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On a New Method in the Theory of Superconductivity.

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Summary. — A generalization of the method elaborated by the author for the theory of superconductivity is presented. It is shown that the original Fröhlich model possesses the property of superconductivity. The ground state and its fermion excitations are considered.

After the discovery of the isotope effect it is generally accepted that interaction between electrons and the lattice should play an important role in superconductivity.

We shall show in the present paper that such a system really does exhibit superconductive properties.

Some very interesting investigations of a system of electrons interacting with a phonon field have been performed ([1-4]) along this line. In the present paper it will be shown that by extending the method previously proposed by us for the study of superfluidity one may develop a consistent theory of the superconductive state. In particular this theory yields results which confirm those of Bardeen's theory ([3]).

For sake of simplicity we shall base our considerations on the model proposed by FRÖHLICH ([1]); Coulomb interaction is not introduced explicitly, the

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([1]) H. FRÖHLICH: *Phys. Rev.*, **79**, 845 (1950); *Proc. Roy. Soc.* (*London*), A **215**, 291 (1952).

([2]) J. BARDEEN: *Rev. Mod. Phys.*, **23**, 261 (1951); *Handb. der Phys.* (Berlin, 1956), **15**, 274; J. BARDEEN and D. PINES: *Phys. Rev.*, **99**, 1140 (1950).

([3]) J. BARDEEN, L. N. COPPER and J. R. SCHRIEFFER: *Phys. Rev.*, **106**, 162 (1957).

([4]) D. PINES: *Phys. Rev.* (in print).

dynamical system being characterized by the Hamiltonian (*)

$$(1)\qquad \begin{cases} H_F = \sum\limits_{k,s} E(k)\overset{+}{a}_{ks}a_{ks} + \sum\limits_{q}\omega(q)\overset{+}{b}_q b_q + H' \\ H' = \sum\limits_{\substack{k,q,s \\ k'-k=q}} g\left\{\dfrac{\omega(q)}{2V}\right\}^{\frac{1}{2}}\overset{+}{a}_{ks}a_{k's}\overset{+}{b}_q + \text{c. c.} \end{cases}$$

where $E(k)$ is the electron energy, $\omega(q)$ the phonon energy, g a coupling constant and V the volume.

As is now well known, conventional perturbation theory expressed in powers of the coupling constant is not valid due to the circumstance that the electron-phonon interaction, despite its smallness, is very significant near the Fermi surface.

Thus as a preliminary step we shall perform a canonical transformation on the basis of the following considerations.

First of all it will be noted that matrix elements corresponding to virtual « particle » creation in vacuum always involve energy denominators of the form

$$\{\varepsilon(k_1) + \ldots + \varepsilon(k_{2s}) + \omega(q_1) + \ldots + \omega(q_{2r})\}^{-1}$$

in which $\varepsilon(k) \sim |E(k) - E_F|$ is the particle energy of an electron ($E(k) > E_F$), or of a hole ($E(k) < E_F$) which becomes small near the Fermi surface.

Such denominators are in general not « dangerous » and integration over the momenta $k_1, \ldots, k_{2s}, q_1, \ldots, q_{2r}$ does not lead to divergences, *an exception being the case of virtual creation of a single pair without phonons.* In virtue of the conservation law the momenta of this pair will be oppositely directed and the energy denominator

$$\{2\varepsilon(k)\}^{-1}$$

will then be « dangerous » during integration.

It may also be mentioned that the particle spins will likewise have opposite directions.

Thus in choosing a canonical transformation one should keep in mind the *necessity of ensuring mutual compensation of graphs leading to virtual creation in vacuum of a pair of particles with opposite momenta and spins* (+).

(*) The unit system in which $\hbar = 1$ is employed here.

(+) It is to be stressed that in the ordinary perturbation theory applied directly to the normal state these graphs cannot appear because of conservation of the number of real electrons.

But if we mix electron and hole states by a canonical transformation the conservation principle does not help and such graphs appear.

It might be pertinent to point out the analogy between the present situation and that encountered in our theory of superfluidity (5) of a non-ideal Bose gas; in the latter case the same role was played by virtual creation from the condensate of a pair of particles with momenta $\pm k$. In this theory (5) we employed a linear transformation of Bose amplitudes which « mixes » b_q with $\bar{b}_{-q}$.

Generalizing this transformation we introduce in the case under consideration new Fermi amplitudes α

$$\alpha_{k0} = u_k a_{k,\frac{1}{2}} - v_k \overset{+}{a}_{-k,-\frac{1}{2}}$$

$$\alpha_{k1} = u_k a_{-k,-\frac{1}{2}} + v_k \overset{+}{a}_{k,\frac{1}{2}}$$

or

$$a_{k,\frac{1}{2}} = u_k \alpha_{k0} + v_k \overset{+}{\alpha}_{k1}$$

$$a_{-k,-\frac{1}{2}} = u_k \alpha_{k1} - v_k \overset{+}{\alpha}_{k0}$$

where u_k, v_k are real numbers which are related as follows

$$u_k^2 + v_k^2 = 1 .$$

It is not difficult to verify that this transformation retains all commutation properties of Fermi operators and is therefore canonical. It may also be noted that it is a generalization of the usual transformation employed to introduce creation and annihilation operators for holes inside the Fermi surface or for electrons outside this surface.

Indeed, if we put

$$u_k = 1 , \quad v_k = 0 \qquad (E(k) > E_F)$$

$$u_k = 0 , \quad v_k = 1 \qquad (E(k) < E_F)$$

we obtain

$$\alpha_{k0} = a_{k,\frac{1}{2}} , \quad \alpha_{k1} = a_{-k,-\frac{1}{2}} \qquad (E(k) > E_F)$$

$$\alpha_{k0} = - \overset{+}{a}_{-k,-\frac{1}{2}} , \quad \alpha_{k1} = \overset{+}{a}_{k,\frac{1}{2}} \qquad (E(k) < E_F)$$

so that α_{k0} for example, will be the annihilation operator of an electron having momentum k and spin $\frac{1}{2}$ outside the Fermi sphere and the annihilation operator of a hole with a momentum $-k$ and spin $-\frac{1}{2}$ inside it.

(5) N. N. BOGOLJUBOV: *Journ. of Phys. USSR*, **9**, 23 (1947); *Vestnik MGU*, no. 7, 43 (1947).

In the general case, when $(u_k, v_k) \neq (0, 1)$ superposition of a hole and electron is encountered.

Returning to the Fröhlich Hamiltonian we note that for technical reasons it will be more convenient for us not to be tied up with the relation

$$\sum_{k,s} \overset{+}{a}_{ks} a_{ks} = N_0 ,$$

where N_0 is the total number of electrons, but to proceed in a manner which is usual in such cases, and choose a parameter λ which plays the role of a chemical potential.

Thus instead of H_F we shall deal with the Hamiltonian

$$H = H_F - \lambda N . \tag{2}$$

The value of parameter λ will be found from the condition that in the state under consideration

$$\overline{N} = N_0 . \tag{3}$$

Transforming H to new Fermi amplitudes we get

$$\begin{aligned} H &= V + H_0 + H_{\text{int}} \\ H_{\text{int}} &= H_1 + H_2 + H_3 , \end{aligned}$$

where V is a constant

$$\begin{aligned} V &= 2 \sum E(k) v_k^2 - 2\lambda \sum v_k^2 \\ H_0 &= \sum (E(k) - \lambda)(u_k^2 - v_k^2)(\overset{+}{\alpha}_{k0}\alpha_{k0} + \overset{+}{\alpha}_{k1}\alpha_{k1}) \end{aligned}$$

and

$$H_1 = \sum_{\substack{k,k' \\ (k'-k=q)}} g \sqrt{\frac{\omega(q)}{2V}} \{ u_k v_{k'} \overset{+}{\alpha}_{k0} \overset{+}{\alpha}_{k'1} + u_{-k} v_{-k'} \overset{+}{\alpha}_{-k'0} \overset{+}{a}_{-k1} + u_{k'} v_k \alpha_{k1} \alpha_{k'0} + u_{-k'} v_{-k} \alpha_{-k'1} \alpha_{-k0} \} \overset{+}{b}_q + \text{c. c.}$$

$$H_2 = \sum_{\substack{k,k' \\ (k'-k=q)}} g \sqrt{\frac{\omega(q)}{2V}} \{ u_k u_{k'} \overset{+}{\alpha}_{k0} \alpha_{k'0} + u_{-k} u_{-k'} \overset{+}{\alpha}_{-k'1} \alpha_{-k'1} - v_k v_{k'} \overset{+}{\alpha}_{k'1} \alpha_{k1} - v_{-k} v_{-k} \overset{+}{\alpha}_{-k0} \alpha_{-k0} \} \overset{-}{b}_q + \text{c. c.}$$

$$H_3 = 2 \sum (E(k) - \lambda) u_k v_k (\alpha_{k1}\alpha_{k0} + \overset{+}{\alpha}_{k0}\overset{+}{\alpha}_{k1}) .$$

We now introduce the occupation numbers

$$\nu_{k0} = \overset{+}{\alpha}_{k0}\alpha_{k0} , \qquad \nu_{k1} = \overset{+}{\alpha}_{k1}\alpha_{k1} ,$$

for the new quasi particles created by the operators.

Evidently the state

$$C_\nu = \prod_k \delta(\nu_{k0})\,\delta(\nu_{k1})\,,$$

with zero ν will be a « non-interaction vacuum » that is, a state for which

$$H_0 C_\nu = 0\,.$$

It may be pointed out that λ should be close to E_F since in the absence of interaction $\lambda = E_F$. The expression

$$\varepsilon(k) = (E(k) - \lambda)(u_k^2 - v_k^2)$$

thus should vanish on a surface close to the Fermi surface.

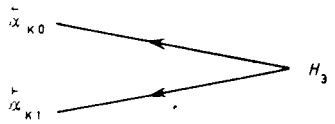


Fig. 1.

We can now see that as regards the criterion given above a « dangerous process » will be one of virtual creation of a pair of quasi-particles ν_{k0}, ν_{k1} in vacuum without phonons since the corresponding energy denominator will be

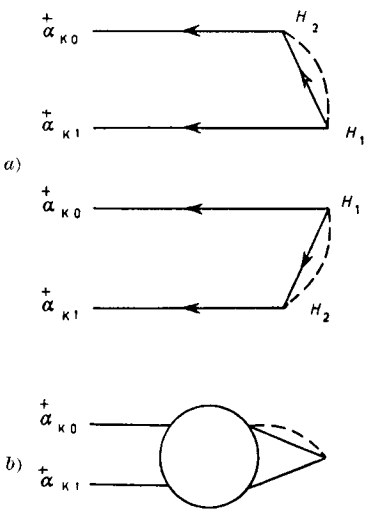
$$\{2\varepsilon(k)\}^{-1}\,.$$

The Hamiltonian H_3 directly leads to this type of process; for the vacuum this Hamiltonian yields the graph (*) shown in Fig. 1. The joint action of H_1, H_2 can also yield this process.

Thus, for example, in the second order in the coupling constant g we have the graphs shown in Fig. 2*a*.

At higher orders graphs of the type shown in Fig. 2*b* are obtained; the circle denotes a connected part which cannot be divided into two connected parts linked only by two lines of the single pair under consideration.

In virtue of the principle of compensation of dangerous graphs proposed above we are obliged to equate to zero the sum of



. Fig. 2.

(*) Application of the graph technique to the many body problem can be found in a detailed paper by HUGENHOLTZ (6).

(6) N. M. HUGENHOLTZ: *Physica*, **23**, 481 (1957).

the contributions from the graph in Fig. 1 and from those in Fig. 2. An equation for u_k, v_k is thus obtained.

It will now be unnecessary to take into account the graphs in Fig. 1 and 2 (and their conjugates) and hence no expressions involving dangerous energy denominators will appear in the perturbation theory expansions.

We now set up an equation for u_k, v_k in the second order. In this approximation the graphs in Fig. 1 should be compensated by those in Fig. 2.

We get

$$2(E(k) - \lambda) u_k v_k + \Omega_k = 0 ,$$

where Ω_k is the coefficient of $\overset{+}{\alpha}_{k0} \overset{+}{\alpha}_{k1} C_\nu$ in the expression

$$- H_2 H_0^{-1} H_1 C_\nu .$$

In the expanded form we finally obtain

$$\{\tilde{E}(k) - \lambda\} u_k v_k = (u_k^2 - v_k^2) \frac{1}{2V} \sum_{k'} g^2 \frac{\omega(k - k')}{\omega(k - k') + \varepsilon(k) + \varepsilon(k')} u_{k'} v_{k'} , \tag{4}$$

where

$$\tilde{E}(k) = E(k) - \frac{1}{2V} \sum g^2 \frac{\omega(k - k')}{\omega(k - k') + \varepsilon(k) + \varepsilon(k')} (u_{k'}^2 - v_{k'}^2) . \tag{5}$$

Without leaving the limits of the chosen approximation we can replace

$$\varepsilon(k) = \{E(k) - \lambda\}(u_k^2 - v_k^2)$$

in the denominator of the right hand part by

$$\tilde{\varepsilon}(k) = \{\tilde{E}(k) - \lambda\}(u_k^2 - v_k^2) .$$

Assuming

$$\tilde{E}(k) - \lambda = \xi(k) ,$$

we may then rewrite the equation obtained in the following form

$$\xi(k) u_k v_k = (u_k^2 - v_k^2) \frac{1}{2(2\pi)^3} \int g^2 \frac{\omega(k - k')}{\omega(k - k') + \tilde{\varepsilon}(k) + \tilde{\varepsilon}(k')} u_{k'} v_{k'} \, \mathrm{d}\boldsymbol{k}' . \tag{6}$$

This equation evidently possesses the trivial solution

$$uv = 0 , \qquad (u, v) = (0, 1) ,$$

which corresponds to the « normal state ». However it also possesses a different type of solution which goes over into the trivial one on moving away from the Fermi surface.

Putting

$$C(k) = \frac{1}{(2\pi)^3}\int g^2 \frac{\omega(k-k')}{\omega(k-k') + \tilde{\varepsilon}(k) + \tilde{\varepsilon}(k')} u_k v_{k'}\, \mathrm{d}\boldsymbol{k}'$$

we find from (6)

$$u_k^2 = \frac{1}{2}\left\{1 + \frac{\xi}{\sqrt{C^2+\xi^2}}\right\}, \qquad v_k^2 = \frac{1}{2}\left\{1 - \frac{\xi}{\sqrt{C^2+\xi^2}}\right\}, \tag{7}$$

whence

$$u_k v_k = \frac{1}{2}\frac{C(k)}{\sqrt{C^2(k) + \xi^2(k)}}, \qquad \tilde{\varepsilon}(k) = \frac{\xi^2(k)}{\sqrt{C^2(k) + \xi^2(k)}}.$$

Thus our equation reduces to the following

$$C(k) = \frac{1}{2(2\pi)^3}\int g^2 \frac{\omega(k-k')}{\omega(k-k') + \tilde{\varepsilon}(k) + \tilde{\varepsilon}(k')} \frac{C(k')}{\sqrt{C^2(k') + \xi^2(k')}}\, \mathrm{d}\boldsymbol{k}'. \tag{8}$$

It should be noted that this equation possesses a peculiar feature: for $g^2 \to 0$ the solution C tends to zero like $\exp[-A/g^2]$, A being a positive constant because near the surface, $\xi(k) = 0$ and the integral in the right hand part of (8) becomes logarithmically divergent if $C = 0$.

Under these conditions it is not difficult to derive the asymptotic form of the solution for small values of g:

$$C(k) = \tilde{\omega} \exp\left[-\frac{1}{\varrho}\right] \frac{1}{2}\int_{-1}^{1} \frac{\omega\{k_0\sqrt{2(1-t)}\}}{\omega\{k_0\sqrt{2(1-t)}\} + |\xi(k)|}\, \mathrm{d}t, \tag{9}$$

where

$$\varrho = \frac{g^2}{2\pi^2}\left(\frac{k^2}{\mathrm{d}\tilde{E}(k)/\mathrm{d}k}\right)_{k=k_0}, \qquad \tilde{E}(k_0) = \lambda,$$

$$\ln \tilde{\omega} = \int_0^{\infty} \ln \frac{1}{2\xi} \frac{\mathrm{d}}{\mathrm{d}\xi}\left\{\frac{1}{2}\int_{-1}^{1} \frac{\omega\{k_0\sqrt{2(1-t)}\}}{\omega\{k_0\sqrt{2(1-t)}\} + \xi}\, \mathrm{d}t\right\}^2 \mathrm{d}\xi. \tag{10}$$

Taking into account the auxiliary condition (3) and the expressions (7), (9) found for u, v it may be seen that

$$k_0 = k_F.$$

It is furthermore evident that the corrections to expression (5) which result from the substitution of u_k an v_k with the « normal » values

$$(11)\qquad \begin{cases} u_k = \theta_G(k) = \begin{cases} 1\,, & |k| > k_F \\ 0\,, & |k| < k_F \end{cases} \\ v_k = \theta_F(k) = \begin{cases} 0\,, & |k| > k_F \\ 1\,, & |k| < k_F \end{cases} \end{cases}$$

will be exponentially small.

Thus, without loss of precision we may replace by $\tilde{E}(k)$ in formula (10) the corresponding expression for the normal state and we may interpret the factor

$$\frac{1}{2\pi^2}\left(\frac{k^2}{\mathrm{d}\tilde{E}/\mathrm{d}k}\right)_{k=k_F} = \frac{1}{V}\left\{\frac{V}{(2\pi)^3}\,\frac{4\pi k^2\,\mathrm{d}k}{\mathrm{d}E}\right\}_0$$

as the relative density $\mathrm{d}n/\mathrm{d}E$ of the number of electron levels in an infinitely narrow energy gap near the Fermi surface. Then

$$(12)\qquad \varrho = g^2\frac{\mathrm{d}n}{\mathrm{d}E}\,.$$

We shall now proceed to calculate the ground state energy in a second approximation.

From the total H_{int} only H_1 should now be taken into account. We thus assume that for the eigenvalue of H in the ground state

$$V - \langle C_\nu^* H_1 H_0^{-1} H_1 C_\nu\rangle = 2\sum_k \{E(k) - \lambda\} v_k^2 - \frac{1}{V}\sum_{k\neq k'} g^2\,\frac{\omega(k-k')\{u_k^2 v_k^2 + u_k v_k u_{k'} v_{k'}\}}{\omega(k-k') + \varepsilon(k) + \varepsilon(k')}\,.$$

Inserting the expression for u_k, v_k previously found we calculate the difference ΔE between the energy of the ground state and that of the normal state.

We get

$$(14)\qquad \frac{\Delta E}{V} = -\frac{\mathrm{d}n}{\mathrm{d}E}\,\frac{\tilde{\omega}^2}{2}\exp\left[-\frac{2}{\varrho}\right].$$

It is interesting to note that this result is the same as that of BARDEEN and co-workers ([3]). This can easily be seen if Bardeen's parameters ω, V are

chosen as follows

$$2\omega = \tilde{\omega}\,, \qquad V = g^2\,. \tag{15}$$

We shall now set up in the accepted approximation the formula for the energy of an elementary excitation. For this purpose we consider the excited state

$$C_1 = \overset{+}{\alpha}_{k0} C_v$$

and apply to it the perturbation theory in the usual manner. For the energy of an elementary excitation with momentum k we obtain the following expression

$$E_e(k) = \varepsilon(k) - \langle C_1^* H_{\text{int}} (H_0 - \varepsilon(k))^{-1} H_{\text{int}} C_1 \rangle_{\text{connected}}$$

which in the expanded form is

$$E_e(k) = \tilde{\varepsilon}(k) \left\{ 1 - \frac{g^2}{V} \sum_{k'} \omega(k-k') \frac{u_k^2 u_{k'}^2 + v_k^2 v_{k'}^2}{[\omega(k-k') + \varepsilon(k')]^2 - \varepsilon^2(k)} \right\} + \frac{g^2}{V} 2u_k v_k \sum_{k'} \frac{\omega(k-k')(\omega(k-k') + \varepsilon(k'))}{[\omega(k-k') + \varepsilon(k')]^2 - \varepsilon^2(k)} u_{k'} v_{k'}\,. \tag{16}$$

The first term which is proportional to $\tilde{\varepsilon}(k)$ does not possess any singular properties and vanishes on the Fermi surface. However on this surface the second term is

$$\frac{g^2}{V} 2u_k v_k \sum_{k'} \frac{\omega(k-k')}{\omega(k-k') + \varepsilon(k')} u_{k'} v_{k'} = 2u_k v_k C(k) = C(k_F) = \tilde{\omega} \exp\left[-\frac{1}{\varrho}\right].$$

Thus the energy values of states with fermion excitations are separated from the energy of the ground state by the gap

$$\Delta = \tilde{\omega} \exp\left[-\frac{1}{\varrho}\right]. \tag{17}$$

It should be mentioned that an expression of the same type as (17) is contained in BARDEEN's paper and is interpreted there as the energy required for the destruction of a « pair ».

It is interesting to note that contrary to Bardeen's theory our « vacuum » or the lowest energy state is formed by fermions. These fermions characterized by creation and annihilation amplitudes α^+ and α correspond to a kind of superposition of the electron and the hole.

Consider now the « ground current state », that is, a state possessing the lowest energy among possible estates with a given momentum $\boldsymbol{P}$.

Thus our task is to determine the eigenvalue of H with the auxiliary condition

$$\sum_{k,s} \boldsymbol{k} \overset{+}{a}_{ks} a_{ks} = \boldsymbol{P} .$$

Instead of doing this we shall introduce in the usual manner not only the scalar parameter λ, but also a vector parameter $\boldsymbol{u}$ which plays the role of a mean velocity, and take the complete Hamiltonian in the form

$$H = H_{F2} - \lambda \sum_{k,s} \overset{+}{a}_{ks} a_{ks} - \sum (\boldsymbol{uk}) \overset{+}{a}_{ks} a_{ks} = \tag{18}$$

$$= \sum_{k,s} \{E(k) - (\boldsymbol{u} \cdot \boldsymbol{k}) - \lambda\} \overset{+}{a}_{ks} a_{ks} + \sum_{q} \omega(q) \overset{+}{b}_{q} b_{q} + H_{\text{int}} .$$

The value of $\boldsymbol{k}$ is determined from the condition

$$\sum_{k,s} \boldsymbol{k} \overset{+}{a}_{k} a_{ks} = \boldsymbol{P} .$$

Since we have been dealing only with a small area in the vicinity of the Fermi surface we may put, for sake of simplicity,

$$E(k) = \frac{k^2}{2m} + \mathcal{D} , \qquad \mathcal{D} = E_F - \frac{k_F^2}{2m}$$

and in the final formulae we may assume

$$m = \left(\frac{k}{\mathrm{d}E/\mathrm{d}k} \right)_{k=k_F} .$$

However in this case

$$E(k) - (\boldsymbol{uk}) = E(\boldsymbol{k} - m\boldsymbol{u}) - \frac{mu^2}{2}$$

and therefore if we perform in momentum space the translation

$$\boldsymbol{k} \to \boldsymbol{k} + \boldsymbol{u}m , \qquad a_{ks} \to a_{k+mu,s} \tag{19}$$

and make the substitution

$$\lambda \to \lambda + \frac{mu^2}{2} ,$$

the hamiltonian (18) will have the same form as (2) and will not contain the vector $\boldsymbol{u}$. We again arrive at the case of a ground state with zero momentum. Thus there is no necessity of carrying out a special investigation of the current flow state; it should be sufficient to apply the inverse transformation of (19) to the formulae previously obtained.

In this manner, for example, one may verify that the energy of the ground current state with a mean velocity $\boldsymbol{u}$ differs from the energy of a ground state with no current by $N(mu^2/2)$.

Excitations are separated from the energy of the ground current state by the gap

$$\Delta_u = \Delta - \boldsymbol{k}_F \boldsymbol{u} > \Delta - k_F |\boldsymbol{u}| .$$

Hence if

$$\boldsymbol{k}_F |\boldsymbol{u}| < \Delta ,$$

the current state will nevertheless be stable with respect to excitations, despite the fact that the energy of this state is greater than that of the non-current state (neglecting the effects of the magnetic field).

We thus see that superconductivity is really an intrinsic property of the considered model.

Some additional remarks may be made.

In order to be able to restrict our considerations to asymptotic approximations it was necessary to assume that the parameter ϱ was small. However V. V. TOLMAČEV and S. V. TJABLIKOV (7) have shown, by applying a method which does not assume the smallness of ϱ, that for $\varrho > \frac{1}{2}$ the velocity of sound is imaginary. This means that the lattice is unstable. If the lattice is so rigid that the electron-phonon interaction does not appreciably affect the phonon energy, the parameter ϱ should be small. Already for $\varrho = \frac{1}{4}$, $\exp[-1/\varrho]$ equals 1/55. In our opinion this explains the small magnitude of the energy gap and hence of the critical temperature.

It may also be pointed out that if a Coulomb interaction term is explicitly introduced in the Fröhlich Hamiltonian, summation over electron-hole graphs of the Gell-Mann–Brueckner type should be performed in order to ensure the appearance of screening.

One can thus easily verify that Coulomb interaction counteracts the appearance of superconductivity (*).

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(7) V. V. TOLMAČEV and S. V. TJABLIKOV: *Žu. Éxper. Teor. Fiz.* (in print).

(*) *Note added in proof.* – The detailed study of the Coulomb interaction shows (8) that its efficiency is essentially reduced due to its « long range » in momentum space.

(8) N. N. BOGOLJUBOV, D. V. SHIRKOV and V. V. TOLMAČEV: preprint of the Joint Institute for Nuclear Research.

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In conclusion the author considers it a pleasant duty to express his appreciation to D. N. ZUBAREV, V. V. TOLMAČEV, S. V. TJABLIKOV and YU. A. TSERKOVNIKOV for valuable discussions.

RIASSUNTO (*)

Si presenta una generalizzazione del metodo elaborato dall'autore per la teoria della superconduttività. Si dimostra che il modello originale di Fröhlich possiede la proprietà di essere superconduttivo. Si prendono in considerazione lo stato fondamentale e le eccitazioni dei suoi fermioni.

(*) *Traduzione a cura della Redazione.*