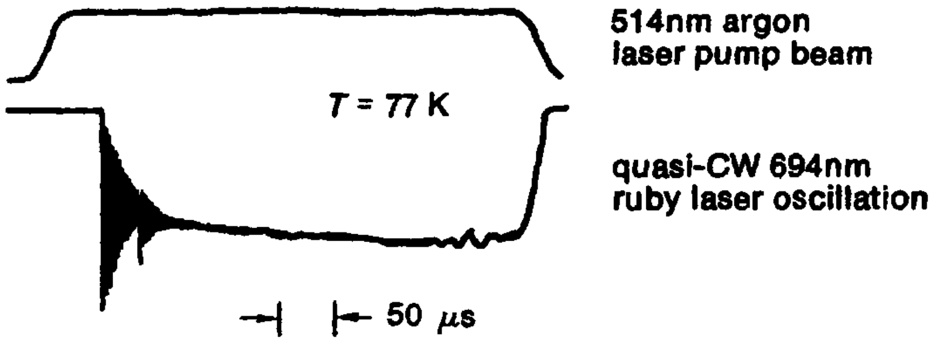
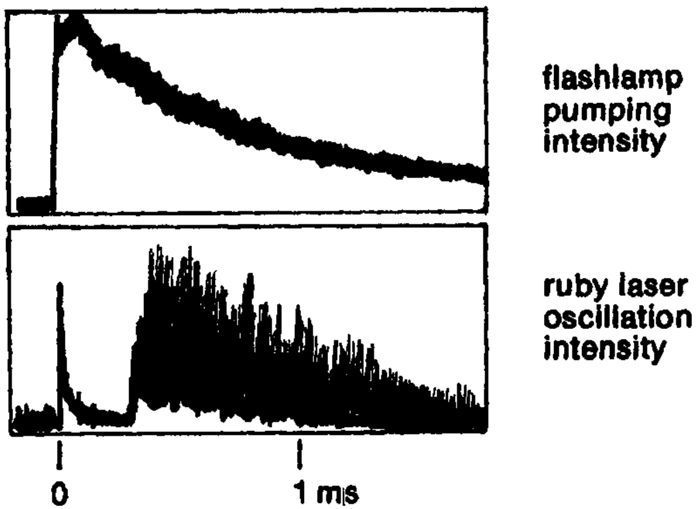
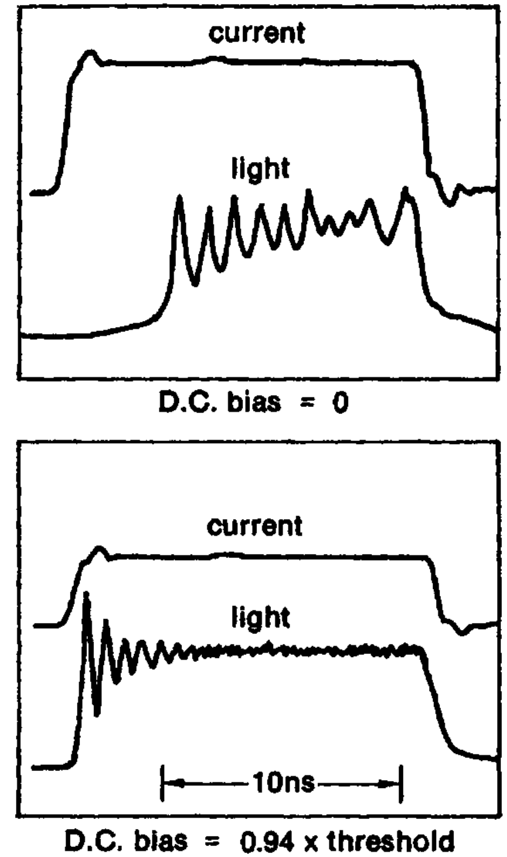
Lasers
lecture 7

Czesław Radzewicz

laser dynamics – simple observations





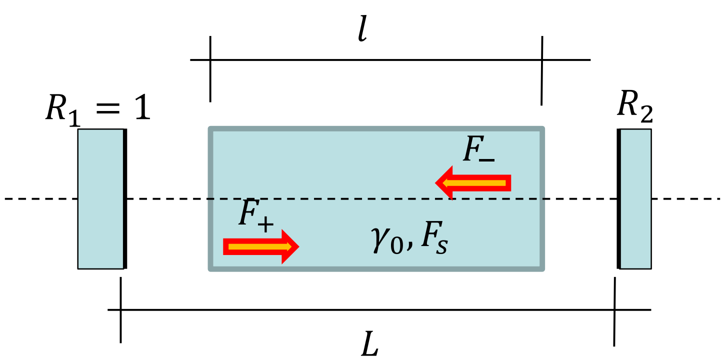


laser diodowy

Nd:YAG

from:
A. E. Siegman, *Lasers*, University Scvience Books, 1986

slow laser dynamics – long times



assumptions:

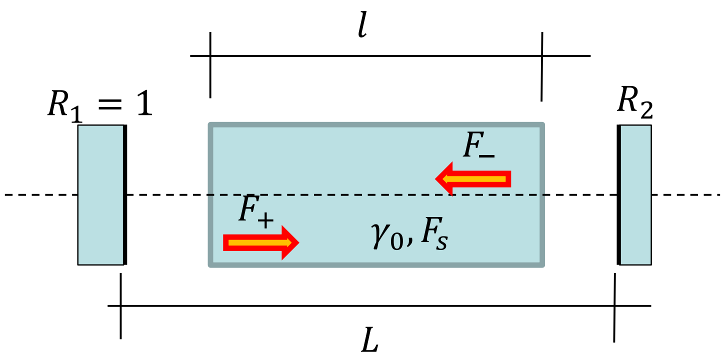
- „closed" resonator
- slow changes of the inter-cavity intensity: $\left| \frac{dF_{int}}{dt} \Big/ F_{int} \right| \ll {}^1/_{\tau_p}, {}^1/_{\tau_{21}}$

Two coupled physical systems: gain medium and laser resonator. The energy is in the medium (population inversion) or in the cavity (energy of the electro-magnetic wave)

from lecture 2: $\left(\frac{\partial}{\partial z} + \frac{1}{v_g}\frac{\partial}{\partial t}\right) I = \gamma I$, assume $v_g = c$ to get

$$\left.\begin{aligned} \frac{\partial F^+}{\partial z} + \frac{1}{c}\frac{\partial F^+}{\partial t} &= \gamma F^+ \\ -\frac{\partial F^-}{\partial z} + \frac{1}{c}\frac{\partial F^-}{\partial t} &= \gamma F^- \end{aligned}\right\} \Rightarrow \left\{\begin{aligned} &\frac{\partial}{\partial z}(F^+ - F^-) + \frac{1}{c}\frac{\partial}{\partial t}(F^+ + F^-) = \gamma(F^+ + F^-) \\ &\frac{\partial}{\partial t}(F^+ + F^-) = c\gamma(F^+ + F^-) \end{aligned}\right.$$

slow laser dynamics – long times, 2



$$\frac{\partial}{\partial t}(F^+ + F^-) = c\gamma(F^+ + F^-)$$

$$\int_0^L \frac{\partial}{\partial t}(F^+ + F^-)\, dz = c\gamma \int_0^L (F^+ + F^-)\, dz$$

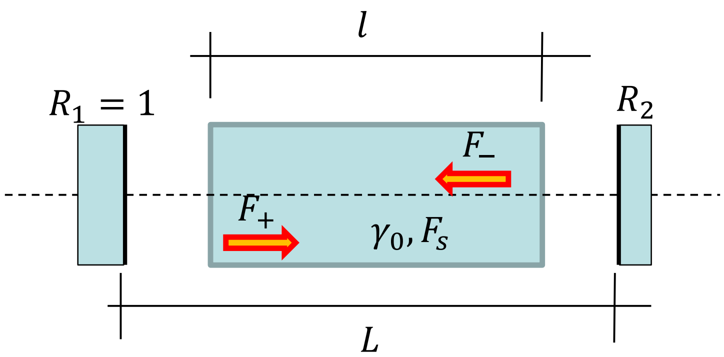
$$L\frac{d}{dt}(F^+ + F^-) = lc\gamma(F^+ + F^-)$$

notice that $L(F^+ + F^-)$ iq is proportional to the number of photons inside the laser cavity and thus

$\frac{dq}{dt} = \chi c\gamma q$ - the rate of change of the photon number due to amplification

where $\chi \equiv \frac{l}{L}$ is a geometrical scaling factor

slow laser dynamics – long times, 3



$$\frac{dq}{dt} = \frac{lc\gamma}{L} q$$

- the rate of change of the photon number due to amplification

we have to account for the photon losses due to mirror transmission:

$dq = -(1 - R_2)q,\ dt = {}^{2L}/_{c}$

so

$$\frac{dq}{dt} = -\frac{q}{\tau_p}, \qquad \tau_p \equiv {}^{2L}/_{c(1 - R_2)}$$

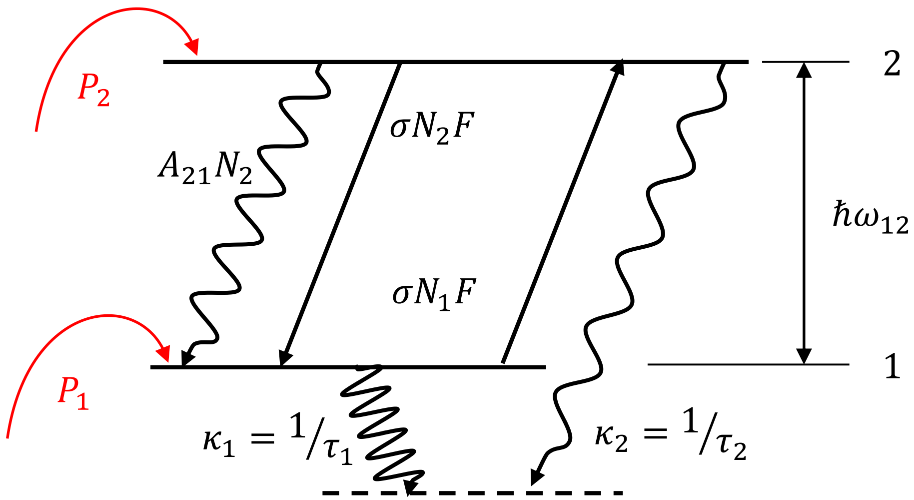
τ_p - photon lifetime (inside the „cold"=no gain cavity

and finally

$$\frac{dq}{dt} = \chi c\gamma q - \frac{1}{\tau_p} q$$

note: in a resonator with internal losses we have to include those losses – this lowers τ_p value

laser dynamics – an universal model of the gain medium



populations:

$$\frac{dN_2}{dt} = -(\kappa_2 + A_{21})N_2 - \sigma(N_2 - N_1)F + P_2$$

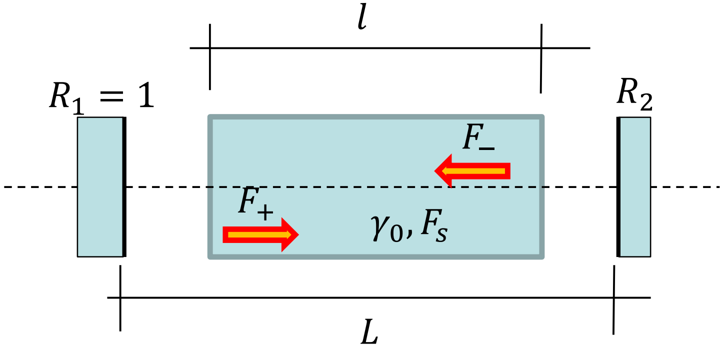
$$\frac{dN_1}{dt} = -\kappa_1 N_1 + A_{21}N_2 + \sigma(N_2 - N_1)F + P_1$$

photon flux

$$\frac{dF}{dt} = \frac{lc\sigma}{L}(N_2 - N_1)F - \frac{1}{\tau_p}F$$

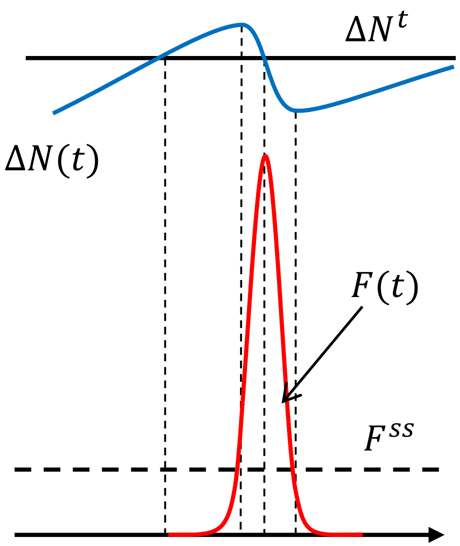
these equations have to be integrated to find the laser time evolution

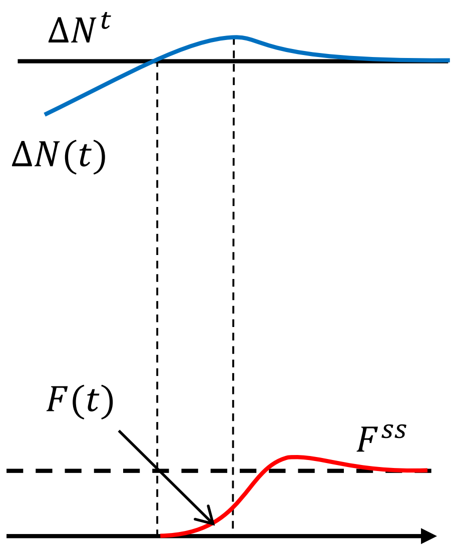
laser dynamics – some initial guesses



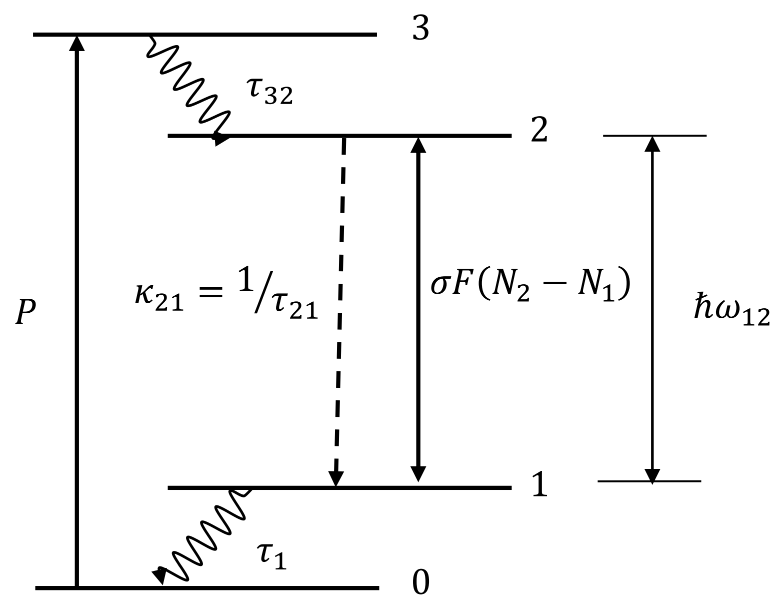
turn-on effects must depend on the ratio of the two characteristic time scales :

$$\tau_{21} = {}^{1}/_{A_{21}} \gg \tau_p \qquad\qquad \tau_{21} \leq \tau_p$$





laser dynamics with a perfect 4-level model



assume $\tau_{32} = \tau_1 \cong 0$. Then $N_3 = N_1 \cong 0$ and the laser dynamics equations are reduced to:

$$\boxed{\begin{aligned} \frac{dN_2}{dt} &= -\kappa_{21}N_2 - \sigma F N_2 + P \\ \frac{dF}{dt} &= \chi c \sigma N_2 F - \frac{1}{\tau_p}F \end{aligned}}$$

Note that P does not correspond to the parameter with the same symbol defined in lecture 4. here we assume that the pumping process does not deplete the ground state population.

laser stability

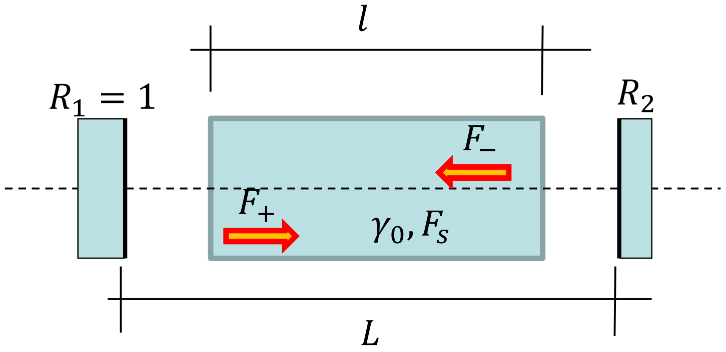
$$\frac{dN_2}{dt} = -\kappa_{21}N_2 - \sigma F N_2 + P$$

$$\frac{dF}{dt} = \chi c\sigma N_2 F - \frac{1}{\tau_p}F$$

assume that there exist stationary solutions $\bar{F}$ and $\overline{N_2}$ of the eqs. describing laser:

$$\frac{dN_2}{dt} = -\kappa_{21}\overline{N_2} - \sigma\overline{N_2}\bar{F} + P = 0$$

$$\frac{dF}{dt} = \chi c\sigma\overline{N_2}\bar{F} - \frac{1}{\tau_p}\bar{F} = 0$$



we add a small perturbation:

$$F = \bar{F} + \epsilon$$

$$N_2 = \overline{N_2} + \eta$$

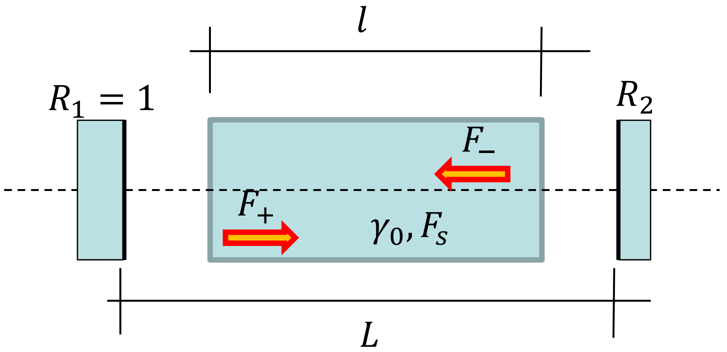
and plug in F and N_2 into lasers dynamics equations

$$\frac{d}{dt}(\overline{N_2} + \eta) = -\kappa_{21}(\overline{N_2} + \eta) - \sigma(\bar{F} + \epsilon)(\overline{N_2} + \eta) + P \qquad (1)$$

$$\frac{d}{dt}(\bar{F} + \epsilon) = \chi c\sigma(\overline{N_2} + \eta)(\bar{F} + \epsilon) - \frac{1}{\tau_p}F \qquad (2)$$

laser stability, 2

$$-\kappa_{21}\overline{N_2} - \sigma\overline{N_2}\bar{F} + P = 0$$
$$\chi c\sigma\overline{N_2}\bar{F} - \frac{1}{\tau_p}\bar{F} = 0$$



eq. (2):

$$\frac{d}{dt}(\bar{F} + \epsilon) = \frac{d\epsilon}{dt} = \chi c\sigma(\overline{N_2} + \eta)(\bar{F} + \epsilon) - \frac{1}{\tau_p}(\bar{F} + \epsilon) = \underbrace{\left(\chi c\sigma\overline{N_2}\bar{F} - \frac{1}{\tau_p}\bar{F}\right)}_{0}\epsilon + \chi c\sigma(\bar{F}\eta + \epsilon\eta)$$

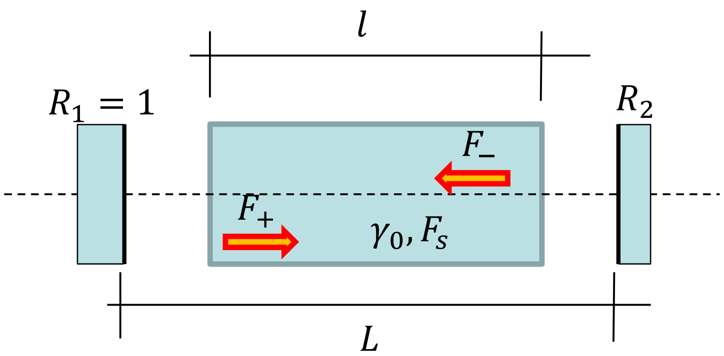
we neglect a small bilinear term $\chi c\sigma\epsilon\eta$ i and get

$$\frac{d\epsilon}{dt} = \chi c\sigma\bar{F}\eta$$

eq. (1):

$$\frac{d}{dt}(\overline{N_2} + \epsilon) = \frac{d\eta}{dt} = -\kappa_{21}(\overline{N_2} + \eta) - \sigma(\overline{N_2} + \eta)(\bar{F} + \epsilon)$$
$$= -\kappa_{21}\overline{N_2} \underbrace{- \sigma\overline{N_2}\bar{F} + P}_{0} - \kappa_{21}\eta - \sigma\bar{F}\eta - \sigma\overline{N_2}\epsilon - \sigma\epsilon\eta =$$
$$= \underbrace{\left(-\kappa_{21} - \sigma\bar{F} + \frac{P}{\overline{N_2}}\right)}_{0}\eta - \frac{P}{\overline{N_2}}\eta - \sigma\overline{N_2}\epsilon =$$
$$= -\frac{P}{\overline{N_2}}\eta - \sigma\overline{N_2}\epsilon$$

laser stability, 3



we end up with two copupled nonlinear differential equations :

$$\frac{d\epsilon}{dt} = \chi c\sigma\bar{F}\eta \qquad (1)$$

$$\frac{d\eta}{dt} = -\frac{P}{\overline{N_2}}\eta - \sigma\overline{N_2}\epsilon \qquad (2)$$

calculate η form the first one and plug it into the second

$$\eta = \frac{1}{\chi c\sigma\bar{F}}\frac{d\epsilon}{dt}$$

$$\frac{1}{\chi c\sigma\bar{F}}\frac{d^2\epsilon}{dt^2} + \frac{P}{\chi c\sigma\bar{F}\overline{N_2}}\frac{d\epsilon}{dt} + \sigma\overline{N_2}\epsilon = 0$$

which turns out to be a harmonic oscillator equation

$$\frac{d^2\epsilon}{dt^2} + \gamma'\frac{d\epsilon}{dt} + {\omega_0}^2\epsilon = 0$$

note: the parameter γ' does not signify gain coefficient!!!

$$\gamma' = \frac{P}{\overline{N_2}} > 0\,, \qquad {\omega_0}^2 = \sigma^2 c\chi\overline{N_2}\bar{F}$$

solution:

$$\epsilon(t) = Ae^{-\frac{\gamma'}{2}t}e^{-i\omega t}$$

$$\omega = \sqrt{{\omega_0}^2 - {\gamma'^2}/_4}^{\,2}$$

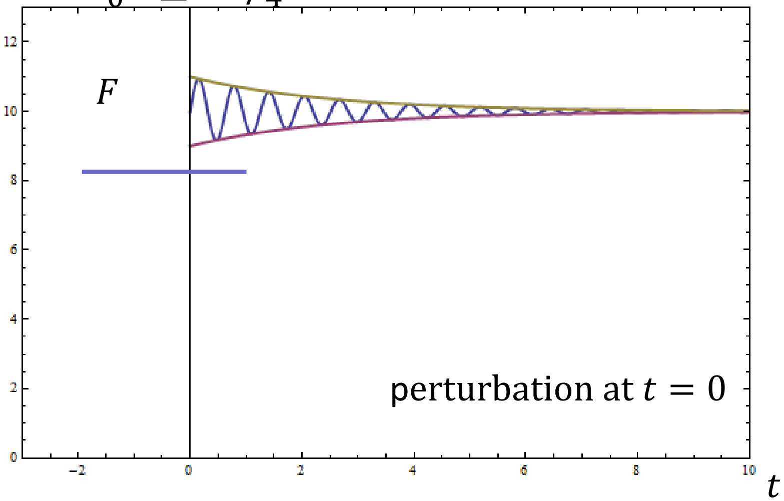
laser stability, 4

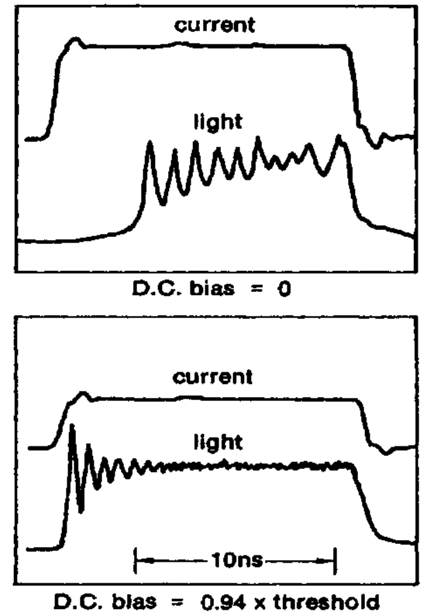
solution:

$$\epsilon(t) = Ae^{-\frac{\gamma'}{2}t}e^{-i\omega t}$$

$$\omega = \sqrt{{\omega_0}^2 - {\gamma'^2}/_4}$$

- ${\omega_0}^2 \geq {\gamma'^2}/_4 \Rightarrow \omega$ is real – relaxation oscillations



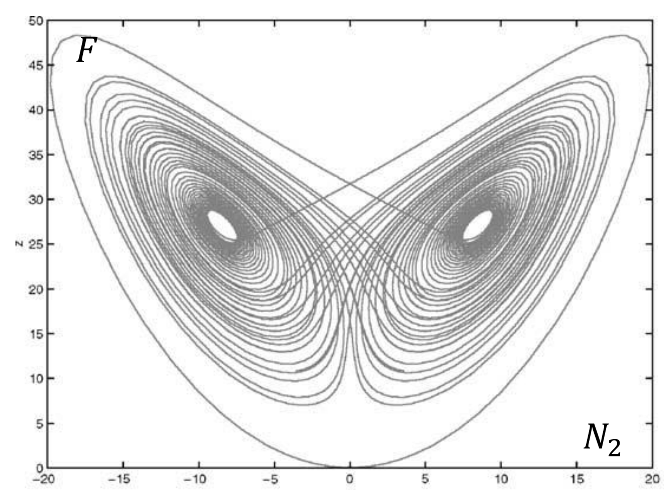


laser diodowy

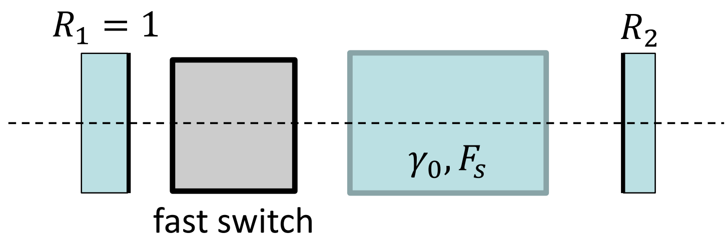
- ${\omega_0}^2 < {\gamma'^2}/_4 \Rightarrow \omega$ is imaginary – laser is not stable (no stationary solutions)

$\omega = ia$, a – positive real number; $\epsilon(t) = Ae^{(a-\gamma'/2)t}$

deterministic chaos

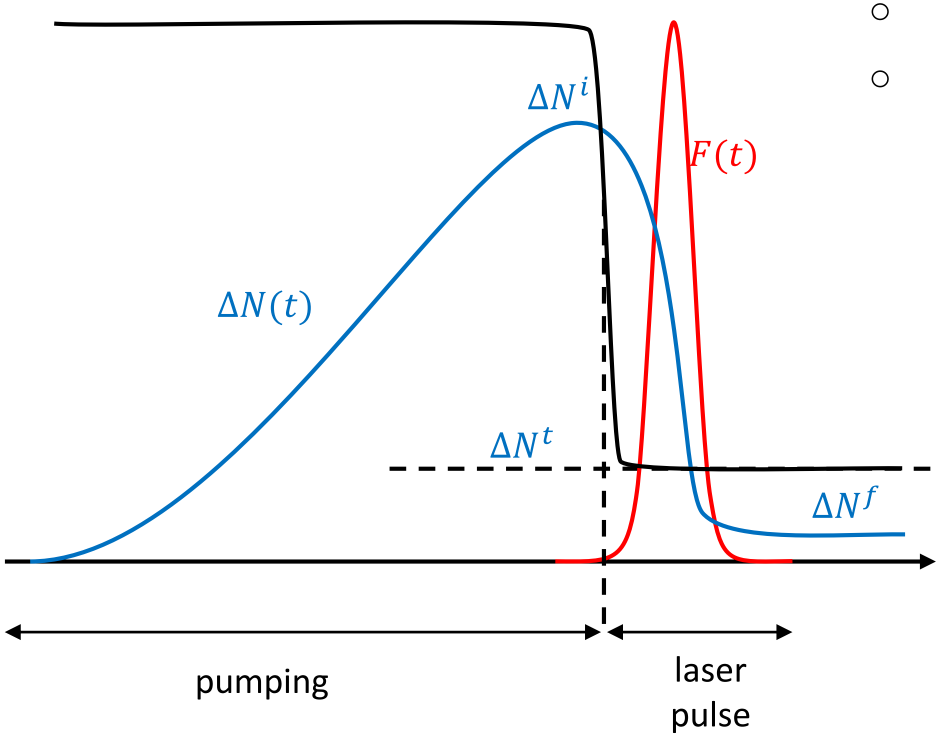


Q-switching



time sequence:

- the switch is closed, the gain medium is pumped
- population inversion exceeds the threshold value for open switch cavity, no lasing because the switch is closed
- maximum of population inversion – the switch is opened, laser action starts
- the laser pulse saturates gain and destroys population inversion
- the pulse ends
- go to the beginning



important parameters:

- gain coefficient at the time of switch opening

  $\gamma^i = \sigma \Delta N^i$

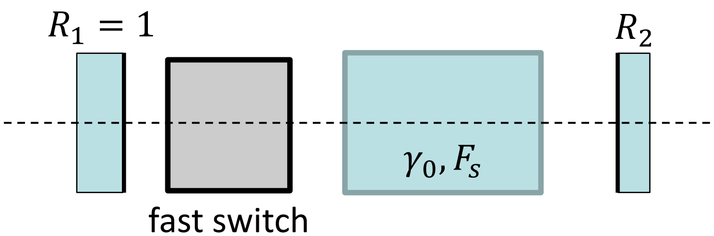
- threshold gain coefficient

$$\gamma^t = \sigma \Delta N^t = -\frac{1}{2l} \ln R_1 R_2 + a$$

- final population inversion

$$\Delta N^f$$

Q-switching; formal analysis



laser dynamics equations

$$\frac{dN_2}{dt} = -\kappa_{21}N_2 - \sigma N_2 F + P \qquad (1)$$

$$\frac{dF}{dt} = \chi c\sigma N_2 F - \frac{1}{\tau_p}F \qquad (2)$$

we can simplify by assuming a short (nanosecond) pulse.
then we can neglect spontaneous emission and pumping which do not influence the populations during the pulse
in effect, we have

$$\frac{dN_2}{dt} = -\sigma N_2 F \qquad (3)$$

$$\frac{dF}{dt} = \chi c\sigma N_2 F - \frac{1}{\tau_p}F \qquad (4)$$

new variables:

$$x = \frac{F}{\chi c \Delta N^t}$$

$$y = \frac{N_2}{\Delta N^t}$$

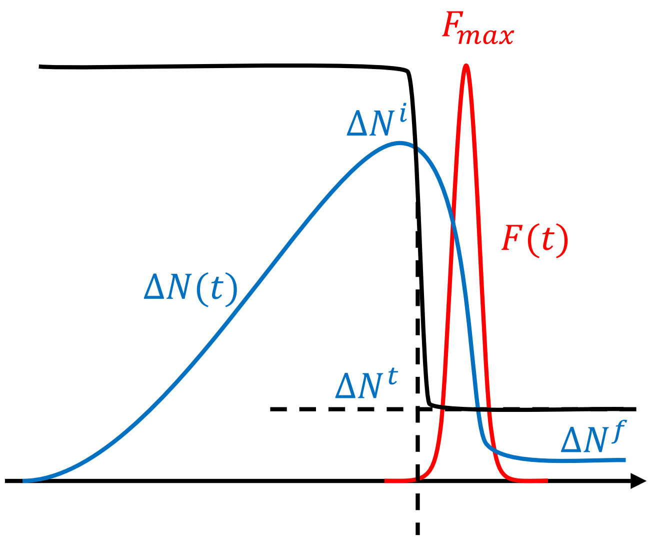
$$\tau = \chi c \gamma^t t$$

lead to:

$$\boxed{\begin{aligned} \frac{dx}{d\tau} &= (y-1)x \\ \frac{dy}{d\tau} &= -xy \end{aligned}}$$

nonlinear coupled equations – we need numerical integration to retrieve $N_2(t)$ and $F(t)$

Q-switching; formal analysis 2



formal integration. from the eqs.:

$$\frac{dx}{d\tau} = (y-1)x$$

$$\frac{dy}{d\tau} = -xy$$

we get

$$dx = \left(\frac{1}{y} - 1\right)dy \qquad (1)$$

integrate (1):

❖ to the maximum intensity

$$\int_0^{x_{max}} dx = \int_{y^i}^{1} \left(\frac{1}{y} - 1\right) dy$$

$$x_{max} = y^i - 1 - \ln y^i =$$

$$= \frac{\Delta N^i - \Delta N^t}{\Delta N^t} - \ln\left(\frac{\Delta N^i}{\Delta N^t}\right)$$

$$\boxed{\lim_{\Delta N^i \gg \Delta N^t} x_{max} = \frac{\Delta N^i}{\Delta N^t}}$$

❖ to the end of pulse

$$\int_0^{0} dx = \int_{y^i}^{y^f} \left(\frac{1}{y} - 1\right) dy$$

$$0 = y^i - y^f - \ln\left(\frac{y^f}{y^i}\right) =$$

$$= \frac{1}{\Delta N^t}\left(\Delta N^i - \Delta N^f\right) - \ln\left(\frac{\Delta N^f}{\Delta N^i}\right)$$

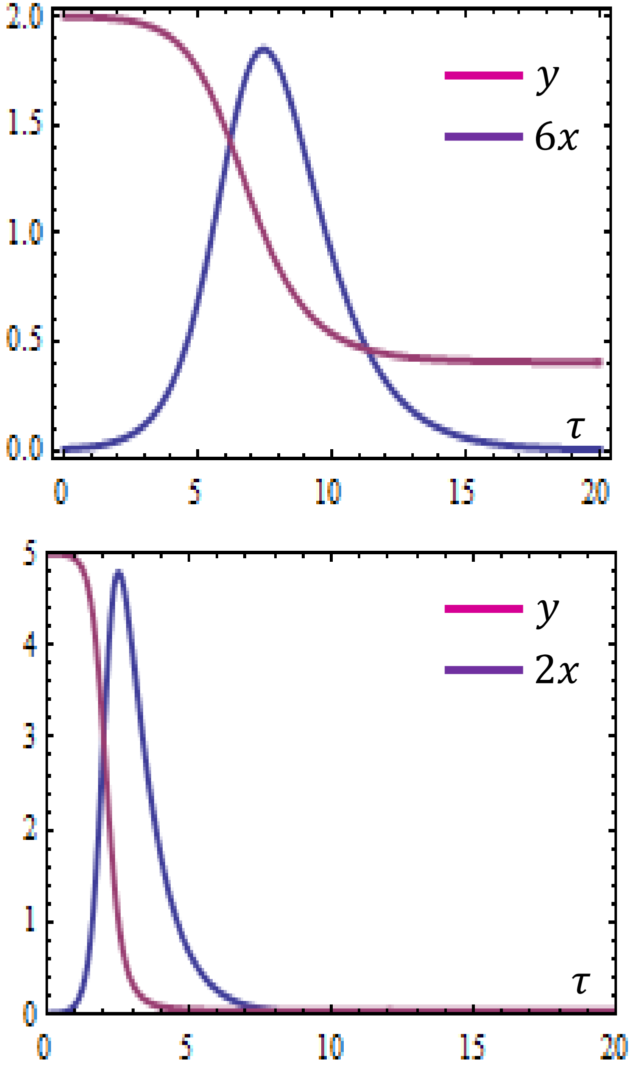
efficiency

$$\eta = \frac{\Delta N^i - \Delta N^f}{\Delta N^i}$$

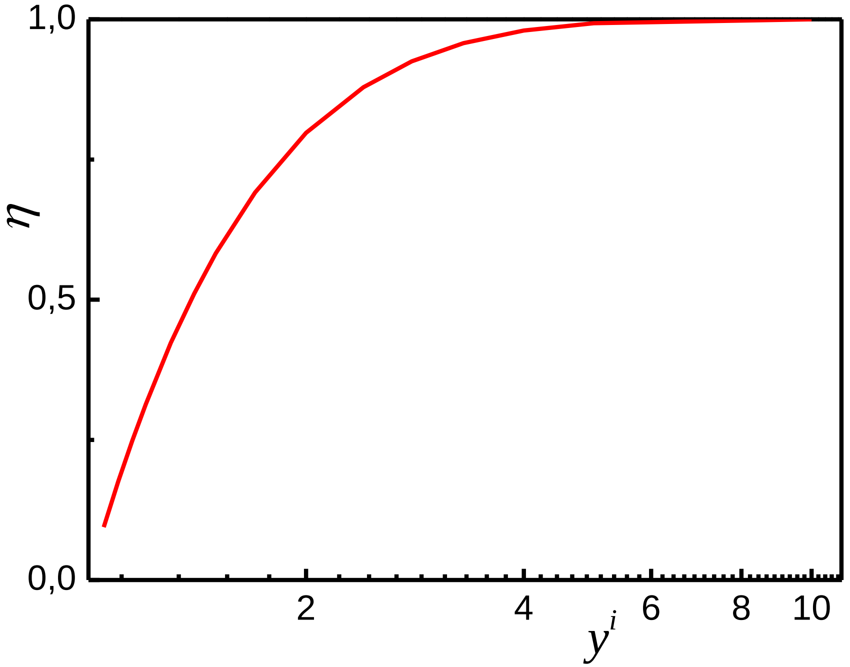
$$\boxed{\lim_{\Delta N^i \gg \Delta N^f} \eta = 1}$$

Q-switching; numerical calculations

numerical integration of the Q-switch laser eqs.
$(x(0) = 0.001)$



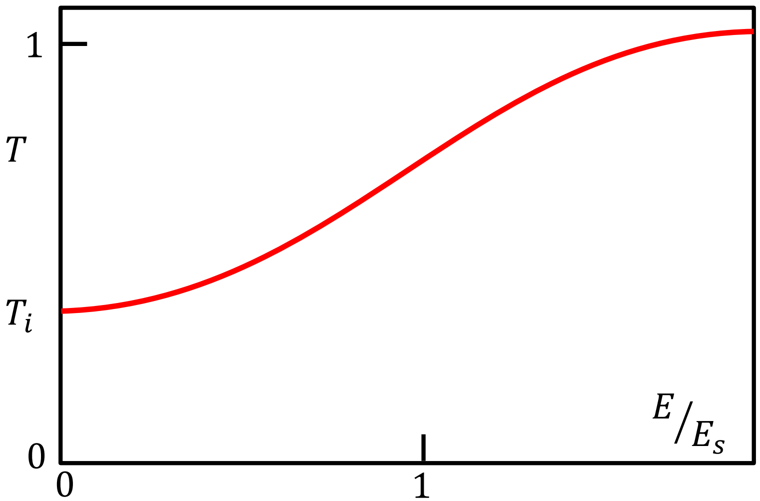
laser efficiency vs initial population inversion



note: if the lower level of the laser transition has lifetime longer than the pulse duration we have to modify the equations accordingly

Q-switching - methods

- passive - the cavity losses are lowered when an absorber inside the cavity is saturated
- active – the switch is opened by an external trigger

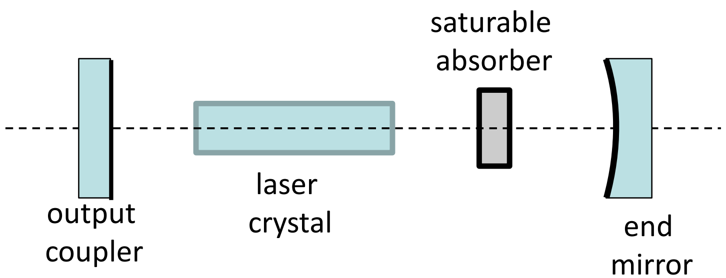


from lecture 2:

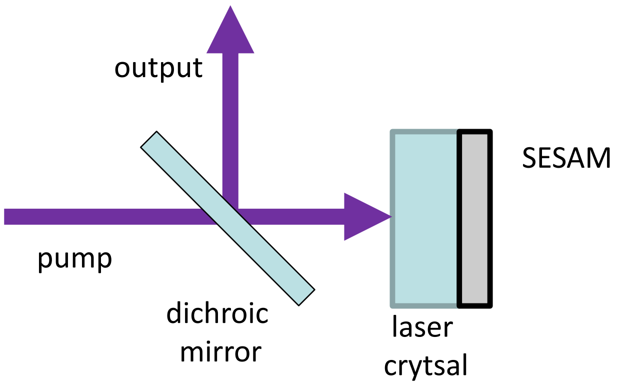
$F_s \equiv \frac{1}{\sigma_{01}}$ - saturating photon flux $\left[\frac{1}{\mathrm{cm}_2}\right]$

$E_s = \hbar\omega F_s$ - saturating energy fluence $\left[\frac{\mathrm{J}}{\mathrm{cm}_2}\right]$

laser construction

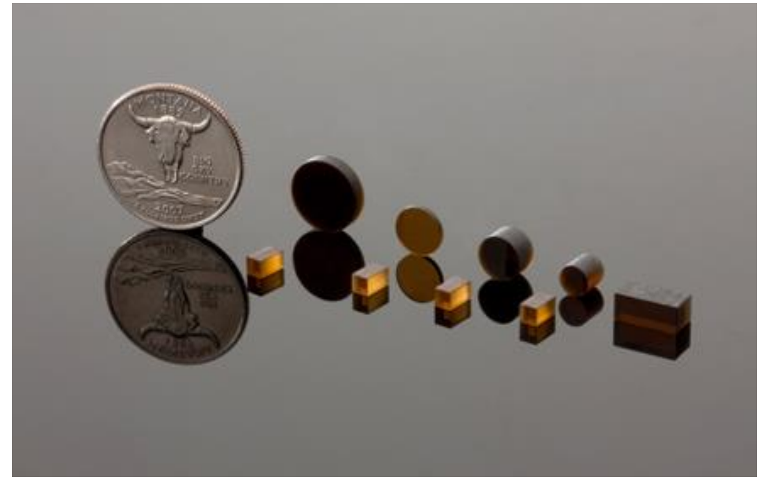


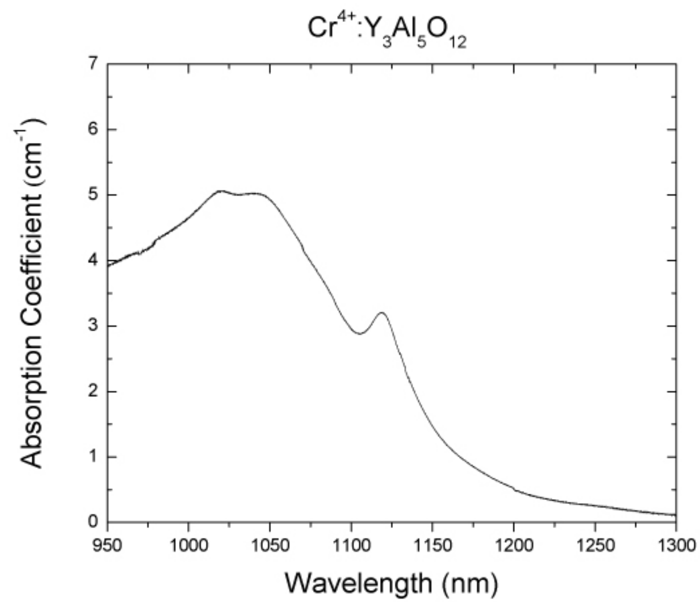
micro-chip laser



Q-switching – an example of saturable absorber

Chromium Doped Yttrium Aluminum Garnet (Cr^{4+}:YAG	
symetria	kubiczny
domieszkowani (%atomów)	0.5÷3
próg zniszczenia (MW/cm^2)	500
czas życia fluorescencji (μs)	3.4
przekrój czynny na emisję (cm^2)	$8.2\cdot10^{-19}$
przewodność cieplna (W/m K)	12
współczynnik załamania	1.82 dla λ=800nm
twardość (Mohs)	8.5
gęstość (g/cm^3)	4.56
moduł Younga (GPa)	282





saturating energy fluence

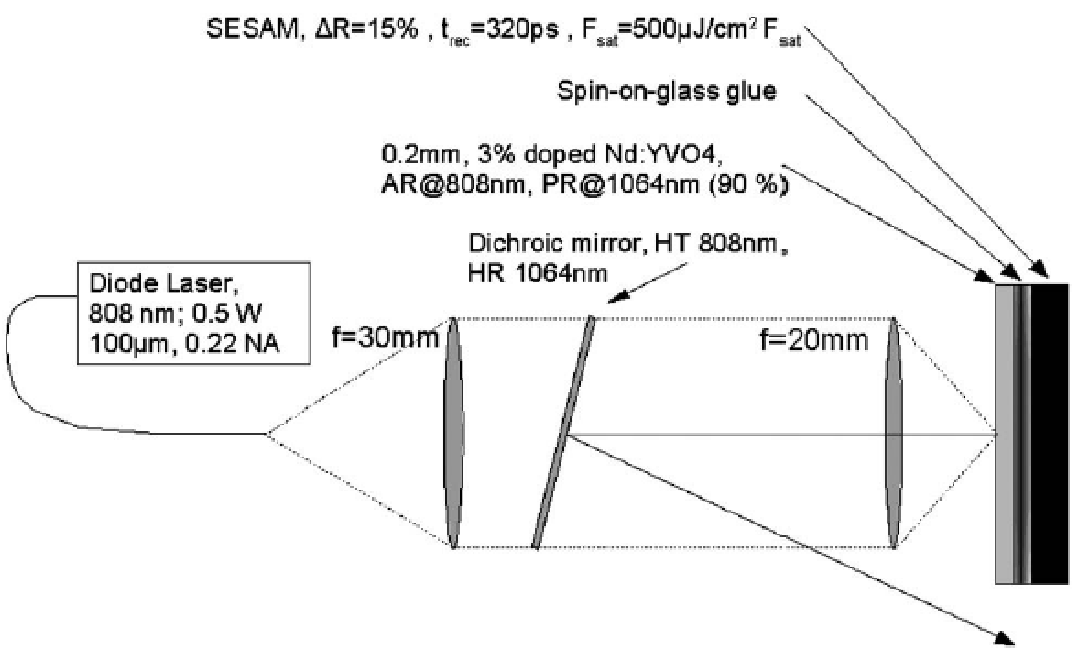
$$E_s = {}^{\hbar\omega}/_{\sigma_{01}} \cong 0.24\left[\frac{\mathrm{J}}{\mathrm{cm_2}}\right]$$

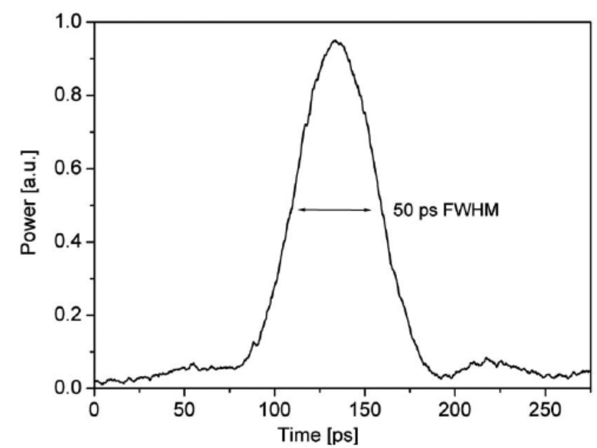
Q-switching, an example of a micro-chip laser

High-pulse-energy passively Q-switched quasi-monolithic microchip lasers operating in the sub-100-ps pulse regime

D. Nodop,[1] J. Limpert,[1,*] R. Hohmuth,[2] W. Richter,[2] M. Guina,[3] and A. Tünnermann[1]

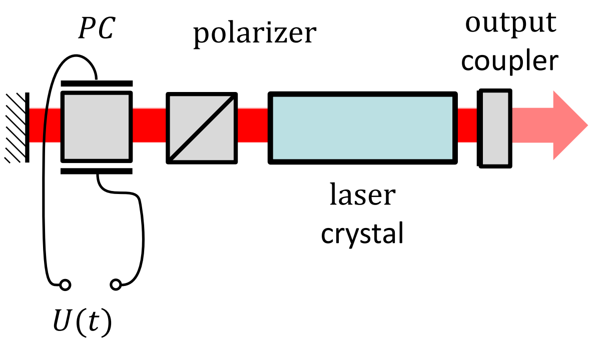
We present passively Q-switched microchip lasers with items bonded by spin-on-glass glue. Passive Q-switching is obtained by a semiconductor saturable absorber mirror. The laser medium is a Nd:YVO_4 crystal. These lasers generate pulse peak powers up to 20 kW at a pulse duration as short as 50 ps and pulse repetition rates of 166 kHz. At 1064 nm, a linear polarized transversal and longitudinal single-mode beam is emitted. To the best of our knowledge, these are the shortest pulses in the 1 μJ energy range ever obtained with passively Q-switched microchip lasers. The quasi-monolithic setup ensures stable and reliable performance.

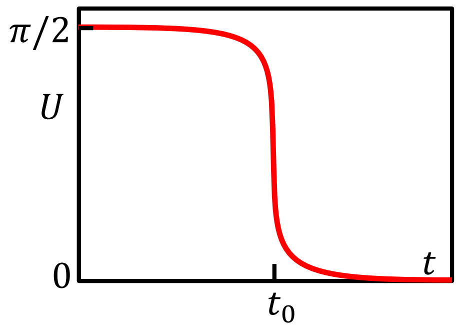




active Q-switching

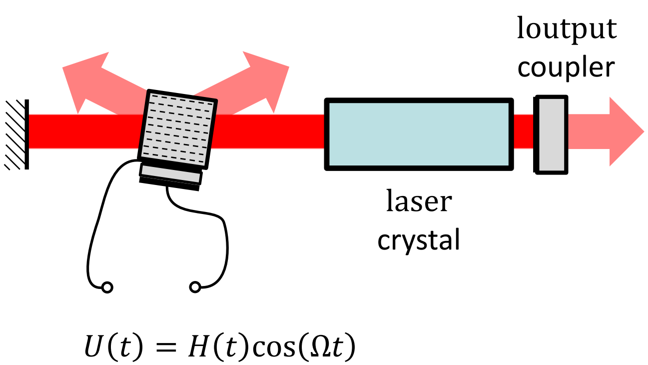
- electro-optic cell (Pockels Cell, PC)

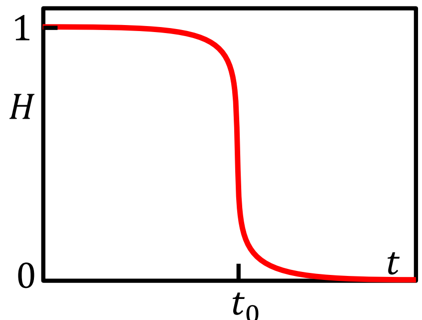




properties:
- speed – ns
- repetition rate - hundreds of kHz
- average power - moderate

- akusto-optic modulator





properties:
- speed –ten/hundreds of ns
- repetition rate - MHz
- average power - large

active Q-switching – examples

PULSELAS®-A Series • Actively Q-Switched Lasers @ 1064 nm				
Model	**PULSELAS-A-1064-300**	**PULSELAS-A-1064-500**	**PULSELAS-A-1064-600-HP**	**PULSELAS-A-1064-1000-HP**[1)]
Wavelength (nm)	1064	1064	1064	1064
Energy / Pulse (µJ, typ.)	20 @ 1 kHz	40 @ 1 kHz	50 @ 1 kHz	100 @ 1 kHz
Average Power (mW) @ Max. Rep. Rate	typ. 300 @ 20 kHz	typ. 500 @ 25 kHz	typ. 600 @ 25 kHz	typ. 700 @ 10 kHz
Pulse Width (ns)	0.7 - 1.0	1.0 - 1.5	1.5 - 2.0	1.5 - 3.0
Repetition Rate (kHz)	0 - 20	0 - 25	0 - 25	0 - 10
Beam Profile	TEM_{00}	TEM_{00}	TEM_{00}	TEM_{00}
Polarization Ratio	> 100:1	> 100:1	> 100:1	> 100:1
Beam Diameter (mm)	0.3	0.3	0.3	0.3
Beam Divergence (mrad Full Angle)	typ. 3	typ. 3	typ. 3	typ. 3
Power Instability (% rms, 1 hour)	< 3	< 3	< 3	< 3

	Evolution-15	**Evolution-30**	**Evolution-45**	**Evolution-HE**
Wavelength (nm)	527	527	527	527
Pulse Repetition-Rate (kHz)		1 to 10		1 (factory set)[1]
Average Output Power (W)	12 at 1 kHz 15 at 5 kHz 15 at 10 kHz	20 at 1 kHz 30 at 5 kHz 30 at 10 kHz	28 at 1 kHz 45 at 5 kHz 45 at 10 kHz	45 at 1 kHz 75 at 5 kHz 75 at 10 kHz
Energy-Per-Pulse (mJ)	12 at 1 kHz 3 at 5 kHz 1.5 at 10 kHz	20 at 1 kHz 6 at 5 kHz 3 at 10 kHz	28 at 1 kHz 9 at 5 kHz 4.5 at 10 kHz	45 at 1 kHz 15 at 5 kHz 7.5 at 10 kHz
Typical Pulse Width (nsec)(FWHM)	<300 at 1 kHz	<250 at 1 kHz	<250 at 1 kHz	<150 at 1 kHz
Pulse-to-Pulse Energy Stability (% rms)		<1		<1
Polarization Ratio		Horizontal, >100:1		Horizontal, >100:1
Spatial Mode		Multimode		Multimode
Beam Divergence (mrad)(full angle)		<10		<8
Beam Circularity (%)		>80		>80
Nominal Beam Diameter at Output Window (mm)($1/e^2$)		3		3

active Q-switching – examples, 2

Powerlite DLS 9000 Specifications

Description	9010	9020	9030	9050	Plus
Repetition Rate (Hz)	10	20	30	50	10
Energy (mJ)					
1064 nm	2000	1800	1600	1200	3000
532[1] nm	1000	900	800	600	1500
355[2] nm	550	475	400	350	800
266 nm	160	110	90	75	160
Pulsewidth[3] (nsec)					
1064 nm	5-9	5-9	5-9	5-9	5-9
532 nm	4-8	4-8	4-8	4-8	4-8
355 nm	3-7	3-7	3-7	3-7	3-7
266 nm	3-6	3-6	3-6	3-6	3-6
Linewidth[4] (cm^{-1})					
Standard	1	1	1	1	1
Injection Seeded, SLM	0.003	0.003	0.003	0.003	0.003
Divergence[5] (mrad)	0.45	0.45	0.5	0.5	0.45
Beam Pointing Stability[6] (±μrad)	30	30	30	30	30
Beam Diameter	9	9	9	9	12

Zaczynamy od równań dynamiki lasera:

$$\frac{dN_2}{dt} = -\sigma N_2 F$$

$$\frac{dF}{dt} = \chi c\sigma N_2 F - \frac{1}{\tau_p} F$$

zapisanych w nowych zmiennych

$$x = \frac{F}{\chi c \Delta N^t}$$

$$y = \frac{N_2}{\Delta N^t}$$

$$\tau = \chi c \gamma^t t$$

$$\frac{dy}{d\tau} = \frac{dy}{dt}\frac{dt}{d\tau} = \frac{dy}{dN_2}\frac{dN_2}{dt}\frac{dt}{d\tau} = -\frac{1}{\Delta N^t} \quad \sigma\Delta N^t y \quad \chi c\Delta N^t \mathrm{x} \quad \frac{1}{\chi c\gamma^t} = -\frac{\sigma\Delta N^t}{\gamma^t} xy = -xy$$

$$\frac{dx}{d\tau} = \frac{dx}{dt}\frac{dt}{d\tau} = \frac{dx}{dF}\frac{dF}{dt}\frac{dt}{d\tau} = \frac{1}{\chi c\Delta N^t}\left(\sigma\chi c\Delta N^t y \quad \chi c\Delta N^t x - \frac{1}{\tau_p}\chi c\Delta N^t x\right)\frac{1}{\chi c\gamma^t} =$$

$$= \frac{\sigma\chi c\Delta N^t \chi c\Delta N^t}{\chi c\Delta N^t \chi c\gamma^t} xy - \frac{1}{\tau_p}\frac{\chi c\Delta N^t}{\chi c\Delta N^t \chi c\gamma^t} x =$$

$$= \frac{\sigma\Delta N^t}{\gamma^t} xy - \frac{1}{\tau_p}\frac{1}{\chi c\gamma^t} x$$

ale

$\sigma\Delta N^t = \gamma^t$

oraz

$$\frac{1}{\tau_p}\frac{1}{\chi c\gamma^t} = \frac{c(1-R_2)}{2L}\frac{L2l}{lc(1-R_2)} = 1$$

bo $\gamma^t \cong \frac{1-R_2}{2l}$

i

$$\frac{dx}{d\tau} = (y-1)x$$

Ostatecznie:

$$\frac{dx}{d\tau} = (y-1)x$$

$$\frac{dy}{d\tau} = -xy$$