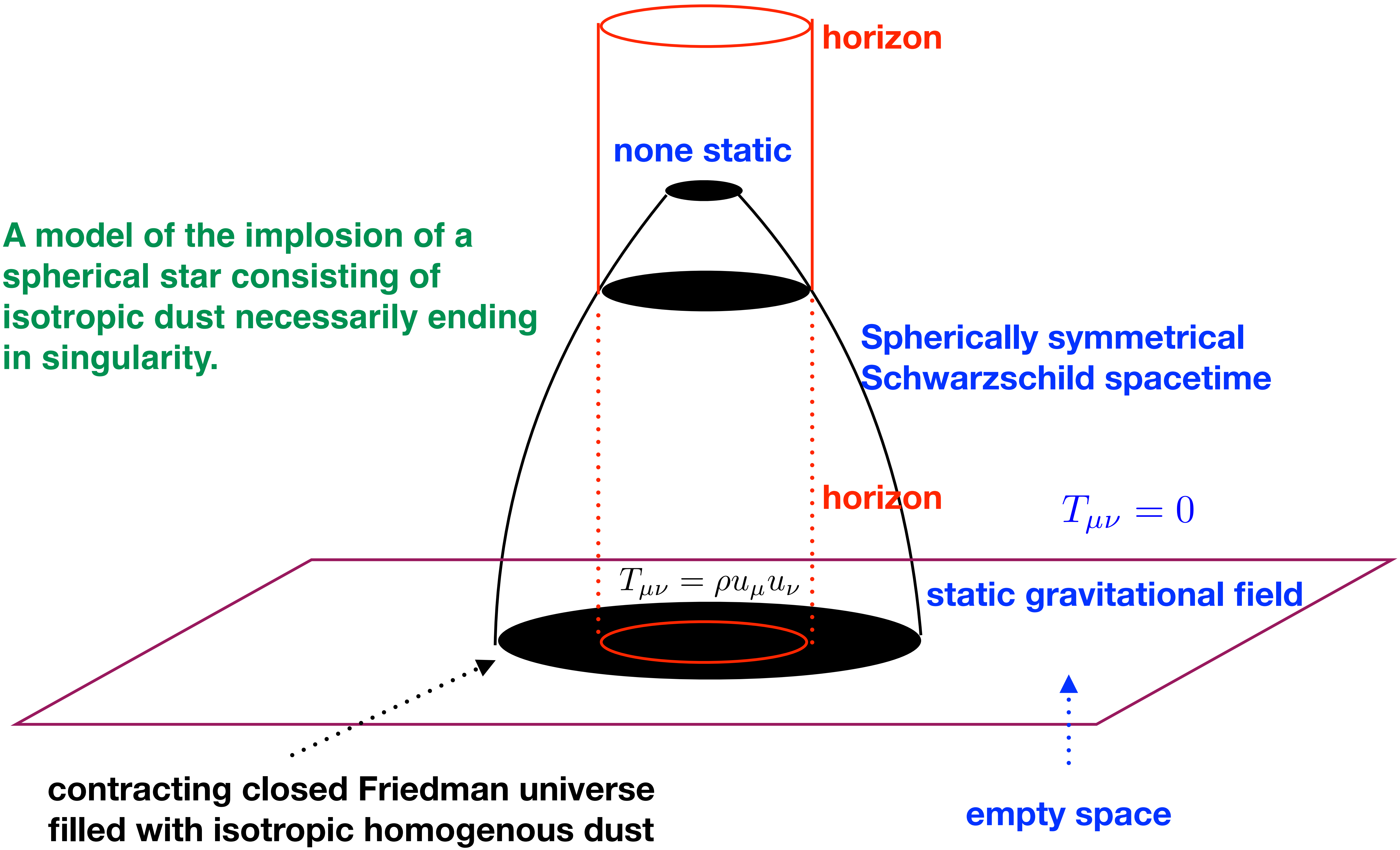

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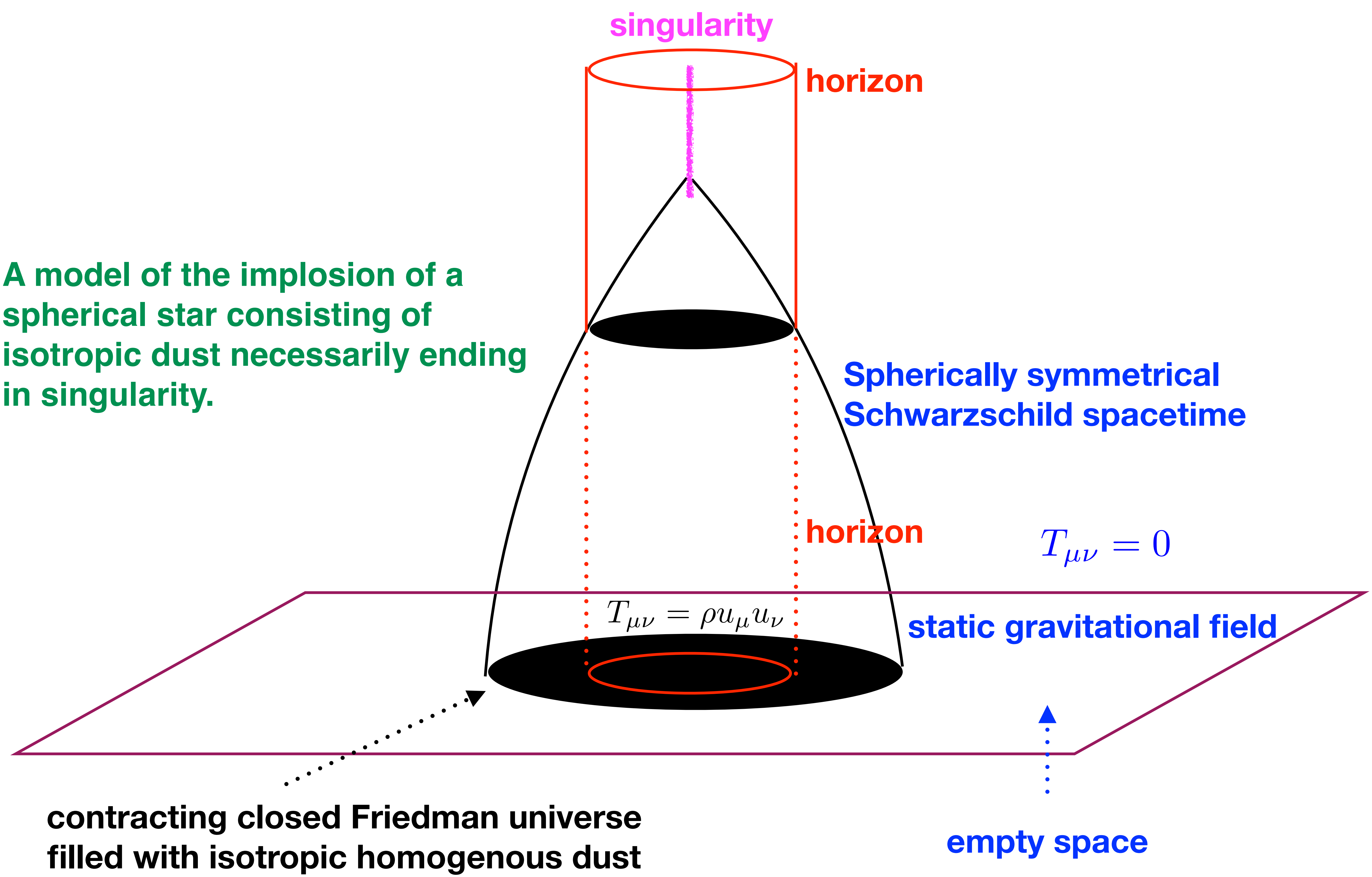

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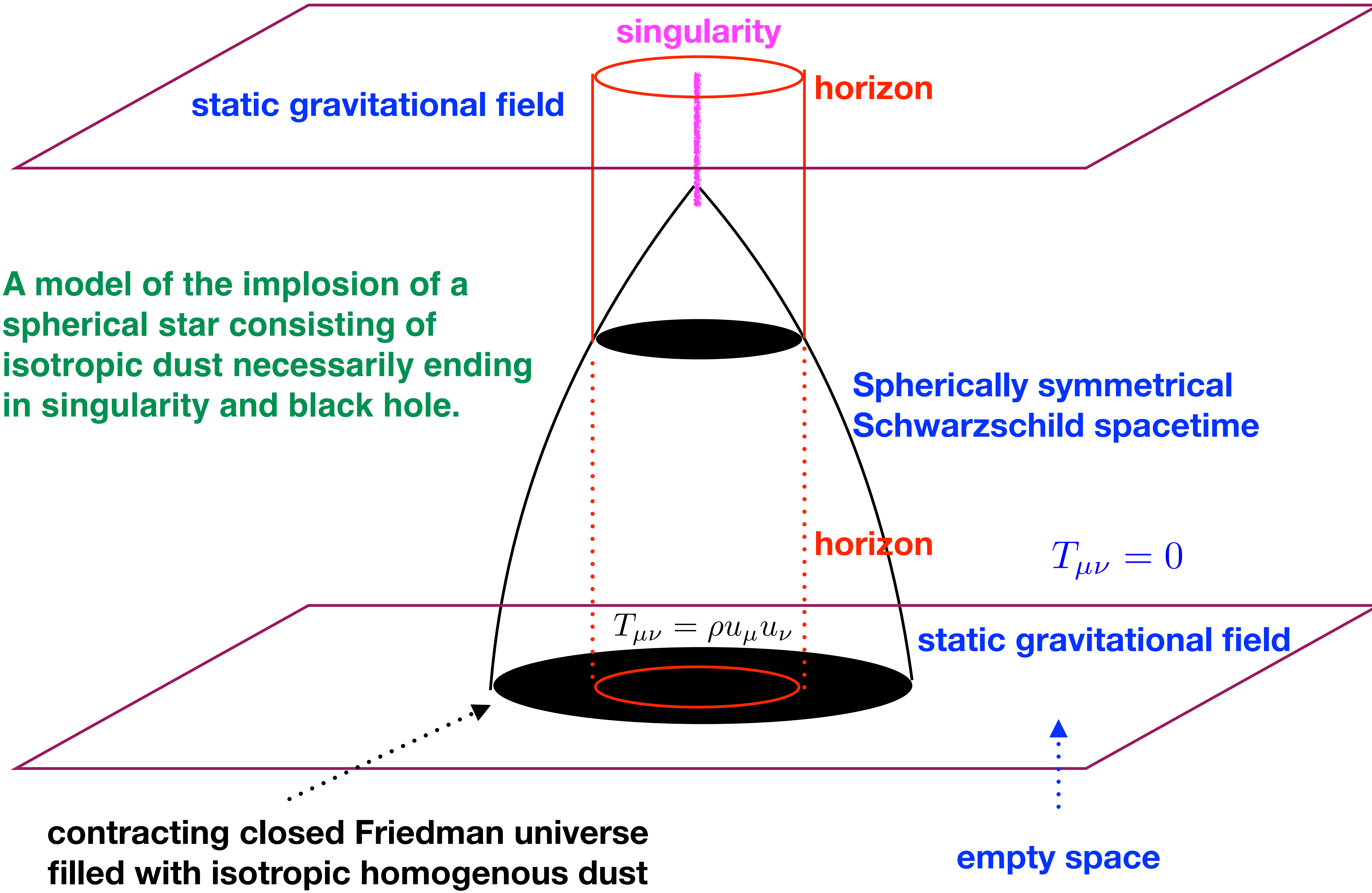


Do random perturbations of spacetime prevent singularity from forming?

Oppenheimer-Snyder model was generalised to non-isotropic dust. However, still homogenous.

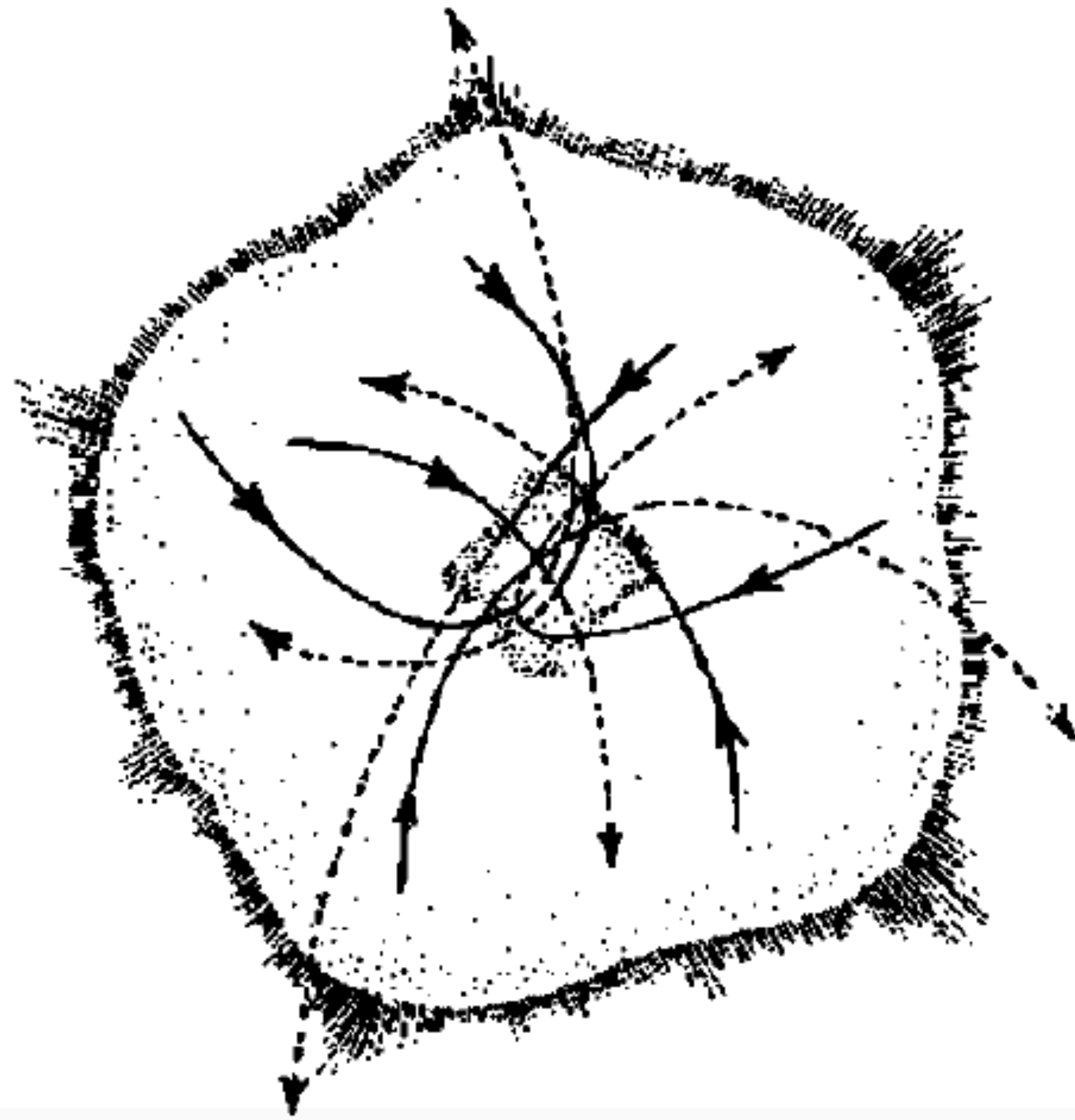
Do random perturbations of spacetime prevent singularity from forming?

Oppenheimer-Snyder model was generalised to non-isotropic dust. However, still homogenous.

Raychaudhuri and Komar showed inevitability of a singularity in the collapse of an irrotational and geodesic fluid provided its expansion is definite (either positive or negative). But what about a rotating perturbation?

Do random perturbations of spacetime prevent singularity from forming?

Models of converting a star's implosion into explosion:
if the imploding star is slightly deformed, its atoms implode in slightly different points, swir around each other, and fly back out



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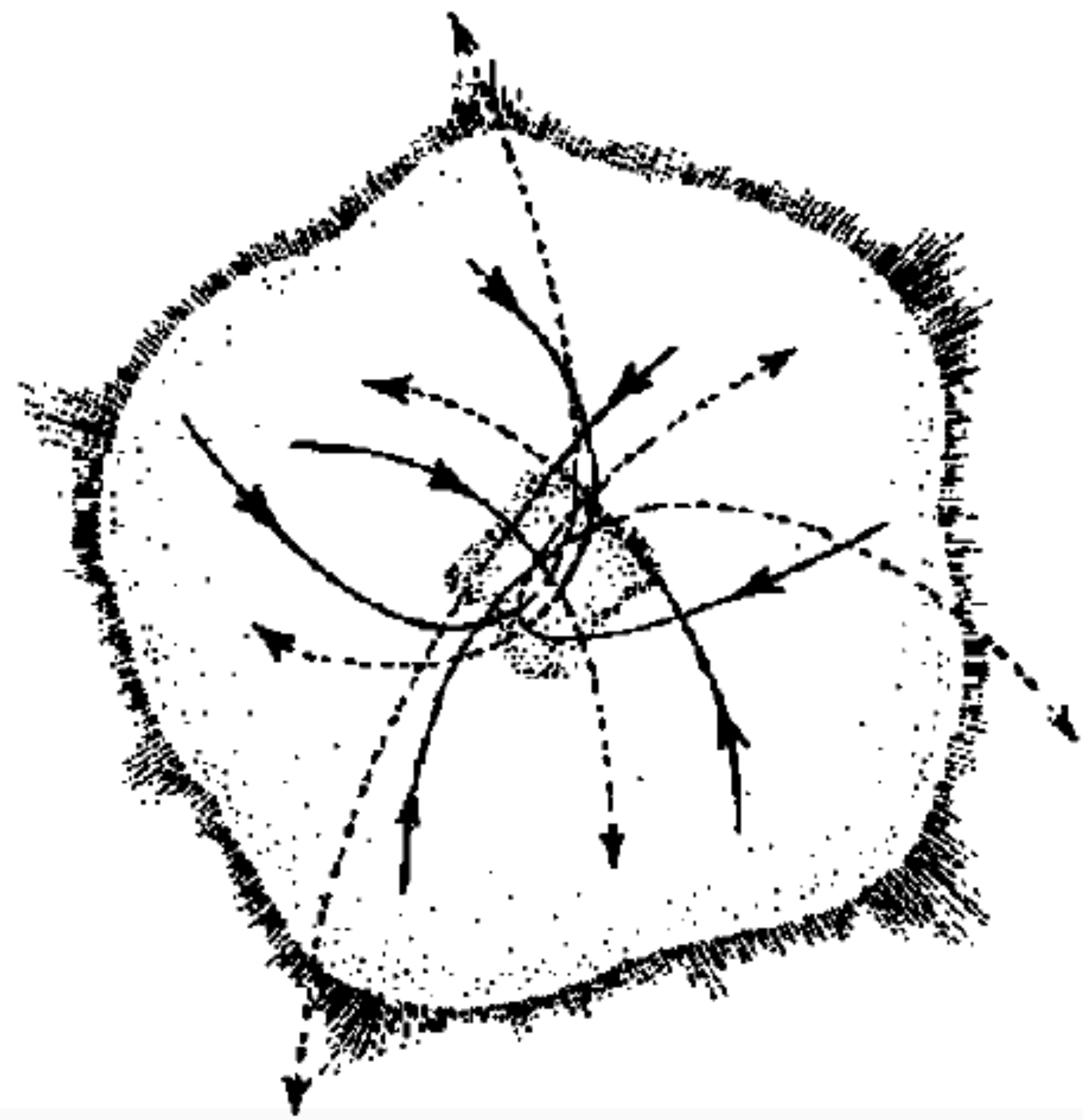


The Khalatnikov-Lifshitz analysis of spacetime singularities allowed by Einstein's equations - random deformations prevented …

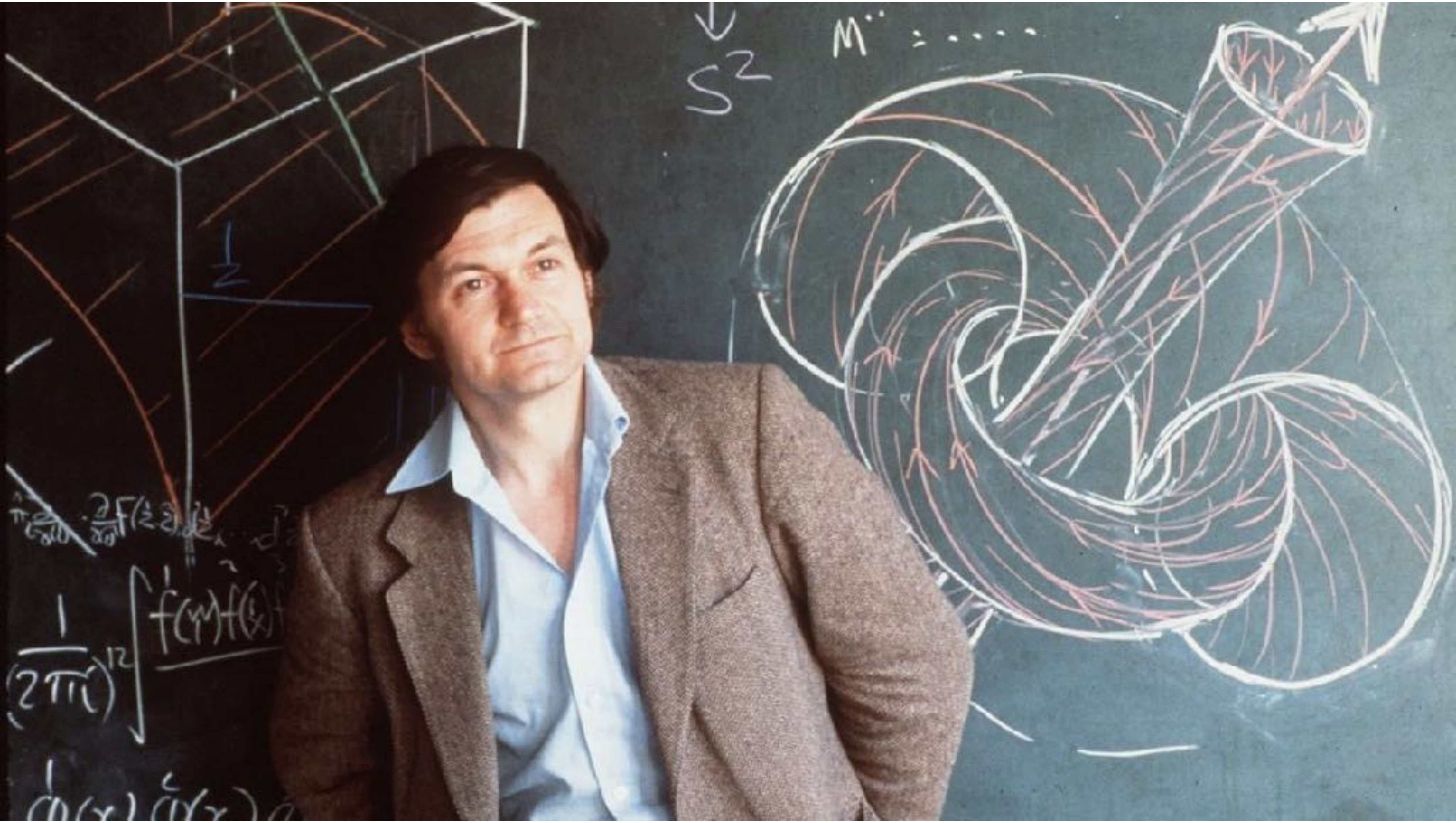
New geometric technics

The relevance of null geodesics, null vector fields in spacetime for Lorentzian geometry was pointed out.

Geometry and dynamics of null geodesic flows in spacetime, generalisation of the Raychaudhuri equation.

Trautman 1958,
Robinson-Trautman 1960,
Bondi-Sachs 1960's

Penrose’s revolution



Sir Roger Penrose in 1980.

GRAVITATIONAL COLLAPSE AND SPACE-TIME SINGULARITIES

Roger Penrose
Department of Mathematics, Birkbeck College, London, England
(Received 18 December 1964)

The discovery of the quasistellar radio sources has stimulated renewed interest in the question of gravitational collapse. It has been suggested

measured by local comoving observers, the body passes <u>within</u> its Schwarzschild radius $r = 2m$. (The densities at which this happens

Can the future have a compact, closed boundary?

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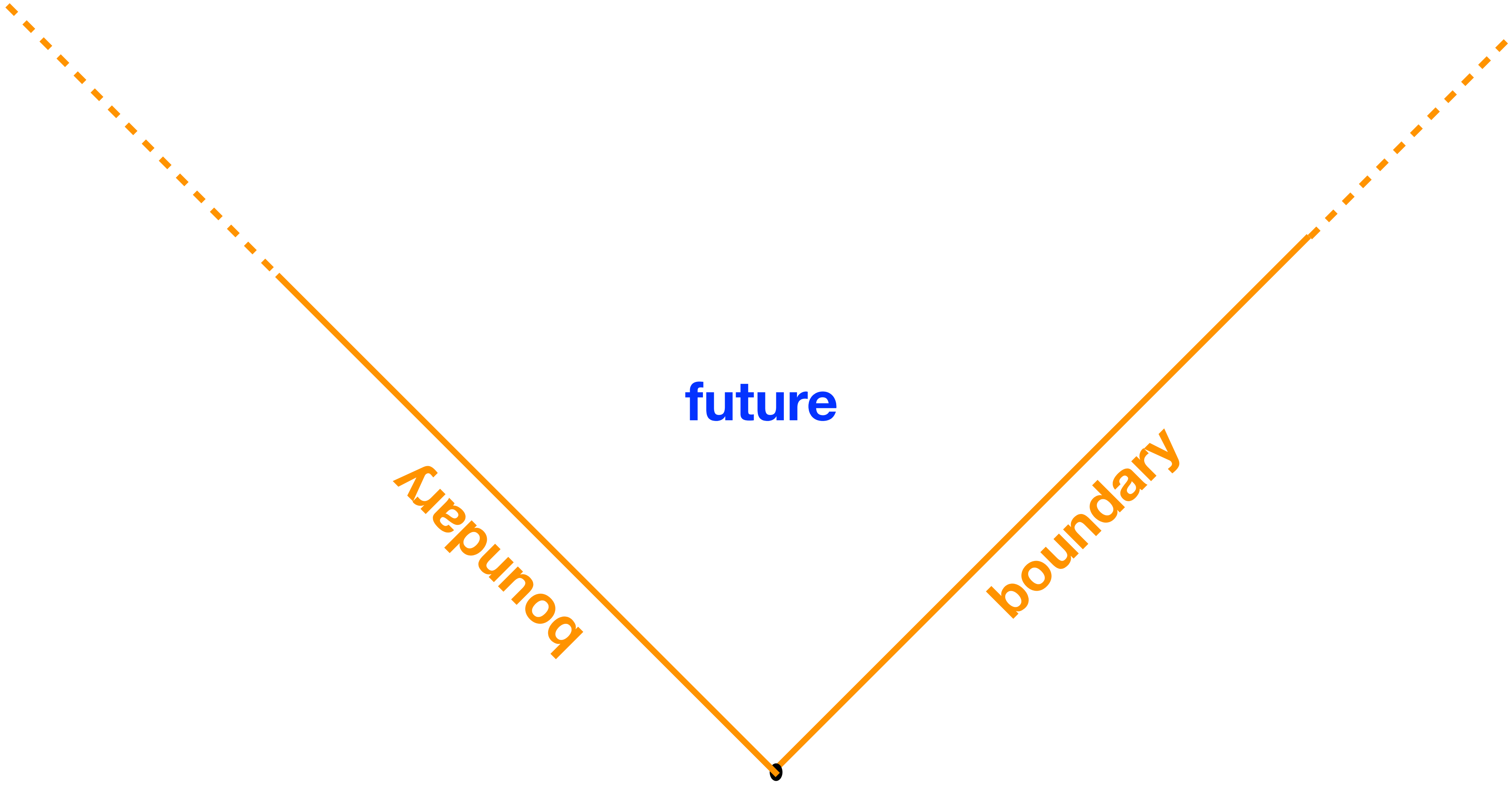

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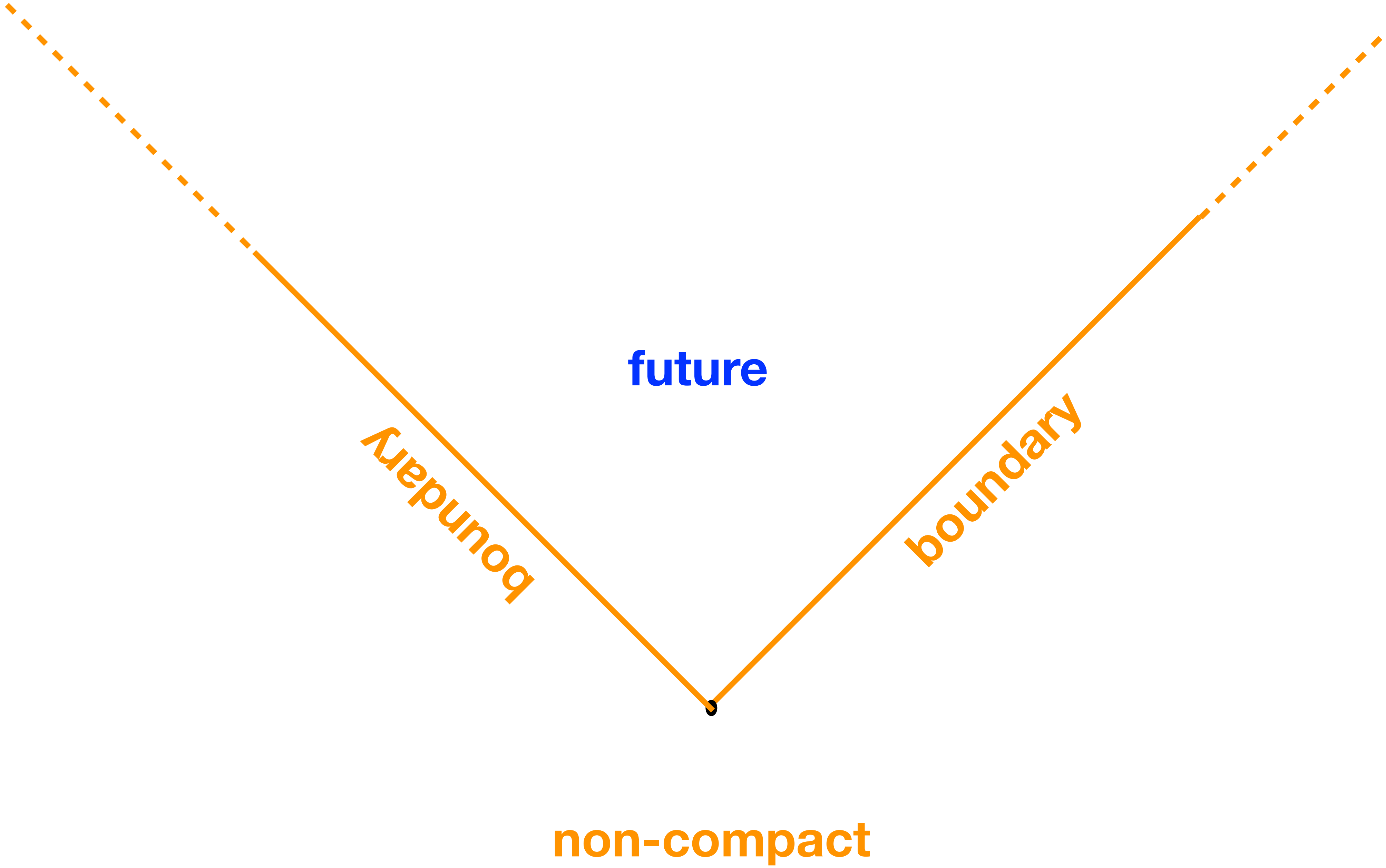

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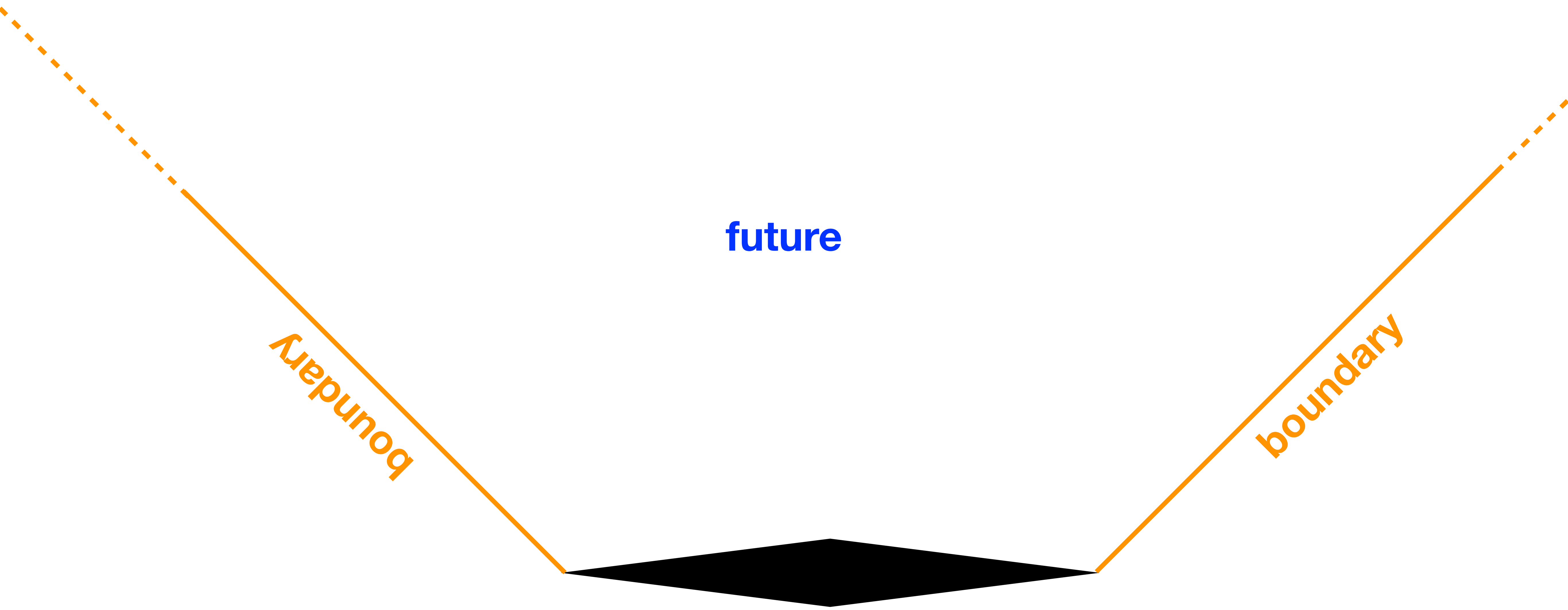

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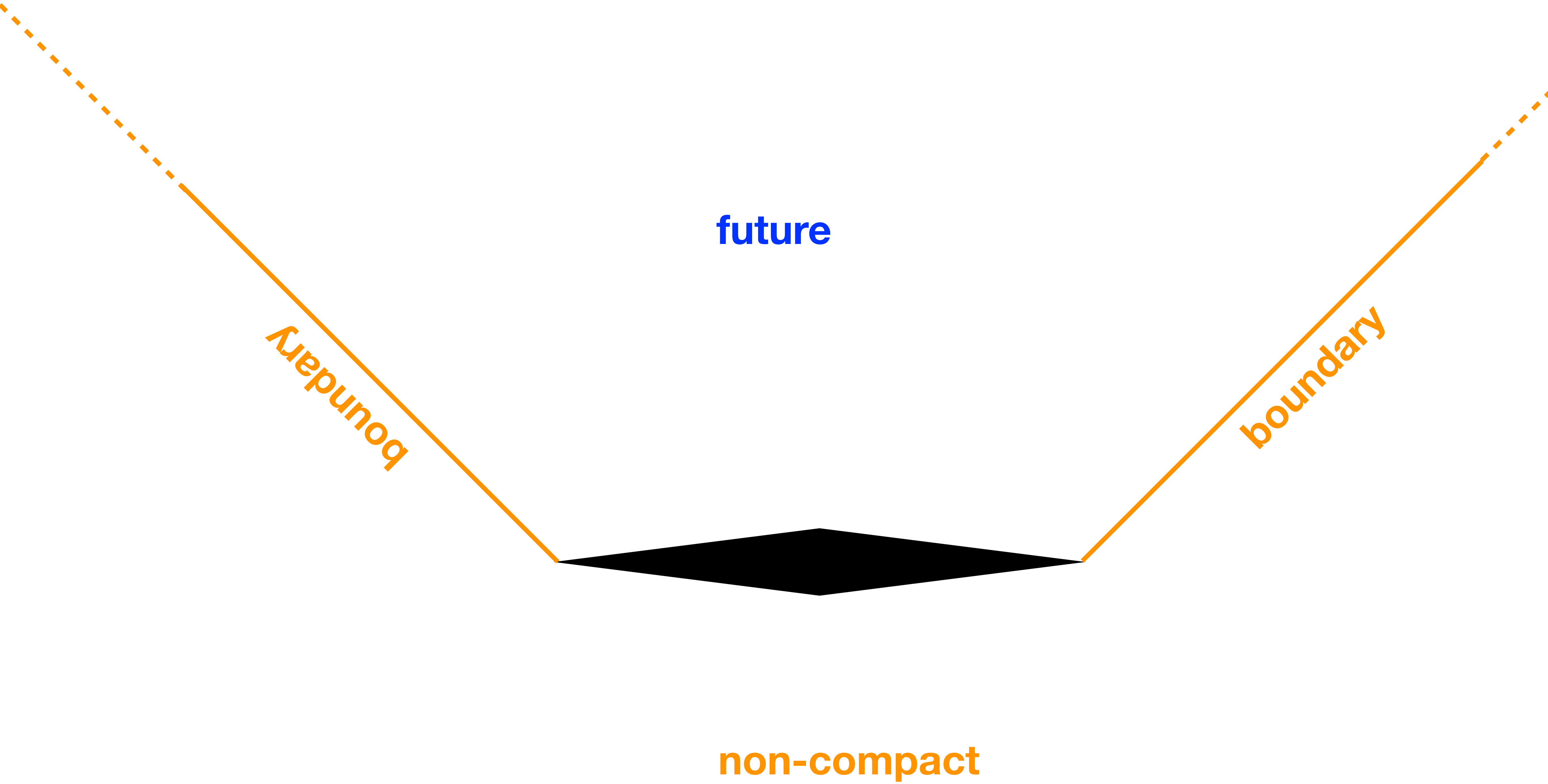
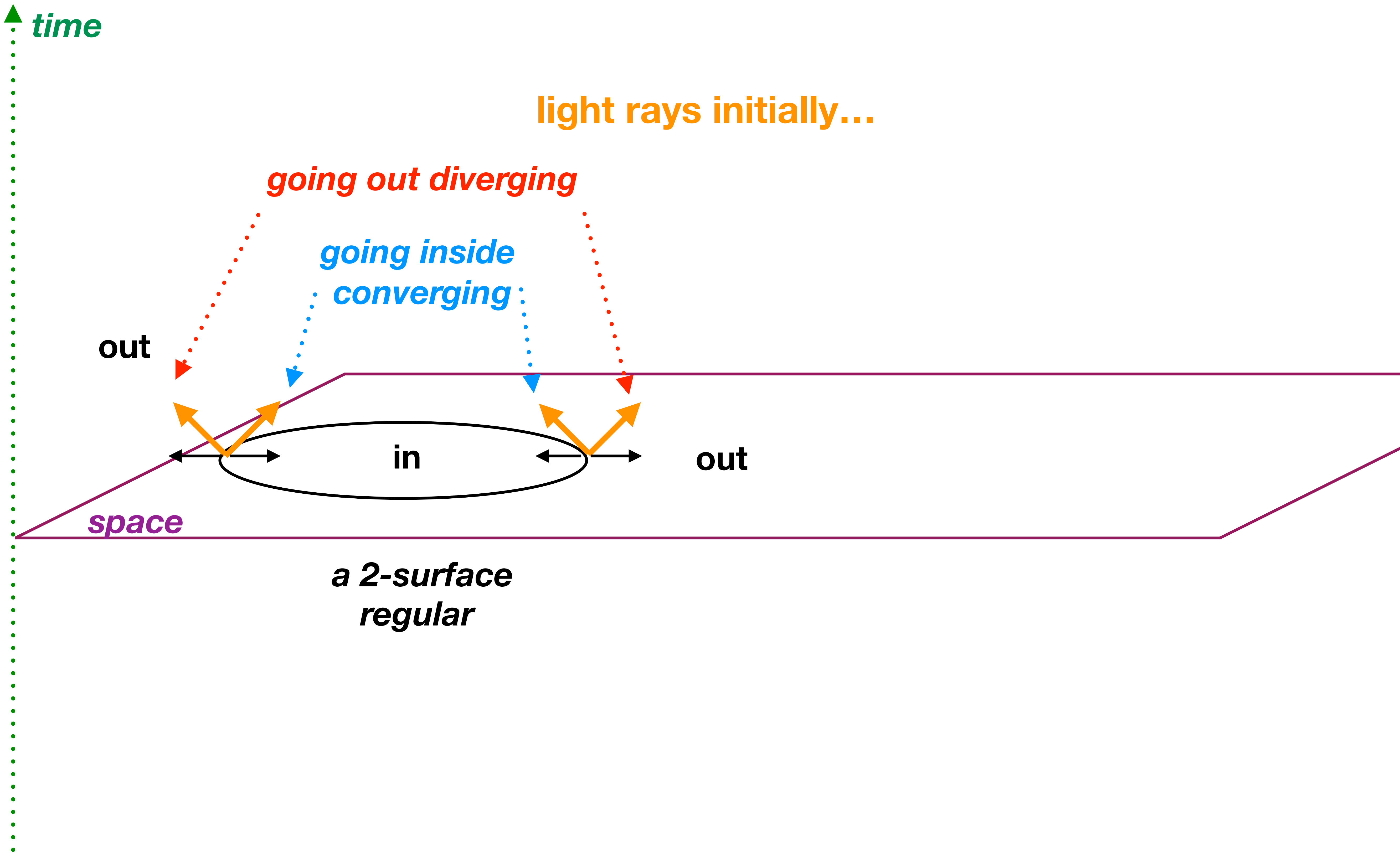


Penrose'e trapped surface's



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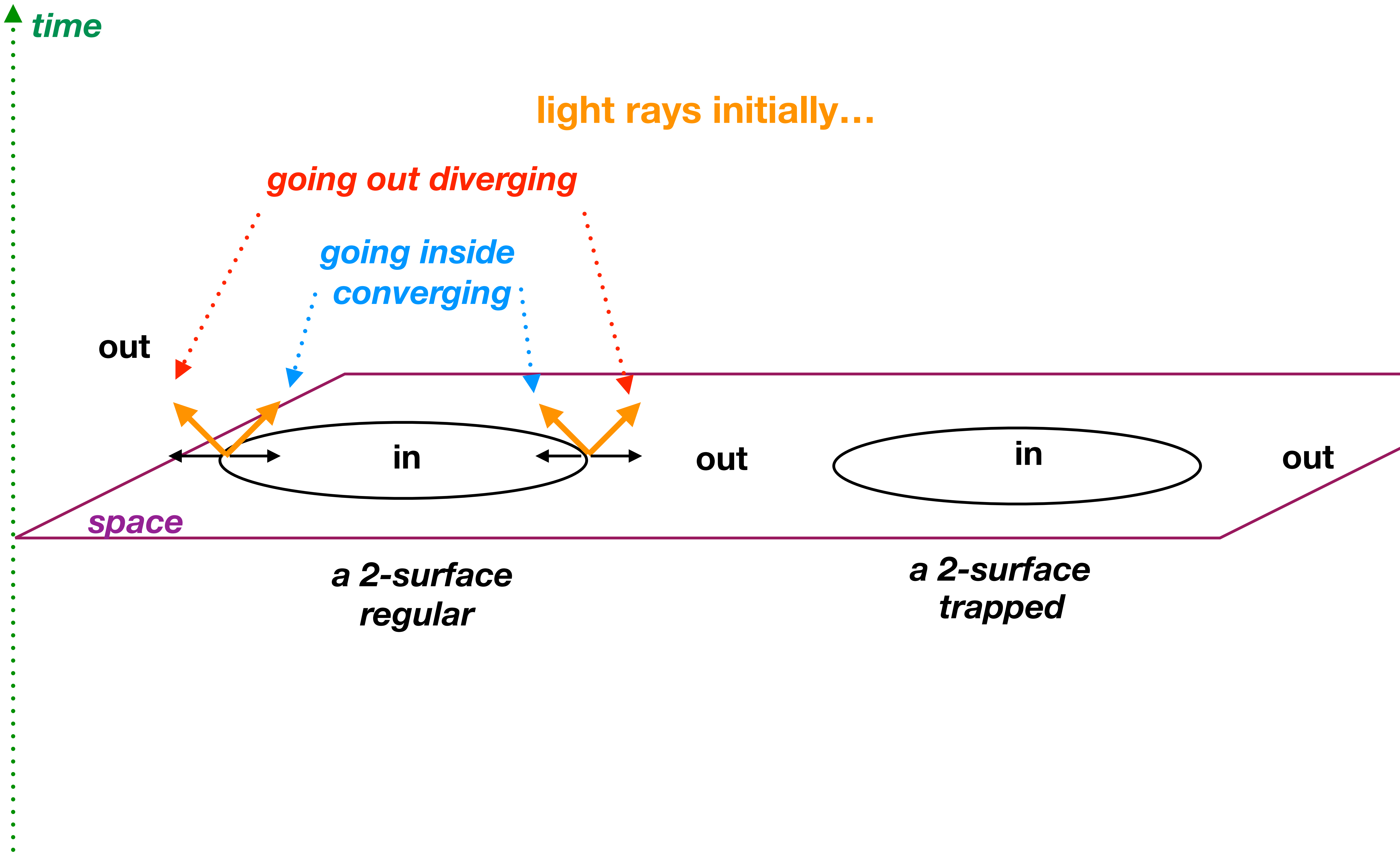

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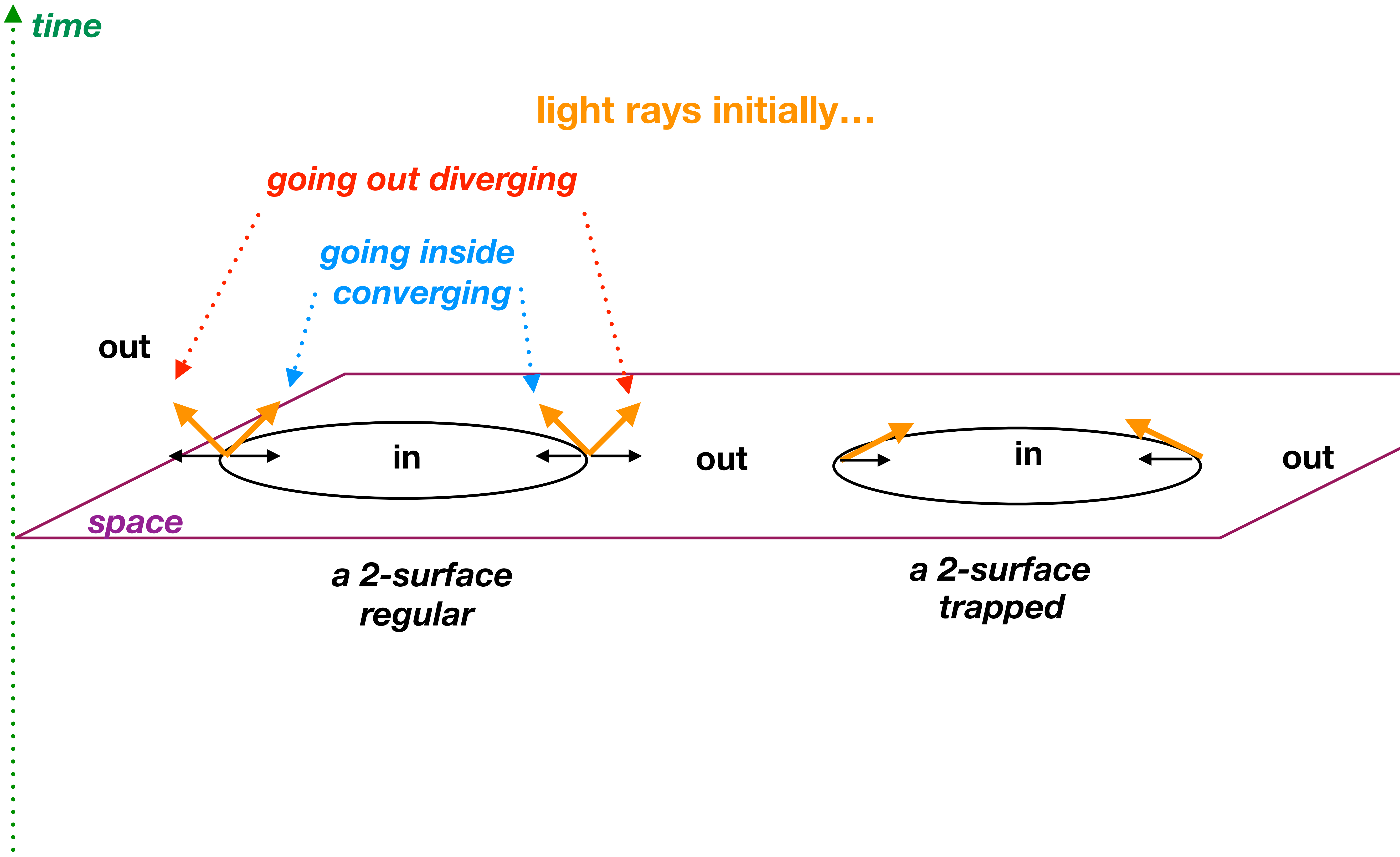

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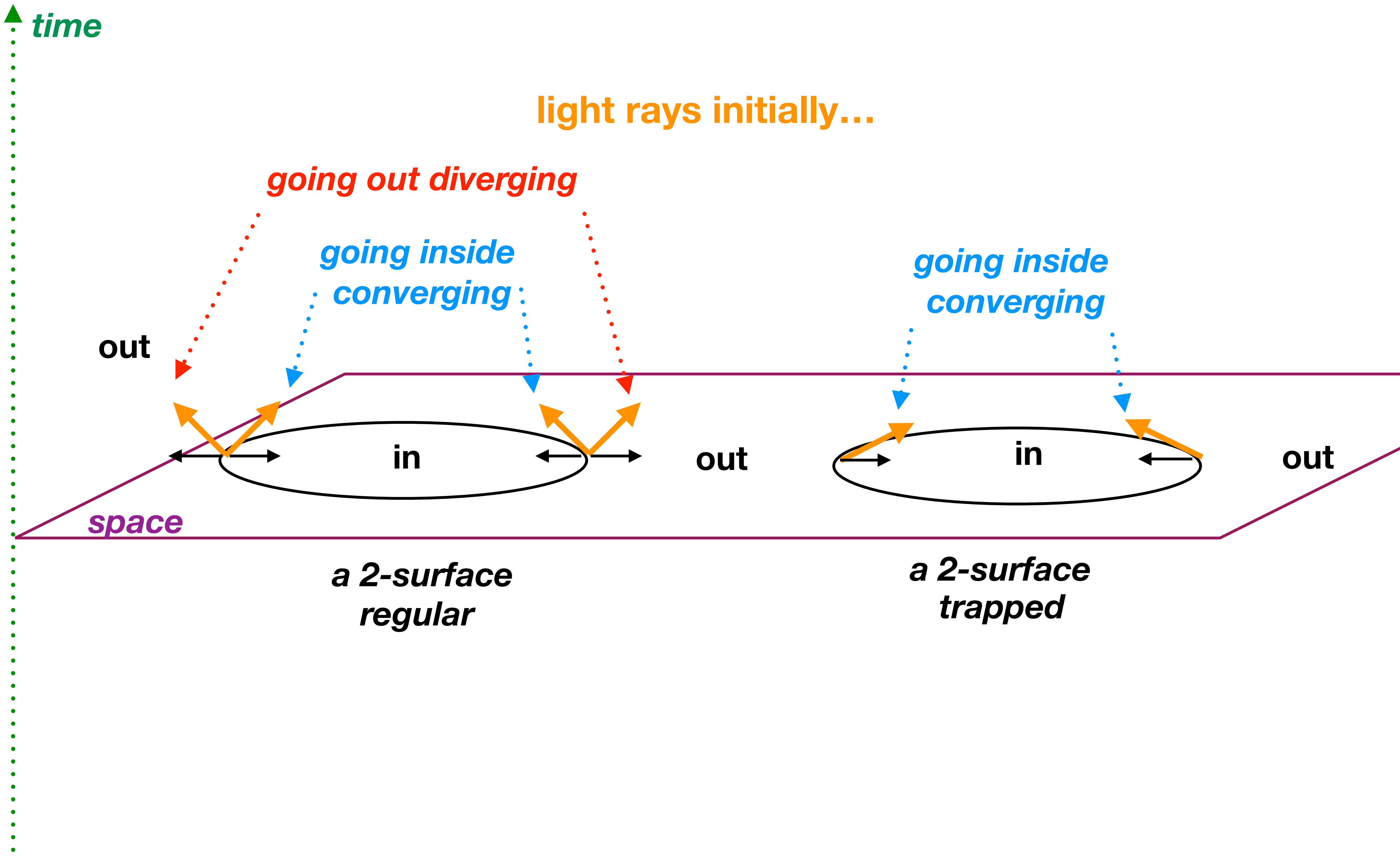

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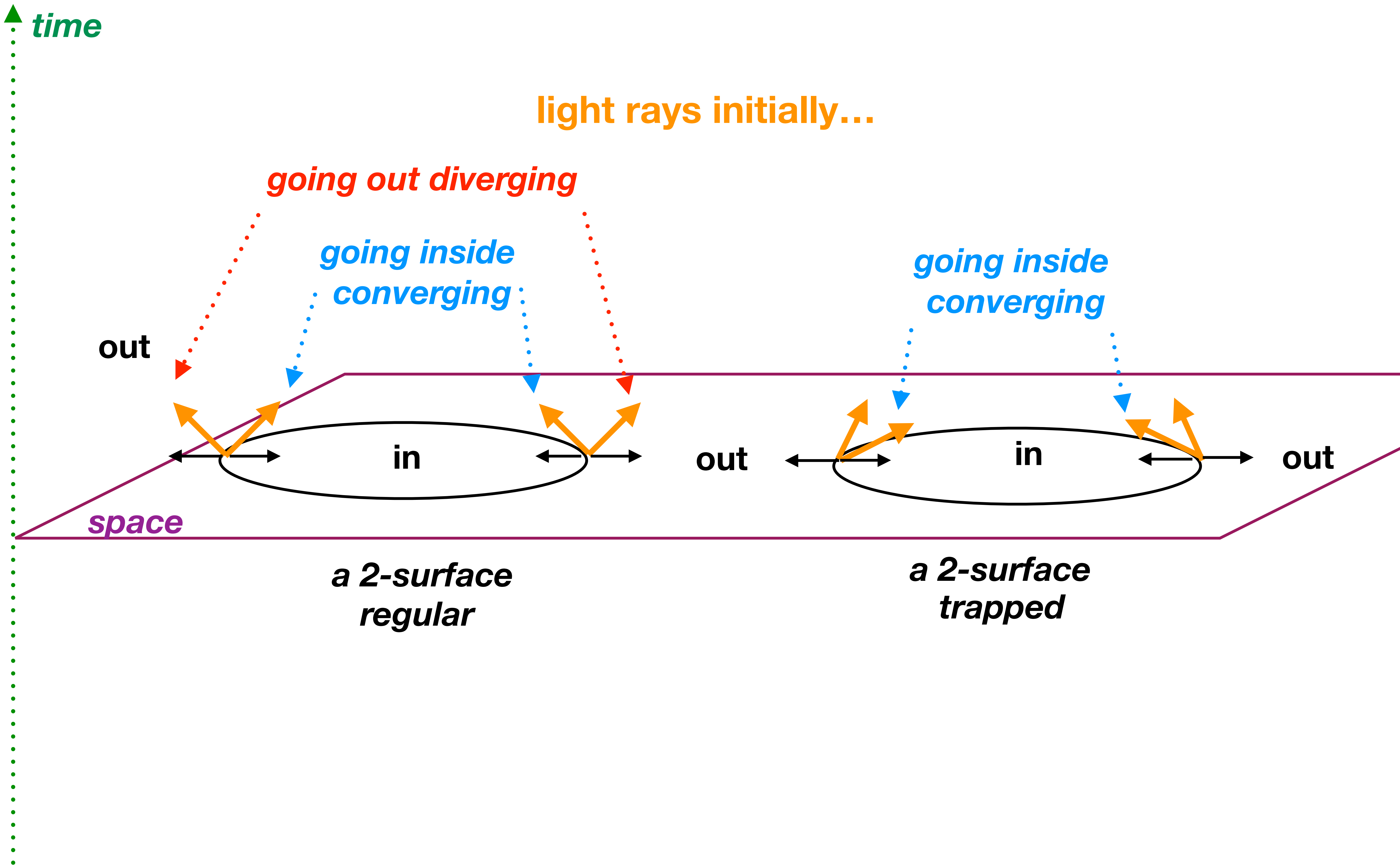

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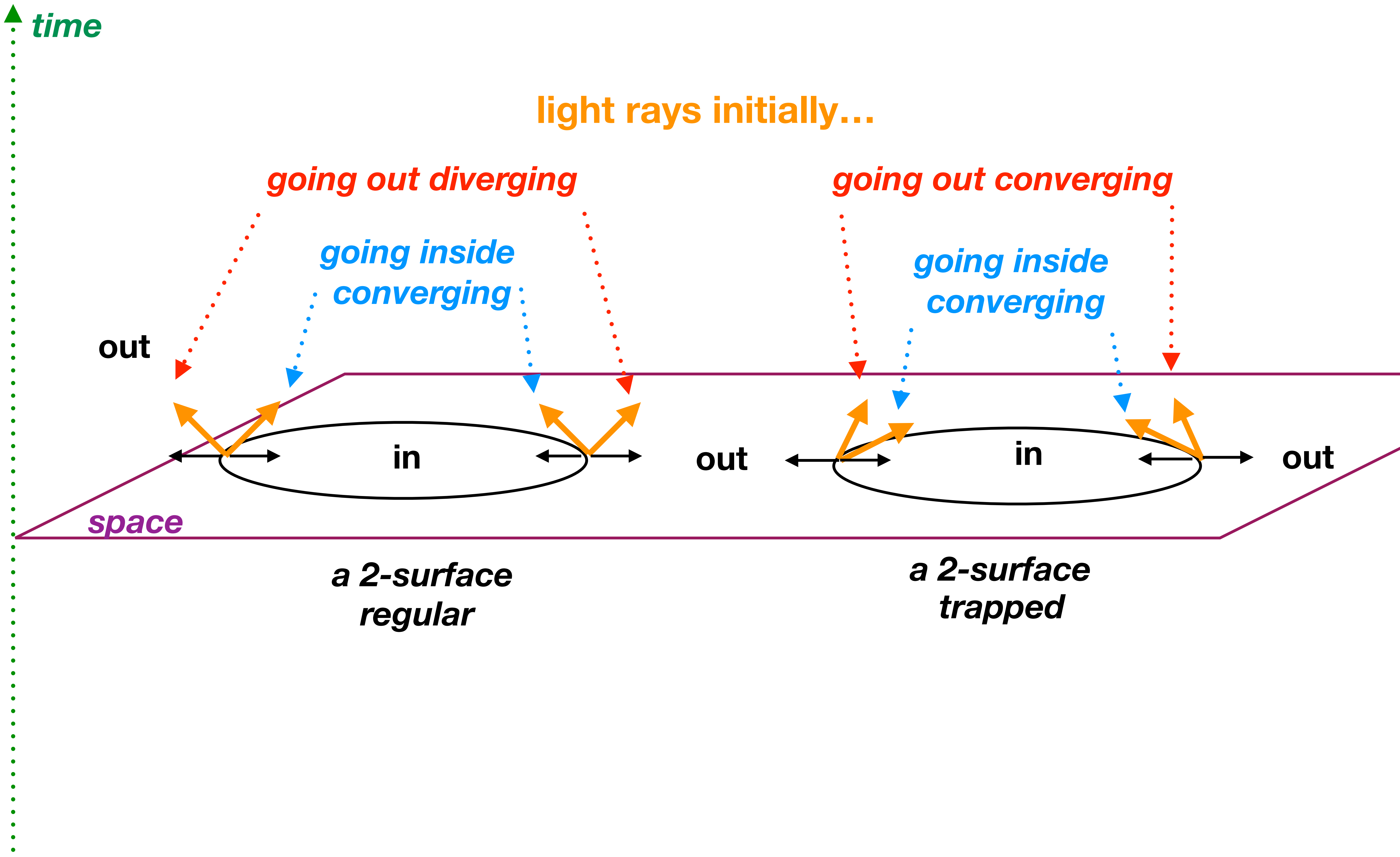
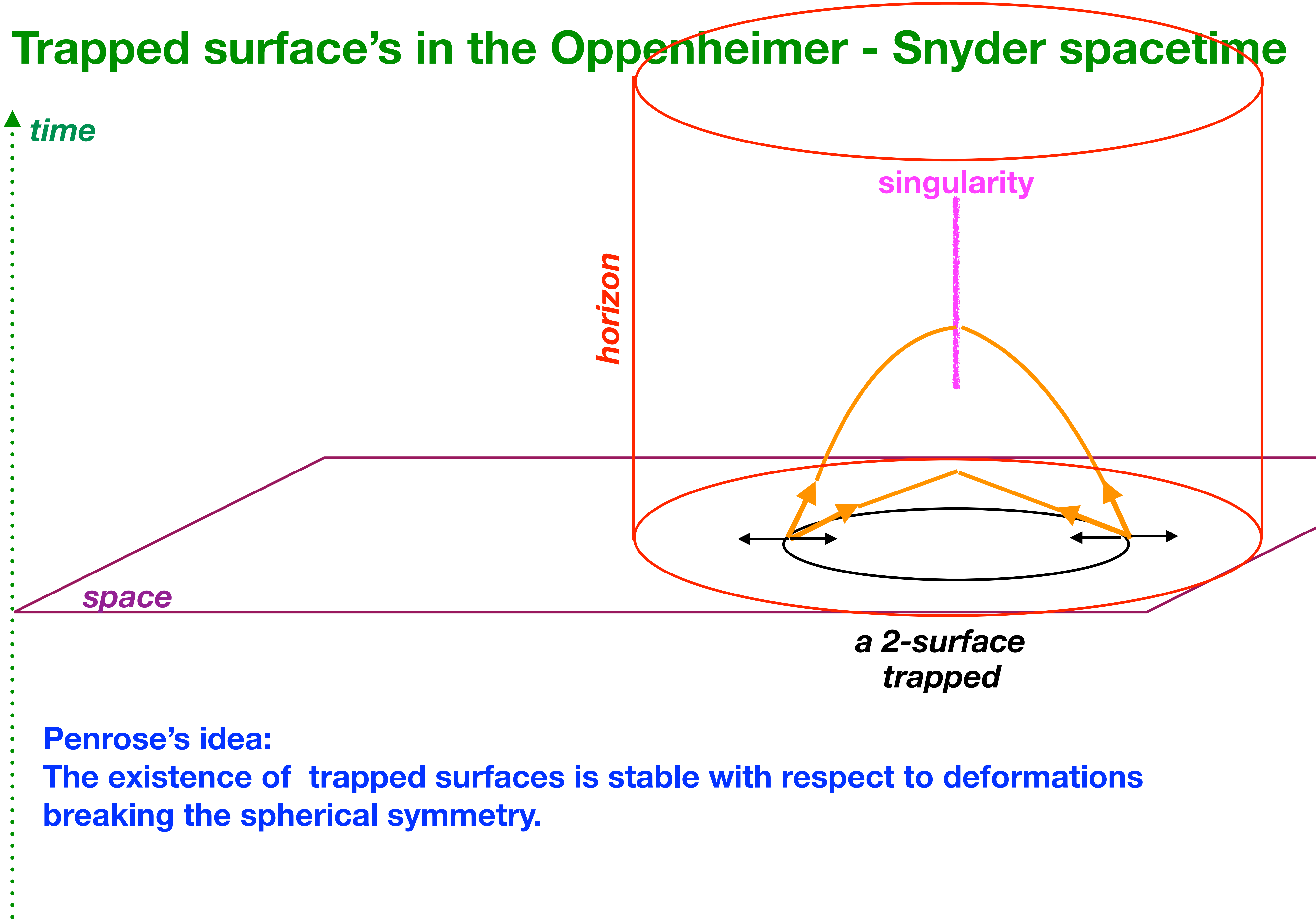
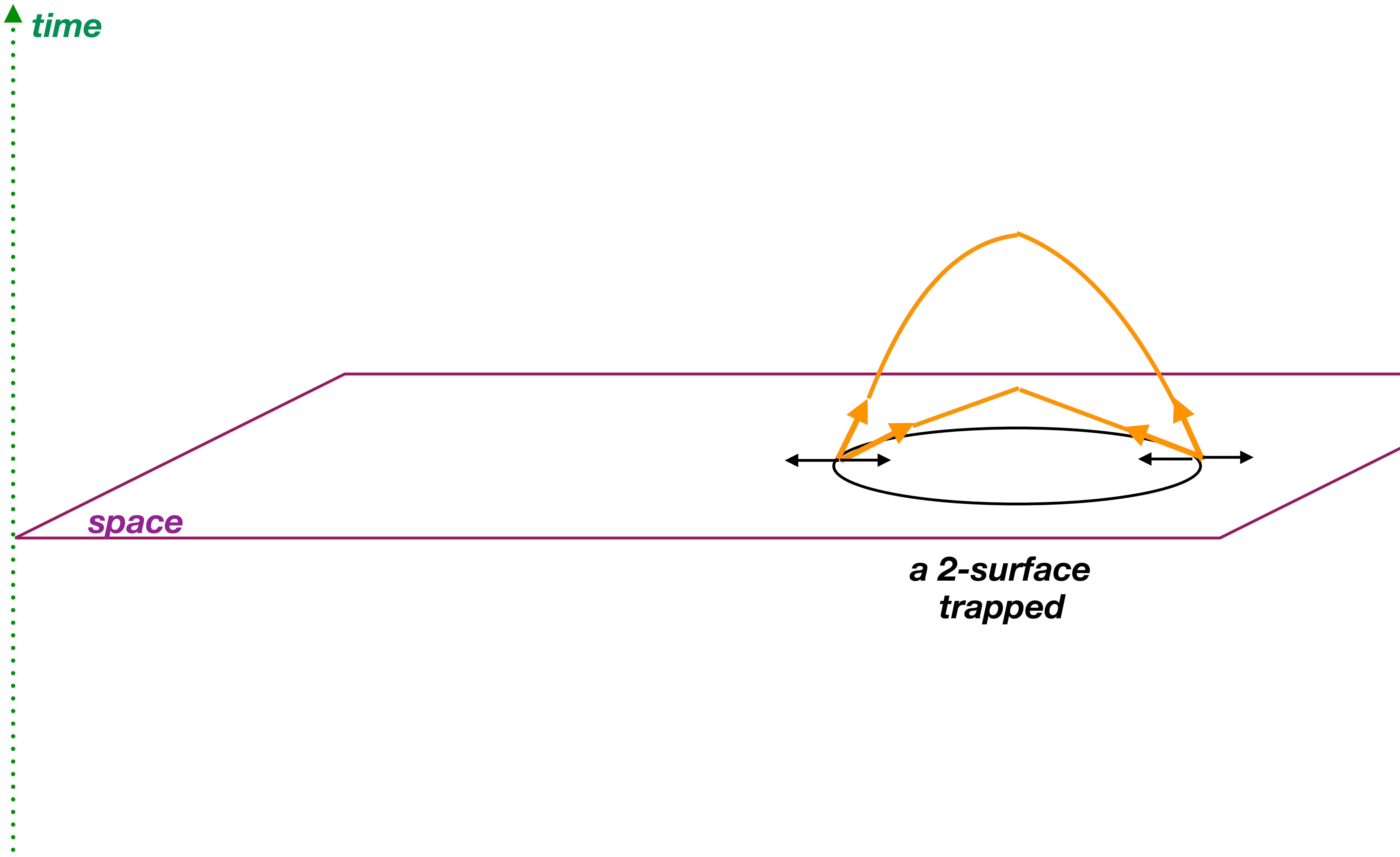


Trapped surface's in the Oppenheimer - Snyder spacetime



Penrose's idea:
The existence of trapped surfaces is stable with respect to deformations breaking the spherical symmetry.

Trapped surface's in non-singular spacetime?



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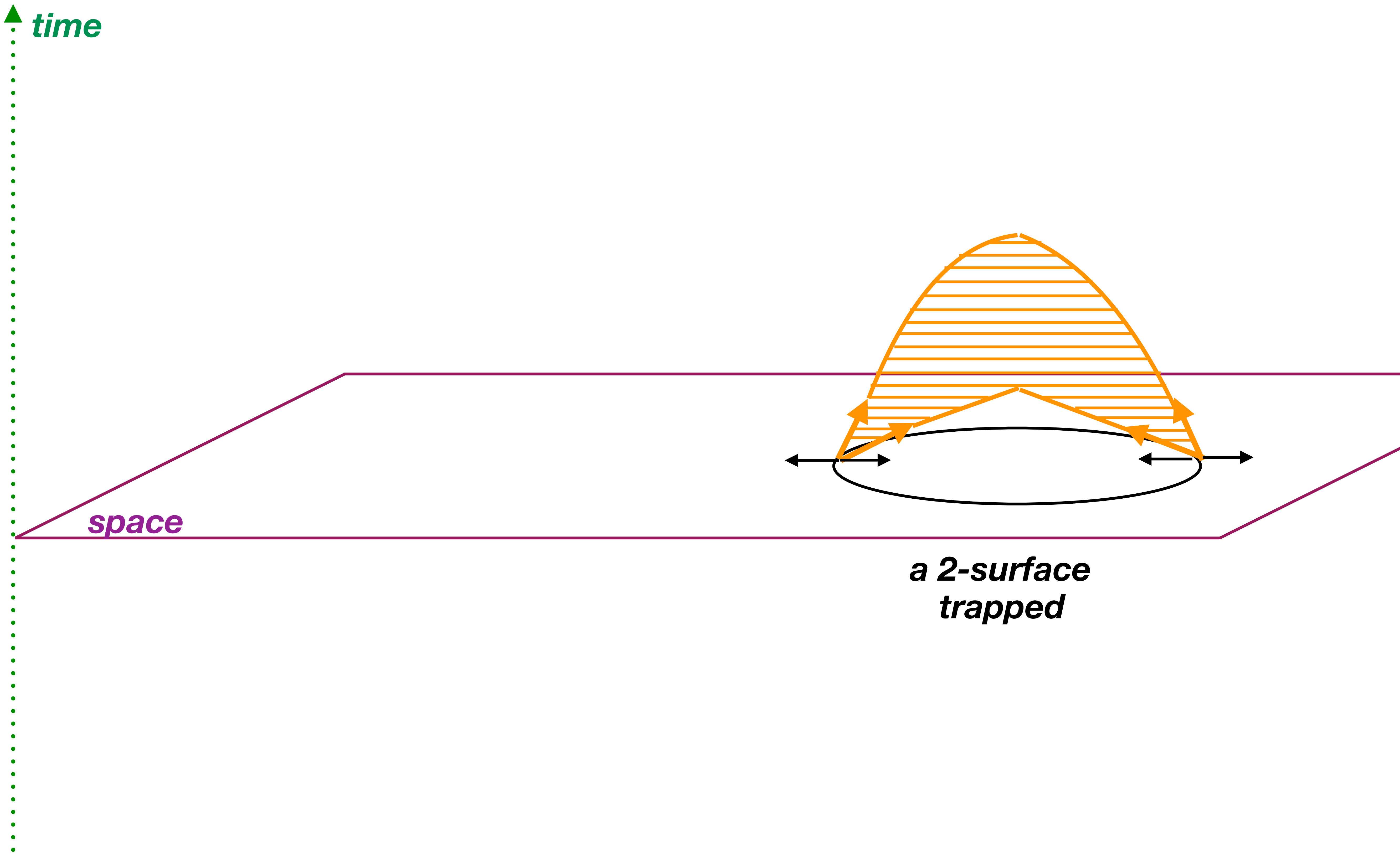

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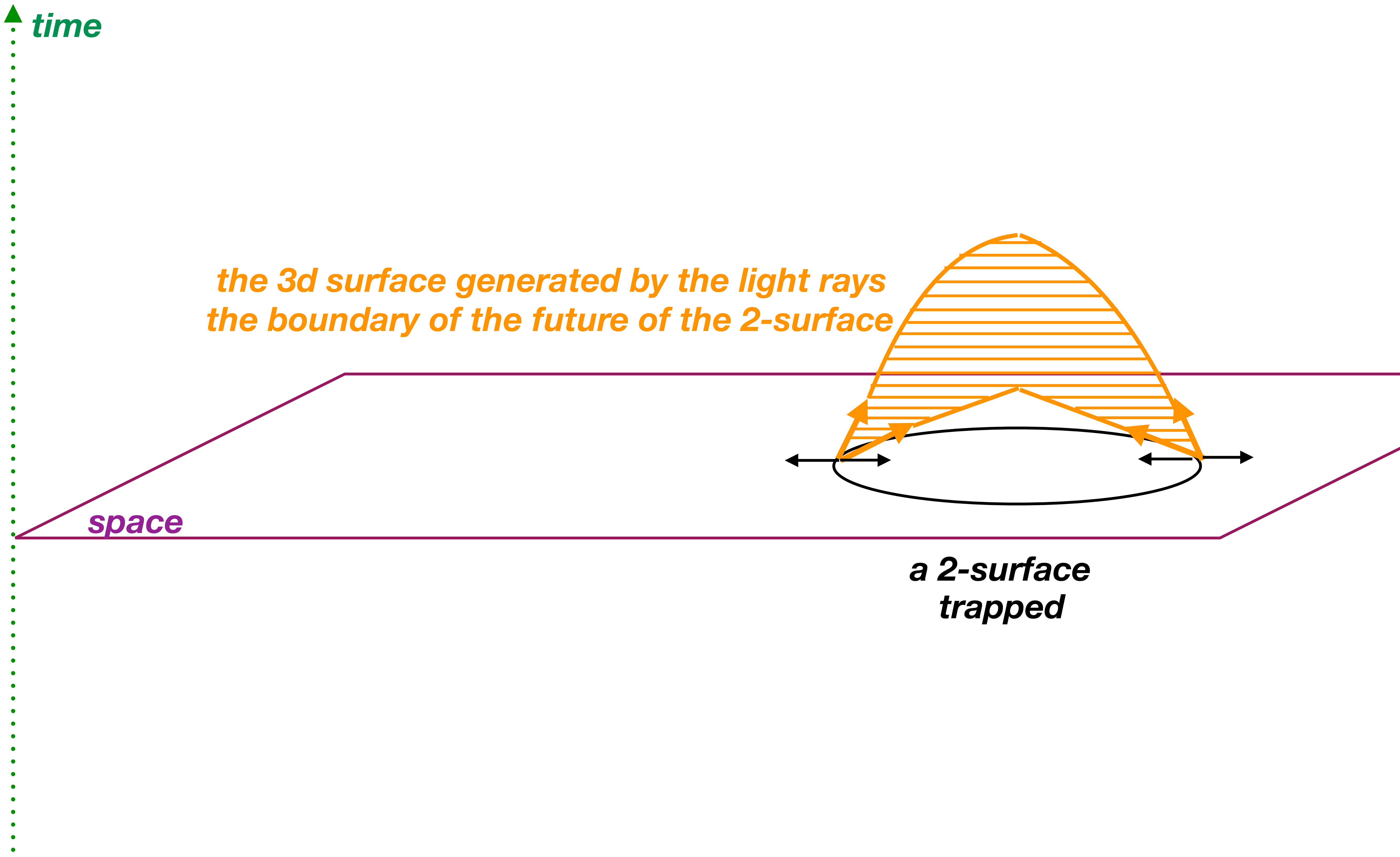

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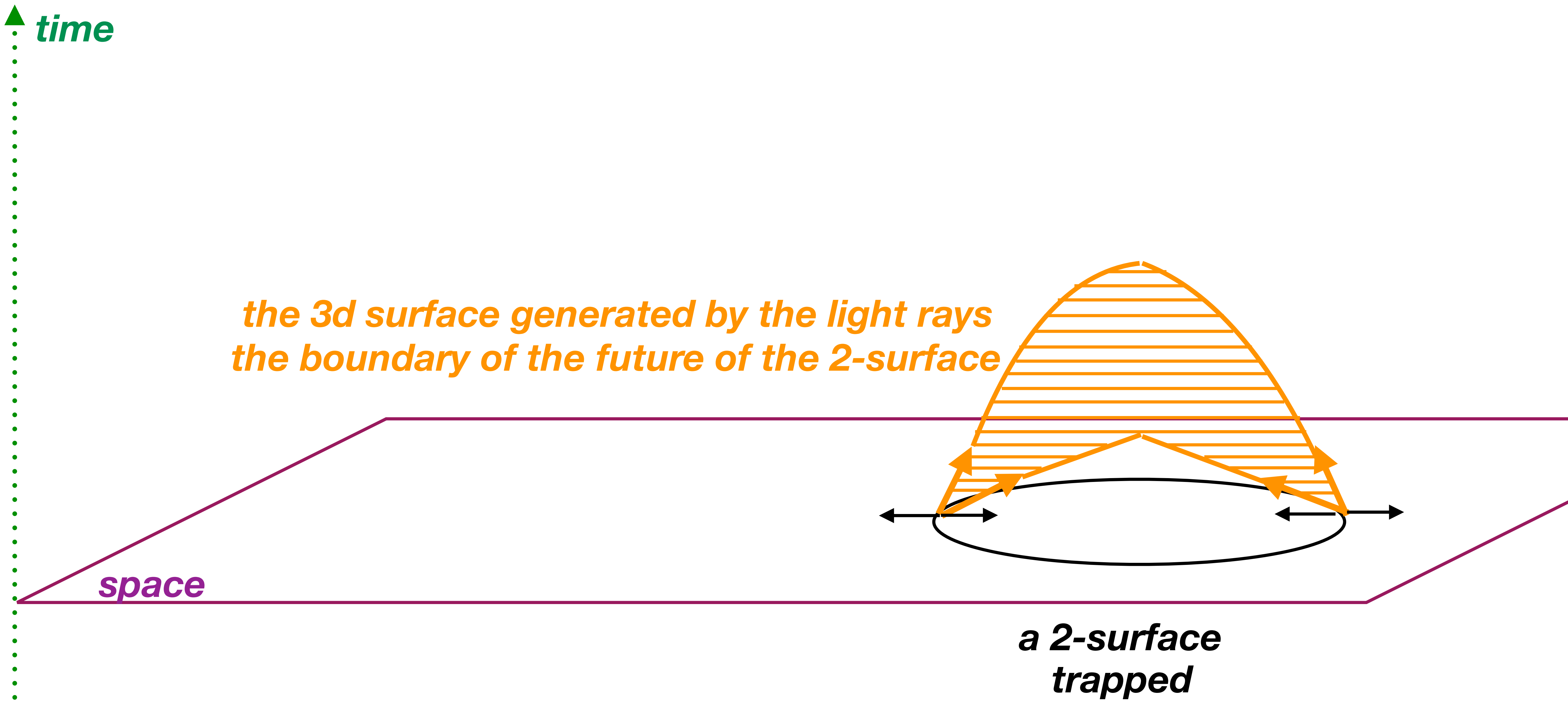

no-singularity
Einstein’s equations
energy positivity

$$T_{\mu\nu}k^{\mu}k^{\nu} \geq 0,$$

for every $k^{\mu}k_{\mu} = 0$

$\Rightarrow$ the surface is achronal, compact and closed

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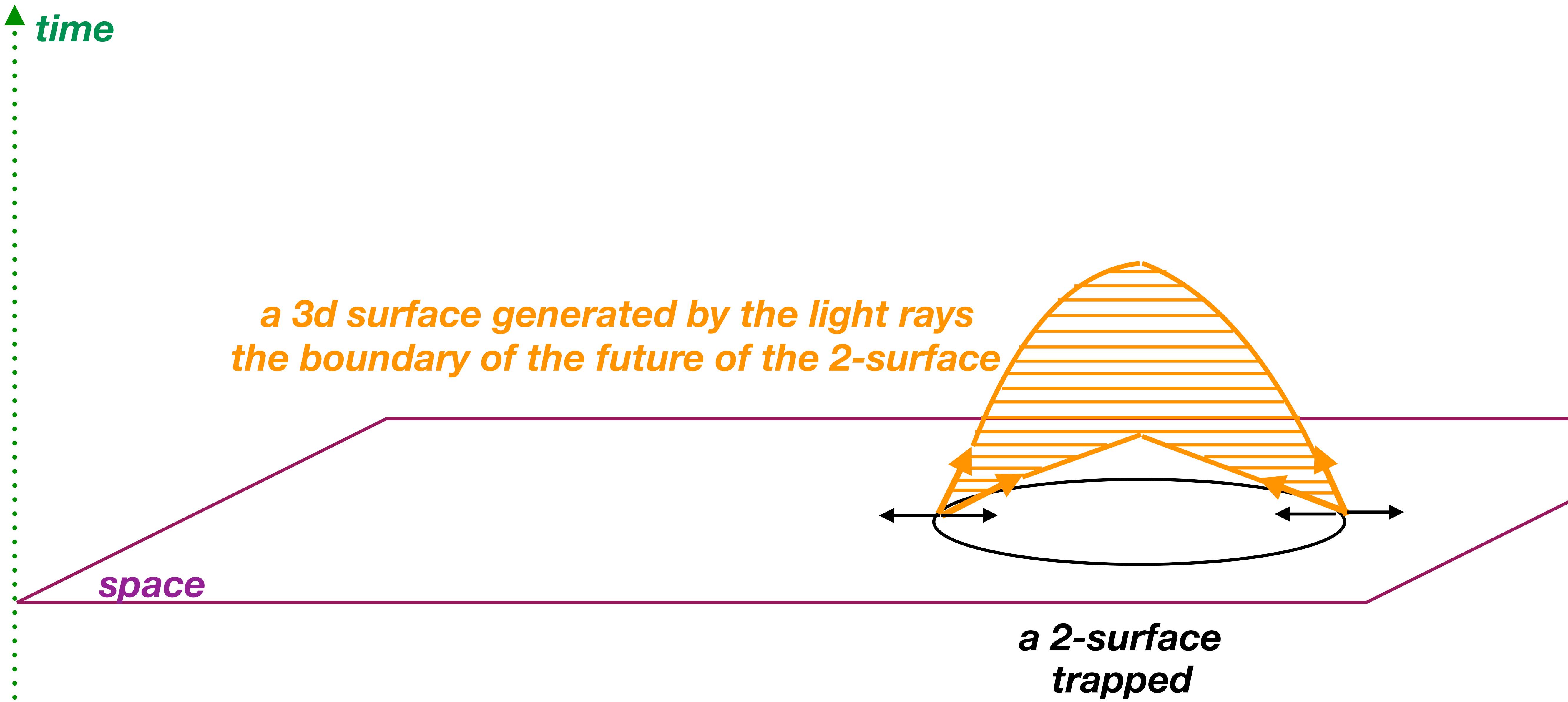

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Uses theory of geodesic flows and conjugate points, generalised to null geodesics

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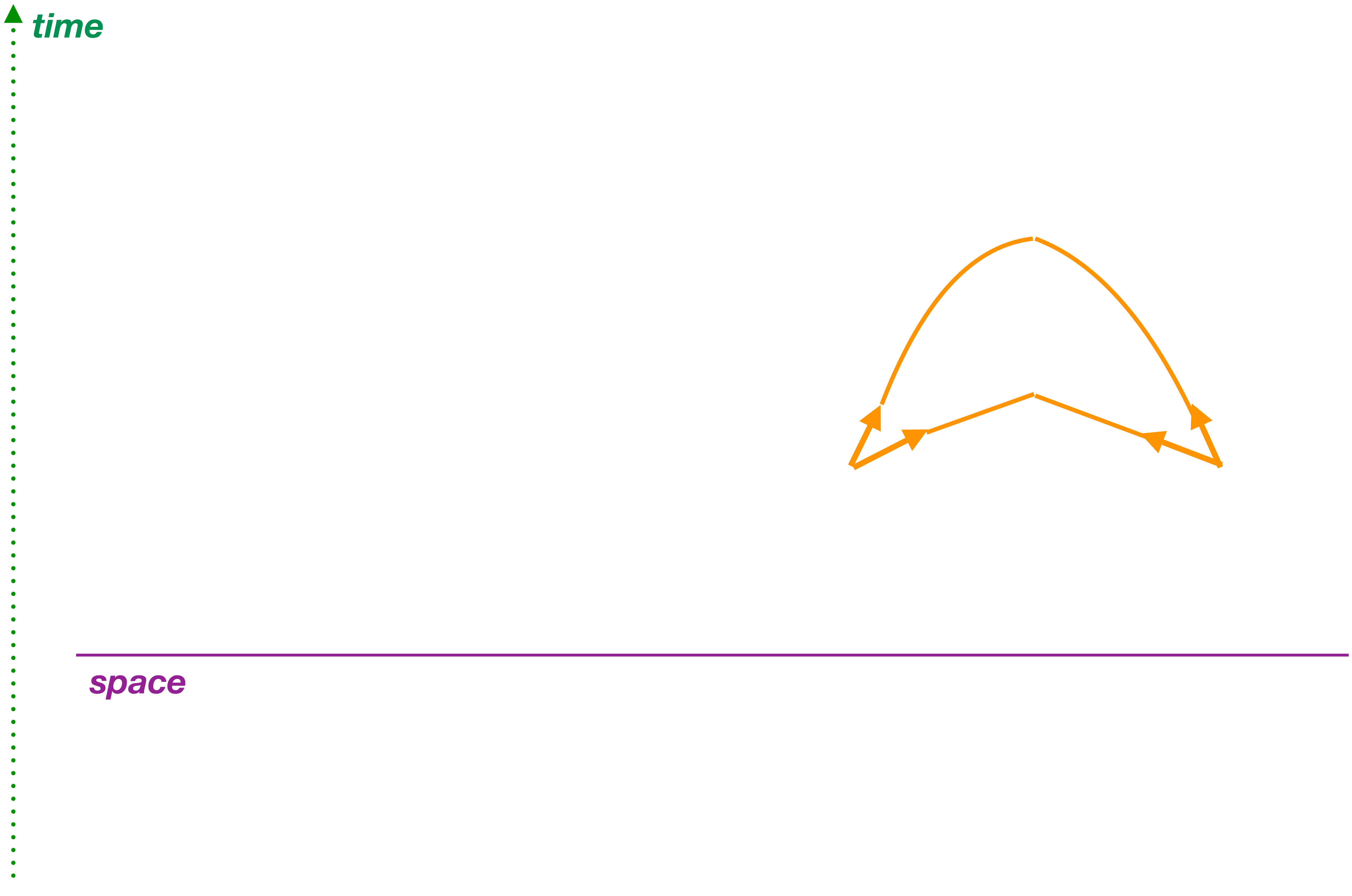

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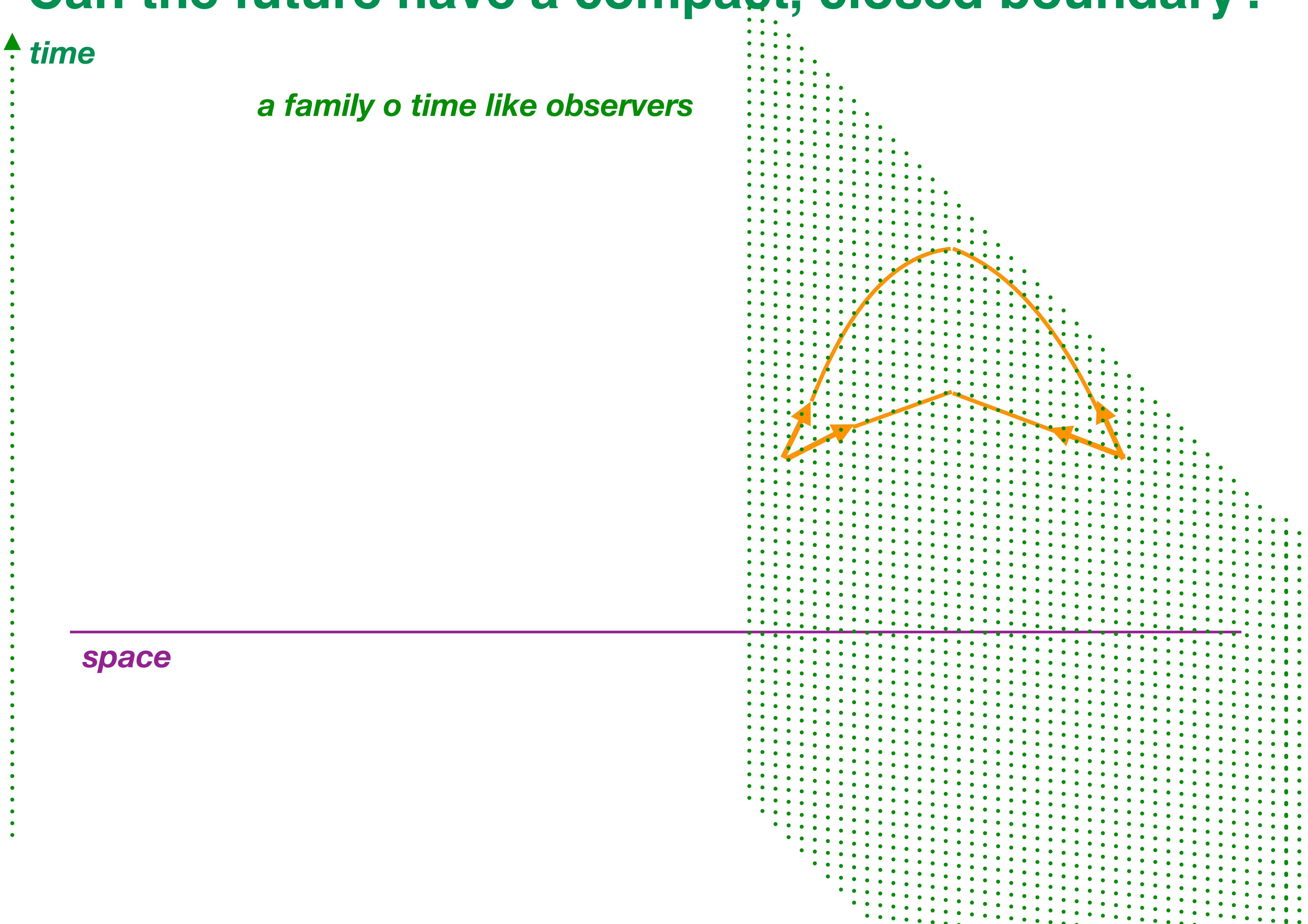

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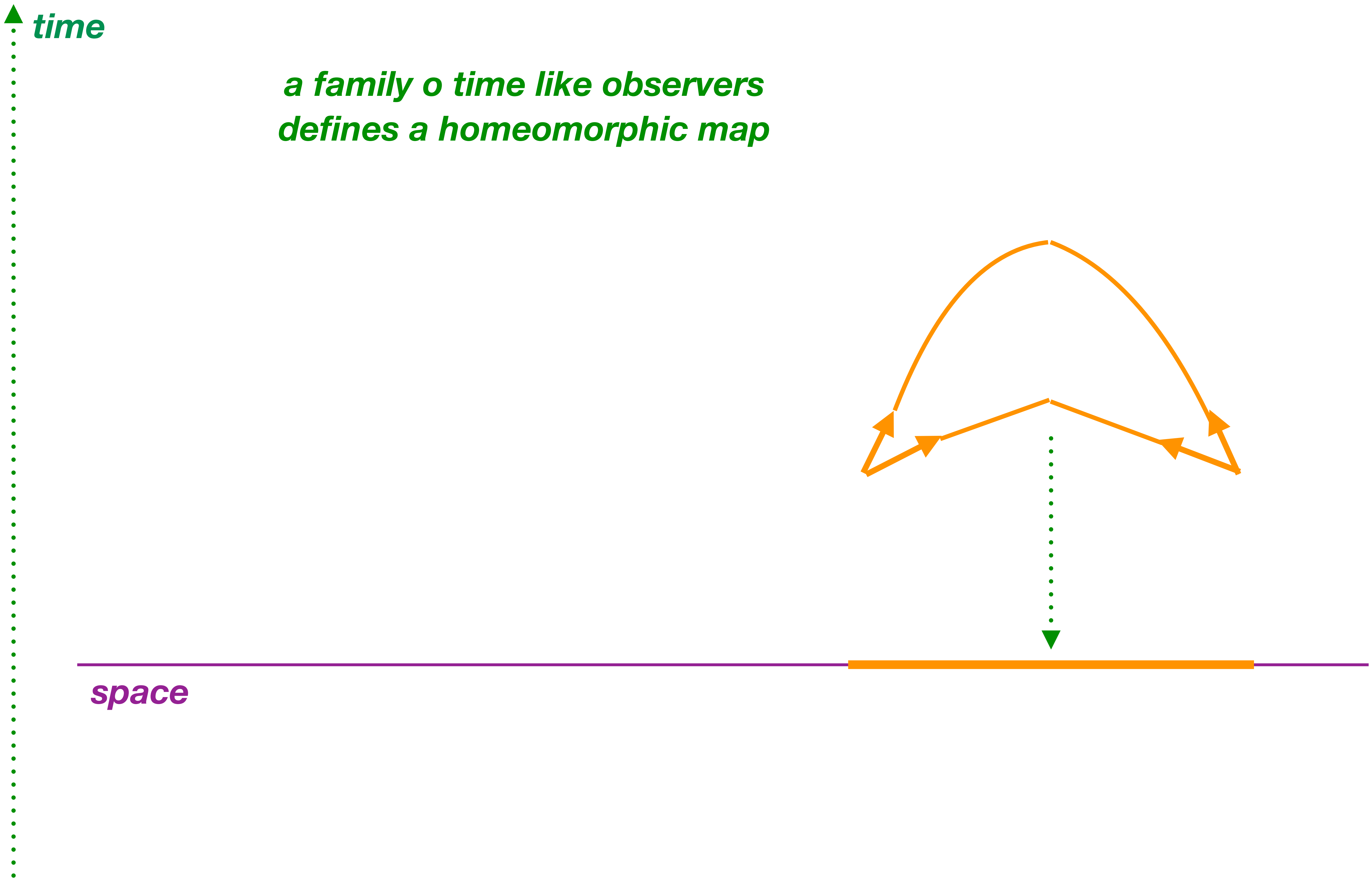

Can the future have a compact, closed boundary?

a family o time like observers
defines a homeomorphic map
of a closed 3d surface onto an open subset
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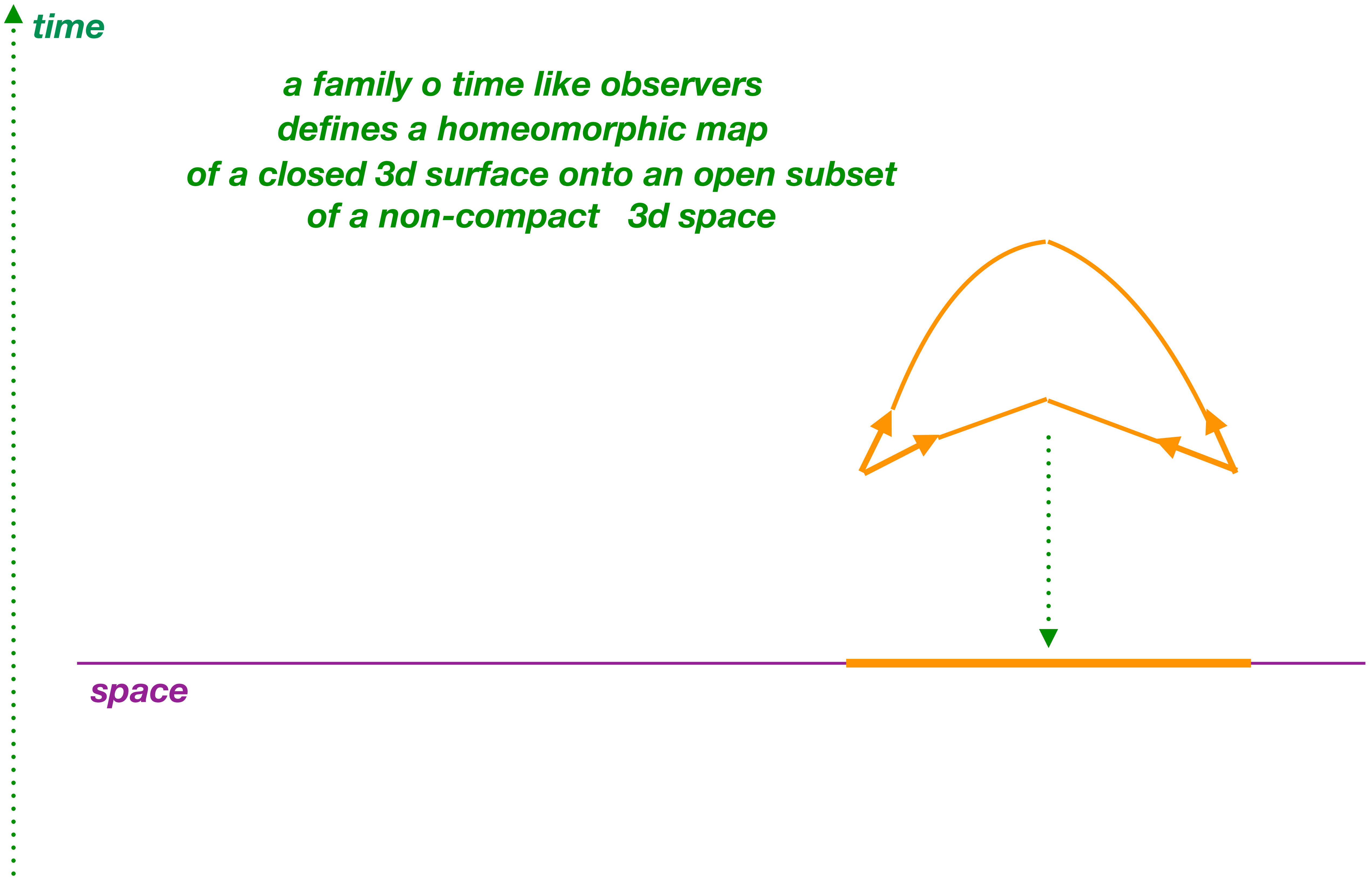

Can the future have a compact, closed boundary?

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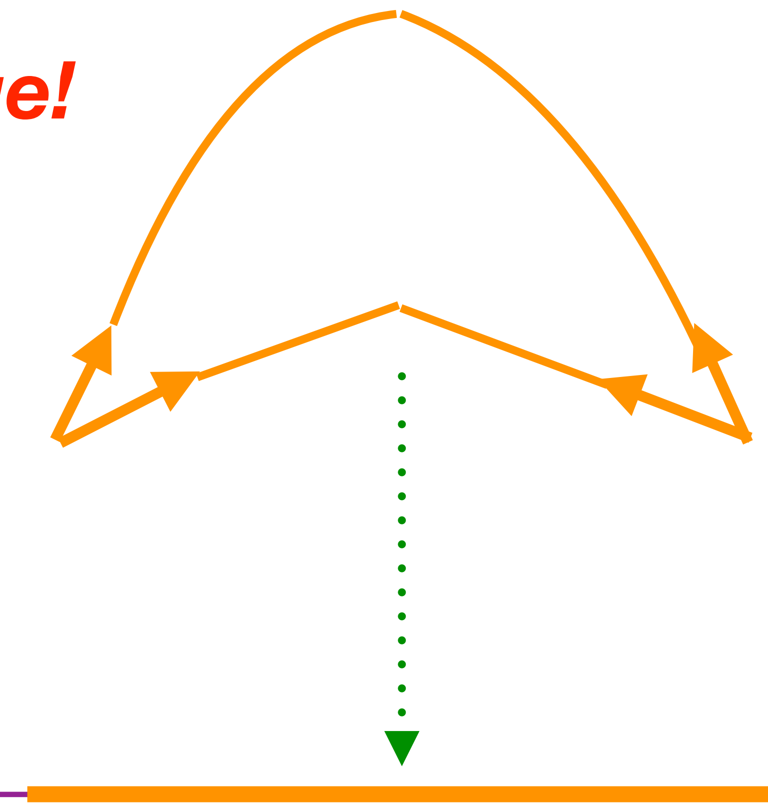
there is no way it can be true!



space

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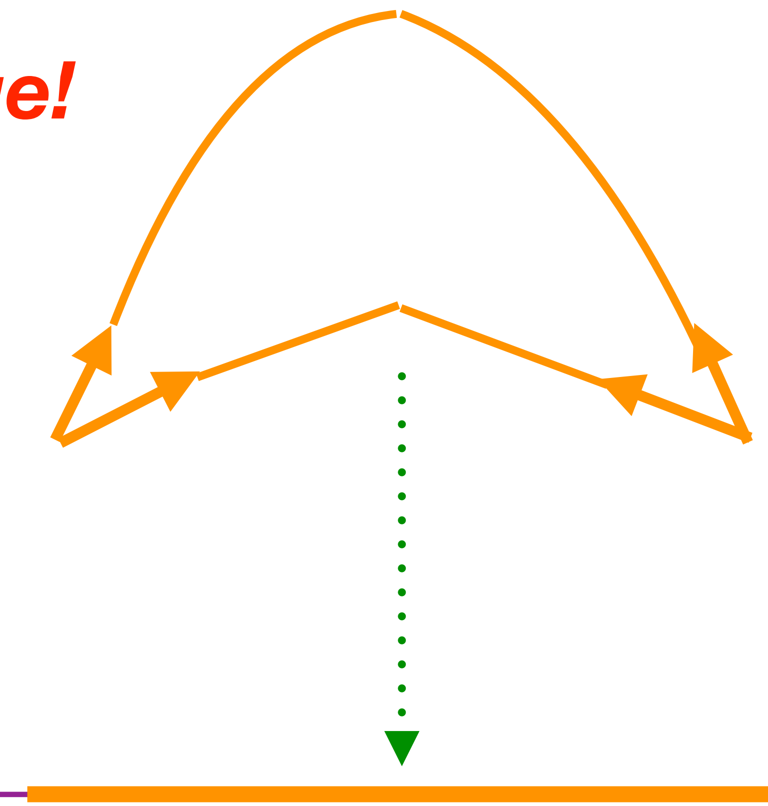

space

Reasonable, however, from the point of view of developing imagination of theoretical physicist, strong assumption:
the existence of a non-compact Cauchy surface

the original drawing by Penrose

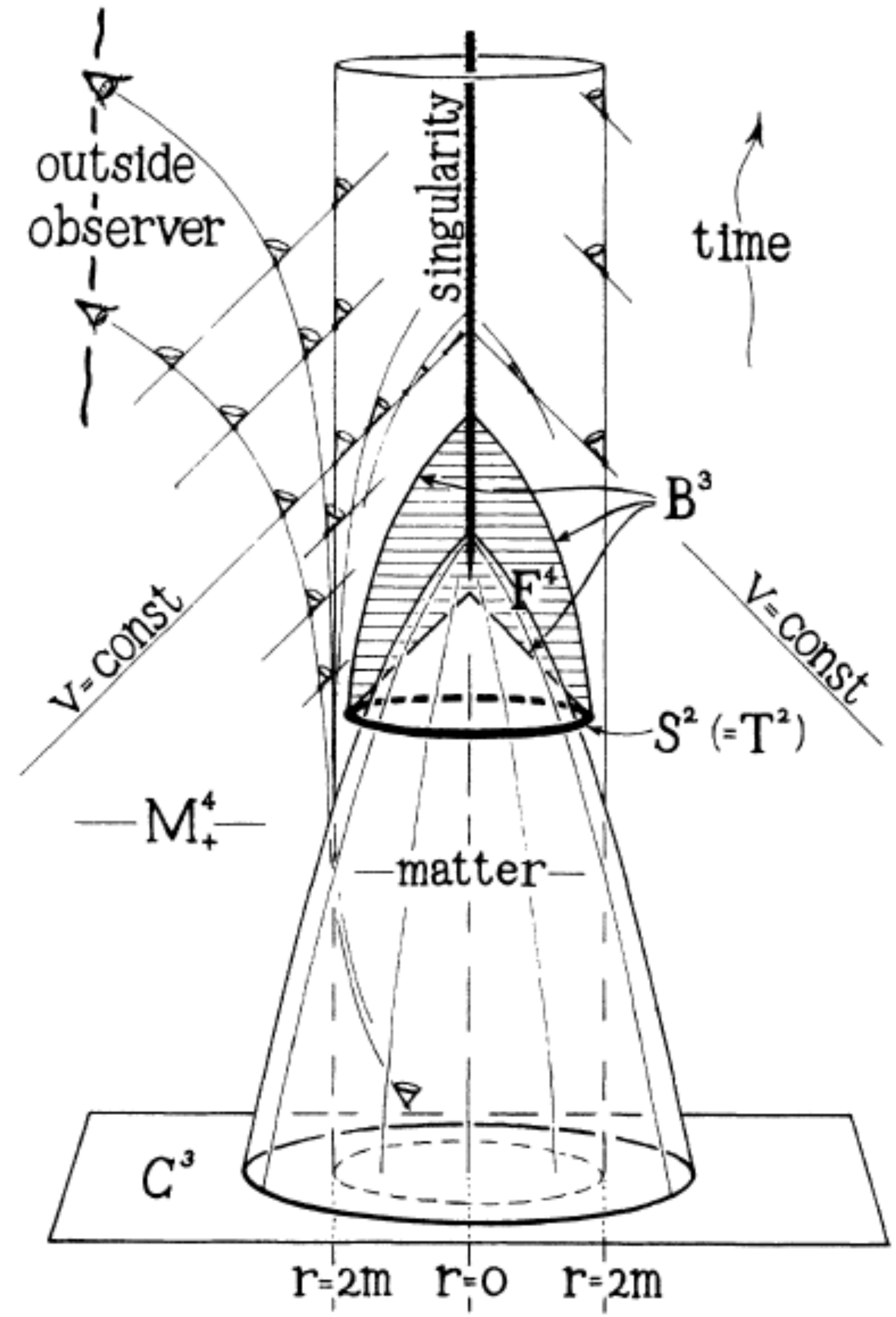

FIG. 1. Spherically symmetrical collapse (one space dimension surpressed). The diagram essentially also serves for the discussion of the asymmetrical case.

The result attracted a lot of researchers, produced lot of generalisations by Penrose himself and newcomers. One of the newcomers was Steven Hawking who dragged the research in his papers toward cosmology. When Penrose and Hawking combined their effort, they have accomplished more than each of them could do individually…

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The singularities of gravitational collapse and cosmology

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A new theorem on space-time singularities is presented which largely incorporates and generalizes the previously known results. The theorem implies that space-time singularities are to be expected if *either* the universe is spatially closed *or* there is an 'object' undergoing relativistic gravitational collapse (existence of a trapped surface) *or* there is a point p whose

Physical conclusion: singularity

Corollary. *A space-time M cannot satisfy causal geodesic completeness if, together with Einstein's equations* (3.5), *the following four conditions hold:*

(3.20) *M contains no closed timelike curves.*

(3.21) *the energy condition* (3.6) *is satisfied at every point,*

(3.22) *the generality condition* (3.10) *is satisfied for every causal geodesic,*

(3.23) *M contains either*

(i) *a trapped surface,*

or (ii) *a point p for which the convergence of all the null geodesics through p changes sign somewhere to the past of p,*

or (iii) *a compact spacelike hypersurface.*

Physical conclusion:

singularity

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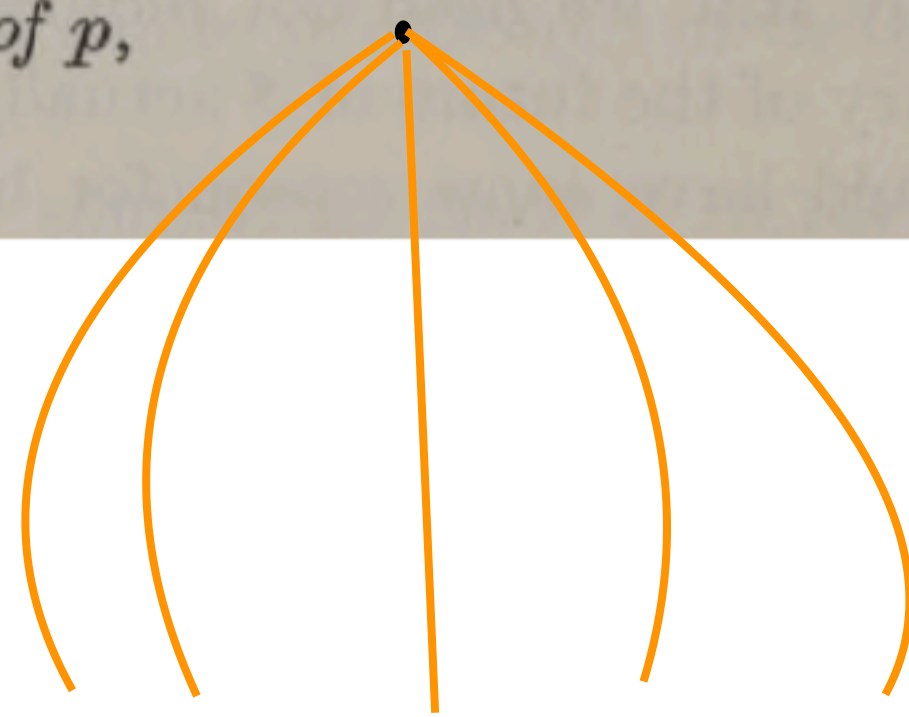

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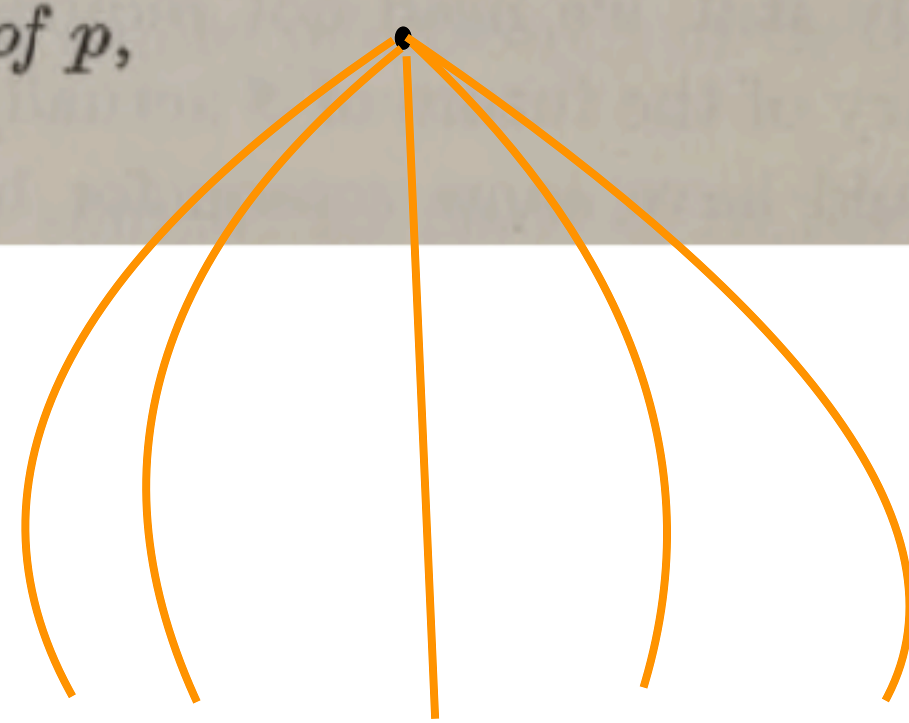
for example a closed universe



The technical reason:

THEOREM. *No space-time M can satisfy all of the following three requirements together:*

(3.1) *M contains no closed timelike curves,*

(3.2) *every inextendible causal geodesic in M contains a pair of conjugate points,*

(3.3) *there exists a future- (or past-) trapped set $S \subset M$.*

Conclusions on the singularities

Singularities of spacetime are realistic consequences of gravitational collapse - it applies to stars, galaxies, and universes.

The singularities amount to incompleteness of some geodesic curves, however, not much more is known about their nature.

Can we see singularities? No, whenever Penrose’s cosmic censorship conjecture is true.

Is the cosmic censorship conjecture true? Yes, provided the Penrose inequality is satisfied. Then, a black hole horizon forms.

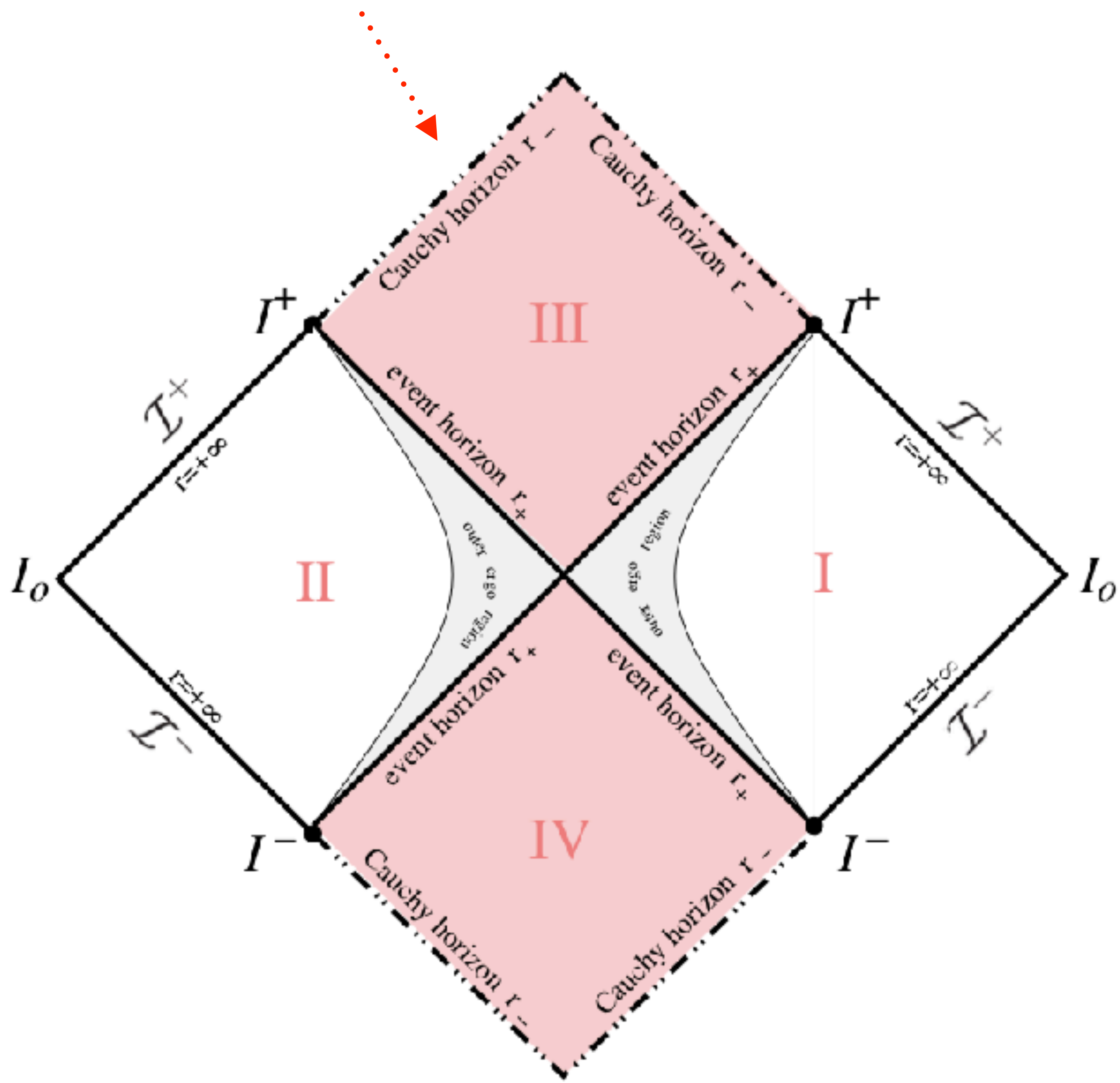
Some of the energy conditions are sensitive to the existence of a positive cosmological constant …

Other major results on black holes by Penrose

General definition of black hole spacetime and an event horizon as the boundary of the past of the future null infinity.

Extraction of energy from black hole on the cost of its angular momentum.

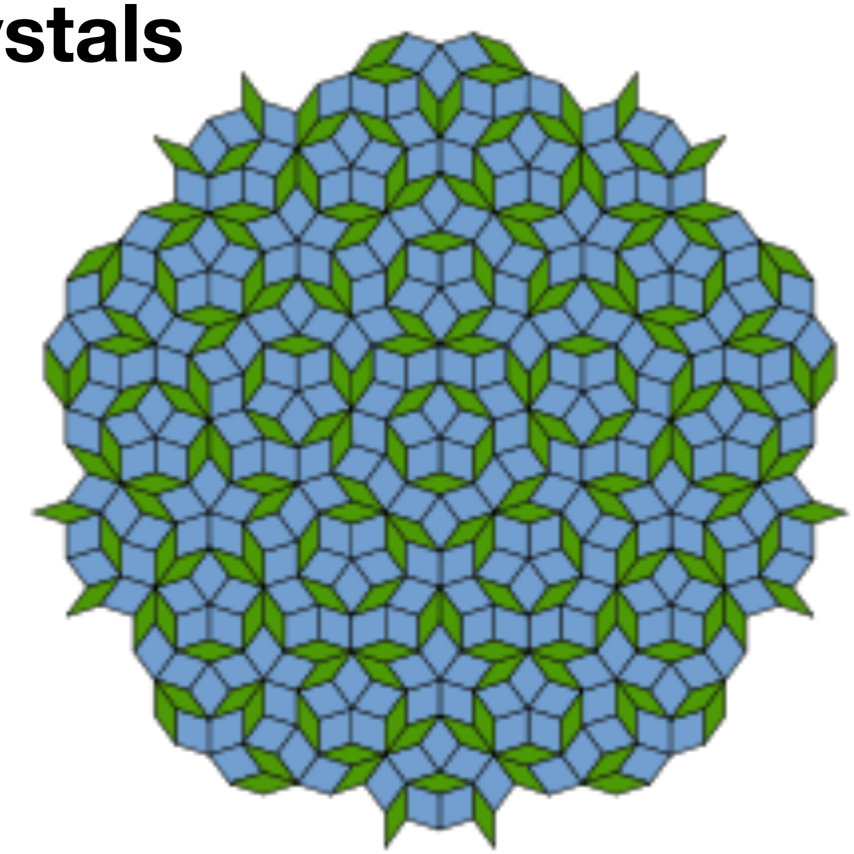
Instability of the inner horizon in the Kerr-Newman black hole with respect to falling in perturbations of matter fields.



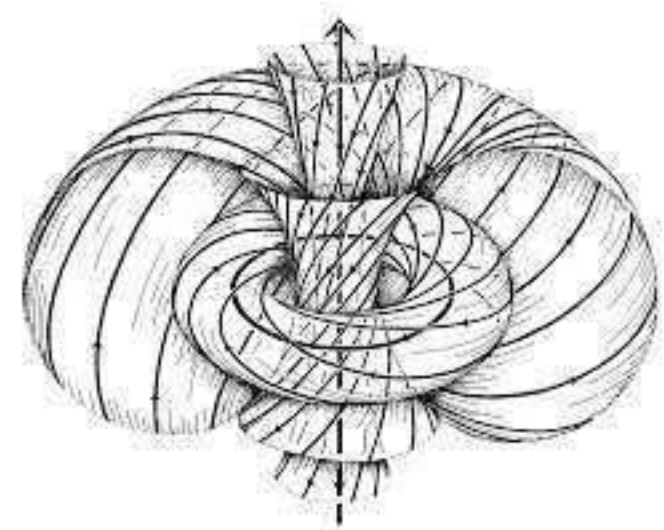
Cauchy horizon r −
Cauchy horizon r −
III
I^+
I^+
$\mathcal{I}^+$
$\mathcal{I}^+$
r=+∞
r=+∞
event horizon r +
event horizon r +
ergo region
ergo region
I_o
II
I
I_o
r=+∞
r=+∞
$\mathcal{I}^-$
$\mathcal{I}^-$
event horizon r +
event horizon r +
IV
I^-
I^-
Cauchy horizon r −
Cauchy horizon r −

Other major results by Penrose

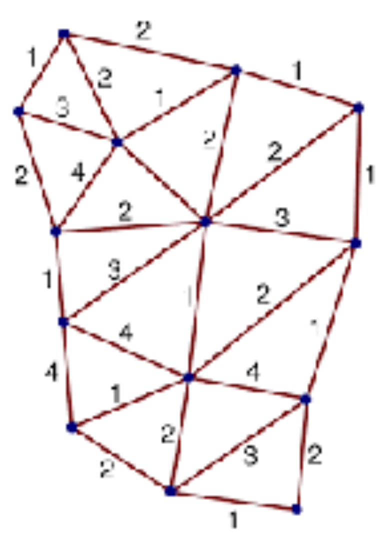
Aperiodic tilling and existence of quasicrystals



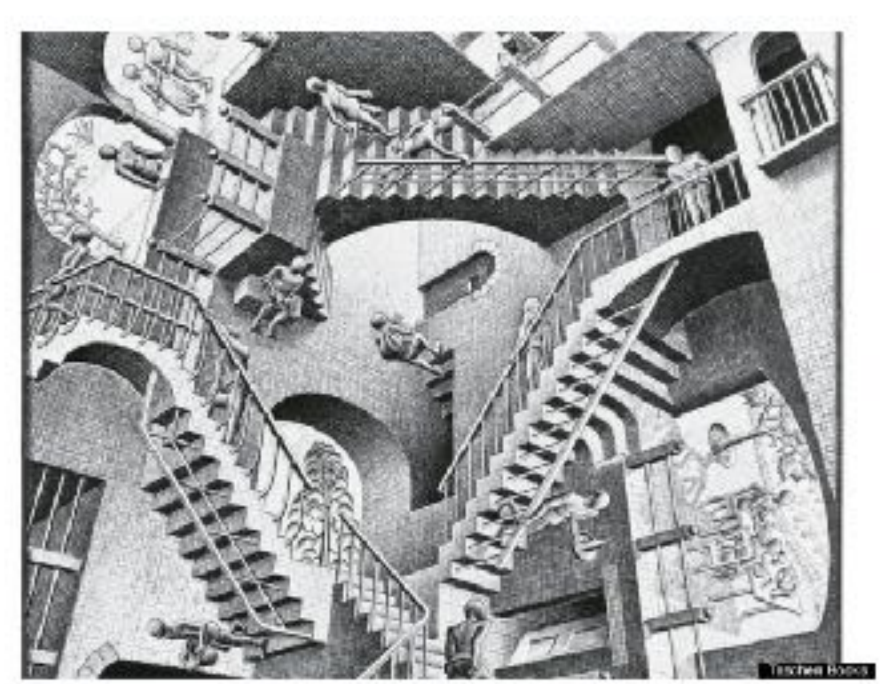
The twistor theory in mathematics



The spin-networks used in quantum models



Impossible objects in painting:



Some early biography of Roger Penrose

Born in Colchester, Essex, Roger Penrose is a son of Margaret (Leathes) and psychiatrist and geneticist Lionel Penrose.[b] His paternal grandparents were J. Doyle Penrose, an Irish-born artist, and The Hon. Elizabeth Josephine Peckover; and his maternal grandparents were physiologist John Beresford Leathes and his wife, Sonia Marie Natanson,[5][6] a Jewish Russian who had left St. Petersburg in the late 1880s.[7][5] His uncle was artist Roland Penrose, whose son with photographer Lee Miller is Antony Penrose.[8][9] Penrose is the brother of physicist Oliver Penrose, of geneticist Shirley Hodgson, and of chess Grandmaster Jonathan Penrose.[10][11]

Penrose spent World War II as a child in Canada where his father worked in London, Ontario.[12] Penrose attended University College School and University College London, where he graduated with a first class degree in mathematics.[10]

Penrose spent the academic year 1956-57 as an Assistant Lecturer at Bedford College, London and was then a Research Fellow at St John's College, Cambridge. He met there Dennis Sciama.