Lasers
lecture 8

Czesław Radzewicz

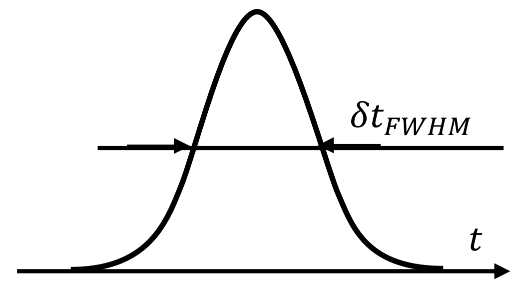
Gaussian pulses

note: do not mistake those for Gaussian beams

a light pulse with a Gaussian envelope $E(t) = Ae^{-at^2}e^{i\omega_0 t}$ $(a > 0)$ can be modified by adding a quadratic phase

$E(t) = Ae^{-at^2}e^{i\omega_0 t + ibt^2} = Ae^{-\Gamma t^2}e^{i\omega_0 t}$, with a single complex parameter $\Gamma = a - ib$ describing both the envelope and nonlinear phase.

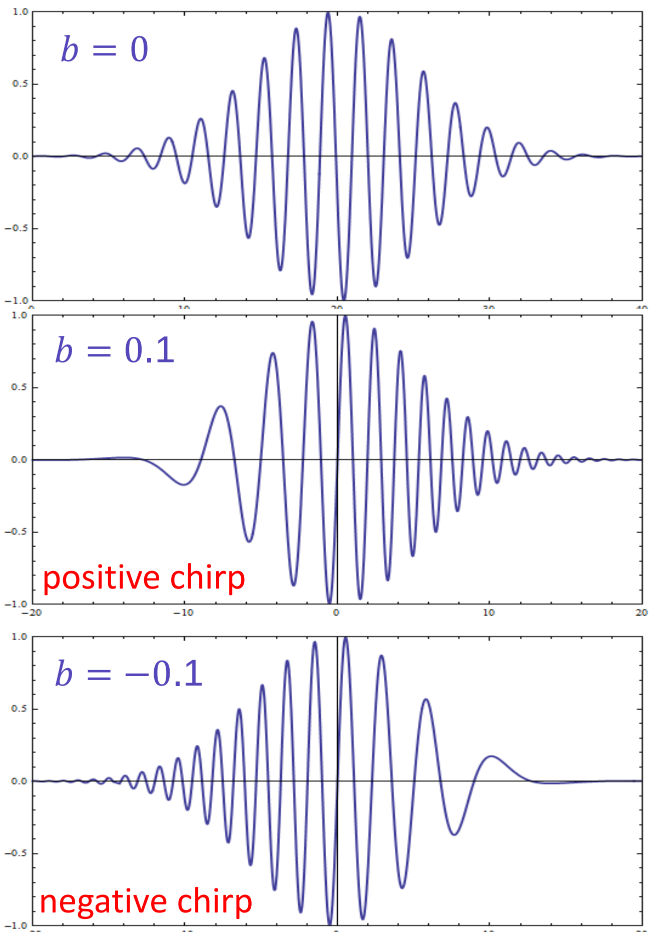
intensity: $I = |E|^2 = A^2 e^{-2at^2}$



$$\delta t(FWHM) = \sqrt{\frac{2\ln 2}{a}}$$

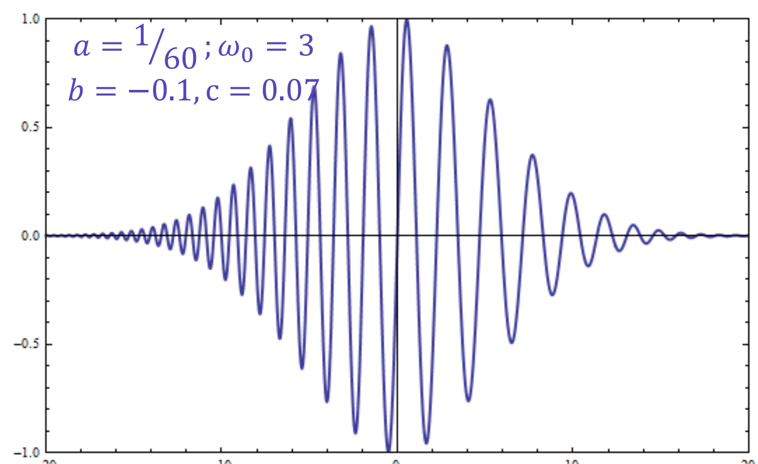
FWHM - Full Width at Half Maximum

$a = {}^{1}/_{60}\,; \omega_0 = 3$



phase and frequency: $\varphi = \omega_0 t + bt^2$, $\omega(t) \equiv \frac{d\varphi}{dt} = \omega_0 + \underbrace{2bt}_{\text{linear chirp}}$

an example of a nonlinear chirp $E(t) = Ae^{-at^2}e^{i(\omega_0 t + bt^2 + ct^3)}$



Gaussian pulses, 2

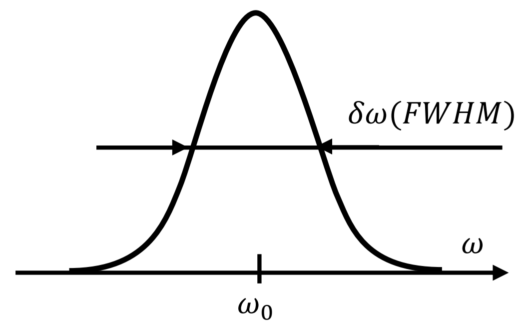
$$\tilde{E}(\omega) \equiv \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} E(t) e^{-i\omega t} dt = \frac{A}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\Gamma t^2} e^{-i(\omega-\omega_0)t} dt$$

lemma: for any complex P, Q if $\mathrm{Re}P > 0$ then

$$\int_{-\infty}^{\infty} e^{-Py^2-2Qy} dy = \sqrt{\frac{\pi}{P}} e^{Q^2/2P}$$

$$\tilde{E}(\omega) = \frac{A}{\sqrt{2\Gamma}} e^{\frac{-(\omega-\omega_0)^2}{4\Gamma}}$$

and thus $\mathrm{I}(\omega) = \left|\tilde{E}(\omega)\right|^2 = \frac{A^2}{2|\Gamma|} e^{\frac{-(\omega-\omega_0)^2}{2|\Gamma|}}$



$$\delta\omega(FWHM) = 2\sqrt{2\ln 2}\sqrt{a^2+b^2}$$

the product of time and frequency uncertainties:

$$\delta t \cdot \delta\omega = 4\ln 2\sqrt{1+(b/a)^2}$$

$$\delta t \cdot \delta\nu = \frac{2\ln 2}{\pi}\sqrt{1+\left(\frac{b}{a}\right)^2} \cong 0.44\sqrt{1+\left(\frac{b}{a}\right)^2}$$

if $b = 0$ we have Fourier limited pulses; their spectra width results solely from finite time duration

note: other envelope shapes result in a slightly different Fourier limit $\delta t \cdot \delta\nu = K$

Shape	$\varepsilon(t)$	K
Gaussian function	$\exp[-(t/t_0)^2/2]$	0.441
Exponential function	$\exp[-(t/t_0)/2]$	0.140
Hyperbolic secant	$1/\cosh(t/t_0)$	0.315
Rectangle	–	0.892
Cardinal sine	$\sin^2(t/t_0)/(t/t_0)^2$	0.336
Lorentzian function	$[1+(t/t_0)^2]^{-1}$	0.142

propagation of a Gaussian pulse in a dispersive system

example: propagation in a medium with a given $n(\omega)$:

$$\tilde{E}(\omega,0) = \frac{A}{\sqrt{2\Gamma}} e^{\frac{-(\omega-\omega_0)^2}{4\Gamma}}$$

$$\tilde{E}(\omega,z) = \tilde{E}(\omega,0) e^{-ik(\omega)z}$$

if k varies slowly in the range of the pulse spectrum then we can write it as a Taylor series up to the quadratic term:

$$k(\omega) = k_0 + k_1(\omega-\omega_0) + \tfrac{1}{2}k_2(\omega-\omega_0)^2 + \cdots; \qquad k_0 = k(\omega_0),\ k_1 = {}^{dk}\!/_{d\omega}\big|_{\omega_0},\ \ k_2 = {}^{d^2k}\!/_{d\omega^2}\big|_{\omega_0}$$

which leads to

$$\tilde{E}(\omega,z) = \tilde{E}(\omega,0) e^{-i\left[k_0 z + k_1 z(\omega-\omega_0) + k_2 z(\omega-\omega_0)^2/2\right]}$$

back to the time domain:

$$E(t,z) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \tilde{E}(\omega,z)\, e^{i\omega t} d\omega = \frac{A}{\sqrt{4\pi\Gamma}} e^{i(\omega_0 t - k_0 z)} \int_{-\infty}^{\infty} e^{\underbrace{-\left[\frac{1}{4\Gamma}+i\frac{k_2 z}{2}\right](\omega-\omega_0)^2}_{\frac{-(\omega-\omega_0)^2}{4\Gamma'}} + i(\omega-\omega_0)t}\, d\omega$$

we still have a Gaussian pulse with a new Γ' parameter

$$\boxed{\frac{1}{\Gamma'} = \frac{1}{\Gamma} + i2k_2 z}$$

we can calculate k_1 and k_2:

$$k = \frac{n(\omega)\omega}{c}$$

$$k_1 \equiv \frac{dk}{d\omega} = \frac{n + \omega\frac{dn}{d\omega}}{c}$$

$$k_2 \equiv \frac{d^2k}{d\omega^2} = \frac{2\frac{dn}{d\omega} + \omega\frac{d^2n}{d\omega^2}}{c}$$

propagation of a Gaussian pulse in a dispersive system, 2

general rule: for a system which has a given spectral phase $\beta(\omega)$:

$$\tilde{E}_{in}(\omega) = \frac{A}{\sqrt{2\Gamma}} e^{\frac{-(\omega-\omega_0)^2}{4\Gamma}}$$

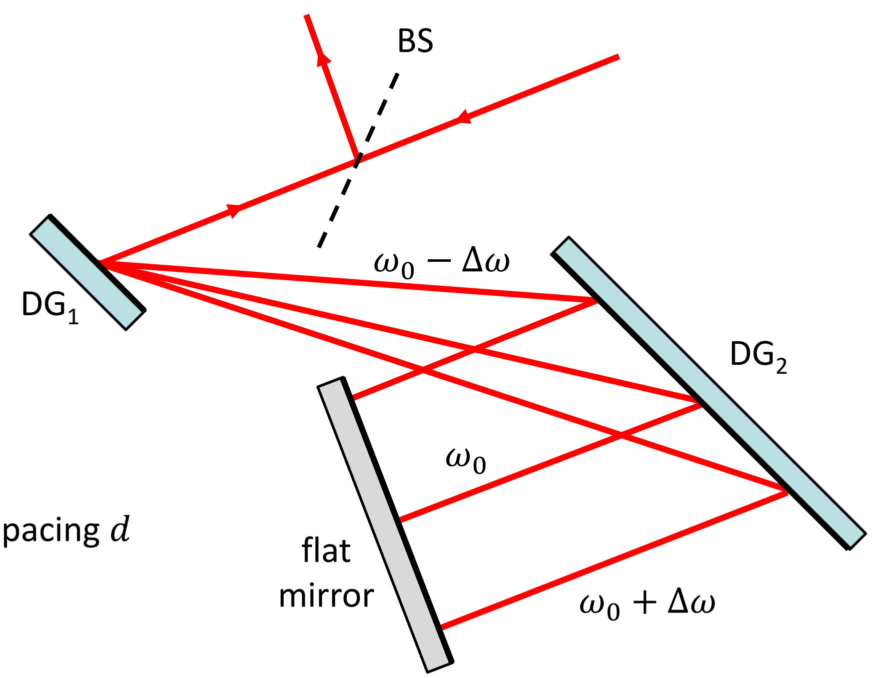
$$\tilde{E}_{out}(\omega) = \tilde{E}(\omega, 0) e^{-i\beta(\omega)}$$

Again, if β varies slowly in the range of the pulse spectrum then we can write it as a Taylor series up to the quadratic term … and we end up with a Gaussian pulse described by a new parameter Γ'; $\frac{1}{\Gamma'} = \frac{1}{\Gamma} + i2\beta_2$, with

$$\boxed{\beta_2 = d^2\beta/d\omega^2|_{\omega_0}}$$

some examples of optical system with non-trivial $\beta(\omega)$:

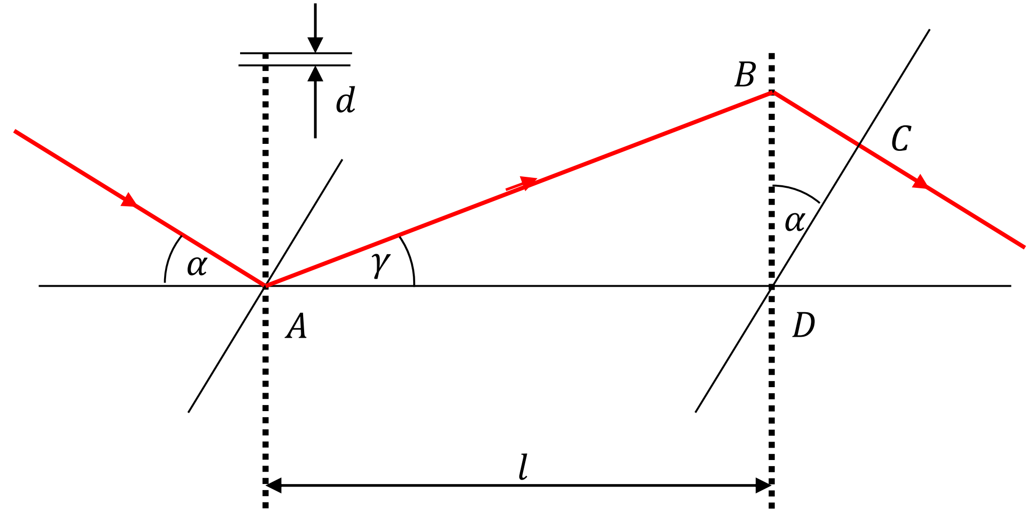
diffraction grating compressor, optical path is frequency dependent.



for a diffraction grating with grove spacing d

$$\sin\alpha + \sin\beta = n\frac{\lambda}{d}$$

diffraction grating compressor



first order diffraction: $\sin\alpha + \sin\gamma = {}^{\lambda}/_{d} = 2\pi \frac{c}{d}\frac{1}{\omega} \Rightarrow \sin\gamma = 2\pi \frac{c}{d}\frac{1}{\omega} - \sin\alpha$

phase (definition): $\beta(\omega) = \frac{\omega}{c} L(\omega)$

with $L(\omega)$ being the optical path: $P(\omega) = AB + BC = \cdots = \frac{l}{\cos\gamma}(1 + \sin\gamma\sin\alpha)$

$P(\omega + d\omega) = \frac{l}{\cos\gamma'}(1 + \sin\gamma'\sin\alpha)$ with a new angle γ' such that: $\sin\gamma' = 2\pi \frac{c}{d}\frac{1}{\omega + d\omega} - \sin\alpha$

for small $d\omega$: $\sin\gamma' \cong \sin\gamma - \frac{2\pi c}{d}\frac{d\omega}{\omega^2}$

$$P(\omega + d\omega) - P(\omega) = \frac{l}{\cos'\gamma}(1 + \sin\gamma'\sin\alpha) - \frac{l}{\cos\gamma}(1 + \sin\gamma\sin\alpha) \cong -2\pi \frac{\sin\alpha}{\cos\gamma}\frac{lc}{d}\frac{d\omega}{\omega^2}$$

$$\frac{dP}{d\omega} \cong -2\pi \frac{\sin\alpha}{d\cos\gamma}\frac{lc}{\omega^2}$$

$$\boxed{\frac{d^2P}{d\omega^2} \cong \frac{4\pi\sin\alpha}{d\cos\gamma}\frac{lc}{\omega^3}}$$

full calculations in : Tracey, IEEE, J. Quant. Electron. QE-5,454 (1969)

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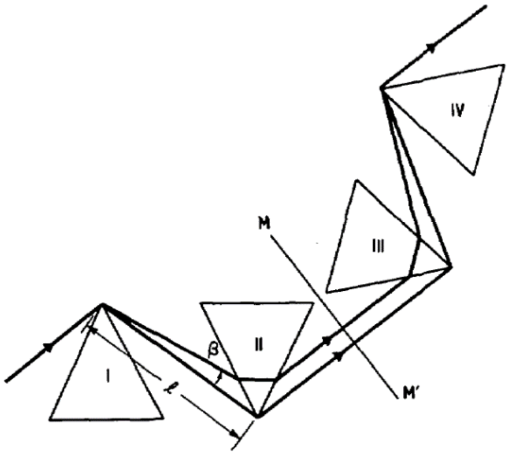
prismatic compressor



Negative dispersion using pairs of prisms

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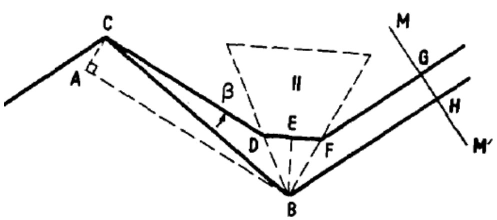
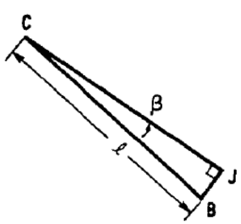
We show that pairs of prisms can have negative group-velocity dispersion in the absence of any negative material dispersion. A prism arrangement is described that limits losses to Brewster-surface reflections, avoids transverse displacement of the temporally dispersed rays, permits continuous adjustment of the dispersion through zero, and yields a transmitted beam collinear with the incident beam.





general formula (P is optical path):

$$\frac{d^2P}{d\lambda^2} = \left[\frac{d^2n}{d\lambda^2}\frac{d\beta}{dn} + \left(\frac{dn}{d\lambda}\right)^2 \frac{d^2\beta}{dn^2}\right]\frac{dP}{d\beta} + \left(\frac{dn}{d\lambda}\right)^2 \left(\frac{d\beta}{dn}\right)^2 \frac{d^2P}{d\beta^2}\,.$$

is simplified upon assumption of Brewster prisms and minimum deviation condition:

$$\frac{d^2P}{d\lambda^2} = 4l\left\{\left[\frac{d^2n}{d\lambda^2} + \left(2n - \frac{1}{n^3}\right)\left(\frac{dn}{d\lambda}\right)^2\right]\sin\beta - 2\left(\frac{dn}{d\lambda}\right)^2\cos\beta\right\}.$$

propagation of a Gaussian pulse in a dispersive system (time domain):

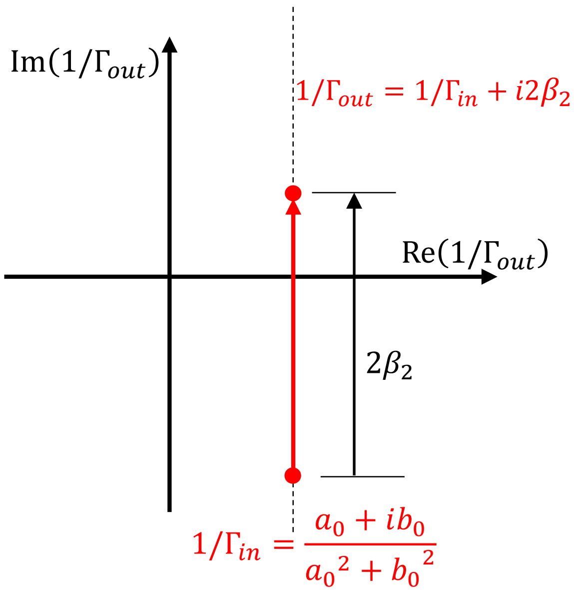
a given spectral phase $\beta(\omega)$ leads to:

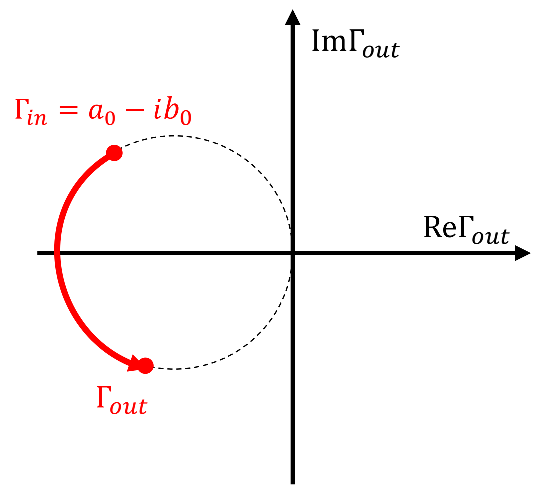
$$\frac{1}{\Gamma_{out}} = \frac{1}{\Gamma_{in}} + i2\beta_2$$
$$\beta_2 = d^2\beta/d\omega^2\Big|_{\omega_0}$$

let's use the notation: $\Gamma_{in} = a_0 - ib_0, \Gamma_{out} = a - ib$

$$\frac{1}{\Gamma_{out}} = \frac{1}{\Gamma_{in}} + i2\beta_2 = \frac{a_0}{{a_0}^2 + {b_0}^2} + i\left(\frac{b}{{a_0}^2 + {b_0}^2} + 2\beta_2\right) = \frac{1}{a - ib}$$

- $\mathrm{Im}\Gamma_{out} = 0$ – Fourier limited pulse
- $\mathrm{Re}\Gamma_{out} = 0$ - $\delta t \to \infty$





a persistent student can finish the calculations:

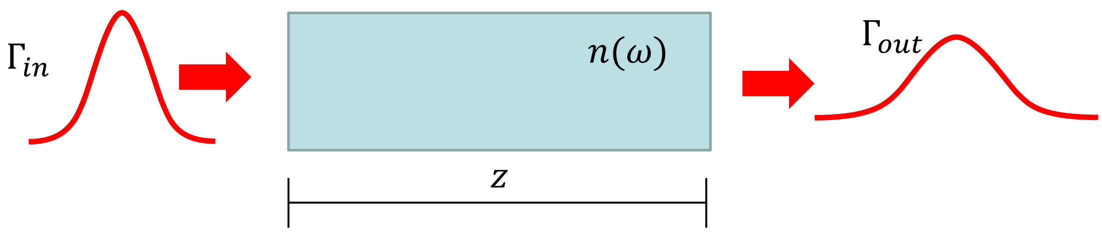
$$a = \frac{a_0}{(1+2\beta_2 b_0)^2+(2\beta_2 a_0)^2}, \quad b = \frac{2\beta_2 a_0 + b_0(1+2\beta_2 b_0)}{(1+2\beta_2 b_0)^2+(2\beta_2 a_0)^2}$$

one can easily type those into computer code

propagation of a Gaussian pulse in a dispersive medium – some facts:

spectral phase $\beta(\omega) = kz$

the second derivative of the phase $\beta_2 = k_2 z$



- let's start with a Fourier limited pulse $\Gamma_{in} = a_0 + i \cdot 0, \;\; a_0 > 0, \;\; b_0 = 0$

  $a = \frac{a_0}{1+(2k_2 z a_0)^2} < a_0, \;\; \delta t = \sqrt{2\ln 2/a} = \sqrt{1+(2k_2 z a_0)^2}\sqrt{2\ln 2/a_0} > \delta t_0$

  - the output pulse is always longer than the input one

  $b = 2k_2 z a_0$

  - the chirp sign depends on k_2

  example: for optical glasses in the visible range we have $k_2 > 0$ – positive chirp (red comes out first)

- the input pulse has non-zero chirp $\Gamma_{in} = a_0 + i \cdot b_0, \;\; a_0 > 0$

  $a = \frac{a_0}{(1+2k_2 z b_0)^2 + (2k_2 z a_0)^2}$ can be either larger or smaller than a_0.

  the result depends on the sign of the product $k_2 b_0$

  - $k_2 b_0 > 0$ gives $a < a_0$ and thus $\delta t > \delta t_0$
  - for $k_2 b_0 < 0$ a is first decreasing and then increasing. we search for the minimum which corresponds to a shortest possible pulse …

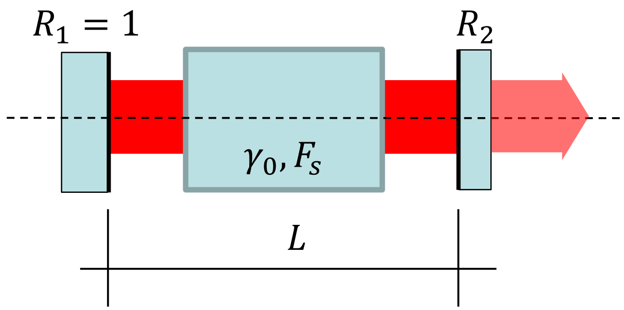
  $$z_{opt} = -\frac{b_0}{2k_2({a_0}^2 + {b_0}^2)}$$

  for a given value of b_0 we can take a medium such that $k_2 b_0 < 0$ and propagate the pulse in the medium over the distance z_{opt} to get the shortest pulse possible.

mode-locking in a laser oscillator:

mode-locking:

$$E_n(t) = A_n \sin(\omega_n t + \varphi_n)$$

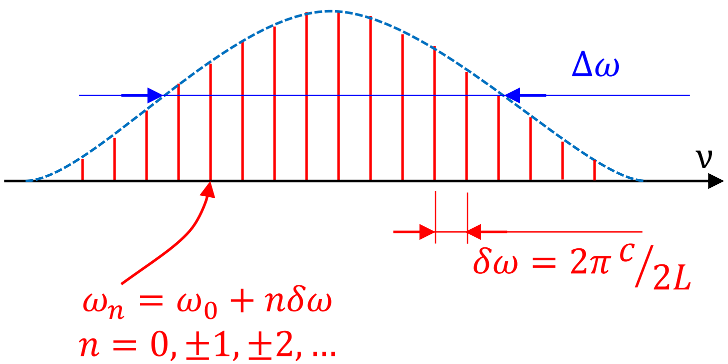


the electrical field of the laser beam is:

$$E(t) = \sum_{n=-N}^{n=N} A_n \sin(\omega_n t + \varphi_n)$$

in a complex notation:

$$E(t) = e^{i\omega_0 t} \sum_{n=-N}^{n=N} A_n e^{i(n\delta\omega t + \varphi_n)}$$

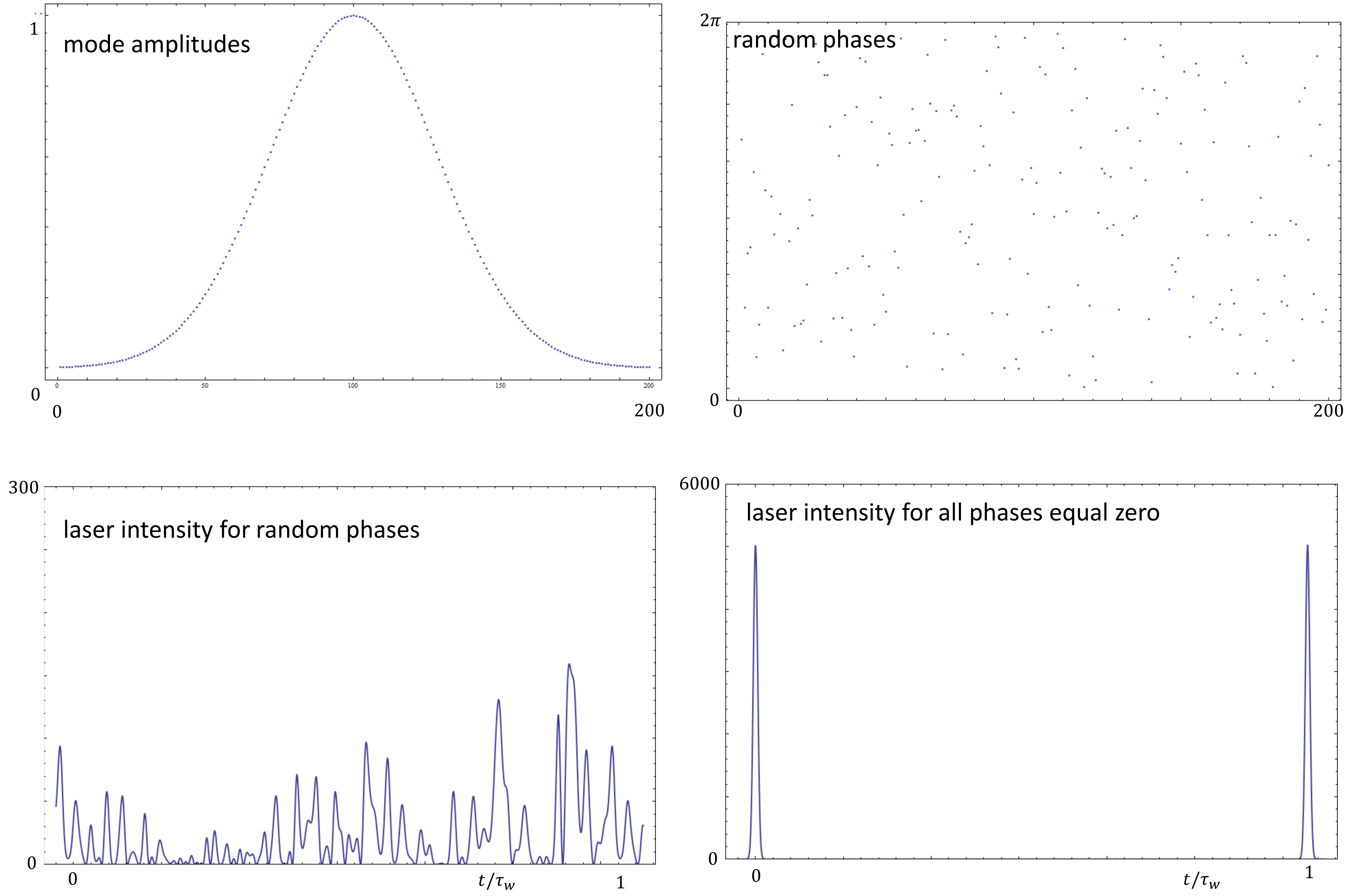


quite different results for different phase relations:

- random phases
- the same phases, e.g. $\varphi_n \equiv 0$

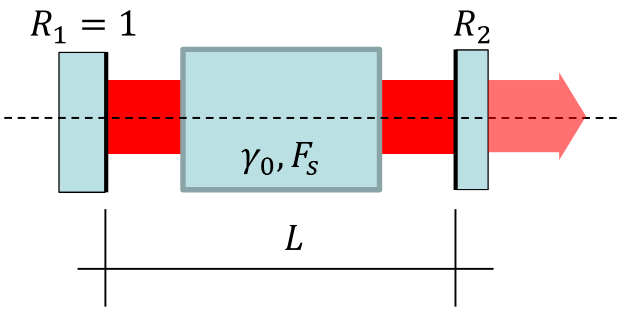
mode-locking a numerical simulations:

200 modes, temporal pictures for a full round-trip time



mode-locking, a simple model with a rectangular spectrum

$2N+1$ modes with the same amplitudes A, the same (zero= phases

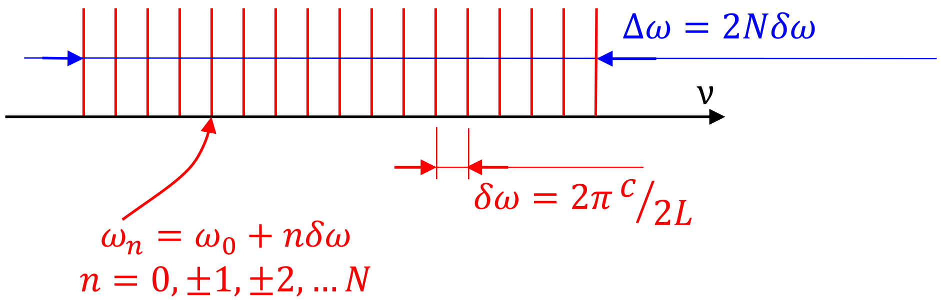


$$E(t) = Ae^{i\omega_0 t} \underbrace{\sum_{n=-N}^{n=N} e^{i(n\delta\omega t)}}_{\text{geometrical series}}$$

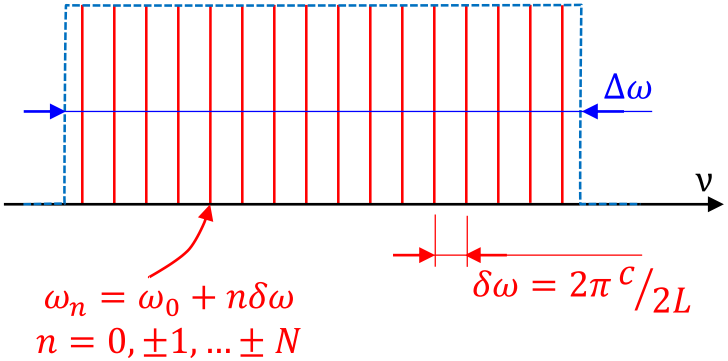
$$E(t) = Ae^{i(\omega_0 - 2N\delta\omega)t} \frac{\sin\left[\left(\frac{2N+1}{2}+1\right)\delta\omega t\right]}{\sin\left(\delta\omega t/_2\right)}$$

intensity:

$$I(t) = A^2 \frac{\sin^2\left[\left(\frac{2N+1}{2}+1\right)\delta\omega t\right]}{\sin^2\left(\delta\omega t/_2\right)}$$



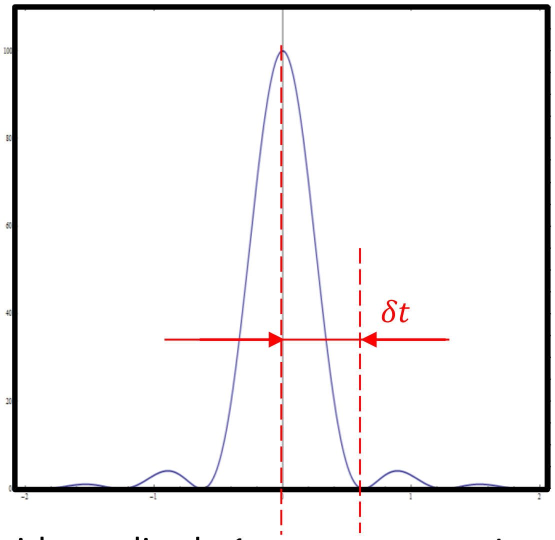
mode-locking, a simple model, 2



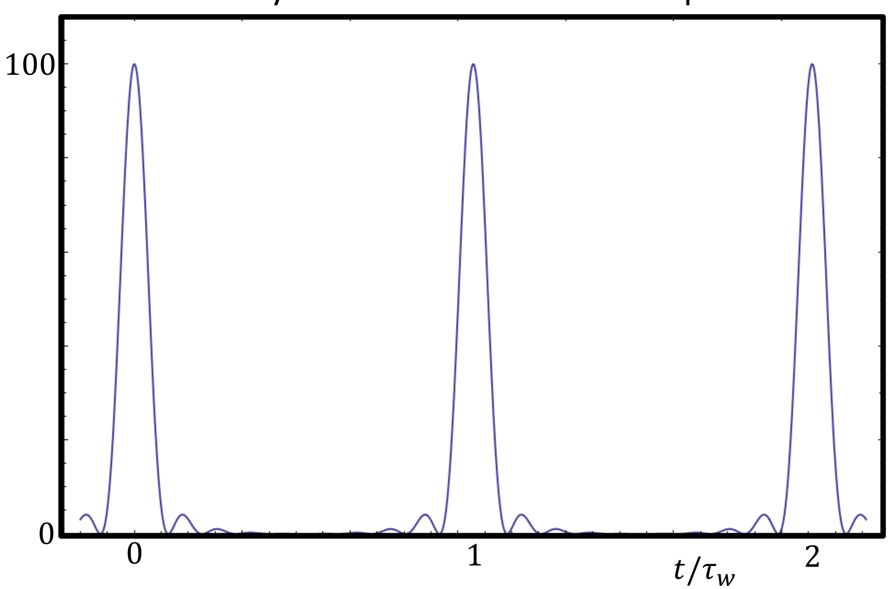
$$I(t) = A^2 \frac{\sin^2\left[\left(\frac{2N+1}{2}\right)\delta\omega t\right]}{\sin^2\left({}^{\delta\omega t}/_{2}\right)}$$

properties:

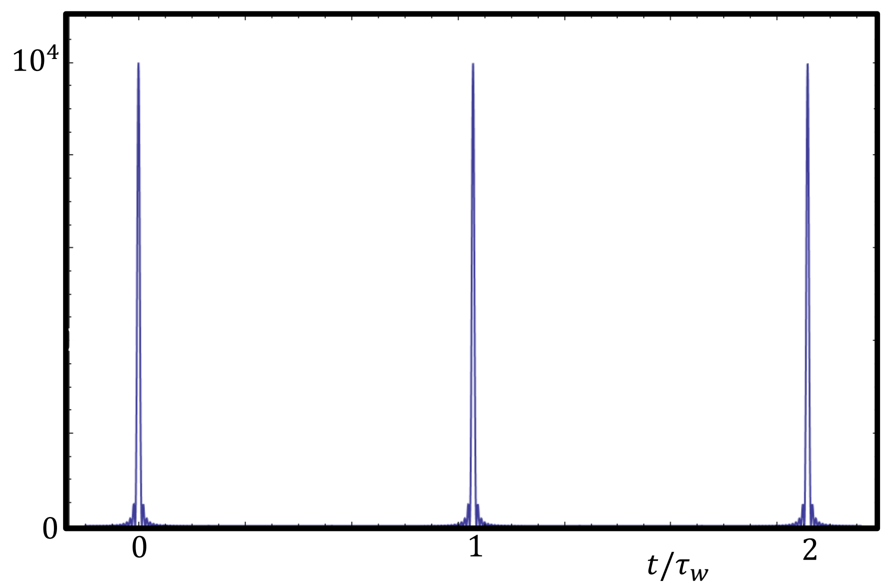
- ❑ $I(0) = I\left(n \cdot \underbrace{\frac{2\pi}{\delta\omega}}_{\tau_w}\right),\ n = 1,2,3 \dots$
- ❑ $\lim_{t\to 0} I(t) = \left(\frac{2N+1}{2} + 1\right)^2$
- ❑ $\delta t = \frac{1}{(2N+1)\delta\omega} \approx \frac{1}{\Delta\omega}$



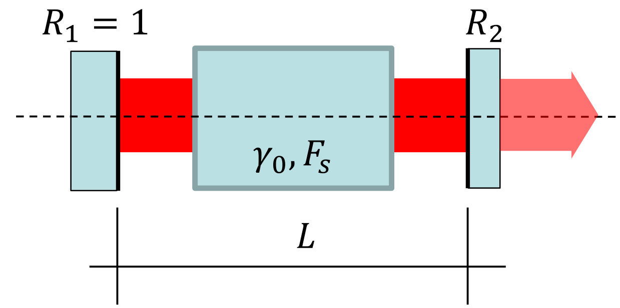
intensity – 10 modes each with amplitude 1



intensity – 100 modes each with amplitude 1



mode-locking; what is inside the cavity



we assume a cavity with no dispersion and perfect mode-locking

$\omega_n = \omega_0 + n\delta\omega, \qquad \mathrm{n} = \pm 1, \pm 2, \ldots \pm N$

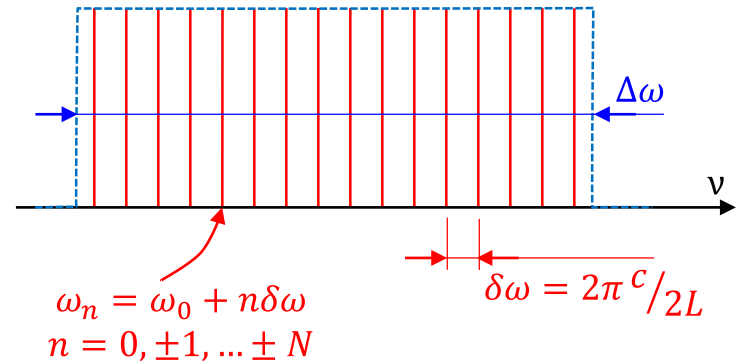
and $k_n = k_0 + n\frac{\pi}{L}$

a „closed” resonator forms a standing wave for each mode

$$E(z,t) = A\sum_{n=-N}^{N} \sin(k_n z)\sin(\omega_n t)$$

$$= \mathrm{A}\sum_{n=-N}^{N} \sin\left[\left(k_0 + n\frac{\pi}{L}\right)z\right]\sin[(\omega_0 + n\delta\omega)t]$$

some calculations using trigonometric formulas …..



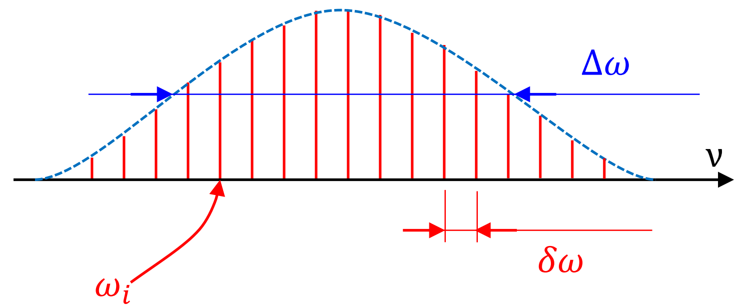
lead to

$$E(z,t) = \frac{1}{2}A\left[\underbrace{\cos(\omega_0 t - k_0 z)\frac{\sin(N+1)x}{\sin{}^{x}/_{2}}}_{\text{pulse propagating in the } +z \text{ direction}} - \underbrace{\cos(\omega_0 t + k_0 z)\frac{\sin(N+1)y}{\sin{}^{y}/_{2}}}_{\text{pulse propagating in the } +z \text{ direction}}\right]$$, gdzie $x = \frac{\pi(z-ct)}{L}$ i $y = \frac{\pi(z+ct)}{L}$

we have short pulse bouncing between the resonator mirrors

mode-locking; the role of intracavity dispersion

for a laser cavity with dispersion the simple relation $\omega_i = i \cdot \frac{c}{2L}$ does not hold. An example; for a cavity filled with a medium with a given dispersion $n(\omega)$ we have $\omega_i = i \cdot \frac{c}{2n(\omega_i)L}$. In the case of a smooth dispersion relation we can expand the last formula into the Taylor series around ω_0:

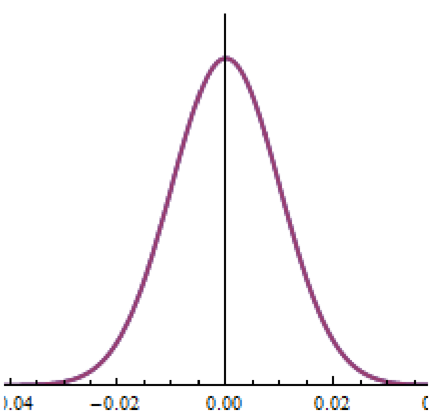


$$\omega_n = \alpha n + \beta n^2 + \frac{\gamma}{2} n^3 + \cdots, \qquad \mathrm{n} = \pm 1, \pm 2, \ldots \pm N$$

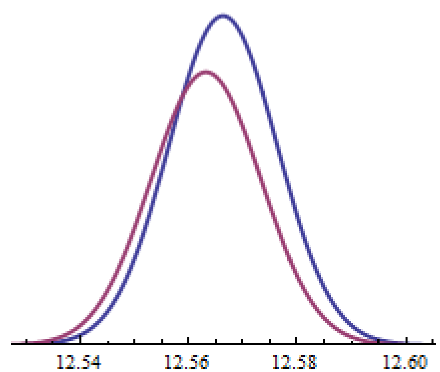
and calculate electric field amplitude

$E(t) = e^{i\omega_0 t} \sum_{n=-N}^{n=N} A_n e^{i\omega_n t}$ and intensity of the laser beam

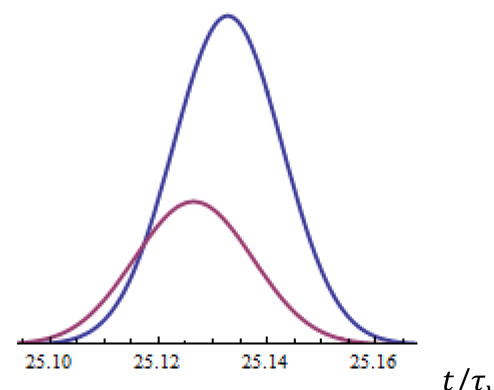
numerical simulations for a Gaussian spectrum: $2N + 1 = 500, \alpha = 1, \beta = 5 \times 10^{-7}, \gamma = 0$



initial pulse



the pulse after 4 round-trips

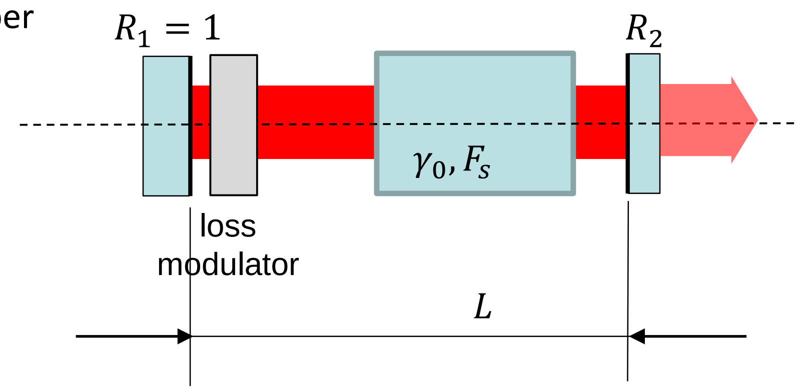


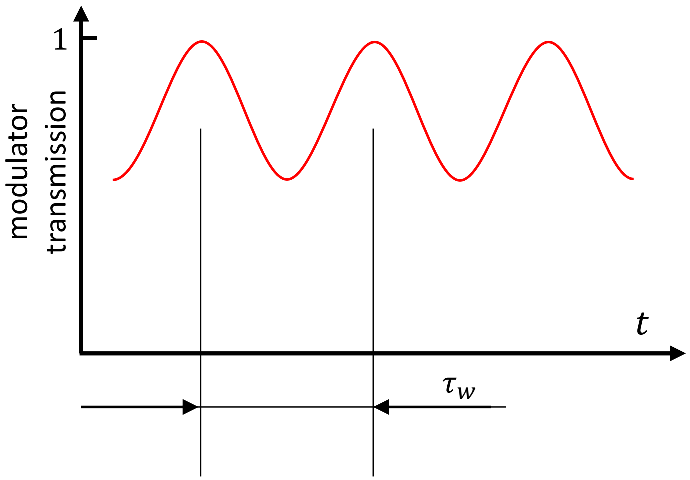
the pulse after 8round-trips

dispersion kills mode-locking!

mode-locking mechanisms

active mode-locking (usually acousto-optic modulator with a standing acoustic wave) driven by an electrical signal with a proper frequency.



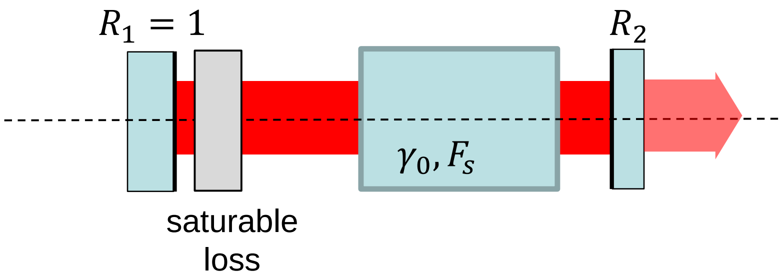


τ_w - round-trip time; $\tau_w = L/v_g$ with v_g being an effective (averaged over the resonator) group velocity

time-dependent losses in the resonator force pulse regime – a pulse transmitted through the modulator when its transmission is maximum experiences minimum loss

mode-locking mechanisms, 2

passive mode-locking, intracavity saturable absorption

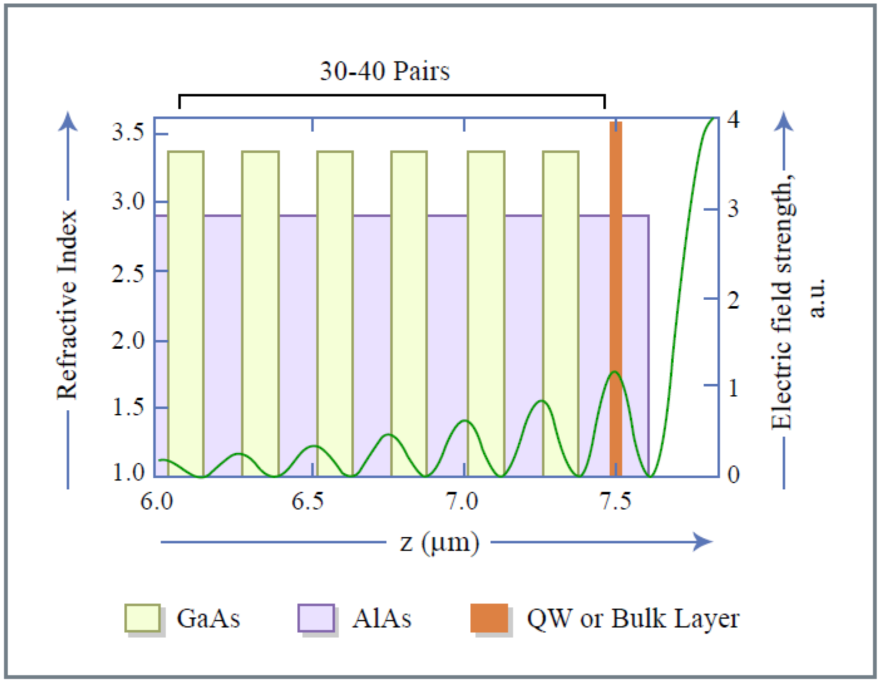


saturable absorber, problems:

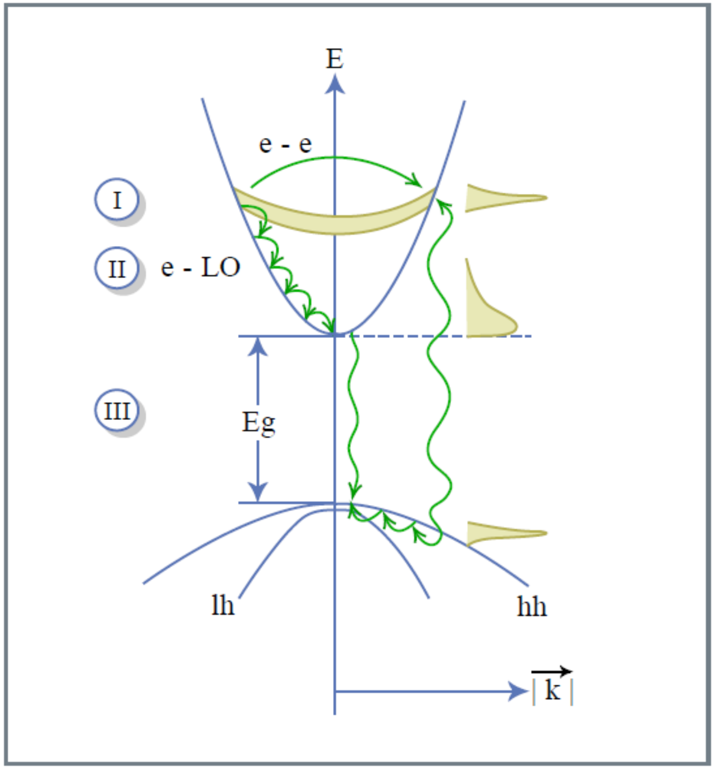
- relaxation speed
- absorber thickness

ssolution: SESAM (Semiconductor Saturable Absorber Mirror)

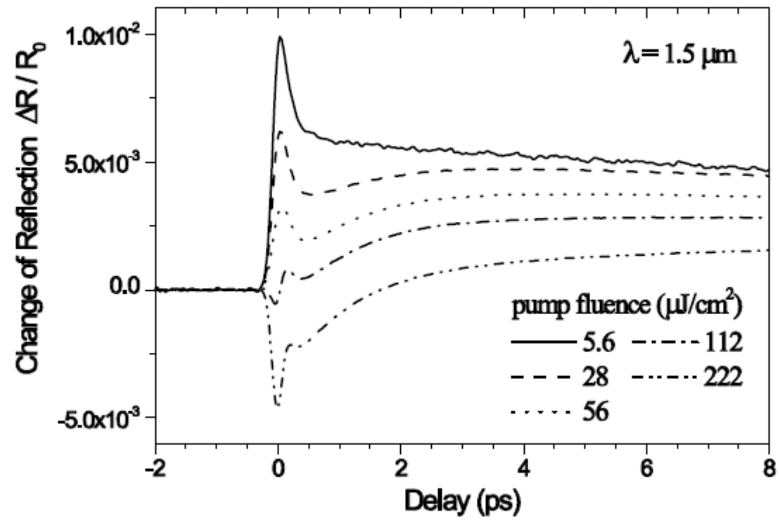
SESAM's structure



carrier dynamics in semiconductors



SESAM - properties



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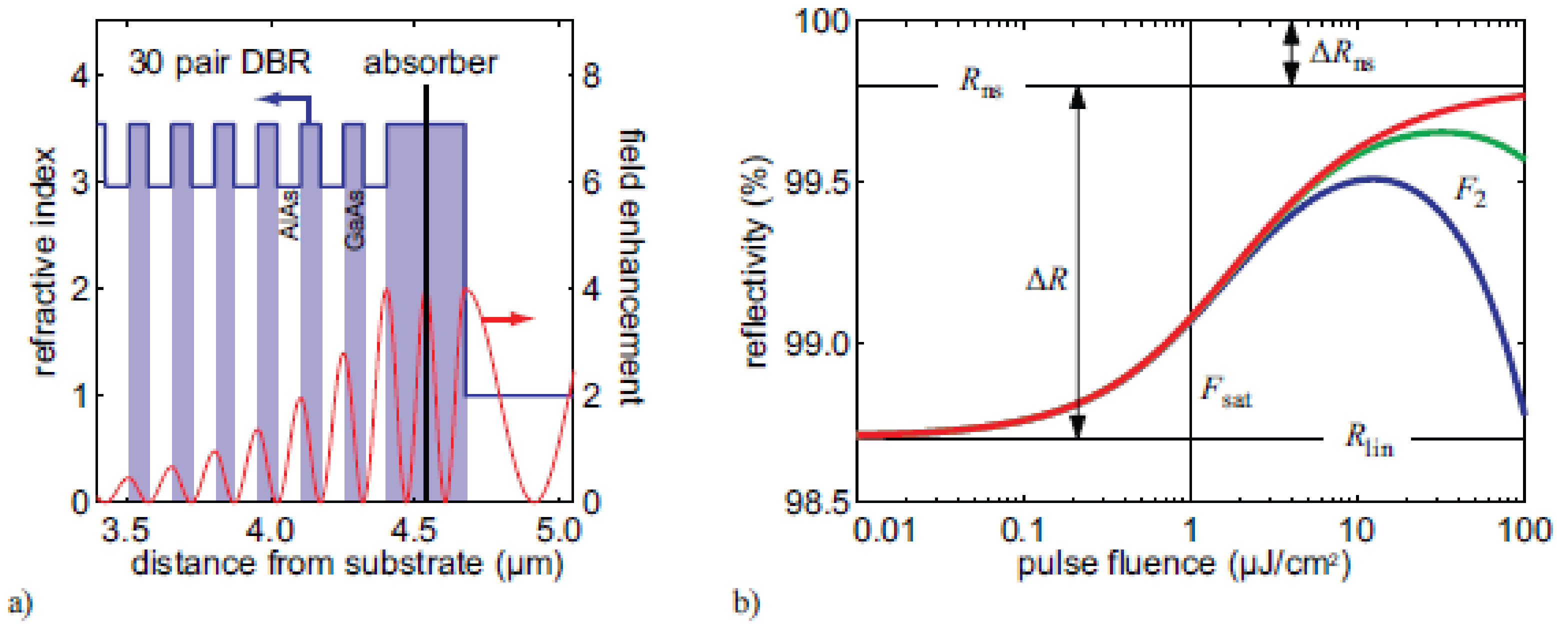

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SESAM in $Ti^{3+}:Al_2O_3$ laser

Self-starting 6.5-fs pulses from a Ti:sapphire laser

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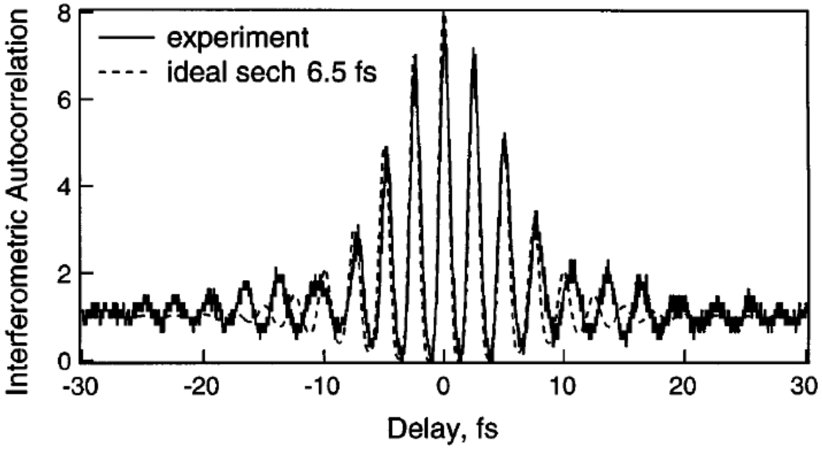

Fig. 1. Interferometric autocorrelation of a self-starting KLM pulse compared with an ideal 6.5-fs pulse at 750 nm.