Lasers
lecture 9

Czesław Radzewicz

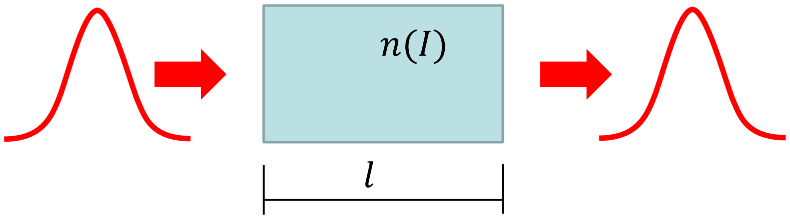
self-phase modulation

For high light intensities the index of refraction is not constant – it depends on light intensity. In most practical cases it suffices to include a term proportional to the light intensity :

$$n(I) = n_0 + n_2 I$$

The values of n_2 coefficient are small. For example, for quartz glass (SiO_2) $n_2 \cong 2 \cdot 10^{-16}\mathrm{cm}^2/\mathrm{W}$.

For very high light intensity $I = \frac{100\mathrm{GW}}{\mathrm{cm}^2} = 10^{11}\mathrm{W/cm}^2$ we have $\Delta n = n_2 I \cong 2 \cdot 10^{-5}$.



Nonlinear index of refraction influences: (1) pulse phase and (2) pulse wavefront.

1. Self-phase modulation

Consider 1-D case (plane wave, or fiber): $E_{in}(t) = E_0 e^{i\omega_0 t}$.
At the output $E_{out}(t, z) = E_0 e^{i(\omega_0 t - kl)}$ with

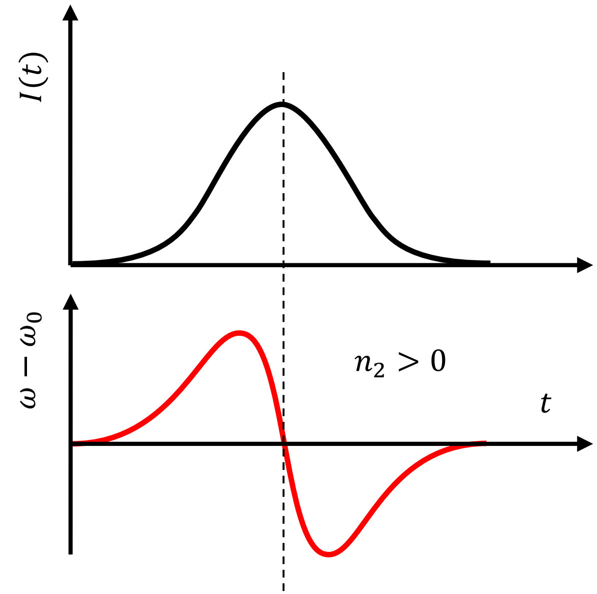
$$k = k(I) = \frac{n(I)\omega_0}{c} = \frac{n_0\omega}{c} + \frac{n_2 I\omega_0}{c} = k_0 + \frac{n_2\omega_0}{c} I$$

which leads to

$$E_{out}(t, z) = E_0 e^{i(\omega_0 t - k_0 z)} e^{i\frac{n_2\omega_0 l}{c} I(t)}$$

resulting in time-dependent phase and frequency

$$\varphi(t) = \omega_0 t + \frac{n_2\omega l}{c} I(t), \qquad \omega(t) = \omega_0 + \frac{n_2\omega l}{c}\frac{dI}{dt}$$



- pulse envelope remains the same
- spectrum is broadened

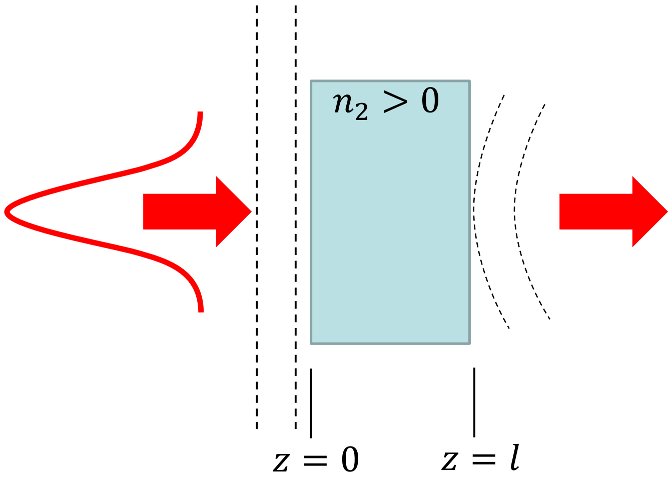
self-focusing

Assume a Gaussian beam with its waist at $z = 0$:

$$E(r, z=0) = E_0 e^{-\frac{r^2}{w_0{}^2}}$$

for $l \ll z_0$ we have

$$E_{out}(r, l) = E(r, z=0)e^{-ikl} = E_0 e^{-\frac{r^2}{w_0{}^2}} e^{-ik_0 l} e^{-i\frac{n_2 \omega_0 l}{c} I(r, z=0)} \qquad (1)$$



If $n_2 > 0$ the phase fronts are delays at the peripheral regions – a plane wavefront bends.

Let's look at the region close to the beam axis

$$I(r, z=0) = I_0 e^{-\frac{2r^2}{w_0{}^2}} \cong I_0 \left(1 - \frac{2r^2}{w_0{}^2}\right)$$

We put this into equation (1)

$$E_{out}(r, l) = \underbrace{E_0 e^{-\frac{r^2}{w_0{}^2}}}_{\text{Gauss}} \underbrace{e^{-ik_0 l} e^{-i\frac{n_2 \omega_0 l}{c} I_0}}_{\text{constant phase}} \underbrace{e^{i\frac{n_2 \omega_0 l I_0}{c} \frac{2r^2}{w_0{}^2}}}_{\text{spherical wavefront}}$$

$$\frac{1}{R_{sf}} = \frac{4 n_2 l}{n_0 w_0{}^2} I_0$$

Kerr Lens

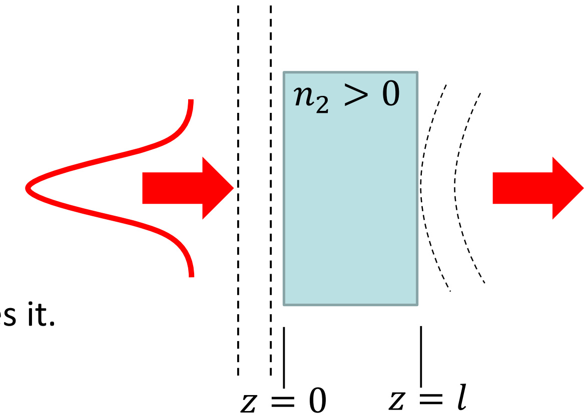
focal length f_K, $1/f_K \propto I$

self-focusing – critical power

from the previous slide

$$E_{out}(r,l) = E_0 \underbrace{e^{-\frac{r^2}{w_0{}^2}}}_{\text{Gauss}} \underbrace{e^{-ik_0 l} e^{-i\frac{n_2\omega_0 l}{c}I_0}}_{\text{constant phase}} \underbrace{e^{i\frac{n_2\omega_0 l I_0}{c}\frac{2r^2}{w_0{}^2}}}_{\text{spherical wavefront}}$$

$$\frac{1}{R_{sf}} = \frac{4n_2 l}{n_0 w_0{}^2} I_0$$



Two effects: diffraction increases beam diameter but self-phocusing decreases it.

Fro lecture 5

$$\frac{1}{R_{diff}} \cong \frac{l}{z_0{}^2} = \frac{\lambda^2 l}{\pi^2 n_0{}^2 w_0{}^4}$$

The beam will self-phocus if

$$R_{sf} < R_{diff} \Leftrightarrow \frac{4n_2 l}{n_0 w_0{}^2} I_0 > \frac{\lambda^2 l}{\pi^2 n_0{}^2 w_0{}^4} \Rightarrow I_0 > I_{cr} = \frac{\lambda^2}{4 n_2 \pi^2 n_0 w_0{}^2}$$

Critical power P_{cr}, for powers higher the P_{cr}

$$P_{cr} = 2\pi I_{cr} \int_0^\infty r e^{-\frac{2r^2}{w_0{}^2}} dr = \frac{\lambda^2}{4\pi n_0 n_2}$$

the beam collapses.

Note: the exact form of self focusing depends on many parameters: pulse duration, dispersion, the nature of medium, …

an example: filaments in air

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Optics Communications 247 (2005) 171–180

OPTICS COMMUNICATIONS

www.elsevier.com/locate/optcom

Range of plasma filaments created in air by a multi-terawatt femtosecond laser

G. Méchain [a,*], C.D'Amico [a], Y.-B. André [a], S. Tzortzakis [a,1], M. Franco [a], B. Prade [a], A. Mysyrowicz [a], A. Couairon [b], E. Salmon [c], R. Sauerbrey [d]

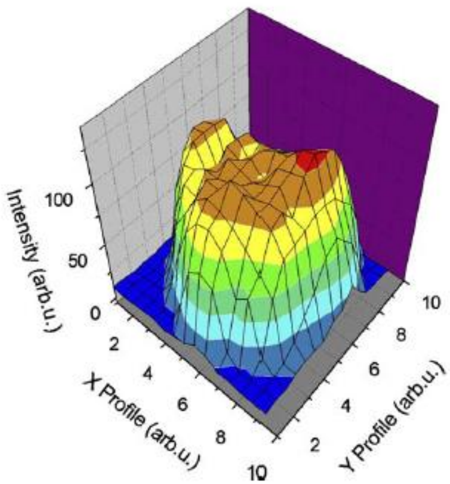
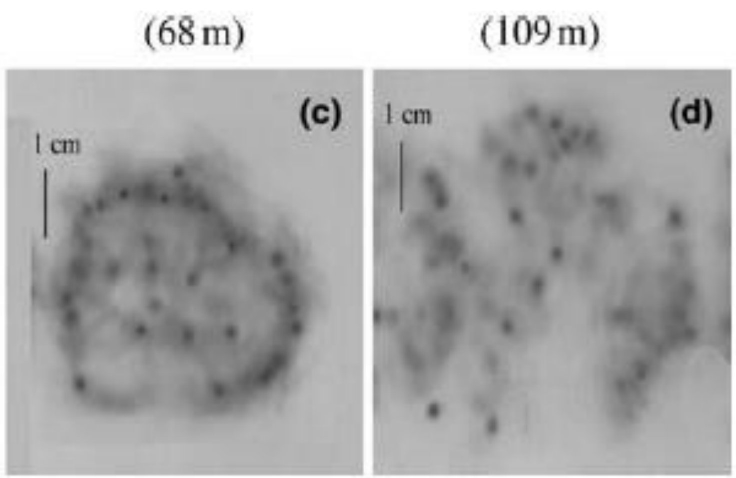


Fig. 1. Profile of the beam. Intensity recorded at the output of the compressor.



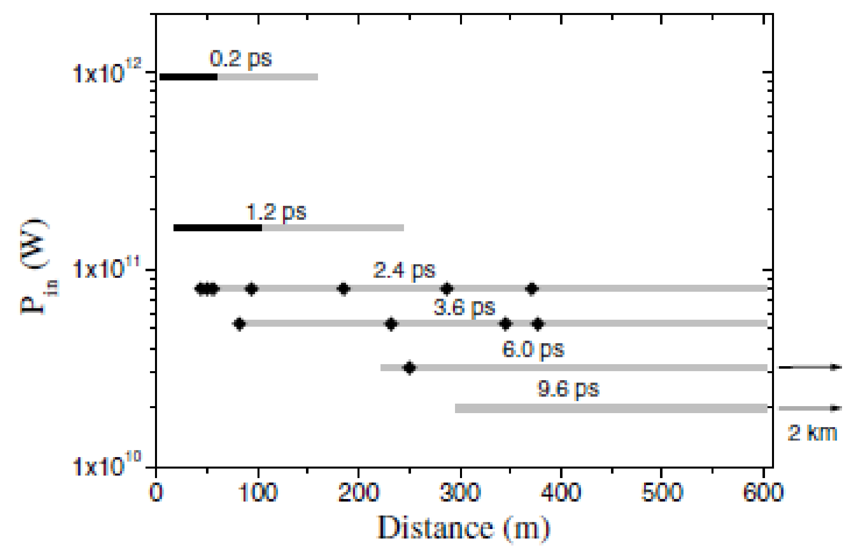
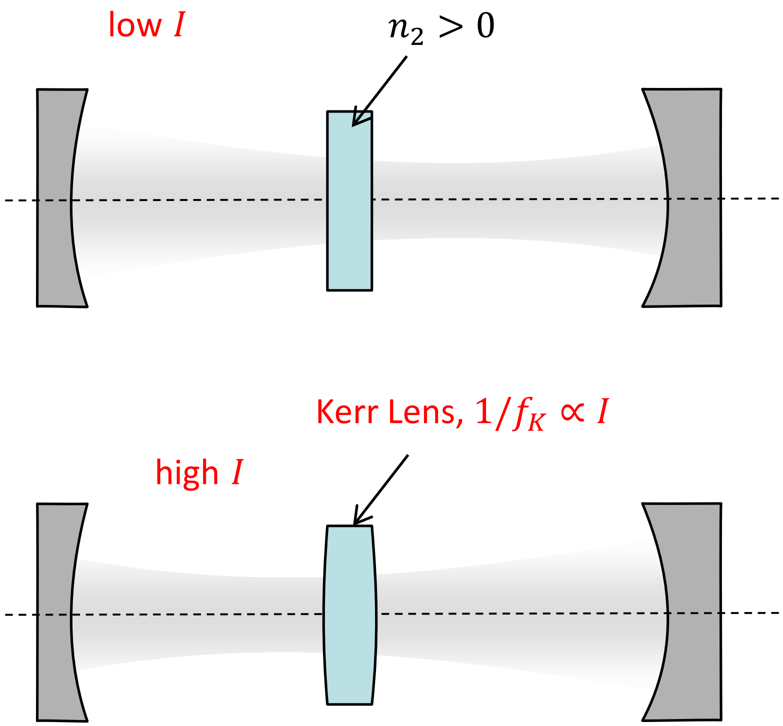


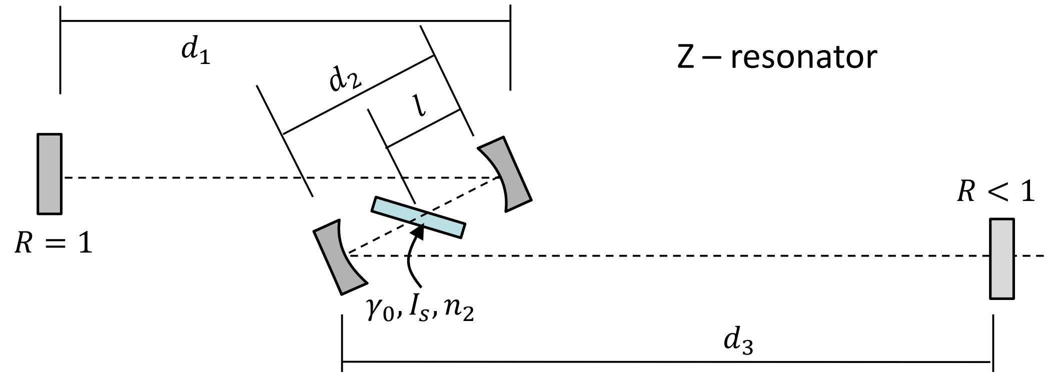
Fig. 8. Evolution of the length of filamentation by varying the initial chirp of the laser pulse. The pulse without chirp has a duration of 100 fs. The black lines and black points refer to locations where air ionisation could be detected, grey lines to distances where bright light channels are observed.

self-focusing inside the laser cavity

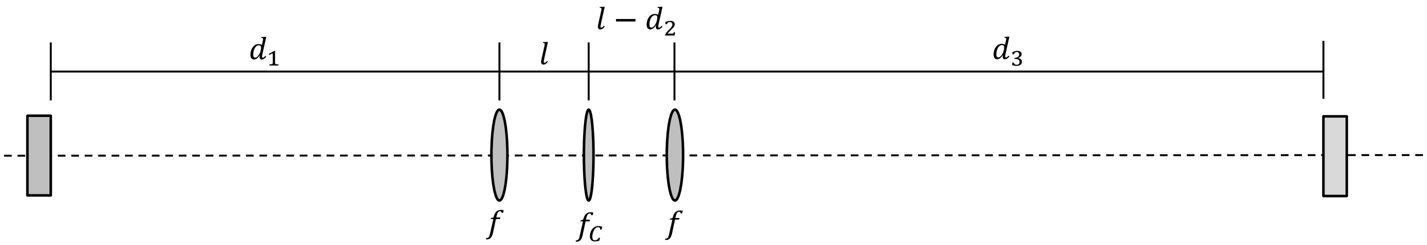
With Kerr lensing any piece of material inside the cavity (for example. a plane parallel glass plate) may act as a lens. The focusing power of the lens scales approx. linearly with the instantaneous light intensity. This extra lens changes the properties of the resonator. If we want to promote short pulse operation of the laser we should make the resonator such that the Kerr lens decreases diffraction losses inside the cavity. Such a cavity mimics a saturable absorber – we call it an artificial saturable absorber.

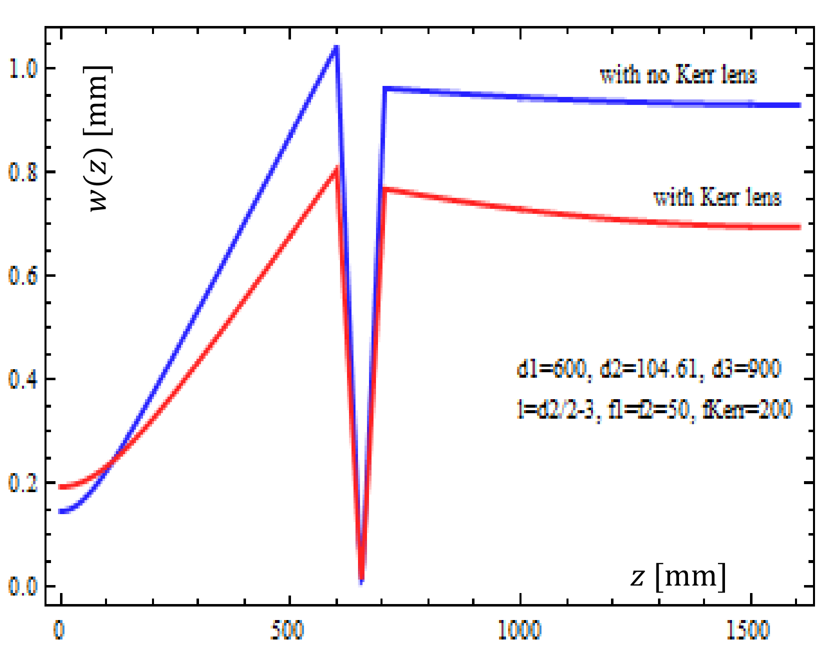


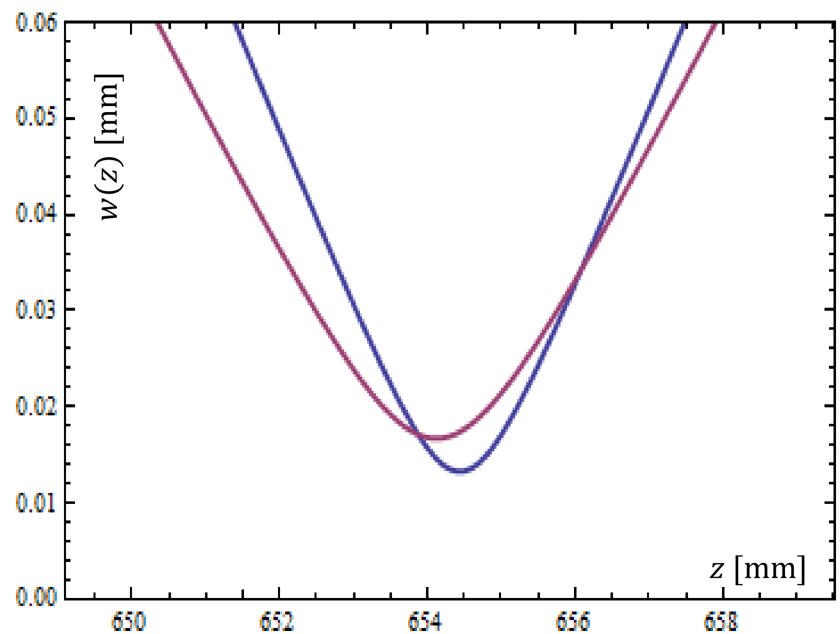
Kerr Lens Mode-locking (KLM)



Z - resonator expanded



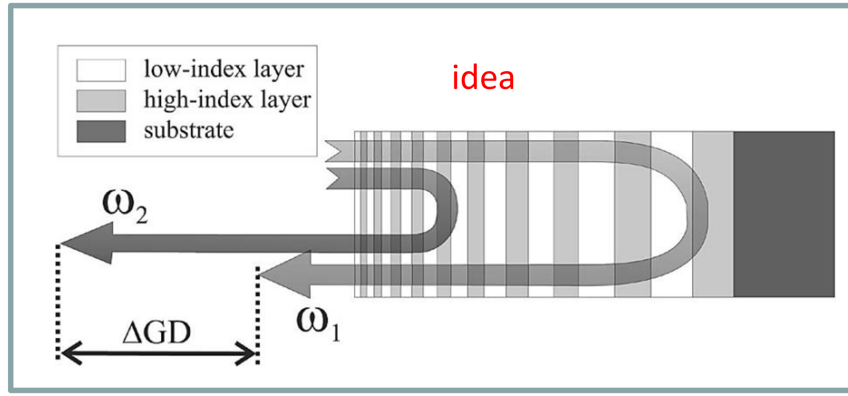


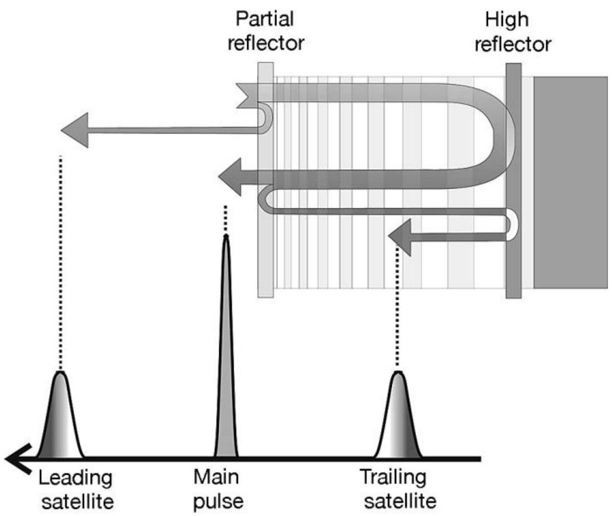


intra-cavity dispersion control

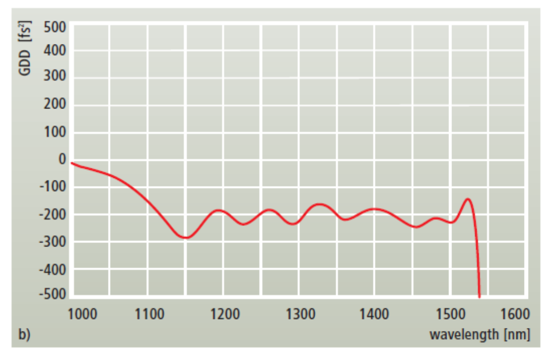
Group delay dispersion (GDD), how can we control it?

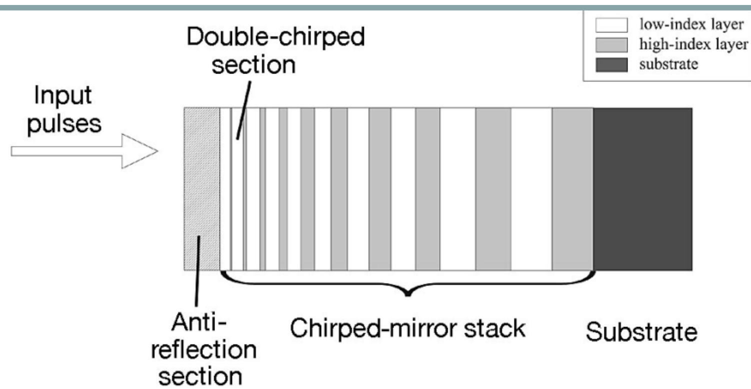
1. chirped mirrors





interference of waves reflecting from different parts of the structure leads to oscillations in reflectivity vs wavelength function





one possible solution – doubly chirped mirrors

intra-cavity dispersion control, 2

Effective mode-locking requires a well defined dispersion of the lasers cavity. We quantize it by group Delay Dispersion $GDD(\omega)$ or $GDD(\lambda)$.

methods to control GDD:

an example from Keller's group
(OL **22**, 1009-1011 (1997))

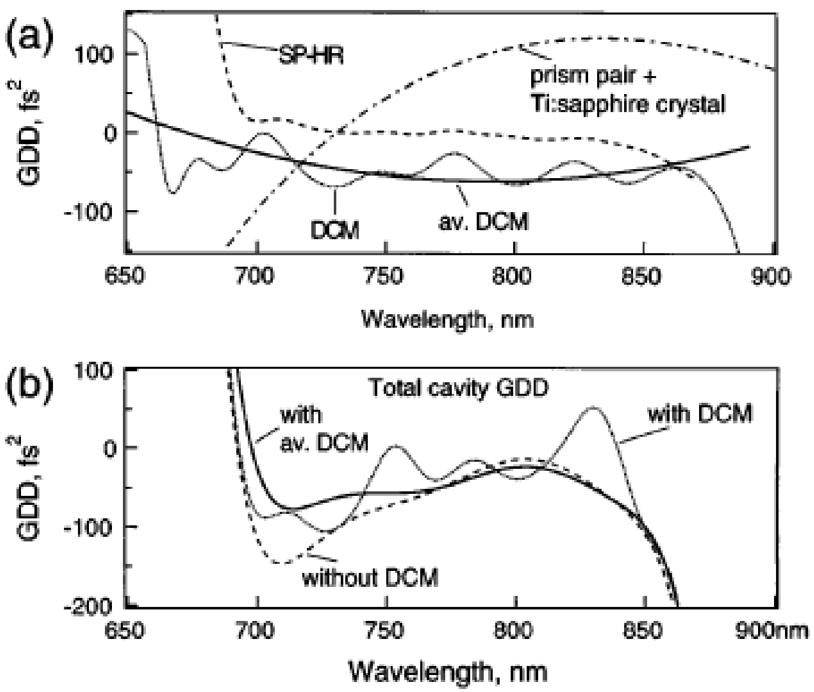
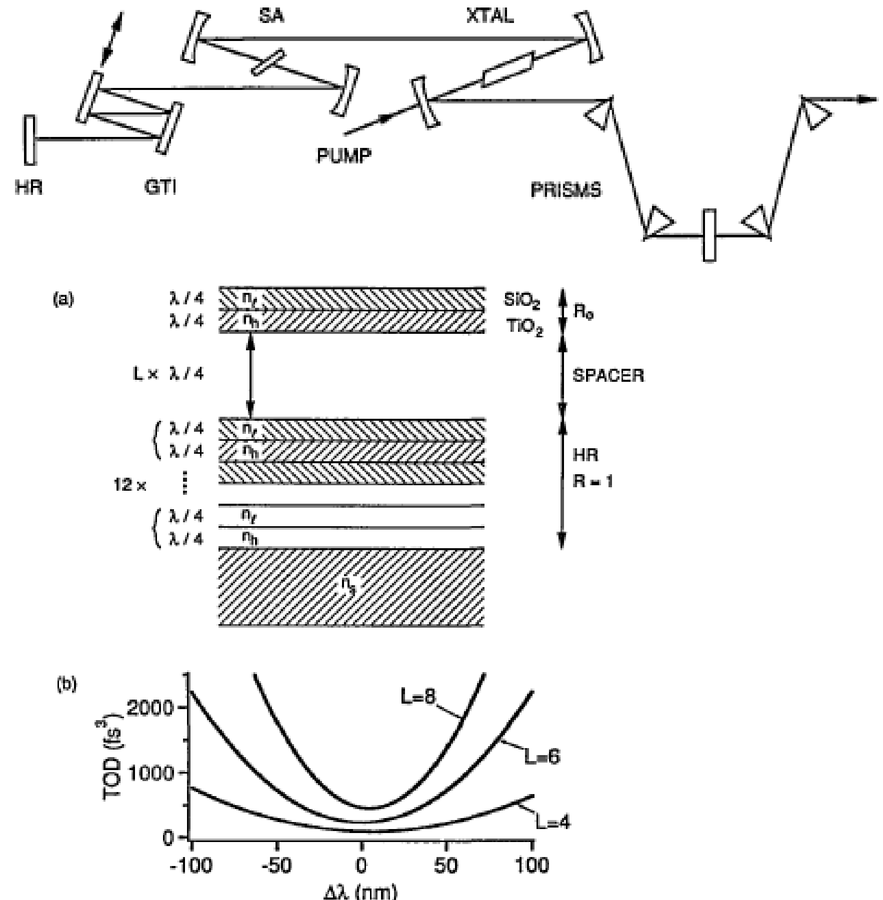


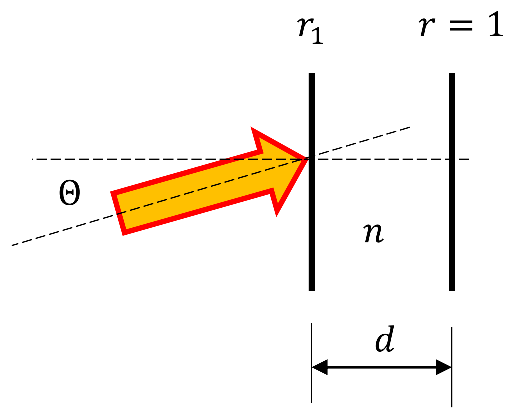
Fig. 3. (a) Group-delay dispersion (GDD) of all intracavity elements: DCM, double-chirped mirror; av., average; SP-HR, Spectra Physics high reflector. (b) Total intracavity GDD. The estimated resolution is ± 5 fs^2.

1. prisms and Gires-Tournois interferometer



— OL **22**, 1608-1610 (1992)

Gires-Tournois interferometer



An analog of the Fabry-Perot interferometer with one mirror fully reflecting $R = 1$. no transmitted beam

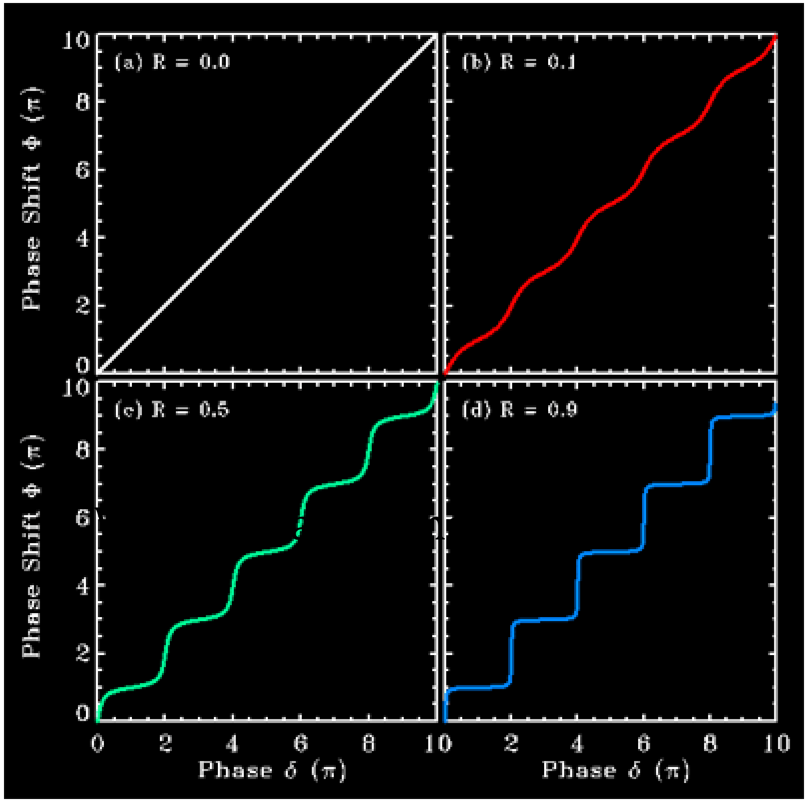
$$r = -\frac{r_1 - e^{-i\delta}}{1 - r_1 e^{-i\delta}}$$

with

$$\delta = \frac{4\pi}{\lambda} nd \cos\Theta$$

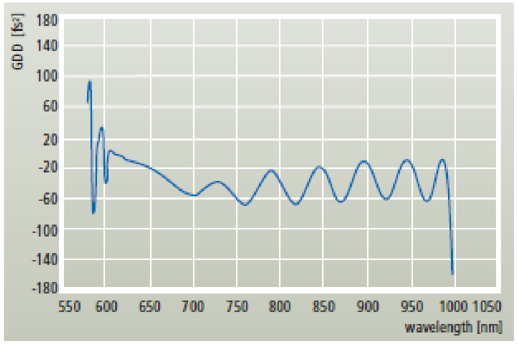
Assuming that r_1 is real we have the phase φ of the reflected wave

$$\tan\left(\frac{\varphi}{2}\right) = \frac{1 + r_1}{1 - r_1} \tan\left(\frac{\delta}{2}\right)$$

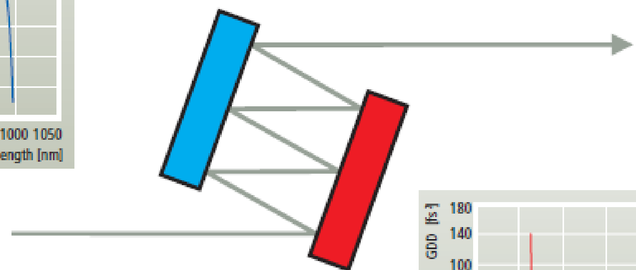


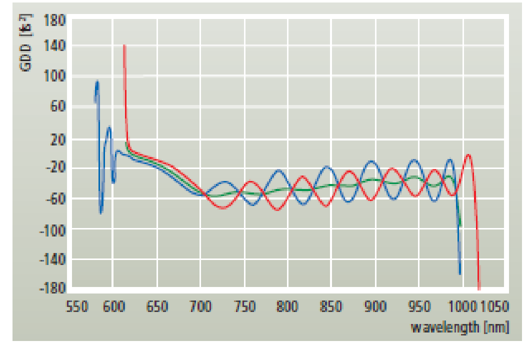
intra-cavity dispersion control, 3

commercial chirped mirrors (Layertech)

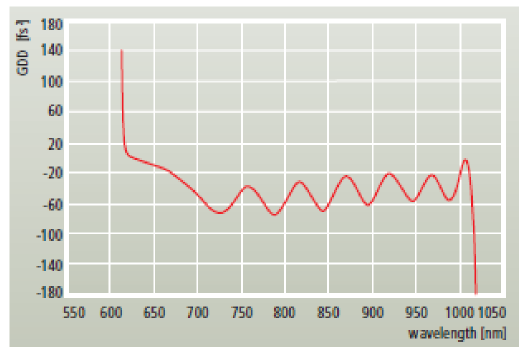


GDD-spectrum of mirror 2





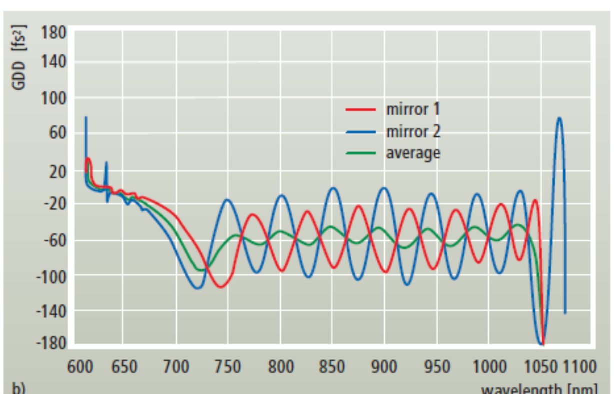
average GDD-spectrum



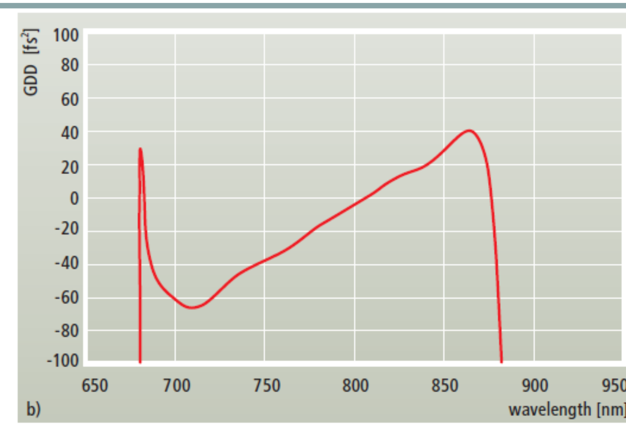
GDD-spectrum of mirror 1

a matched pair of mirrors

Figure 3: Schematic drawing of a negative dispersion mirror pair



broadband negative GDD



custom GDD

selected femtosecond oscillators

Ti:Sapphire (Ti^{3+}:Al_2O_3) crystal.

typical specs

Orientation	Optical axis C normal to rod axis
Ti_2O_3 concentration	0.03-0.25 wt %
Figure of Merit	> 150 (> 300 available on special requests)
Size	up to 20 mm dia and up to 130 mm length
End configurations	flat/flat or Brewster/Brewster
Parallelism	10 arcsec
Surface finishing	10/5 scratch/dig
Wavefront distortion	λ/4 inch

własności fizyczne

Chemical formula	Ti^{3+}:Al_2O_3
Crystal structure	Hexagonal
Lattice constants	a = 4.748, c = 12.957
Density	3.98 g/cm^3
Mohs hardness	9
Thermal conductivity	0.11 cal (°C x sec x cm)
Specific heat	0.10 cal/g
Melting point	2050 °C
Laser action	4-Level Vibronic
Fluorescence lifetime	3.2 μs (T = 300 K)
Tuning range	660-1050 nm
Absorption range	400-600 nm
Emission peak	795 nm
Absorption peak	488 nm
Refractive index	1.76 @ 800 nm

The quality of crystal is quantized by

Figure of Merit: $FOM \equiv \frac{\alpha(\lambda_p)}{\alpha(\lambda_l)}$,

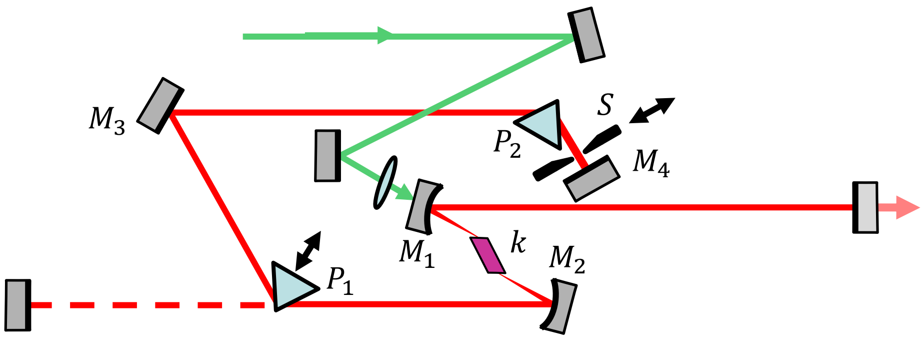
with α standing for absorption coefficient,

Ti:Sap fs oscillators – examples of designs

„old" design Ti:Sap with prisms

advantages: one can tune GDD
one can tune wavelength
high power (several W)

disadvantage: long pulse(50-100fs)



$M_1 - M_4$ - zero GDD mirrors
OC - output coupler
P_1, P_2 - prisms (quartz glass, Brewster)
k - sapphire crystal (Brewster)
S - a slit

Spectra Physics (USA), model Mai Tai eHP

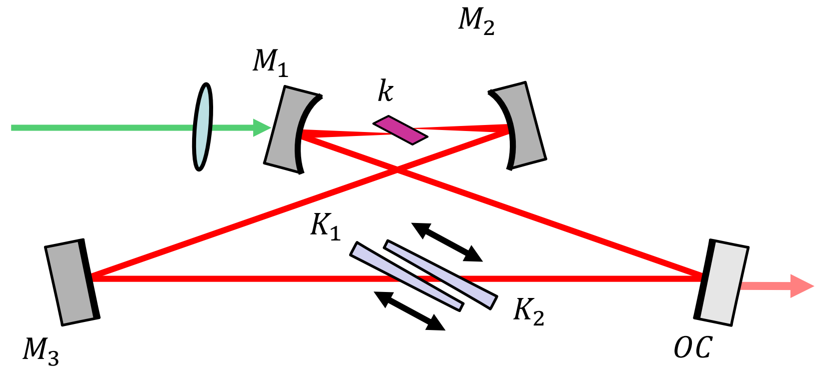
	Mai Tai *e*HP
Peak Power[2]	>450 kW
Pulse Width[2, 3]	<70 fs[9]
Tuning Range[4]	690–1040 nm
Average Power[2]	>2.5 W
Peak Power, Alternative Wavelengths[5]	>70 kW at 690 nm >240 kW at 710 nm >240 kW at 920 nm >38 kW at 1040 nm
Beam Roundness[2]	0.9–1.1
Astigmatism[2]	<10%
Repetition Rate[2, 6]	80 MHz ±1 MHz
Beam Pointing Stability	<50 µrad/100 nm
Noise[2, 7]	<0.15%
Stability[8]	<±1%
Spatial Mode[2]	TEMoo, M^2 <1.1
Polarization[2]	>500:1 horizontal
Beam Divergence[2]	<1 mrad
Beam Diameter (1/e²)[2]	<1.2 m

Ti:Sap fs oscylators – examples of designs, 2

„new" Ti:Sap oscillator design – no prisms

advantage: short pulses

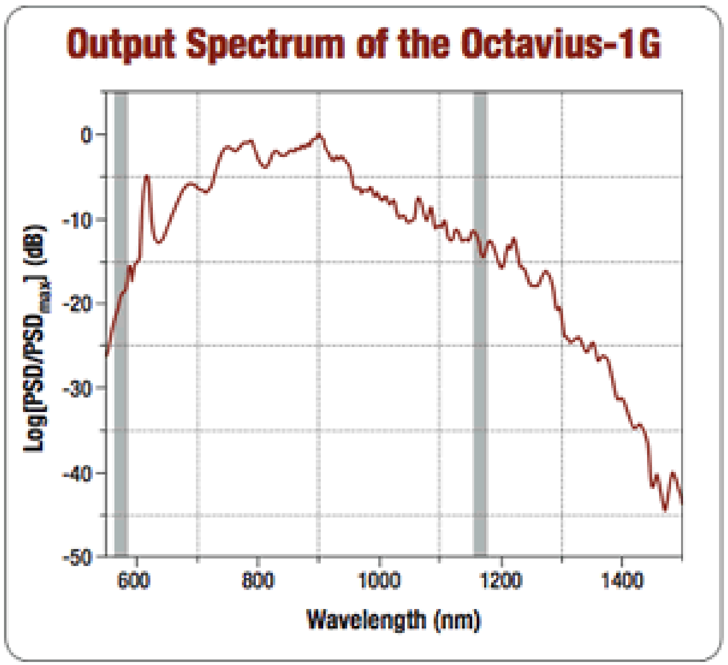
disadvantage: lower power
no tuning



$M_1 - M_3$ - negative GDD mirrors
OC - output coupler
K_1, K_2 - light glass wedges (continuous GDD control)
k - sapphire crystal (Brewster)

IdestaQE (USA), model Octavius

	Ocatvius-1G	Octavius-1G-HP
Pulse width	< 6 fs	
Bandwidth @-10 dB	300 nm	
Average out power	300 mW @6 W pump	750 mW @ 8W pump
Divergence	< 2 mrad	
Polarization	> 90:1	
Power stability over 1h	+/- 1%	
Dimensions	10.0" x 7.7" (255 mm x 196 mm)	

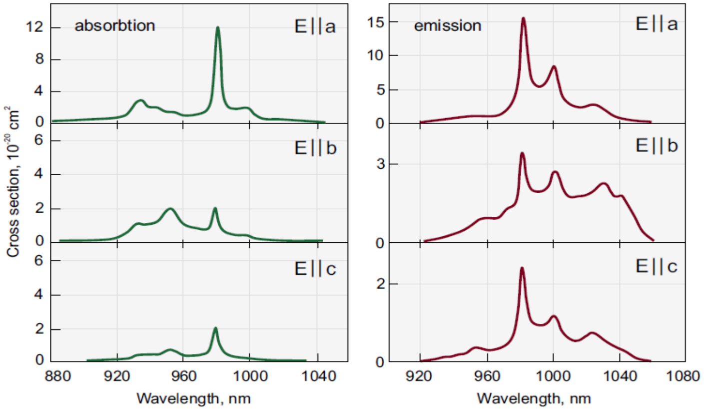


ytterbium fs oscillators – some designs

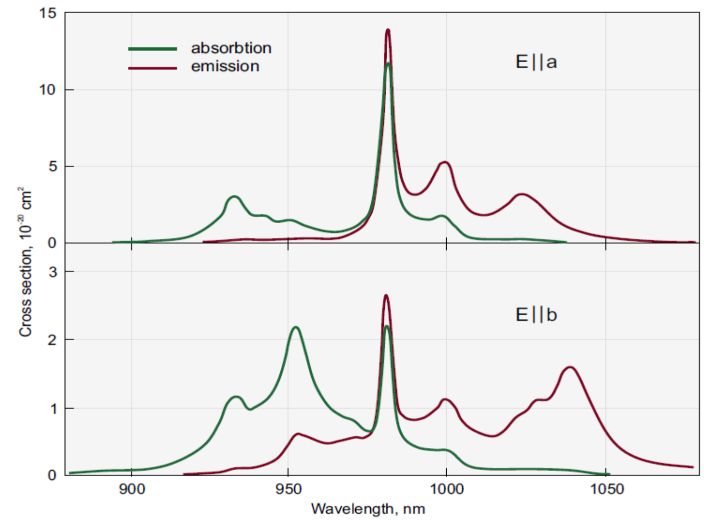
ytterbium doped crystals:

$Yb:KY(WO_4)_2$ - Yb:KYW

$Yb:KGd(WO_4)_2$ - Yb:KGW



Absorption and emission spectrae of Yb(5%):KGW

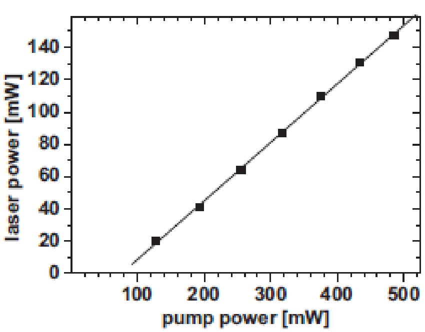


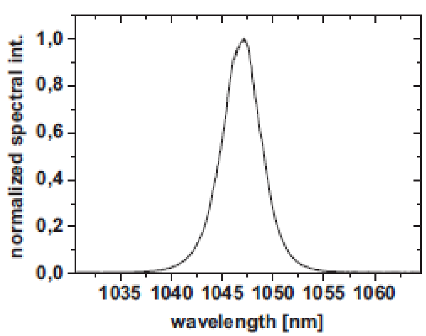
Absorption and emission spectra of Yb(5%):KYW

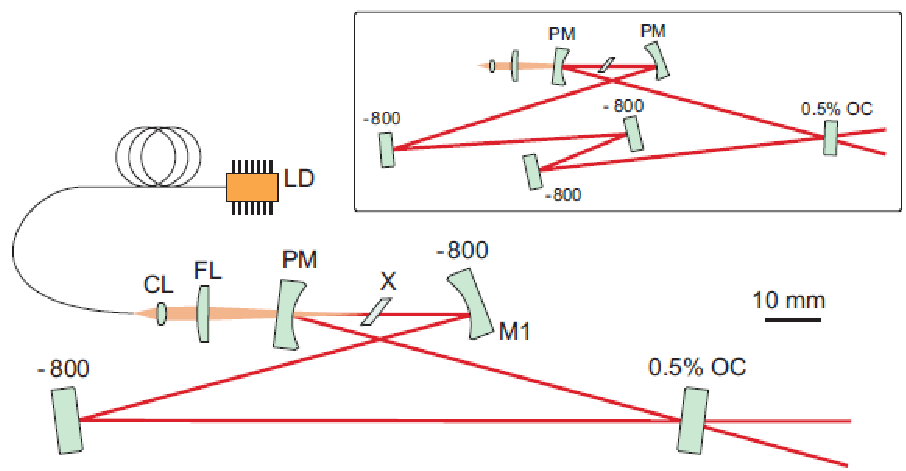
advantages:

- very small Stokes shift – little heat generated
- convenient pump wavelength – 980 nm (laser diodes)
- good quality commercial SESAMs are available

OE **17**, 5630-5635 (2009)







ytterbium fiber fs oscillators

SiO_2 glass fibers doped with Yb:

- classical single mode
- Photonic crystals fibers

pulsed regime is forced by:

- Nonlinear Polarization Evolution, NPE
- similaritons
- ANDi

Generation of 36-femtosecond pulses from a ytterbium fiber laser

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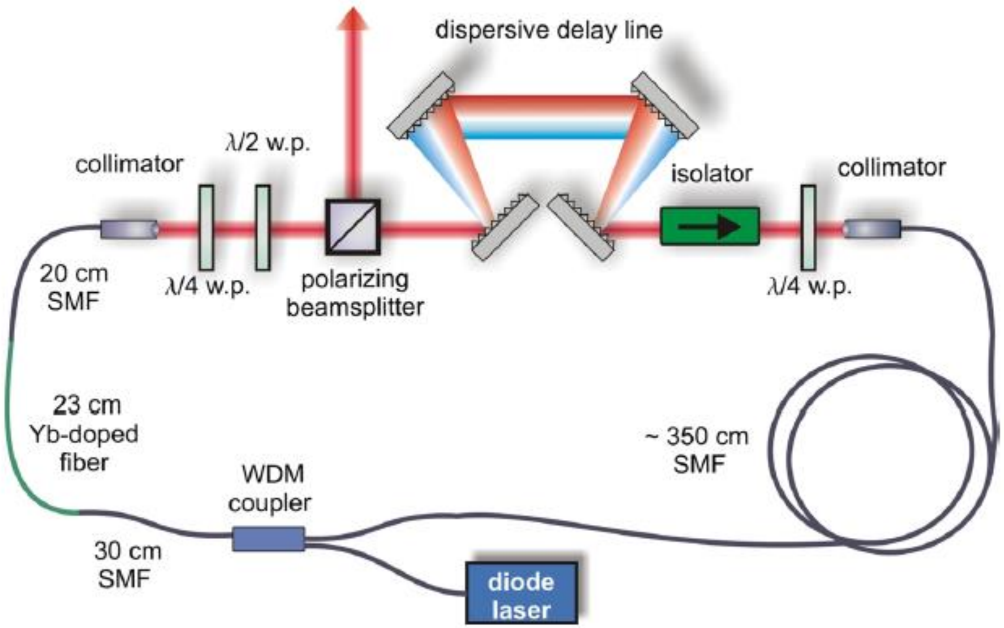
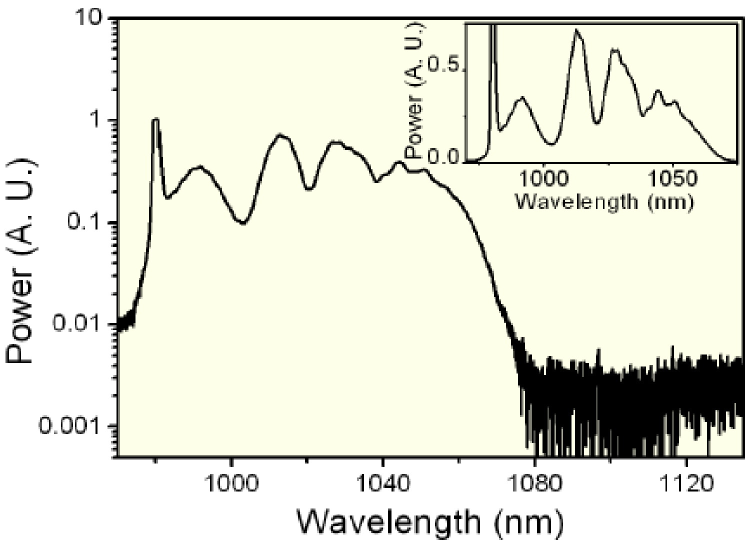




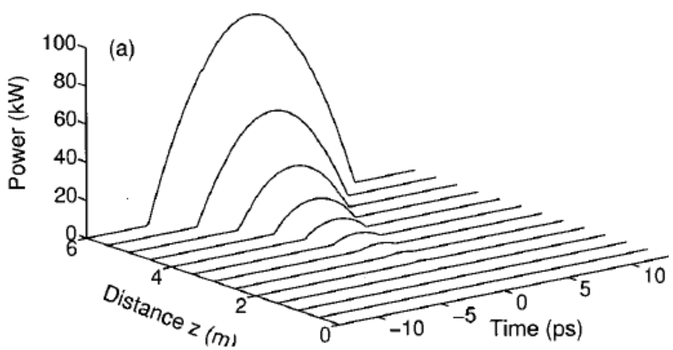
similariton

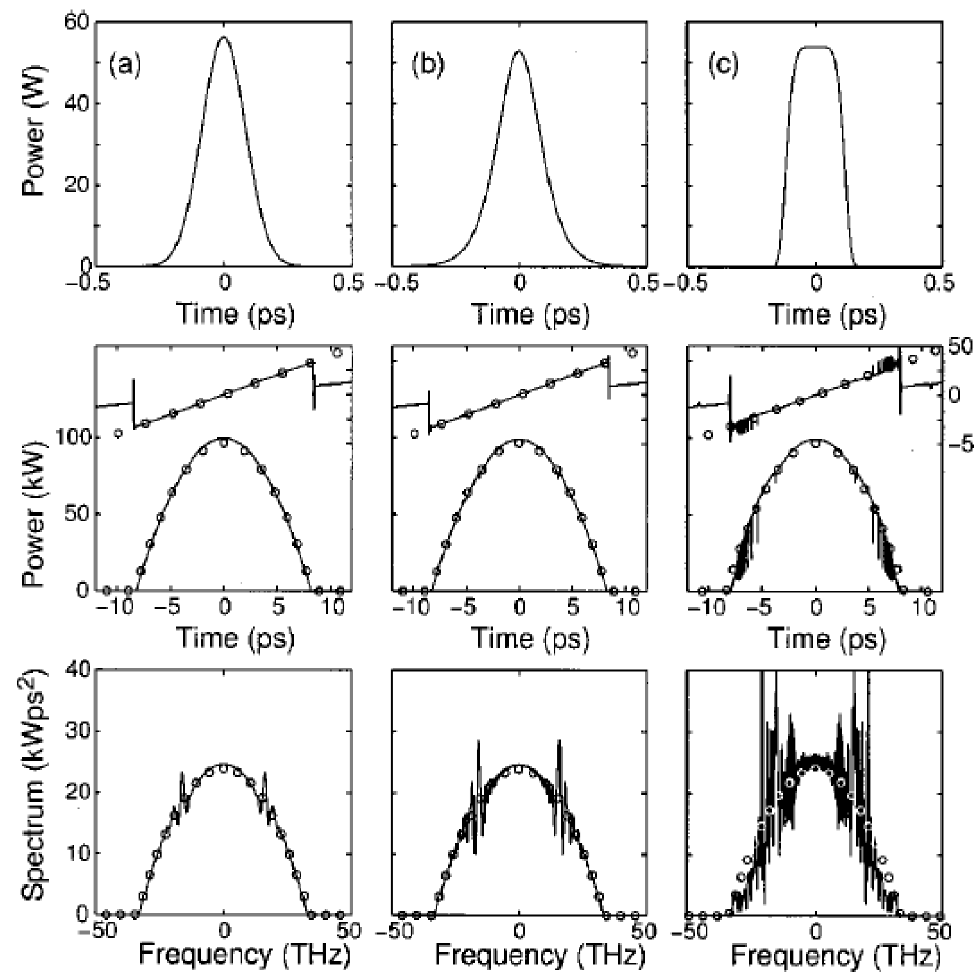
Self-similar propagation of parabolic pulses in normal-dispersion fiber amplifiers

Pulse propagation in a fiber is described by the nonlinear Schrodinger equation:

$$i\frac{\partial\Psi}{\partial z} = \underbrace{\frac{\beta_2}{2}\frac{\partial^2\Psi}{\partial T^2}}_{\substack{\text{dispersion}\\ \text{(the lowest term)}}} - \underbrace{\gamma|\Psi|^2\Psi}_{\substack{\text{self-phase}\\ \text{modulation}}} + \underbrace{i\frac{g}{2}\Psi}_{\text{gain}}$$

Gaussian pulse assumed at the input





ytterbium fiber fs oscillators - designs

All-normal-dispersion femtosecond fiber laser

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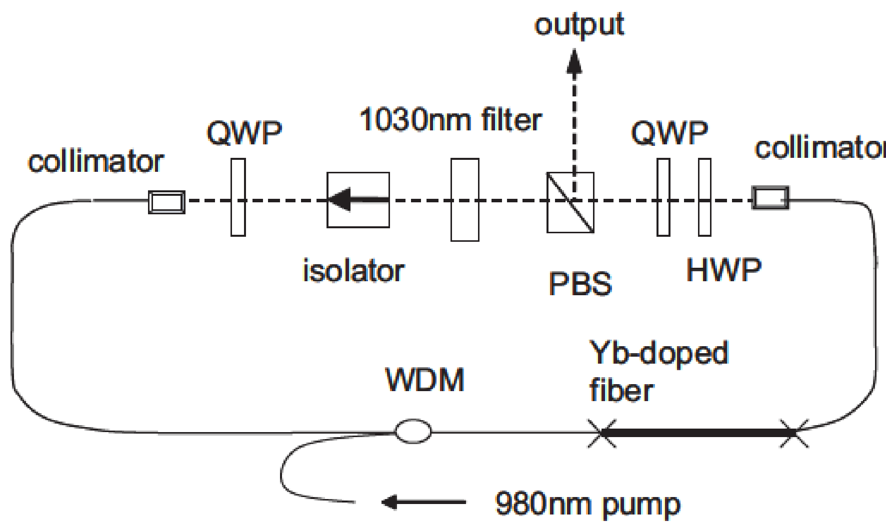
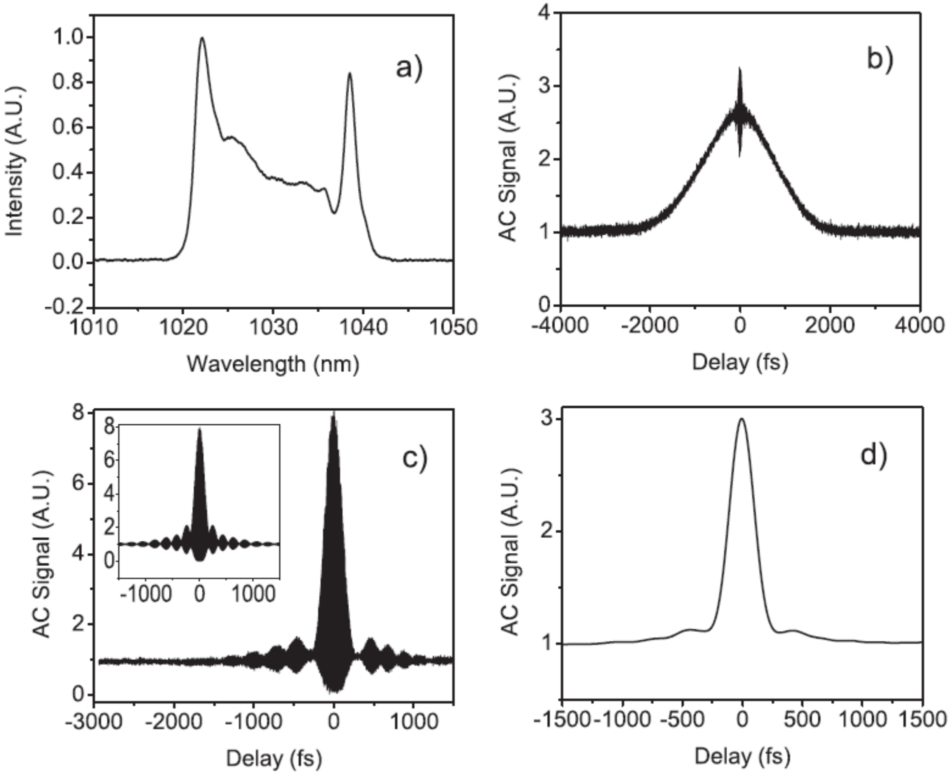
16 October 2006 / Vol. 14, No. 21 / OPTICS EXPRESS 10095

All-Normal-Dispersion (ANDi)
NPE method





170fs, 3 nJ

ytterbium fiber fs oscillators – designs, 2

3500 Vol. 40, No. 15 / August 1 2015 / Optics Letters

Optics Letters

Simple all-PM-fiber laser mode-locked with a nonlinear loop mirror

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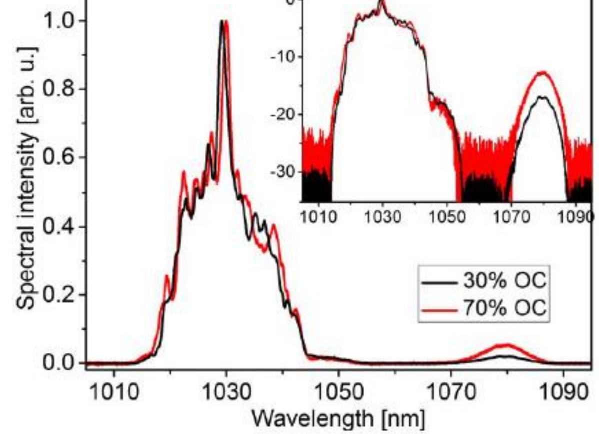
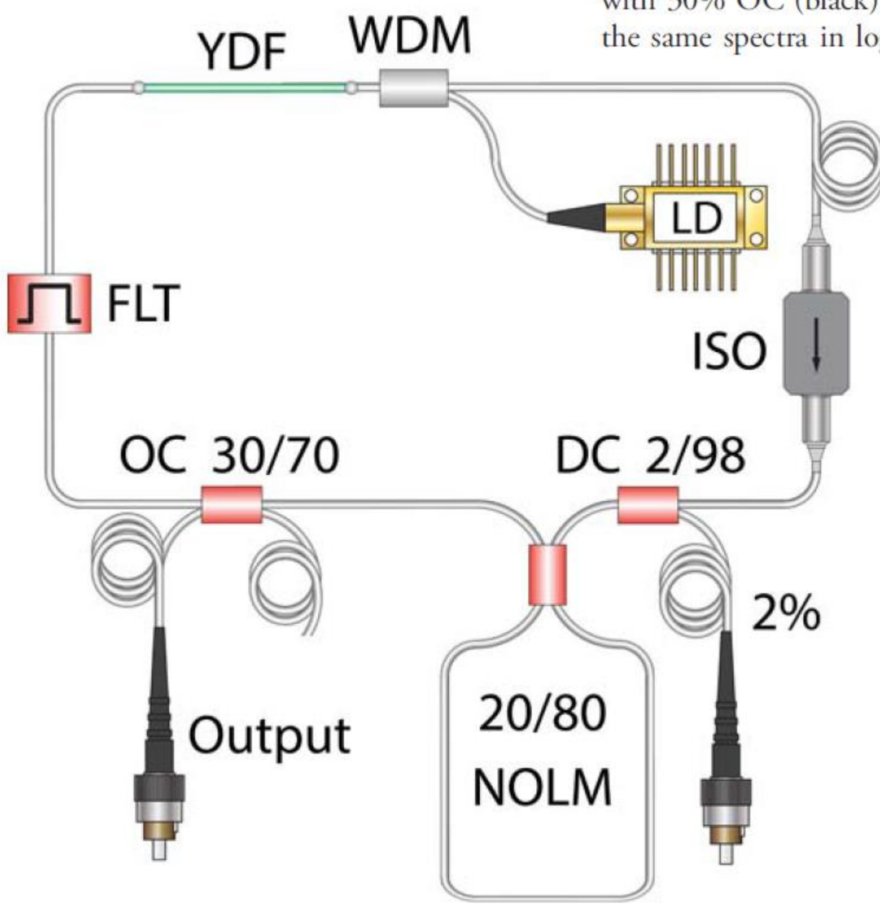
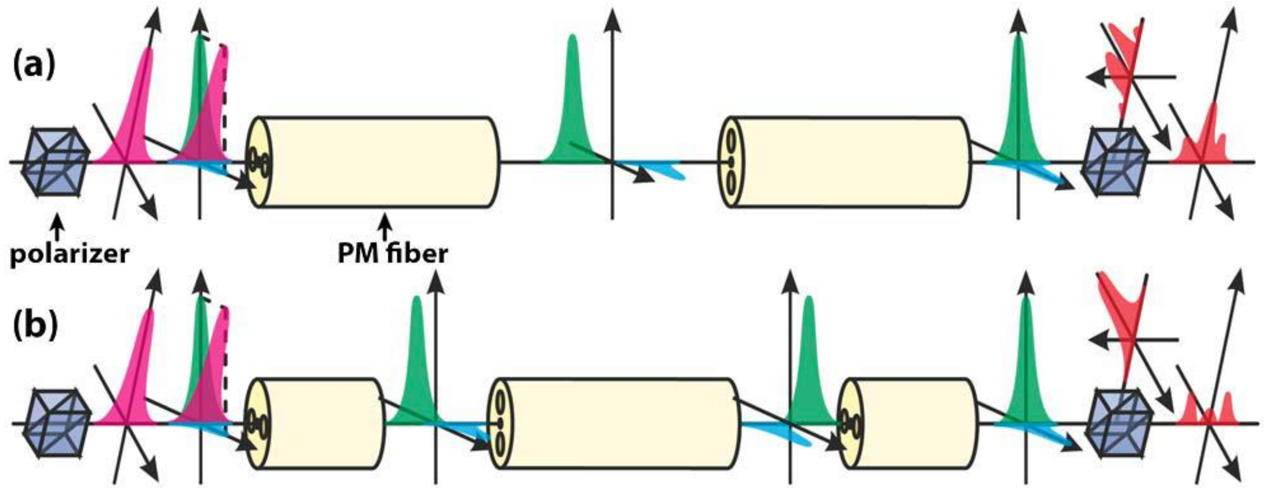


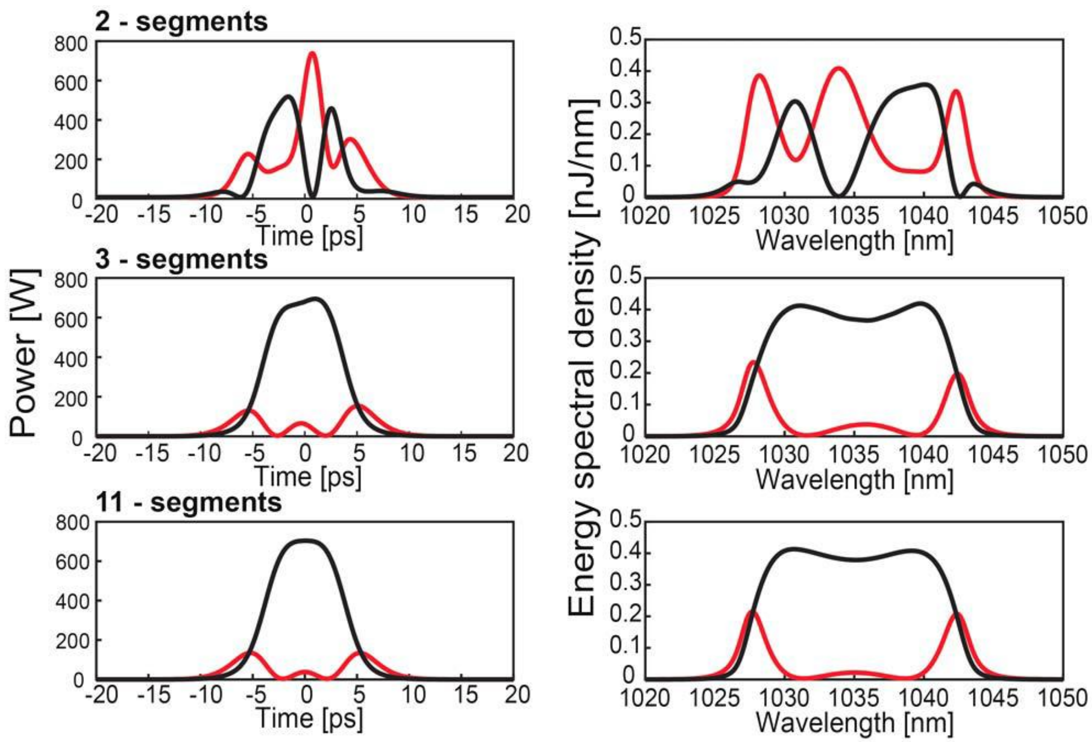
Fig. 3. Output spectra of two different laser designs—configuration with 30% OC (black) and configuration with 70% OC (red). Inset: the same spectra in logarithmic scale.



.5MHz
pulse = 145 fs

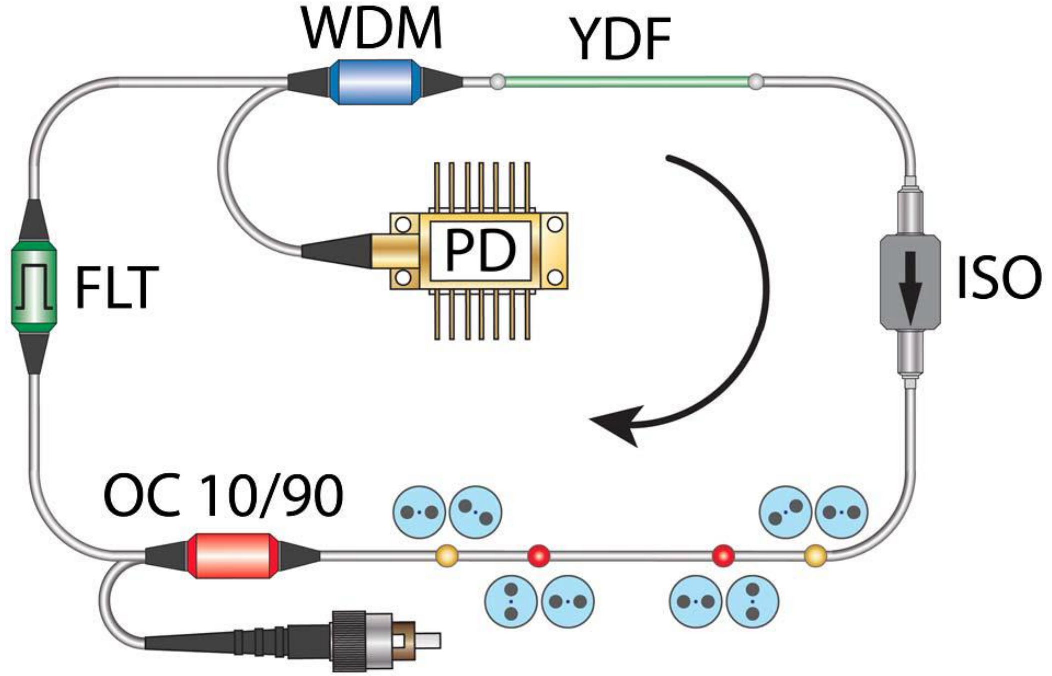
ytterbium fiber fs oscillators – designs, 3

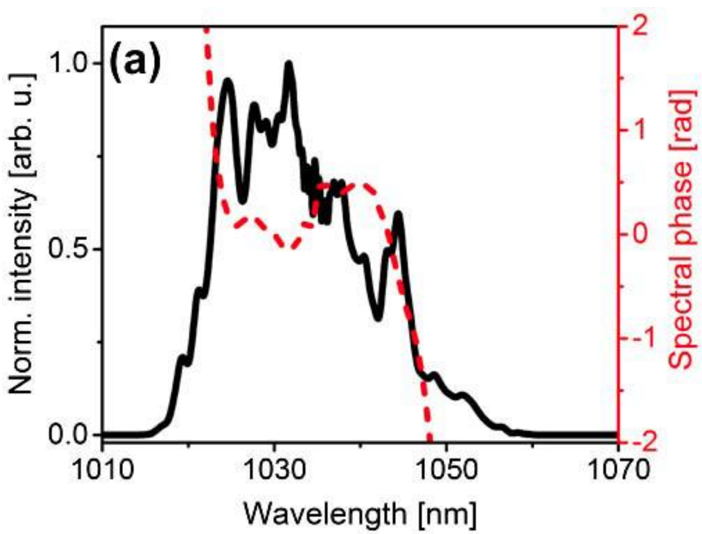


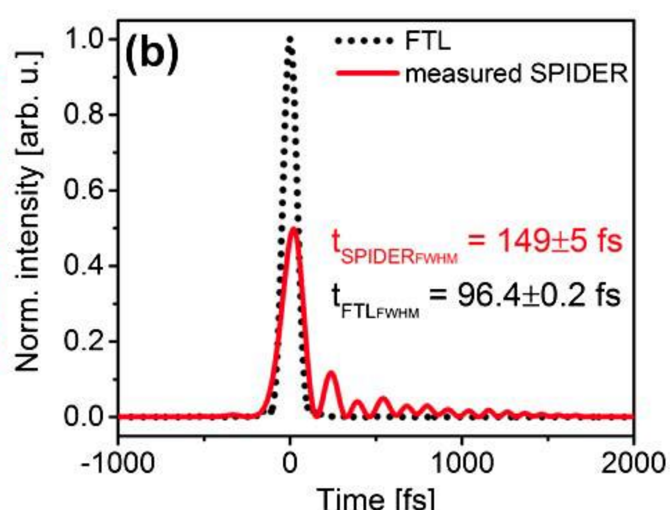


J. Szczepanek et al., *Ultrafast laser mode-locked using nonlinearpolarization evolution in polarization maintaining fibers*, Opt. Lett. 42,575-577 (2017)

ytterbium fiber fs oscillators – designs, 4







J. Szczepanek et al., *Ultrafast laser mode-locked using nonlinearpolarization evolution in polarization maintaining fibers*, Opt. Lett. 42,575-577 (2017)

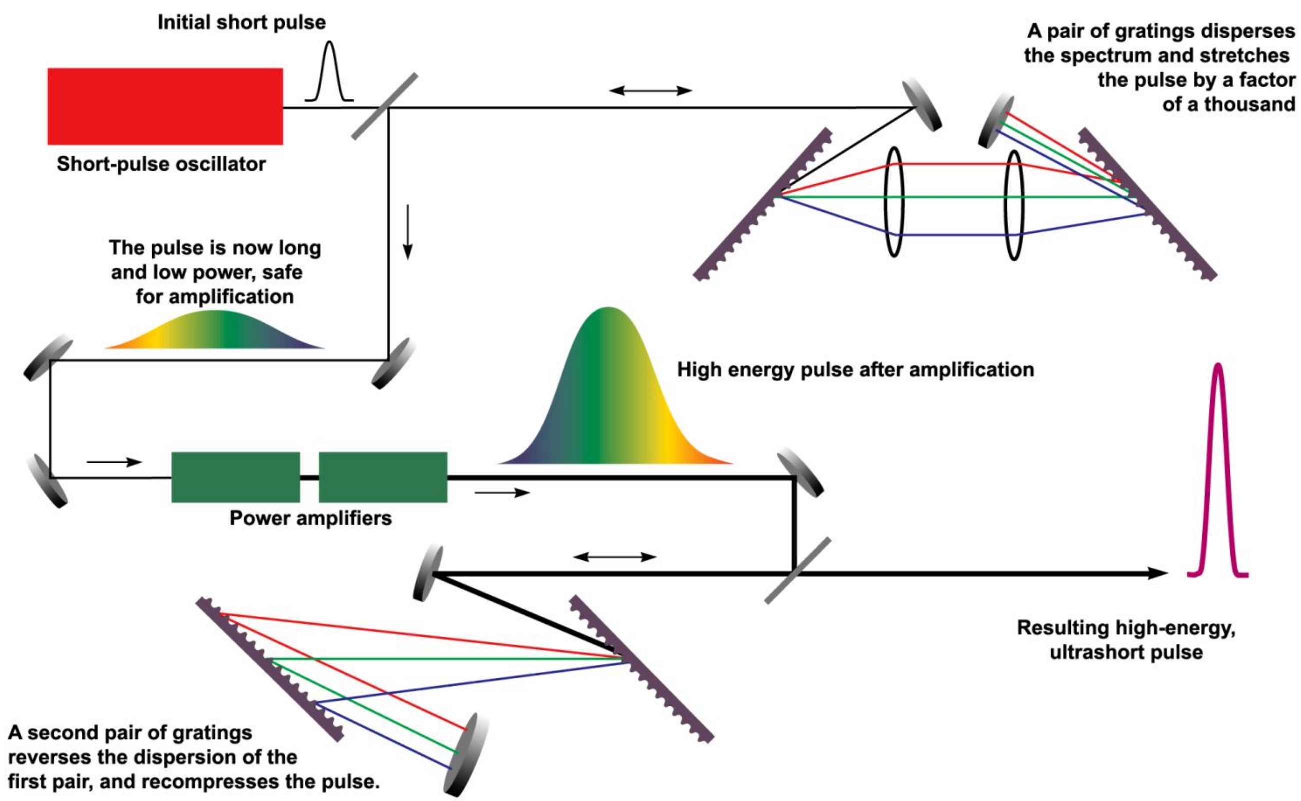
amplification femtosecond pulses

Issues specific for short pulse amplifiers:

- broad band
- high intensities – B inetgral
- high intensities - damage

$$B = \frac{2\pi}{\lambda} \int n_2(z) I(z) dz$$

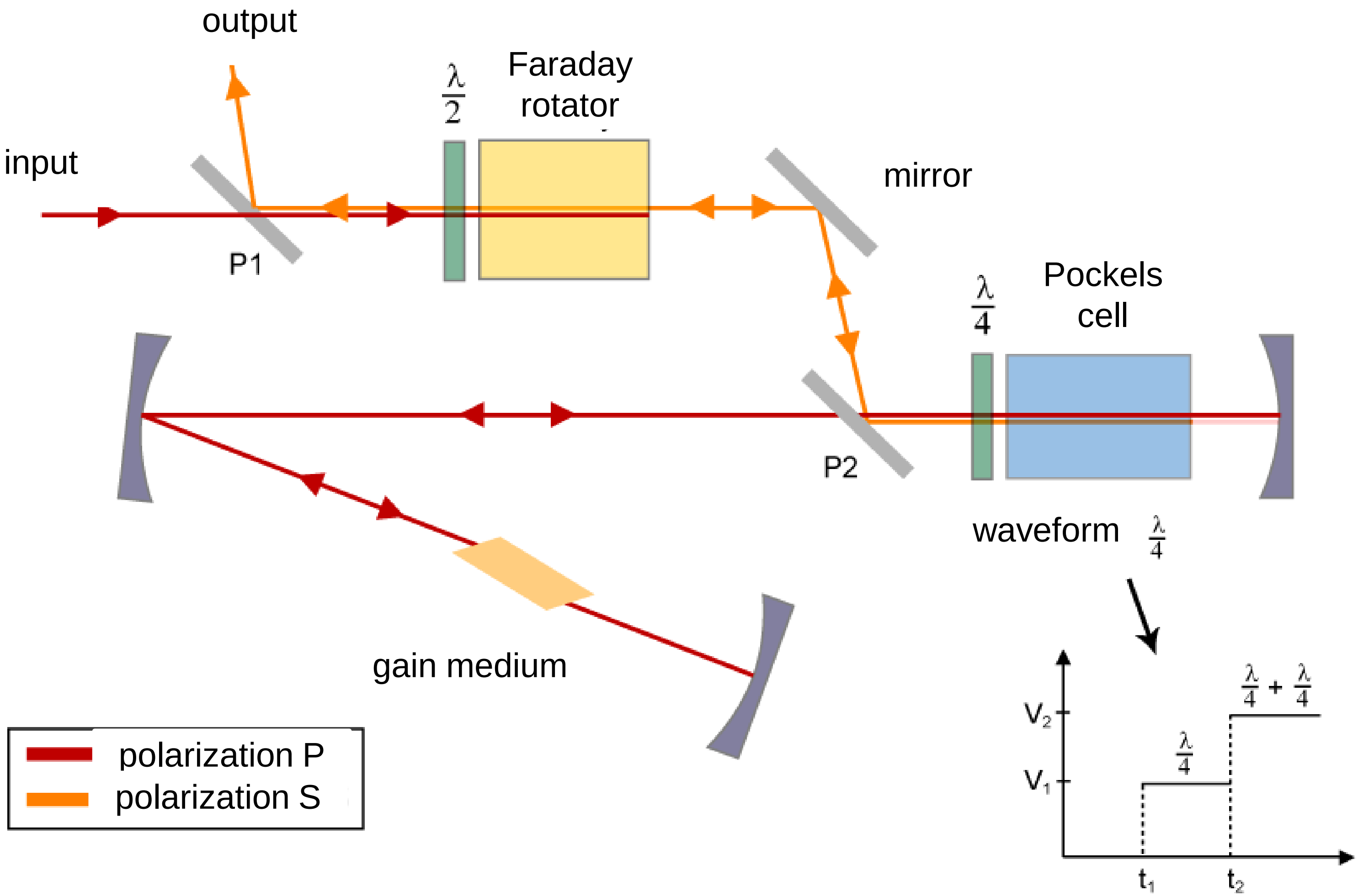
Chirped Pulse Amplification (CPA) technique
D. Strickland and G. Mourou, “Compression of amplified chirped optical pulses”, Opt. Commun. 56, 219 (1985)



nj → mJ, regenerative amplifier

idea:

- an amplifier with a cavity
- the amplified pulse is shorter than the round-trip time
- cycle: capture-amplify-extract



amplifier cavity modelling

- the q parameter equation:

$$q = \frac{Aq + B}{Cq + D}$$

with $\begin{bmatrix} A & B \\ C & D \end{bmatrix}$ - the matrix of elementary cell

G. Fox, T. Li, Resonant Modes in a Maser Interferometer,
Bell Syst. Tech. J. **40**, 453 (1961)

- solution:

$$\frac{1}{q} = -\frac{A - D}{2B} \pm i\frac{\sqrt{1 - \frac{1}{4}(A + D)^2}}{B}$$

- the resonator is stable when

$$-2 \leq A + D \leq 2$$

amplifier cavity modeling – fundamental mode size

method:

- elementary cell R3-R2-R1-R2-R3
- calculate q at mirror R3
- propagate the beam in the resonator till the mirror R1
- separate calculations for the tangential and sagittal planes

R1 = ∞

R2 = 3 m

R3 = 0.75 m

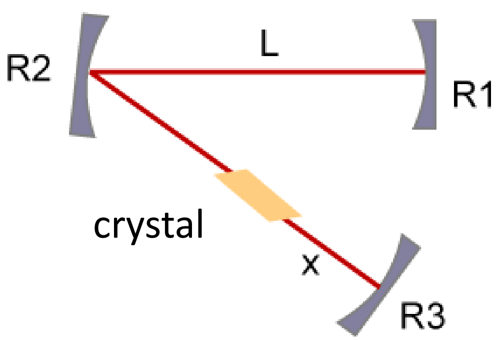
L = 60 cm

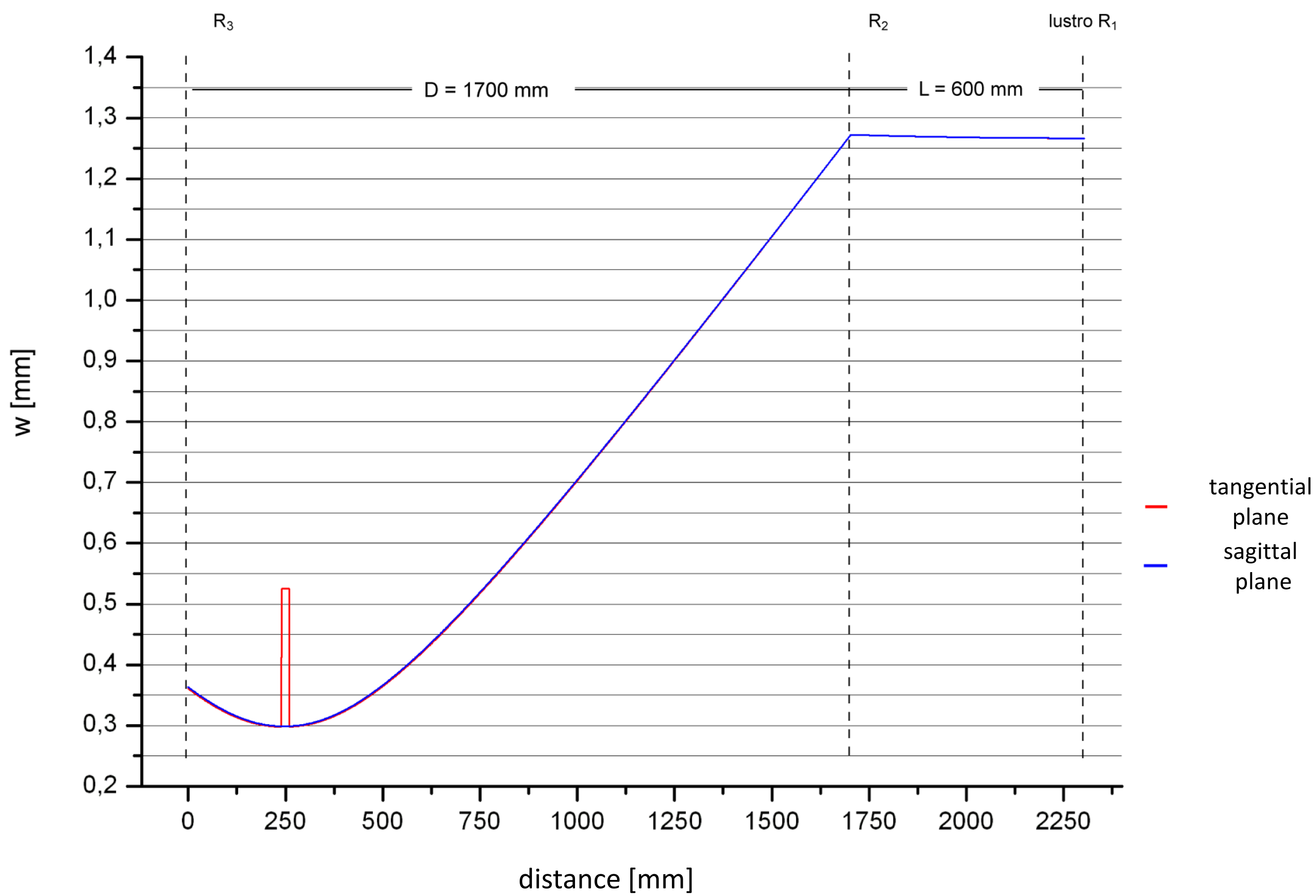
D = 170 cm

d = 2 cm

x = 24 cm

θ=2°50`



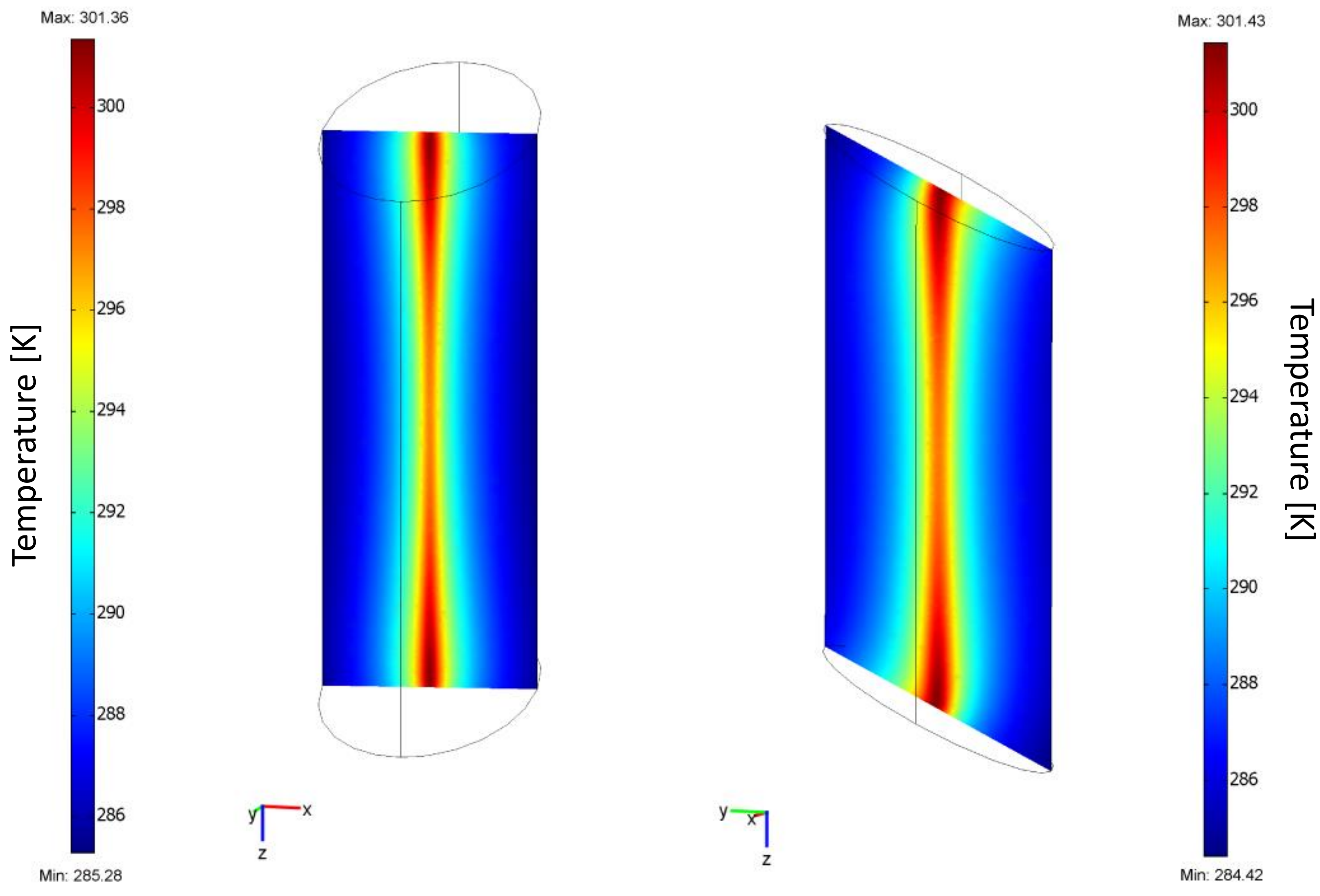


thermal effects in Ti:$Al_2 0_3$ crystal

- more than 34% of the pump power is converted into heat:
- $1 - \eta = \frac{h\nu_p - h\nu_l}{h\nu_p} \approx 0.34$ for $\lambda_p = 527$ nm i $\lambda_l = 800$ nm, η – quantum efficiency
- the heat is generated in a small volume (< 1 mm diameter)
- large temperature gradient
- **thermal lensing, stress-induced birefringence**

thermal effects in Ti:Al_2O_3 crystal, 2

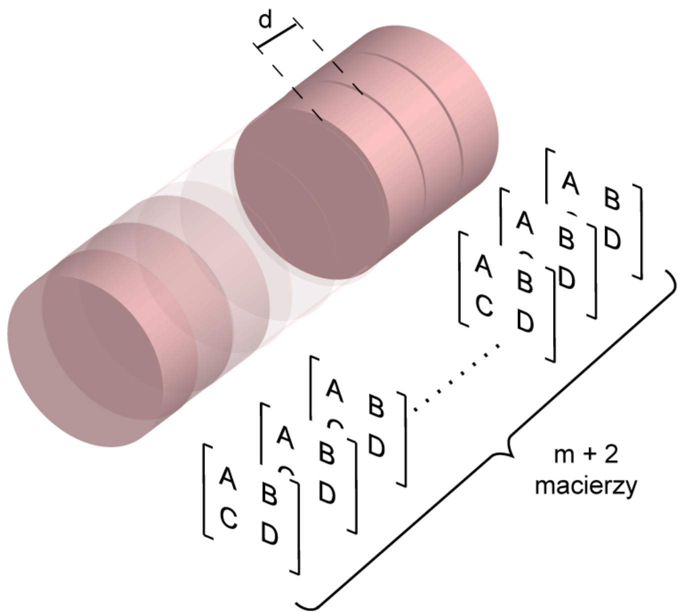
Finite Element Method (FEM) modeling – we know the spatial distribution of heat generated by the pump beam. We know the temperature at the cylindrical surface of the crystal (cooler). The crystal is pumped from both ends.



P = 50 W, w_x=w_y=0.3 mm, T_{cooler} = 283 K

thermal effects in $Ti:Al_2O_3$ crystal, 3

The optical effects are expressed in the ABCD matric language:



from experiment

$$\Delta n = \frac{\partial n}{\partial T}\Delta T + o(\Delta T^2)$$

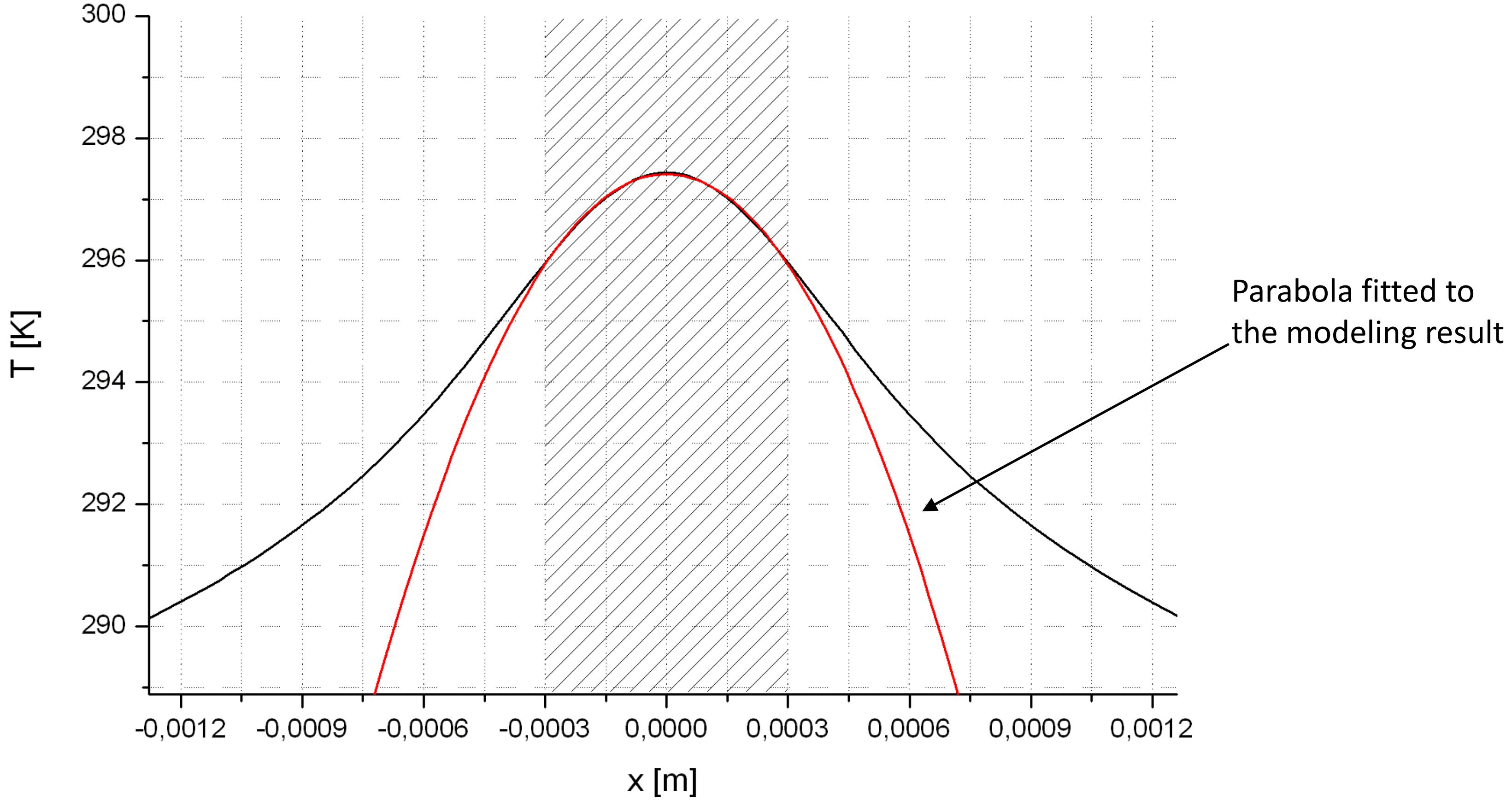
approximate Δn by a parabola

$$n(r) = n_0\left(1 - \frac{r^2}{2b^2}\right)$$

The matrix ABCD for a slice of thickness d:

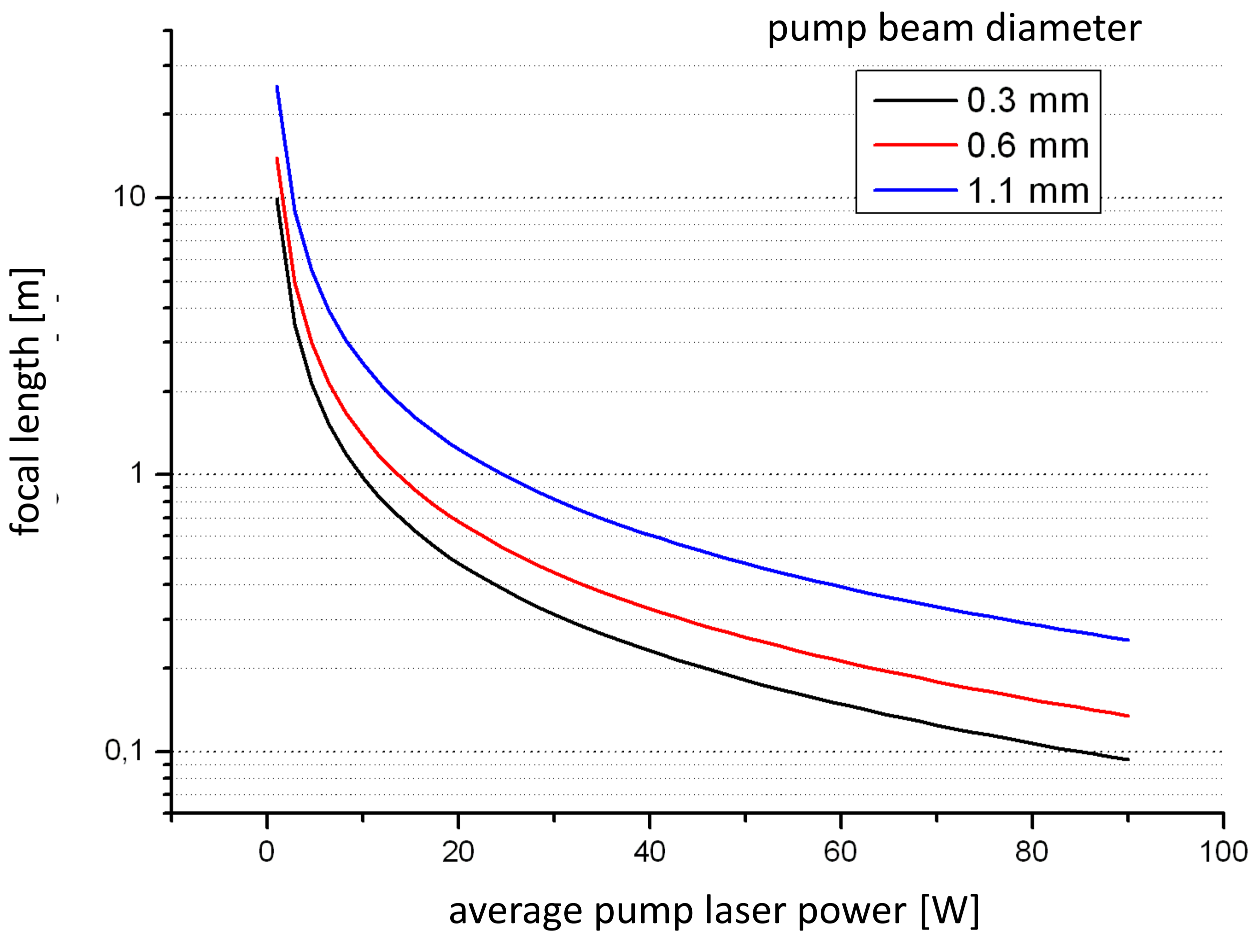
$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \cos\left(\frac{d}{b}\right) & b\sin\left(\frac{d}{b}\right) \\ -\frac{1}{b}\sin\left(\frac{d}{b}\right) & \cos\left(\frac{d}{b}\right) \end{bmatrix}$$

thermal effects in Ti:Al$_2$0$_3$ crystal, 4



thermal effects in Ti:$Al_2 0_3$ crystal, 5

results:



water cooling approx. 283 K

thermal effects in Ti:$Al_2 0_3$ crystal, 6

what happens if we cool the crytsal?

thermal conductivity:

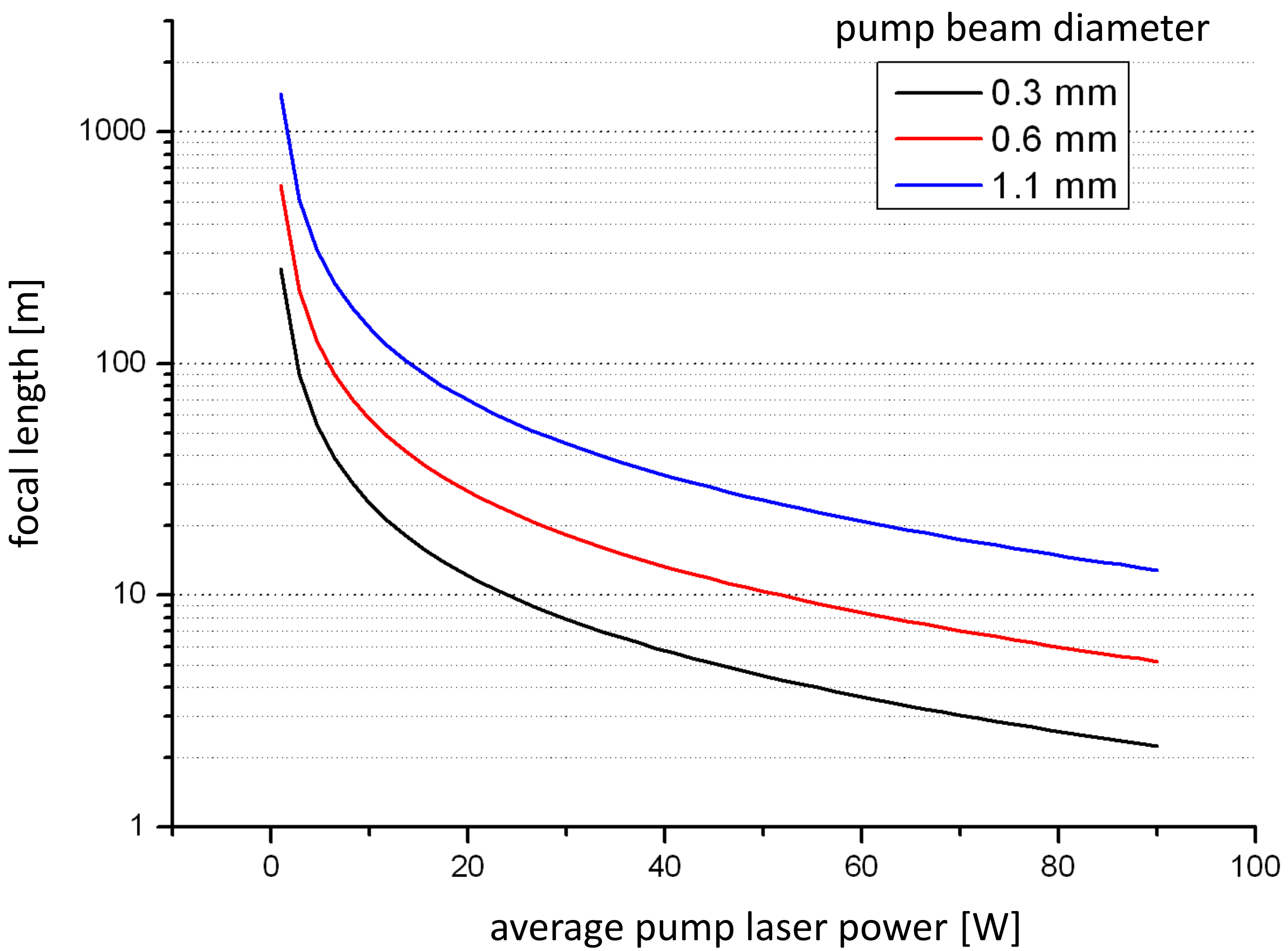
28.6 W·m^{-1}·K^{-1} at 300 K

1094 W·m^{-1}·K^{-1} at 77 K

$$\frac{dn}{dT} = 9.87 \cdot 10^{-6}\ \mathrm{K}^{-1} \text{ at } 300\ \mathrm{K}$$

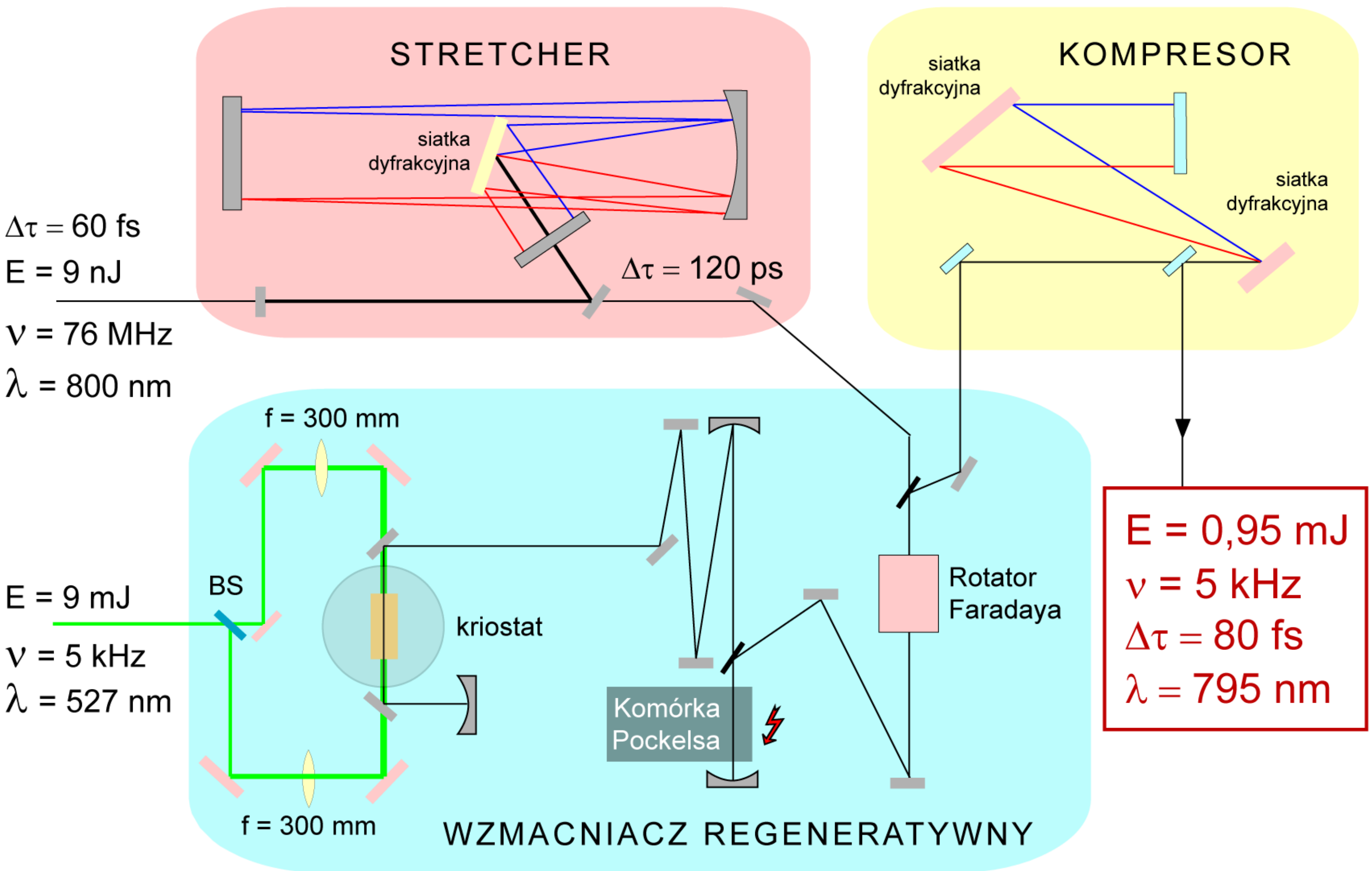
$$\frac{dn}{dT} = 4.48 \cdot 10^{-6}\ \mathrm{K}\text{-}1 \text{ at } 77\ \mathrm{K}$$

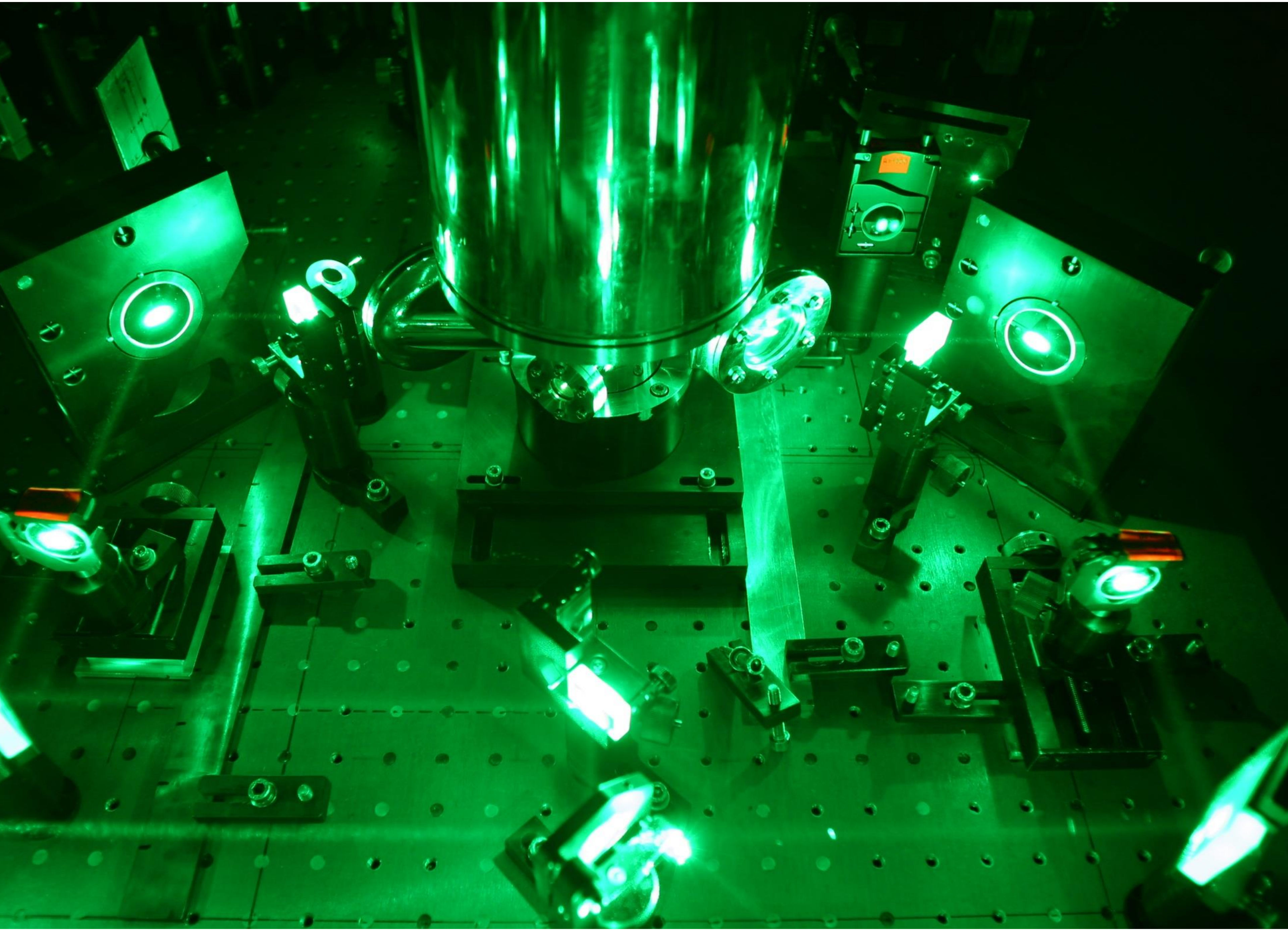
thermal effects in Ti:$Al_2 0_3$ crystal, 6



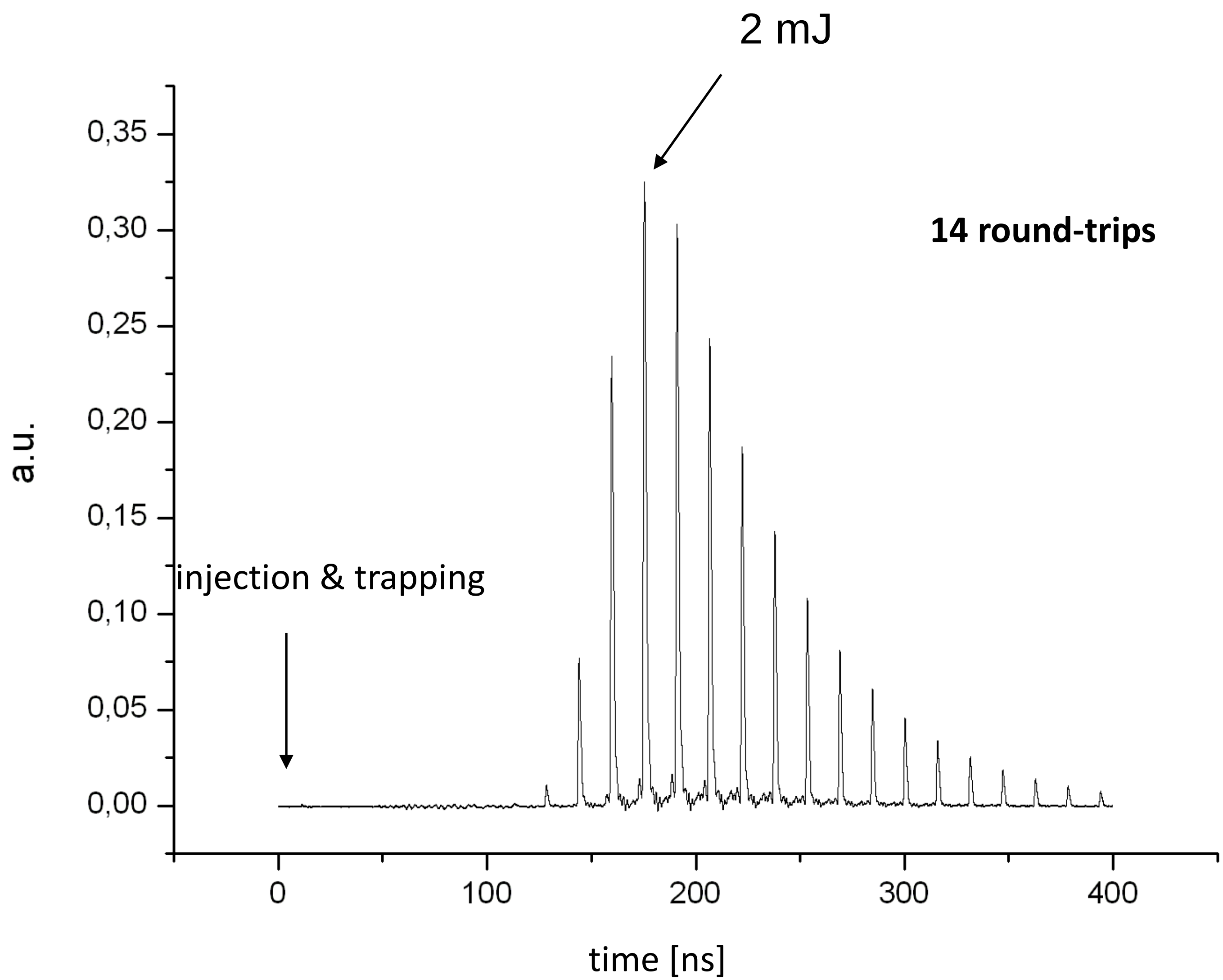
liquid nitrogen
cooling
approx. 283 K

experimental set-up

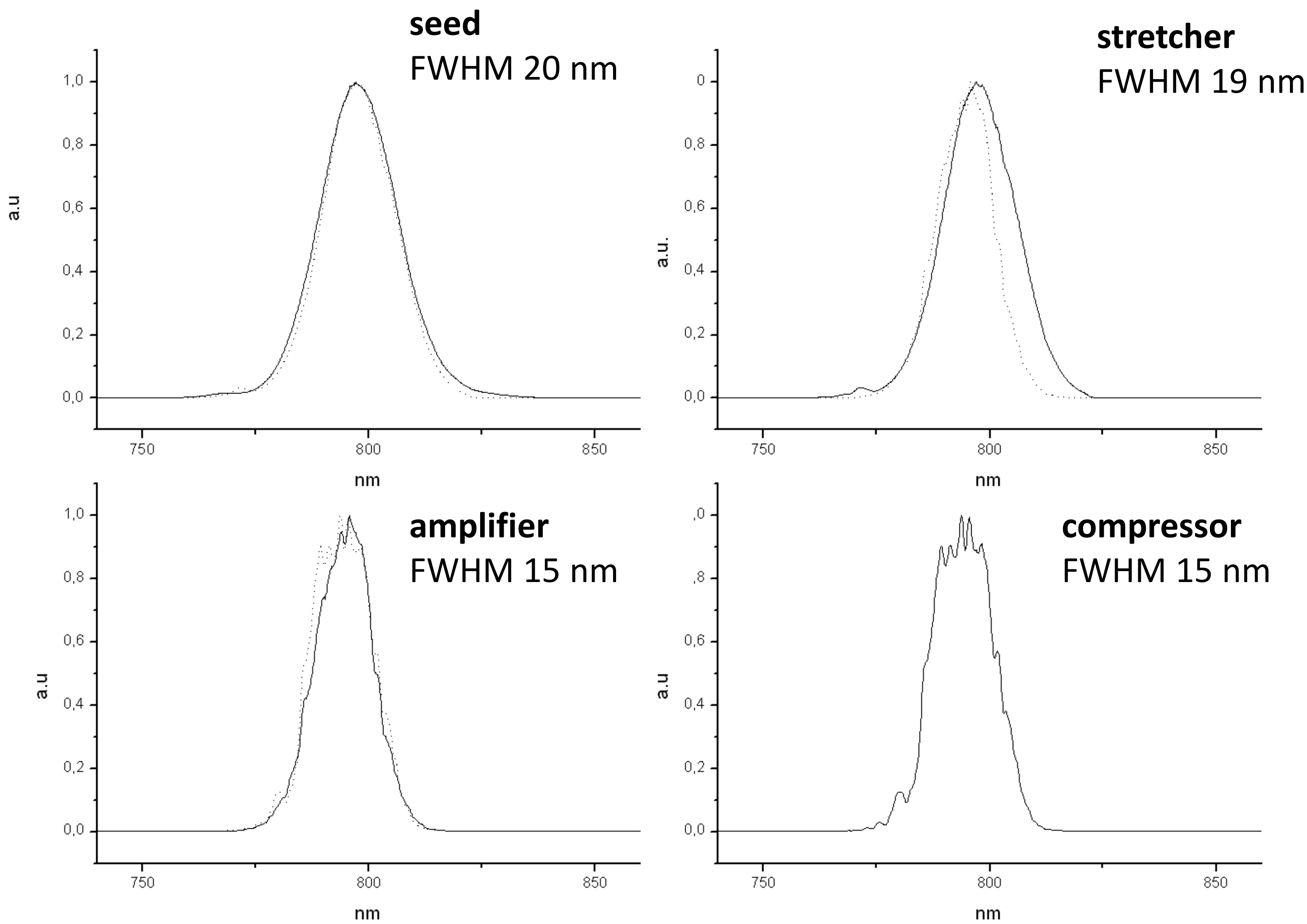




pulse dynamics inside the cavity



spectrum


seed
FWHM 20 nm
stretcher
FWHM 19 nm
amplifier
FWHM 15 nm
compressor
FWHM 15 nm
a.u
a.u.
0,0
0,2
0,4
0,6
0,8
1,0
750
800
850
nm