Lasers

lecture 4

Czesław Radzewicz

Dicke effect

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The Effect of Collisions upon the Doppler Width of Spectral Lines

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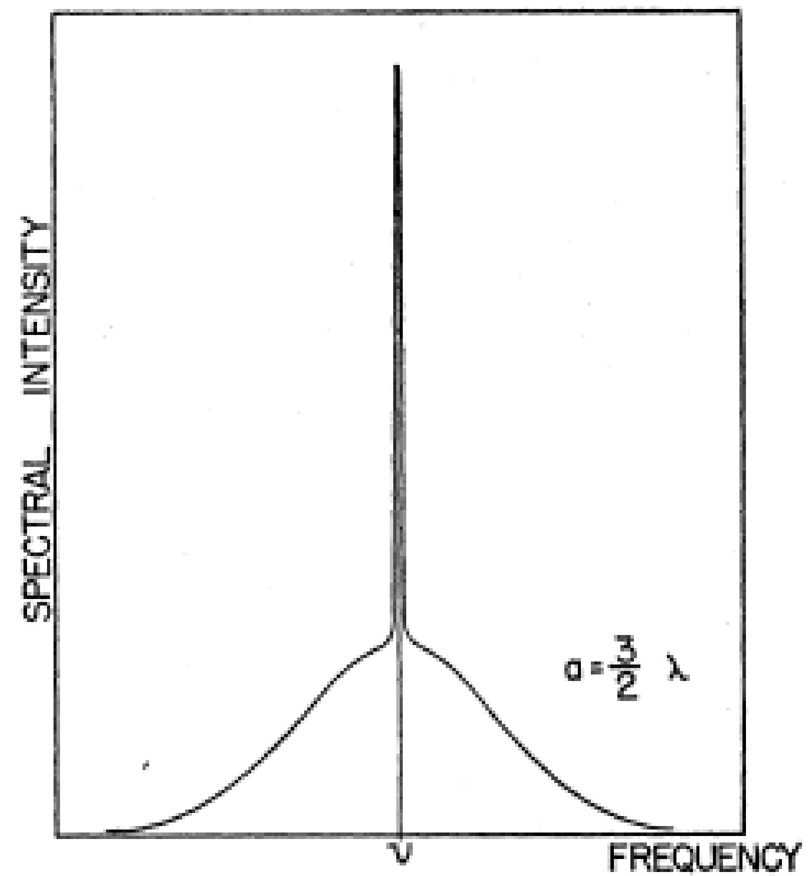
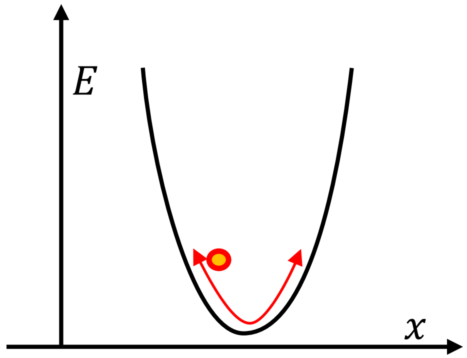
- Atomic gas, collisions change the velocity of atom but do not interrupt the phase of
- assume $a < \lambda$, with a being mean free path of the atom and λ is the wavelength

$$I(\alpha) = I_0 \frac{2\pi D/\lambda^2}{(\alpha - \nu)^2 + (2\pi D/\lambda^2)^2}$$

D – diffusion constant

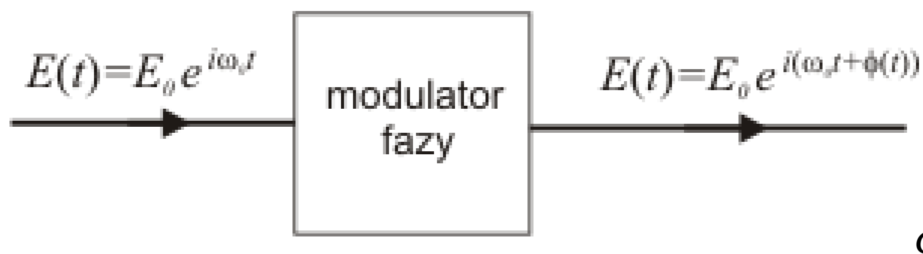


Dicke effect in optical lattice



- an atom is radiating em wave of frequency ω measured at its own reference system
- the atom moves in the x direction: $x(t) = a\sin(\Omega t)$, $\upsilon(t) \equiv \frac{dx}{dt} = \Omega a\cos(\Omega t)$, harmonic oscillations
- classical approach, linear Doppler effect, the observer is located on the x axis
- $\omega'(t) = \left(1 + {}^{\upsilon(t)}/_{c}\right)\omega$ with $\upsilon(t)$ being the velocity of atom
- $\Delta\omega(t) = \omega - \omega'(t) = \frac{\upsilon(t)}{c}\omega = \frac{\Omega a}{c}\frac{2\pi c}{\lambda}\cos(\Omega t) = \frac{2\pi\Omega a}{\lambda}\cos(\Omega t)$ – pure phase modulation of the radiation
- the phase $\phi(t) = \int \omega'(t)dt = \omega t - \frac{2\pi a}{\lambda}\sin(\Omega t)$

Dicke effect in optical lattice, 2



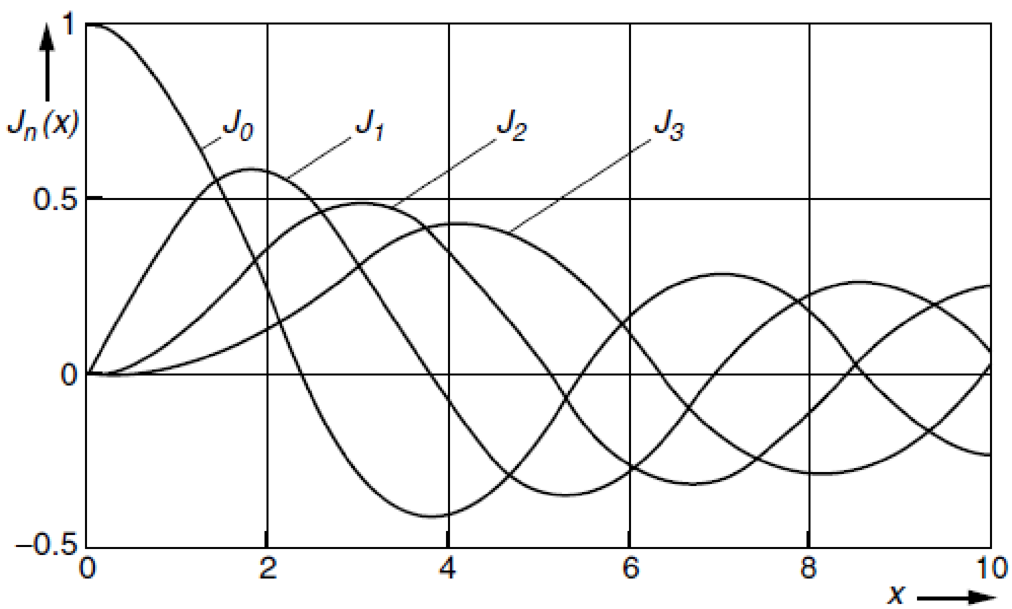
$$\phi(t) = \omega t - \frac{2\pi a}{\lambda}\sin(\Omega t)$$

$$E_{out}(t) = E_0 e^{-i[\omega t - \delta \sin(\Omega t)]\omega t}, \delta = \frac{2\pi a}{\lambda}$$

$$E_{out}(t) = E_0 \sum_{n=-\infty}^{\infty} J_n(\delta) e^{-i(\omega t - n\Omega)t}$$

J_n- Bessel function type 1 order n

If $\delta \ll 1$ then $J_n(\delta) \ll J_0(\delta)$ for $n = 1,2,3 \ldots$.

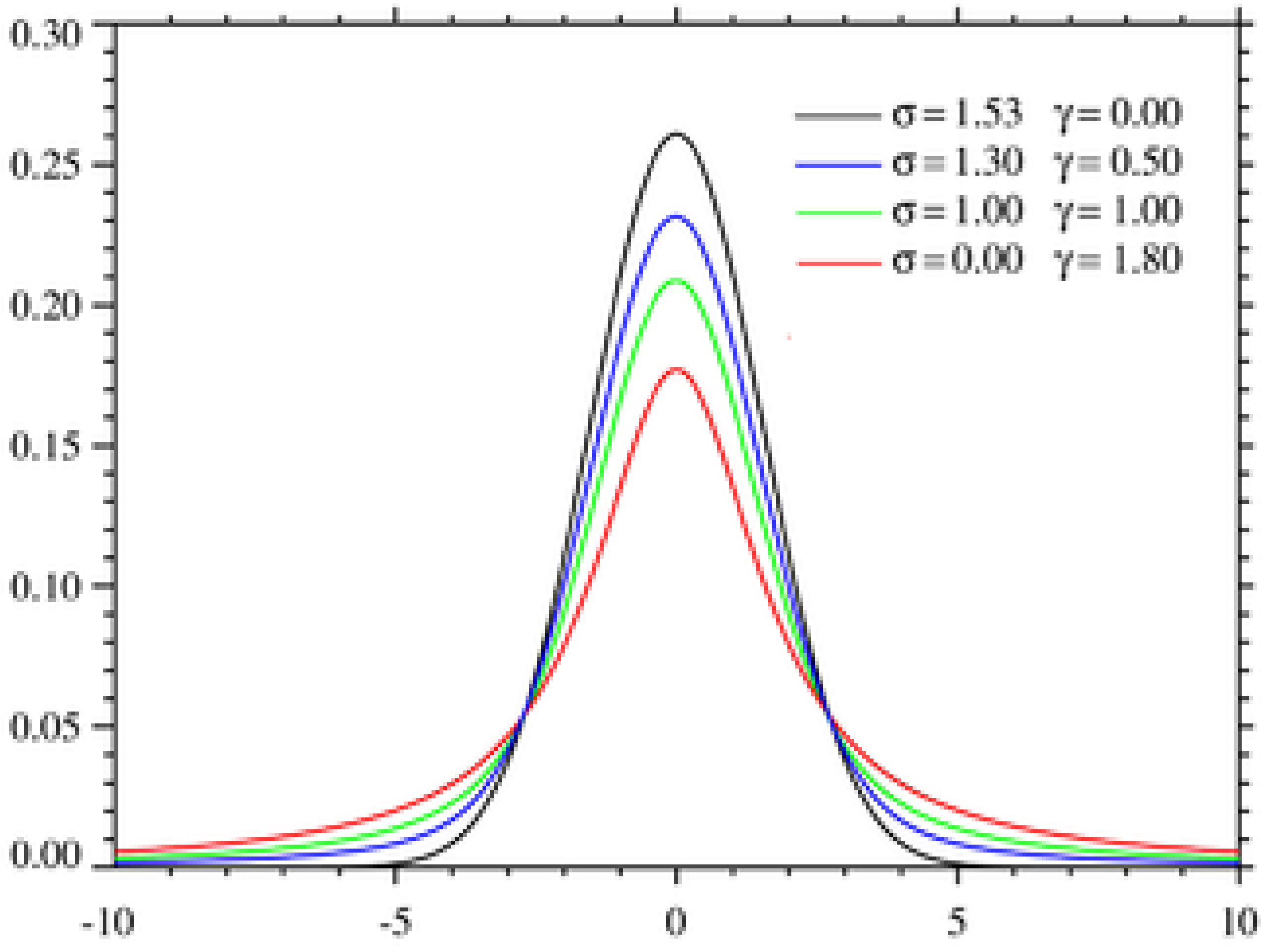


mixed line-broadening, 1

Voigt's profile is a convolution of Lorentz and Gauss functions

$$g_V(x) = \int_{-\infty}^{\infty} dx' G(x';\sigma) L(x-x';\gamma)$$

$$G(x;\sigma) \equiv \frac{e^{-\frac{x^2}{(2\sigma^2)}}}{\sigma\sqrt{\pi}}, \quad L(x;\gamma) \equiv \frac{\gamma}{\pi(x^2+\gamma^2)}$$



mixed line-broadening, 2

absorption coefficient α $\alpha \propto p \times {}^{1}/_{\Delta\nu}$

Doppler broadening: $\alpha \propto p$

pressure broadening: $\alpha \propto p \times {}^{1}/_{p} = \text{const}$

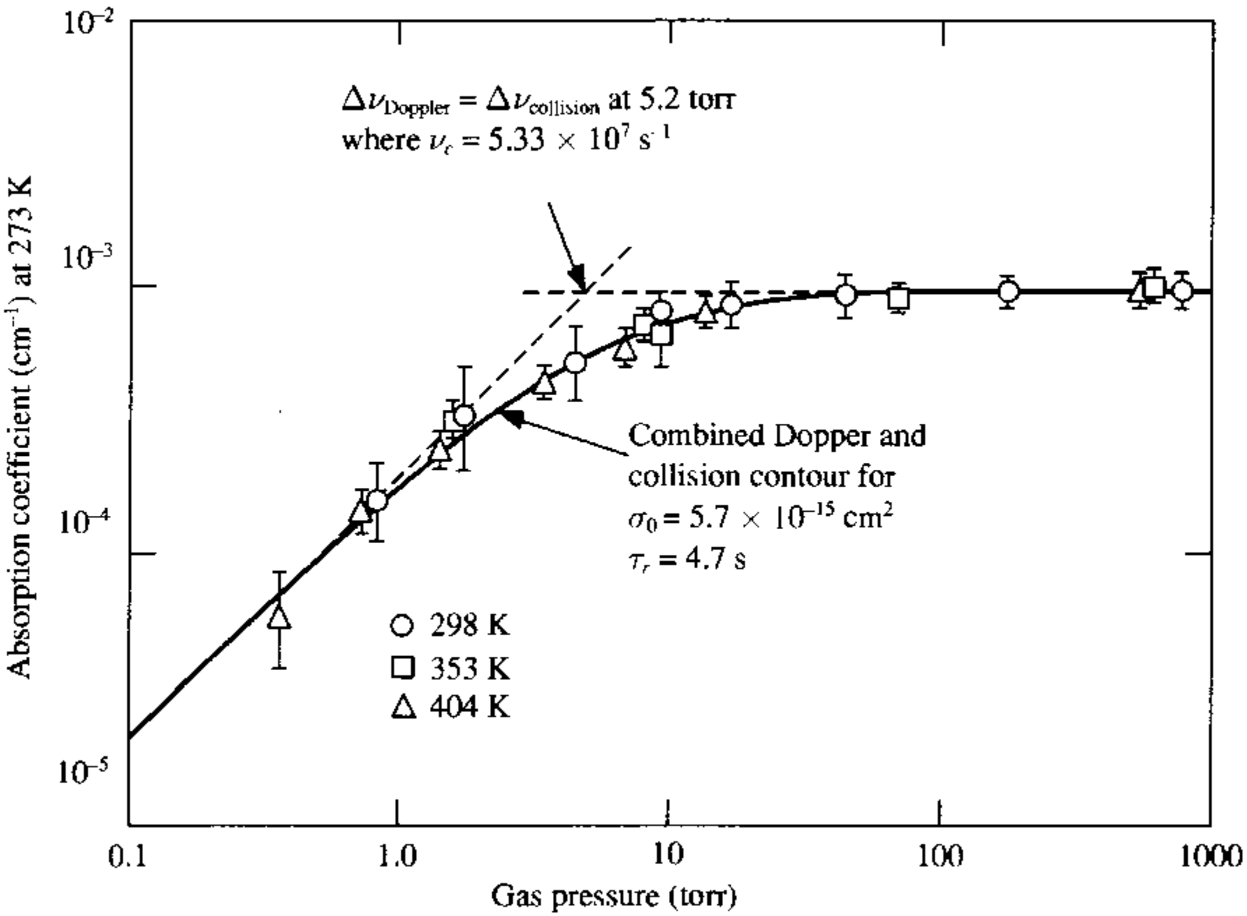


FIGURE 7.9. Absorption coefficient in CO_2 at 10.6 μm as a function of CO_2 pressure. (After E. T. Gerry and D. A. Leonard, *Appl. Phys. Lett.* 8, 227, 1966.)

"typical" linewidths

	effect	gas	liquid	condensed matter
homogenous	natural	0.001Hz-10MHz	n *	n
	atomic collision	5-10MHz/mbar	≈ 300 cm^{-1}	----
	phonons	---	---	≈ 10 cm^{-1}
inhomogeneous	Doppler	50MHz-1GHz	n	---
	Local fields	---	≈ 500 cm^{-1}	1-500 cm^{-1}

*n - negligible

cm^{-1} units are often used in spectroscopy

$$\tilde{\nu} \equiv \frac{1}{\lambda[\mathrm{cm}]}$$

$$\tilde{\nu} \equiv \frac{1}{\lambda}[\mathrm{cm}^{-1}] = \frac{\nu}{c\left[\frac{\mathrm{cm}}{\mathrm{s}}\right]} = 10^{-2}\frac{\nu}{c}$$

numbers: $\lambda = 1\ \mu\mathrm{m} \Leftrightarrow 10\,000\ \mathrm{cm}^{-1}$

for $\lambda = 1\ \mu\mathrm{m}$: $1\ \mathrm{cm}^{-1} = 30\mathrm{GHz}$

gain saturation in media with different line broadening

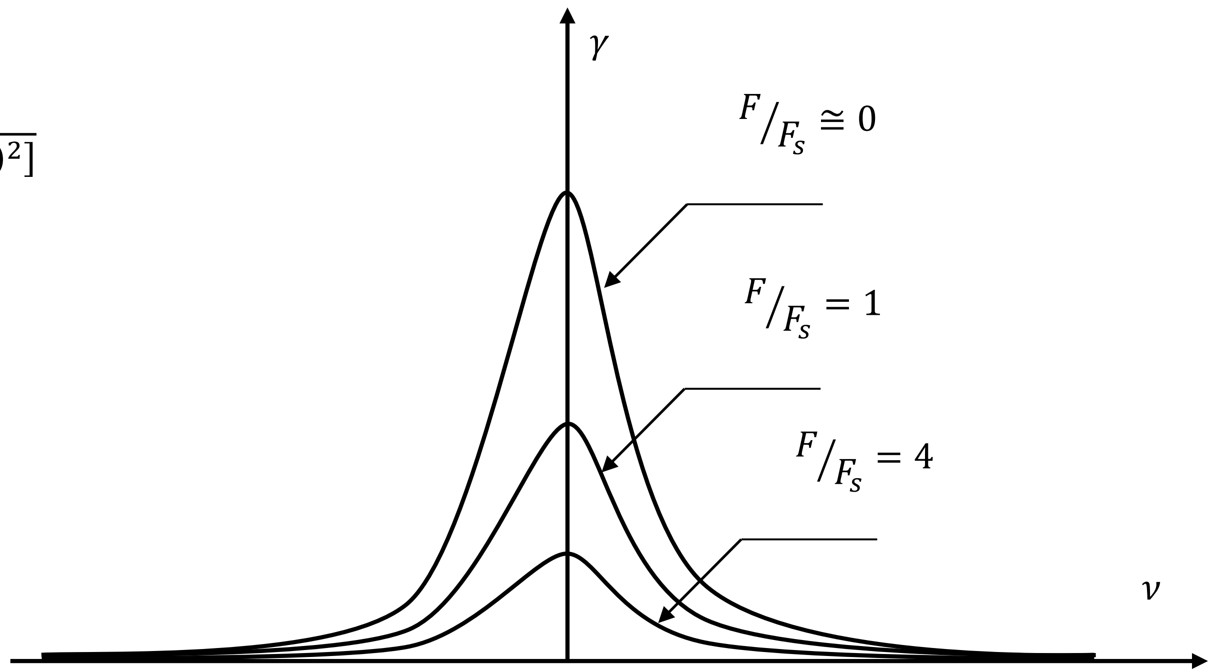
We will concentrate on the case $\tau_p \gg T_1$. Similar reasoning can be extende to other cases as well.

$$\gamma(F) = \frac{\gamma_0}{1 + F/F_s}$$

- Homogenous broadening dominates. As the population inversion decreases the gain drops for all frequencies because all atoms interact with the em wave in the same way. – Saturation requires higher intensities for frequencies far away for the resonance.

$$\gamma_0(\nu) \propto \frac{\Delta\nu}{2\pi[(\nu - \nu_0)^2 + (\Delta\nu/2)^2]}$$

$$\gamma(F, \nu) = \frac{\gamma_0(\nu)}{1 + F/F_s}$$

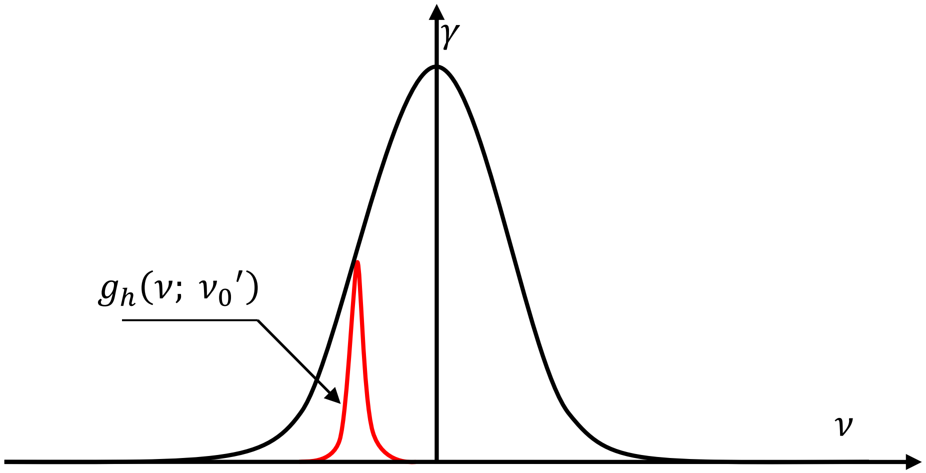


gain saturation in media with different line broadening, 2

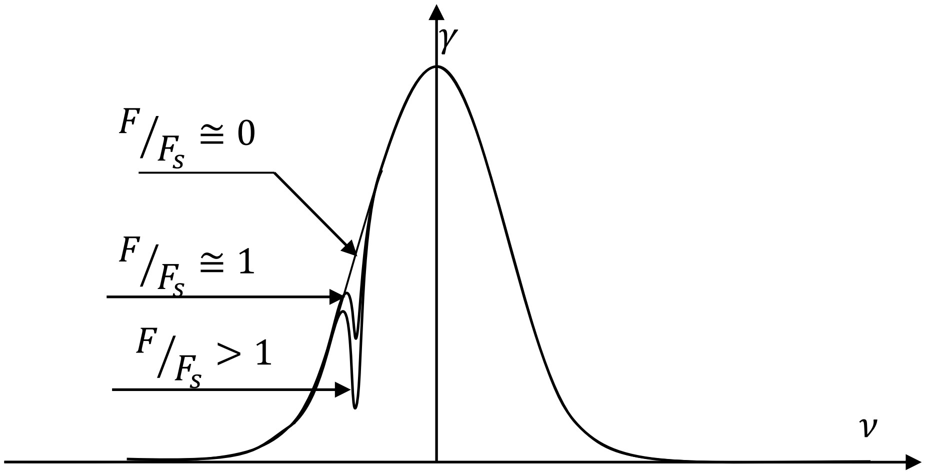
- inhomogeneous broadening dominates. A monochromatic em wave of frequency ν inetracts only with atoms that have their resonant frequencies close to ν (closer than homogenous linewidth). The saturation affects only this selected group of atoms – other groups „do not see" the em field.

$$\gamma_0(\nu) \propto \int_{-\infty}^{\infty} d\nu_0' \underbrace{\frac{\Delta\nu}{2\pi[(\nu-\nu_0')^2+(\Delta\nu/2)^2]}}_{g_h(\nu,\nu_0')} g(\nu_0')$$

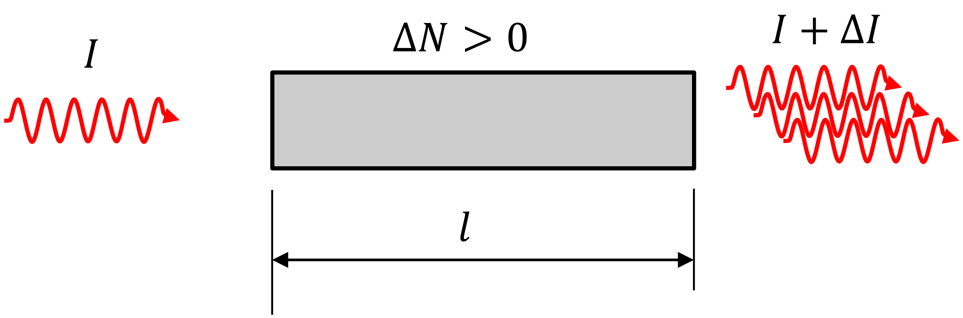
$g_j(\nu,\nu_0')$
homogenously broaden line centered at ν_0'.

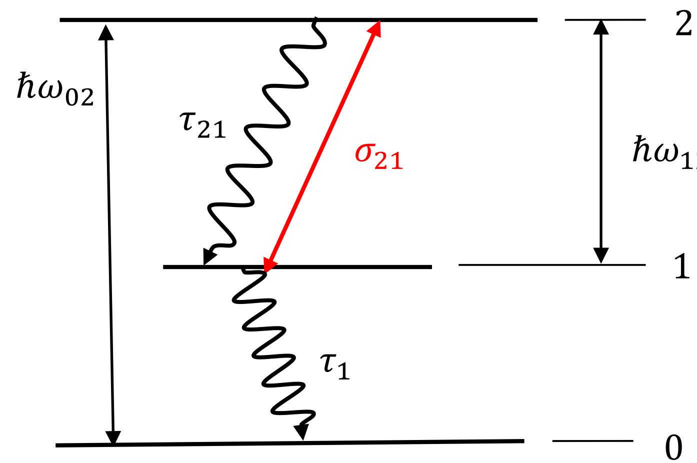


Saturation „burns a hole" in the gain profile. Its width corresponds to the homogenous linewidth. The depth of the hole scales with saturation (em field intensity).



laser amplifier efficiency





Definition:

- surface energy density (energy stored in the amplifier per unit area of its cross-section)

$$\mathcal{E} \equiv \hbar\omega_{12}\Delta N l = \frac{\hbar\omega_{12}}{\sigma}\sigma\Delta N l = E_s \cdot \gamma_0 \cdot l \qquad \text{for } \tau_p \gg \tau_1$$

$$\mathcal{E} \equiv E_s \cdot \gamma_0 \cdot \frac{l}{2} \qquad \text{for } \tau_p \ll \tau_1$$

- surface power density (power that can be extracted from the amplifier per unit area of its cross-section)

$$\mathcal{P} \equiv \frac{\hbar\omega_{12}\Delta N l}{\tau_{21}} = I_s \cdot \gamma_0 \cdot l$$

with

$E_s = \frac{\hbar\omega_{12}}{\sigma_{21}}$, saturating fluence

$I_s = \hbar\omega_{12}/(\sigma_{21}\,\tau_{21})$ saturating intensity

minimum energy density needed to create population inversion ΔN:

$$\hbar\omega_{02}\Delta N$$

useful energy density>

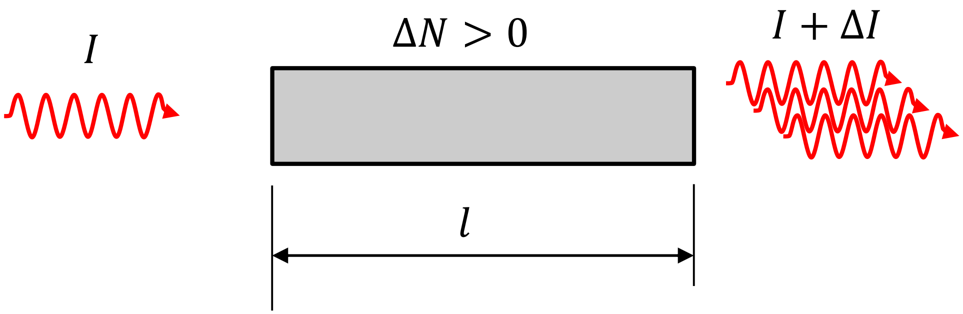
$$\hbar\omega_{12}\Delta N$$

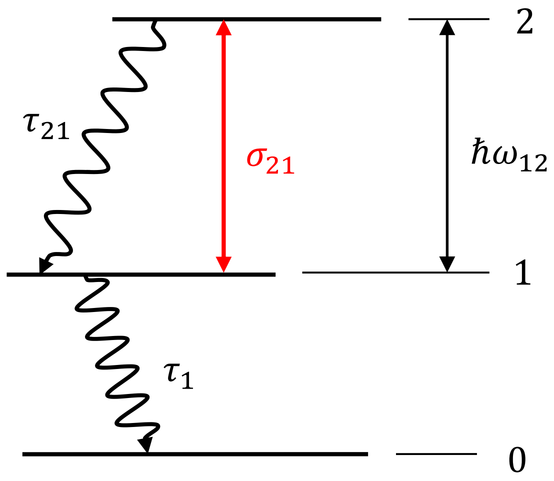
quantum defect

$$\frac{\omega_{02} - \omega_{12}}{\omega_{02}}$$

the smaller the better

laser amplifier efficiency, 2





The definition of efficiency depends on the pulse duration:

- for short pulse $\tau_p << \tau_{21}$ we use surface energy density

$$\eta = \frac{\mathcal{E}_{out} - \mathcal{E}_{in}}{\mathcal{E}}$$

- long pulse $\tau_p \gg \tau_{21}$

The medium can adiabatically follow the photon flux – we should consider intensity. For simplicity, let's assume stationary case

$$\eta = \frac{I_{out} - I_{in}}{\mathcal{P}}$$

„in" and „out" correspond to the input and output of the amplifier, respectively.

long pulse laser amplifier efficiency

A note that is always valid:

- The stronger the saturation the higher the efficiency.

Let's take the eqs. describing long pulse amplifier:

$$\ln\frac{I_{out}}{I_{in}}+\frac{I_{out}-I_{in}}{I_s}=\gamma_0 l$$

calculate

$$I_{out}-I_{in}=I_s\left(\gamma_0 l-\ln\frac{I_{out}}{I_{in}}\right)$$

in deep saturation we have in $I_{out}\cong I_{in}$ and thus

$$I_{out}-I_{in}\cong I_s\gamma_0 l=\mathcal{P}$$

and

$$\eta=\frac{I_{out}-I_{in}}{\mathcal{P}}\cong 1$$

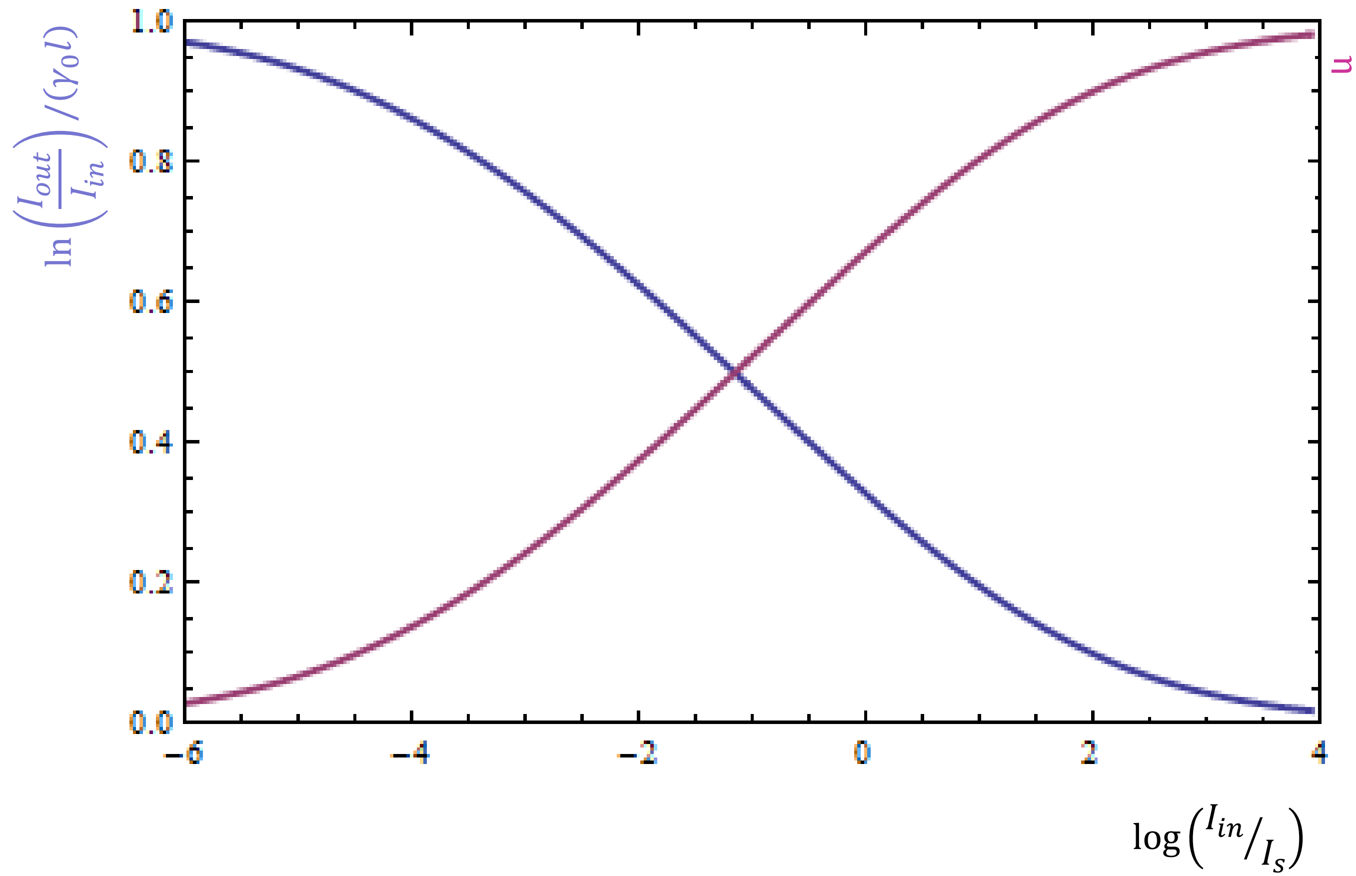
- The other limit (unsaturated amplifier):

$I_{in}, I_{out}\ll I_s \Longrightarrow I_{out}=e^{\gamma_0 l}\, I_{in}$

$$\eta=\frac{I_{out}-I_{in}}{\mathcal{P}}=\frac{\gamma_0 l-\ln\left({}^{I_{out}}/{}_{I_{in}}\right)}{\gamma_0 l}=0$$

long pulse laser amplifier efficiency, 2

numerical modelling
$\gamma_0 l = 4$



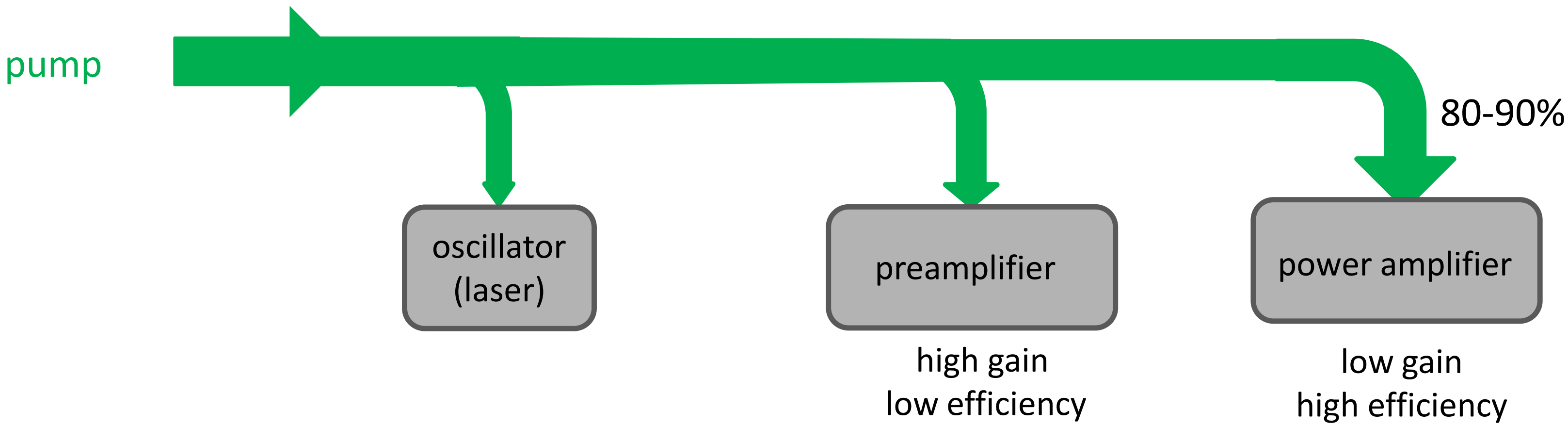
the dilemma of a laser master:
gain or efficiency?

laser amplifier efficiency, practical remarks

Can you eat the cake and keep it? Yes, you can have both in laser amplifiers!

- ns and longer pulses:

<u>MOPA</u> (Master Oscillator Power Amplifier

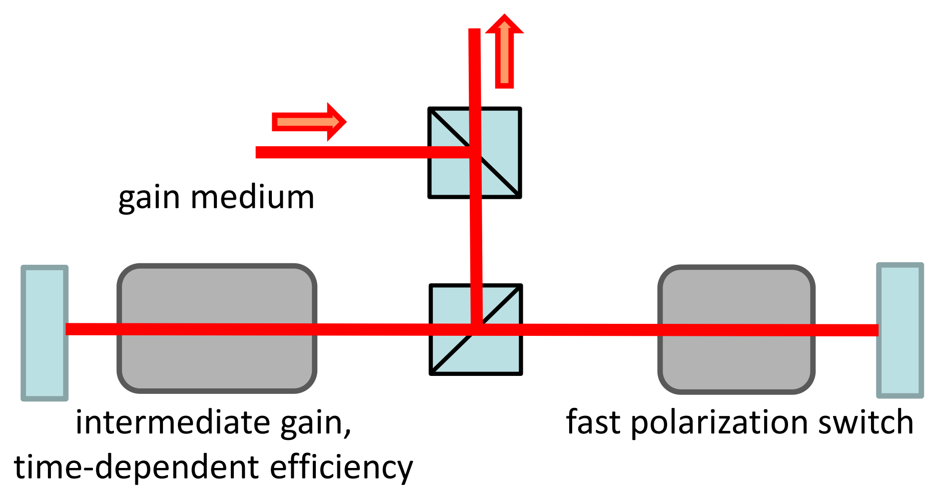


- sub-ns and shorter pulses

<u>regenerative amplifier</u>, pulsed operation

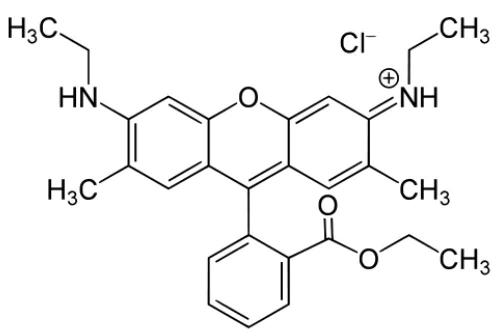
amplifier cycle:

- ❑ pumping of the gain medium
- ❑ seeding – the pulse is locked in the cavity
- ❑ amplification; a few up to few tens of round-trips
- ❑ the pulse is ejected from the cavity

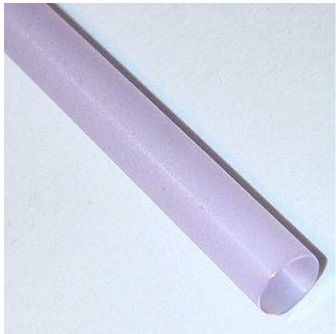


examples of amplifying media

name	formula	$\sigma[10^{-19}\text{cm}^2]$	$\lambda[\mu\text{m}]$	$\tau_{21}[\mu\text{s}]$	$\mathcal{E}[\text{J/cm}^2]$	$\mathcal{P}[10^6\text{W/cm}^2]$
Rhodamine 6G	$C_{28}H_{31}N_2O_3Cl$	2000	$\cong 0.59$	0.022	0.002	0.33
Nd:YAG	$Nd^{3+}{:}Y_3Al_5O_{12}$ 1% - 1.38×10^{20} /cm^3	2.8	1.064	230	0.89	
Ti:Sapphire	$Ti^{3+}{:}Al_2O_3$	3.8	$0.75 \div 1.1$	2.4	0.66	0.2
LiSAF	$Cr^{3+}{:}LiSrAlF_6$	0.5	$0.8 \div 0.9$	67	5.2	0.08
Yb:KYW	$Yb^{3+}{:}KY(WO_4)_2$ 0.5-100%	0.3	$1.03 \div 1.06$	300	7	
alexandrite	$Cr^{3+}{:}BeAl_2O_4$	0.1	0.75	~ 200	26	0.13



structure of Rhodamine 6G molecule


Nd:YAG


alexandrite


Ti:Sapphire

pumping of gain media

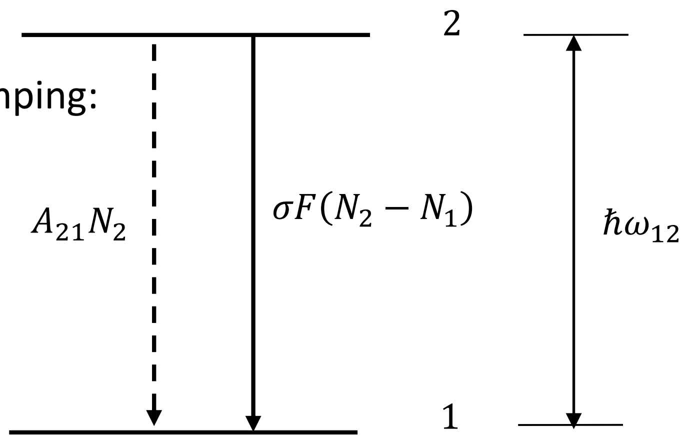
we need population inversion: $N_2 > N_1$. In thermodynamic equilibrium we have Boltzman distribution of the populations: $\frac{N_2}{N_1} = \exp\left(-\frac{\hbar\omega_{12}}{kT}\right) < 1$. Heating of the medium does not work because temperature increase can, at most, equalize the populations. We need to put energy selectively so it results in mowing the atom/ion to the upper level of the laser transition.
The methods:

- electric current
- em radiation – light
- exothermic chemical reaction

2-level system, let's consider optical pumping:

$$\frac{dN_2}{dt} = -A_{21}N_2 - \sigma F(N_2 - N_1)$$

$$N_2 - N_1 = 2N_2 - N$$



$$\frac{dN_2}{dt} = -(A_{21} + 2\sigma F)N_2 + \sigma FN$$

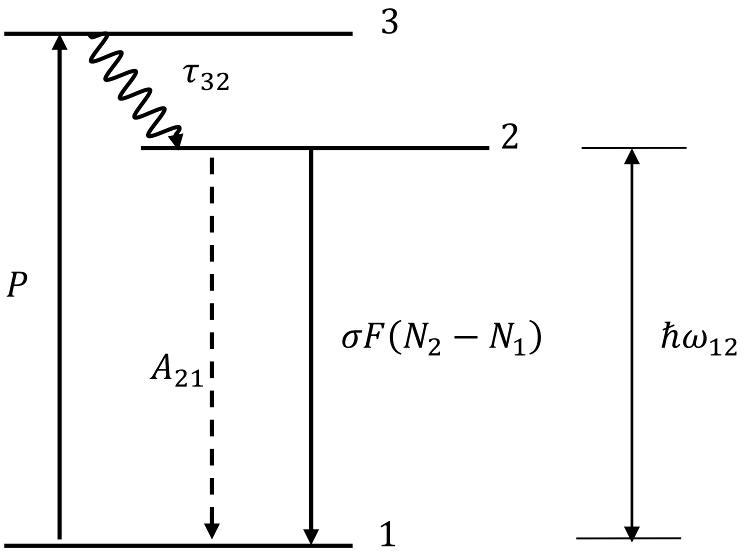
stationary solution:

$$N_2 = \frac{\sigma FN}{(A_{21} + 2\sigma F)}$$

in the high intensity limit

$$\lim_{F\to\infty} N_2 = N/2$$

3-level system



assumptions:

- $\tau_{21} = 1/A_{21}$,
- $\tau_{32} \ll \tau_{21}$
- $\frac{dN_3}{dt} = P \cdot N_1$

rate equations:

$$N_3 = 0$$
$$\frac{dN_2}{dt} = PN_1 - A_{21}N_2 - \sigma F(N_2 - N_1)$$
$$N_1 = N - N_2$$

stationary solutions $\left(\frac{dN_2}{dt} = \frac{dN_1}{dt} = 0\right)$

for small light intensity (we neglect the term $\sigma F(N_2 - N_1)$):

$$N_2 = \frac{P\tau_{21}}{1 + P\tau_{21}} N$$
$$N_1 = \frac{1}{1 + P\tau_{21}} N$$

population inversion:

$$\Delta N_0 = \frac{P\tau_{21} - 1}{1 + P\tau_{21}} N$$

gain possible for :

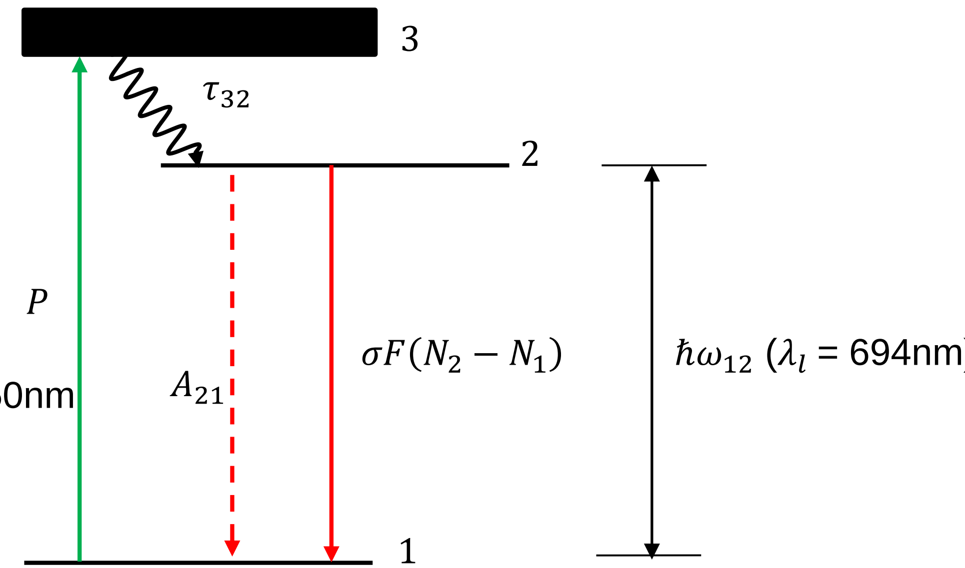
$$\Delta N_0 > 0 \Leftrightarrow P > \frac{1}{\tau_{21}} = A_{21}$$

3-level system, an example

ruby – $Cr^{3+}:Al_2O_3$ chromium concentration 0.05%

$N \cong 2 \times 10^{19}\mathrm{cm}^{-3}$

$\tau_{21} \cong 2 \times 10^{-3}\mathrm{s}$



minimum pump rate:

$$P_{min} = \frac{1}{\tau_{21}} \cong 500\mathrm{s}^{-1}$$

pump power density needed to reach $\Delta N > 0$:

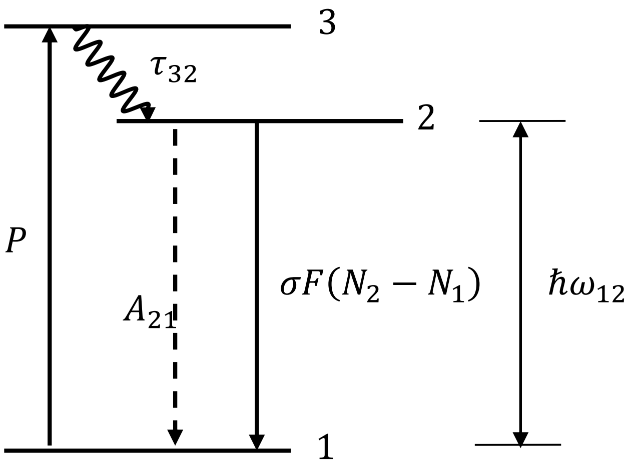
$$\mathcal{P} = P_{min} \cdot N \cdot \hbar\omega_{12} \cong (0.5 \times 10^{-3}\mathrm{s}^{-1})(2 \times 10^{19}\mathrm{cm}^{-3})(3.6 \times 10^{-19}\mathrm{J}) = 3.6\mathrm{kW/cm}^3$$

heat dissipation:

$$\mathcal{P}_{ciepło} = \frac{\omega_{p-}\omega_{21}}{\omega_p}\mathcal{P} \cong 0.8\mathrm{kW/cm}^3$$

pulsed operation

3-level system, saturation



population equations with light

$$\frac{dN_2}{dt} = PN_1 - A_{21}N_2 - \sigma F(N_2 - N_1) = 0$$

$$N_1 = N - N_2$$

gives

$$P(N - N_2) - A_{21}N_2 - \sigma F(2N_2 - N) = 0$$

algebra…

$$\Delta N = \frac{A_{21} + \sigma F}{P + A_{21} + 2\sigma F} N$$

more algebra…

$$\Delta N = \Delta N_0 \frac{1}{1 + F/F_s}$$

remember this formula

$$\boxed{\begin{aligned} \gamma(\nu, I, P) &= \gamma_0 \frac{1}{1 + F/F_s} \\ \gamma_0 &= \sigma(\nu)\frac{P\tau_{21} - 1}{P\tau_{21} + 1}N \end{aligned}}$$

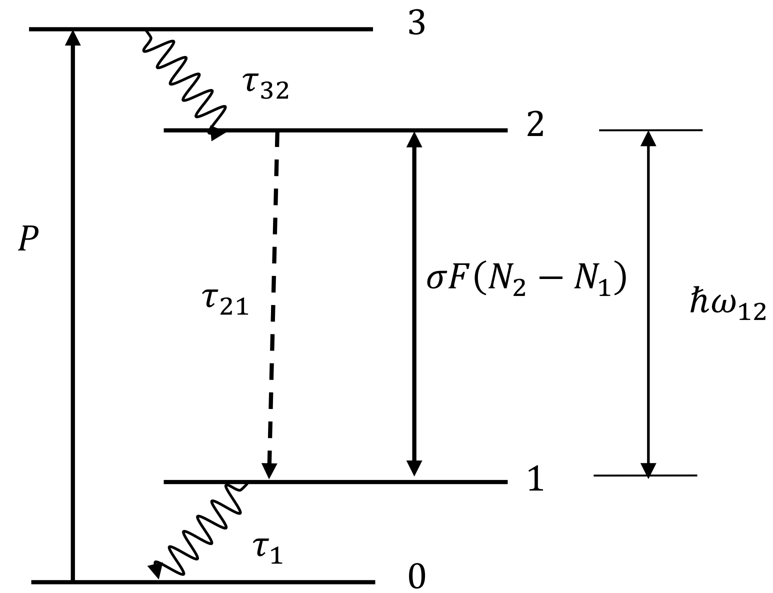
$$F_s(\nu, P) = \frac{1}{\sigma(\nu)\tau_{21}} \frac{1 + P\tau_{21}}{2}$$

$$I_s(\nu, P) = \frac{\hbar\omega_{12}}{\sigma(\nu)\tau_{21}} \frac{1 + P\tau_{21}}{2}$$

4-level system

Assume:

- $\tau_{21} = 1/A_{21}$,
- $\tau_{32} \ll \tau_{21}$
- $\frac{dN_3}{dt} = P \cdot N_1$



rate equations again:

$$N_3 = 0$$

$$\frac{dN_2}{dt} = PN_1 - A_{21}N_2 - \sigma F(N_2 - N_1)$$

$$\frac{dN_1}{dt} = A_{21}N_2 - \frac{1}{\tau_1}N_1\sigma + F(N_2 - N_1)$$

$$N_0 + N_1 + N_2 = N$$

Stationary solutions for small light intensity:

$$\boxed{\Delta N_0 = \frac{P(\tau_{21} - \tau_1)}{1 + P(\tau_{21} + \tau_1)}N}$$

gain possible if :

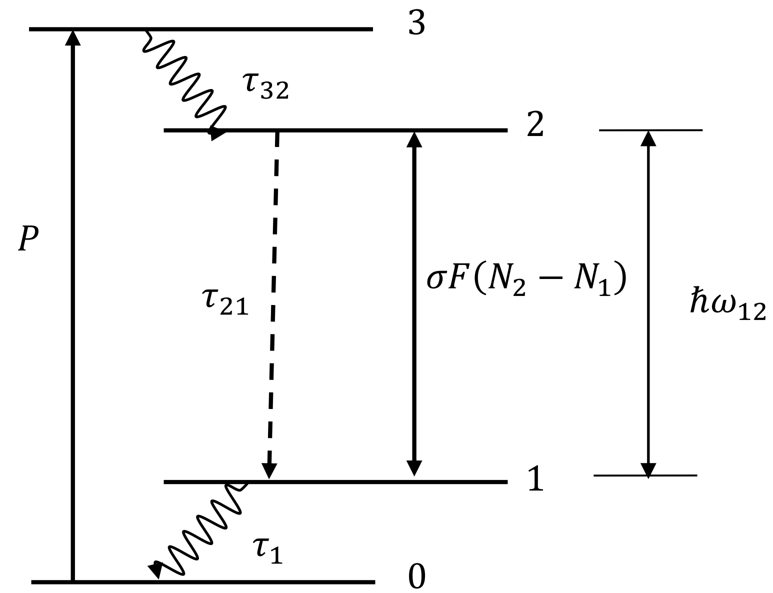
$$\Delta N_0 > 0 \Leftrightarrow \tau_{21} > \tau_1$$

<u>independently of the pumping rate</u>

4-level system, gain saturation

Assumptions:

- $\tau_{21} = 1/A_{21}$,
- $\tau_{32} \ll \tau_{21}$
- $\frac{dN_3}{dt} = P \cdot N_1$



rate eqs.:

$$N_3 = 0$$

$$\frac{dN_2}{dt} = PN_0 - A_{21}N_2 - \sigma F(N_2 - N_1)$$

$$\frac{dN_1}{dt} = A_{21}N_2 - \frac{1}{\tau_1}N_1\sigma + F(N_2 - N_1)$$

$$N_0 + N_1 + N_2 = N$$

Stationary solutions:

$$\gamma(\nu, F, P) = \gamma_0(\nu)\frac{1}{1 + F/F_s}$$

$$\gamma_0(\nu) = \sigma(\nu)\Delta N_0$$

$$F_s(\nu, P) = \frac{1}{\sigma(\nu)}\frac{1 + P(\tau_{21} + \tau_1)}{1 + 2P\tau_1}$$