New symmetries in scalar field theories

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based on

- P.M. Ferreira , B.G., O.M. Ogreid, P. Osland, "New Symmetries of the Two-Higgs-Doublet Model", *Eur.Phys.J.C* 84 (2024) 3, 234, e-Print: 2306.02410
- work in progress

The Two-Higgs Doublet Model (2HDM) in the bilinear notation

$$
\begin{aligned}
V &= m_{11}^2\Phi_1^\dagger\Phi_1 + m_{22}^2\Phi_2^\dagger\Phi_2 - [m_{12}^2\Phi_1^\dagger\Phi_2 + \text{h.c.}] + \\
&\quad \frac{1}{2}\lambda_1(\Phi_1^\dagger\Phi_1)^2 + \frac{1}{2}\lambda_2(\Phi_2^\dagger\Phi_2)^2 + \lambda_3(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2) + \lambda_4(\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1) + \\
&\quad \left\{\frac{1}{2}\lambda_5(\Phi_1^\dagger\Phi_2)^2 + [\lambda_6(\Phi_1^\dagger\Phi_1) + \lambda_7(\Phi_2^\dagger\Phi_2)]\Phi_1^\dagger\Phi_2 + \text{h.c.}\right\},
\end{aligned}
$$

where m_{12}^2 and $\lambda_{5,6,7}$ might be complex.

An alternative notation uses four gauge-invariant bilinears constructed from the doublets (Velhinho 1994, Nagel 2004, Ivanov 2005, Maniatis 2006, Nishi 2006):

$$
\begin{aligned}
r_0 &\equiv \tfrac{1}{2}\left(\Phi_1^\dagger\Phi_1 + \Phi_2^\dagger\Phi_2\right), \\
r_1 &\equiv \tfrac{1}{2}\left(\Phi_1^\dagger\Phi_2 + \Phi_2^\dagger\Phi_1\right) = \text{Re}\left(\Phi_1^\dagger\Phi_2\right), \\
r_2 &\equiv -\tfrac{i}{2}\left(\Phi_1^\dagger\Phi_2 - \Phi_2^\dagger\Phi_1\right) = \text{Im}\left(\Phi_1^\dagger\Phi_2\right), \\
r_3 &\equiv \tfrac{1}{2}\left(\Phi_1^\dagger\Phi_1 - \Phi_2^\dagger\Phi_2\right).
\end{aligned}
$$

The Two-Higgs Doublet Model (2HDM) in the bilinear notation

The potential of may be written as

$$V = M_\mu \, r^\mu + \Lambda_{\mu\nu} \, r^\mu \, r^\nu \,,$$

where

$$r^\mu \equiv (r_0 \,, \, r_1 \,, \, r_2 \,, \, r_3) = (r_0 \,, \, \vec{r}) \,,$$

$$M^\mu \equiv \left(m_{11}^2 + m_{22}^2 \,, \, 2\mathrm{Re}(m_{12}^2) \,, \, -2\mathrm{Im}(m_{12}^2) \,, \, m_{22}^2 - m_{11}^2\right) = (M_0 \,, \, \vec{M}) \,,$$

$$\Lambda^{\mu\nu} \equiv \begin{pmatrix} \frac{1}{2}(\lambda_1 + \lambda_2) + \lambda_3 & -\mathrm{Re}\,(\lambda_6 + \lambda_7) & \mathrm{Im}\,(\lambda_6 + \lambda_7) & \frac{1}{2}(\lambda_2 - \lambda_1) \\ -\mathrm{Re}\,(\lambda_6 + \lambda_7) & \lambda_4 + \mathrm{Re}\,(\lambda_5) & -\mathrm{Im}\,(\lambda_5) & \mathrm{Re}\,(\lambda_6 - \lambda_7) \\ \mathrm{Im}\,(\lambda_6 + \lambda_7) & -\mathrm{Im}\,(\lambda_5) & \lambda_4 - \mathrm{Re}\,(\lambda_5) & -\mathrm{Im}\,(\lambda_6 - \lambda_7) \\ \frac{1}{2}(\lambda_2 - \lambda_1) & \mathrm{Re}\,(\lambda_6 - \lambda_7) & -\mathrm{Im}\,(\lambda_6 - \lambda_7) & \frac{1}{2}(\lambda_1 + \lambda_2) - \lambda_3 \end{pmatrix}$$

$$\Lambda^{\mu\nu} \equiv \begin{pmatrix} \Lambda_{00} & \vec{\Lambda} \\ \vec{\Lambda}^T & \Lambda \end{pmatrix}$$

Global symmetries of the 2HDM

- *Higgs-family symmetries*, unitary transformations mix both doublets,

$$\Phi_i \;\rightarrow\; \Phi_i' = \sum_{j=1}^{2} U_{ij}\,\Phi_j\,, \qquad\qquad U \in U(2)$$

  e.g. Z_2:

$$\Phi_1 \;\rightarrow\; \Phi_1 \quad , \quad \Phi_2 \;\rightarrow\; -\Phi_2\,,$$

  prevents the occurrence of tree-level flavour-changing neutral currents (FCNC).
- *generalized CP (GCP)*, transformations:

$$\Phi_i \;\rightarrow\; \Phi_i' = \sum_{j=1}^{2} X_{ij}\,\Phi_j^*\,, \qquad\qquad X \in U(2)$$

  e.g. “standard" CP transformation (CP1):

$$\Phi_i \;\rightarrow\; \Phi_i^*$$

Global symmetries of the 2HDM

In the bilinear formalism, symmetries are represented by rotations in the 3-dimensional space defined by the vector $\vec{r}$:

$$\vec{r} \to \vec{r}' = S\,\vec{r},$$

where $S \in O(3)$.

$$S_{Z_2} = \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad , \quad S_{CP1} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Global symmetries of the 2HDM

CP2: $\Phi_1 \to \Phi_2^*, \qquad \Phi_2 \to -\Phi_1^*$

$$S_{CP2} = \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix},$$

A parity transformation about the three axes.

S	m_{11}^2	m_{22}^2	m_{12}^2	λ_1	λ_2	λ_3	λ_4	λ_5	λ_6	λ_7
CP1			real					real	real	real
Z_2			0						0	0
U(1)			0					0	0	0
CP2		m_{11}^2	0		λ_1					$-\lambda_6$
CP3		m_{11}^2	0		λ_1			λ_{134}	0	0
$SO(3)$		m_{11}^2	0		λ_1		$\lambda_1 - \lambda_3$	0	0	0

Table 1: Relations between 2HDM scalar potential parameters for each of the six symmetries discussed, $\lambda_{134} \equiv \lambda_1 - \lambda_3 - \lambda_4$.

Running of parameters of the 2HDM

The 1-loop β-functions for the quadratic couplings

$$\beta_{m_{11}^2} = 3\lambda_1 m_{11}^2 + (2\lambda_3 + \lambda_4)\, m_{22}^2 - 3\,\left(\lambda_6^*\, m_{12}^2 + \text{h.c.}\right) - \frac{1}{4}(9g^2 + 3g'^2)\, m_{11}^2 + \beta^F_{m_{11}^2},$$

$$\beta_{m_{22}^2} = (2\lambda_3 + \lambda_4)\, m_{11}^2 + 3\lambda_2\, m_{22}^2 - 3\,\left(\lambda_7^*\, m_{12}^2 + \text{h.c.}\right) - \frac{1}{4}(9g^2 + 3g'^2)\, m_{22}^2 + \beta^F_{m_{22}^2},$$

$$\beta_{m_{12}^2} = -3\left(\lambda_6\, m_{11}^2 + \lambda_7\, m_{22}^2\right) + (\lambda_3 + 2\lambda_4)\, m_{12}^2 + 3\lambda_5\, {m_{12}^2}^* - \frac{1}{4}(9g^2 + 3g'^2) m_{12}^2 + \beta^F_{m_{12}^2},$$

Running of parameters of the 2HDM

and 1-loop β functions for the quartic ones,

$$\begin{aligned}
\beta_{\lambda_1} &= 6\lambda_1^2 + 2\lambda_3^2 + 2\lambda_3\lambda_4 + \lambda_4^2 + |\lambda_5|^2 + 12\,|\lambda_6|^2 \\
&\quad + \frac{3}{8}(3g^4 + g'^4 + 2g^2 g'^2) - \frac{3}{2}\lambda_1(3g^2 + g'^2) + \beta^F_{\lambda_1}, \\
\beta_{\lambda_2} &= 6\lambda_2^2 + 2\lambda_3^2 + 2\lambda_3\lambda_4 + \lambda_4^2 + |\lambda_5|^2 + 12\,|\lambda_7|^2 \\
&\quad + \frac{3}{8}(3g^4 + g'^4 + 2g^2 g'^2) - \frac{3}{2}\lambda_2(3g^2 + g'^2) + \beta^F_{\lambda_2}, \\
\beta_{\lambda_3} &= (\lambda_1 + \lambda_2)(3\lambda_3 + \lambda_4) + 2\lambda_3^2 + \lambda_4^2 + |\lambda_5|^2 + 2\left(|\lambda_6|^2 + |\lambda_7|^2\right) + 8\,\mathrm{Re}\left(\lambda_6\lambda_7^*\right) \\
&\quad + \frac{3}{8}(3g^4 + g'^4 - 2g^2 g'^2) - \frac{3}{2}\lambda_3(3g^2 + g'^2) + \beta^F_{\lambda_3}, \\
\beta_{\lambda_4} &= (\lambda_1 + \lambda_2)\,\lambda_4 + 4\lambda_3\lambda_4 + 2\lambda_4^2 + 4\,|\lambda_5|^2 + 5\left(|\lambda_6|^2 + |\lambda_7|^2\right) + 2\,\mathrm{Re}\left(\lambda_6\lambda_7^*\right) \\
&\quad + \frac{3}{2}g^2 g'^2 - \frac{3}{2}\lambda_4(3g^2 + g'^2) + \beta^F_{\lambda_4}, \\
\beta_{\lambda_5} &= (\lambda_1 + \lambda_2 + 4\lambda_3 + 6\lambda_4)\,\lambda_5 + 5\left(\lambda_6^2 + \lambda_7^2\right) + 2\lambda_6\lambda_7 \\
&\quad - \frac{3}{2}\lambda_5(3g^2 + g'^2) + \beta^F_{\lambda_5}, \\
\beta_{\lambda_6} &= (6\lambda_1 + 3\lambda_3 + 4\lambda_4)\,\lambda_6 + (3\lambda_3 + 2\lambda_4)\,\lambda_7 + 5\lambda_5\lambda_6^* + \lambda_5\lambda_7^* \\
&\quad - \frac{3}{2}\lambda_6(3g^2 + g'^2) + \beta^F_{\lambda_6}, \\
\beta_{\lambda_7} &= (6\lambda_2 + 3\lambda_3 + 4\lambda_4)\,\lambda_7 + (3\lambda_3 + 2\lambda_4)\,\lambda_6 + 5\lambda_5\lambda_7^* + \lambda_5\lambda_6^* \\
&\quad - \frac{3}{2}\lambda_7(3g^2 + g'^2) + \beta^F_{\lambda_7},
\end{aligned}$$

where the β^F_x terms contain all contributions coming from fermions.

Running of parameters of the 2HDM

$$Z_2 \text{ symmetry} \Rightarrow \lambda_6 = \lambda_7 = 0 \Rightarrow \beta_{\lambda_6} = \beta_{\lambda_7} = 0$$

a symmetry-based condition on λ's are preserved by RGE running at the one-loop order.

For the Z_2 model

$$\beta_{\lambda_5} = \left[\lambda_1 + \lambda_2 + 4\lambda_3 + 6\lambda_4 - \frac{3}{2}\left(3g^2 + g'^2\right)\right] \lambda_5$$

$\lambda_5 = 0$ is *a fixed point* of this RG equation: if at any scale $\lambda_5 = 0$, that coupling will remain zero for all renormalization scales. Such fixed points of RG equations are usually fingerprints of symmetries, and indeed that is the case here: if $\lambda_6 = \lambda_7 = 0$, the extra constraint $\lambda_5 = 0$ takes us from Z_2-symmetric model to $U(1)$-symmetric.

Running of parameters of the 2HDM

We have noticed that

$$\left\{ m_{11}^2 + m_{22}^2 = 0 \;\; , \;\; \lambda_1 - \lambda_2 = 0 \;\; , \;\; \lambda_6 + \lambda_7 = 0 \right\}$$

- constitutes a fixed point of the 1-loop RG equations,
- are basis transformation invariants.

$$\begin{aligned}
\beta_{m_{11}^2+m_{22}^2} = &\; 3(\lambda_1 m_{11}^2 + \lambda_2 m_{22}^2) + (2\lambda_3 + \lambda_4)(m_{11}^2 + m_{22}^2) \\
&-3 \left[(\lambda_6^* + \lambda_7^*) m_{12}^2 + \text{h.c.}\right] - \frac{1}{4}(9g^2 + 3g'^2)(m_{11}^2 + m_{22}^2) \\
\beta_{\lambda_1-\lambda_2} = &\; 6\left(\lambda_1^2 - \lambda_2^2\right) + 12\left(|\lambda_6|^2 - |\lambda_7|^2\right) - \frac{3}{2}(\lambda_1 - \lambda_2)(3g^2 + g'^2) \\
\beta_{\lambda_6+\lambda_7} = &\; 6(\lambda_1\lambda_6 + \lambda_2\lambda_7) + (3\lambda_3 + 2\lambda_4)(\lambda_6 + \lambda_7) + 6\lambda_5(\lambda_6^* + \lambda_7^*) \\
&- \frac{3}{2}(\lambda_6 + \lambda_7)(3g^2 + g'^2)
\end{aligned}$$

Running of parameters of the 2HDM

It turns out that

$$\left\{ m_{11}^2 + m_{22}^2 = 0 \;\;,\;\; \lambda_1 - \lambda_2 = 0 \;\;,\;\; \lambda_6 + \lambda_7 = 0 \right\}$$

is also the 2-loop fixed point.

$$\Downarrow$$

Conclusion:
Perhaps there is a symmetry behind the fixed point:

$$\left\{ m_{11}^2 + m_{22}^2 = 0 \;\;,\;\; \lambda_1 - \lambda_2 = 0 \;\;,\;\; \lambda_6 + \lambda_7 = 0 \right\}$$

Running of parameters of the 2HDM

$$V \;=\; M_\mu\, r^\mu \;+\; \Lambda_{\mu\nu}\, r^\mu\, r^\nu$$

The rotation matrix $R_{ij}(U) = \mathrm{Tr}\left(U^\dagger \sigma_i U \sigma_j\right)/2$, and the basis transformations:

$$\vec{M} \to \vec{M}' = R\,\vec{M} \qquad \vec{\Lambda} \to \vec{\Lambda}' = R\,\vec{\Lambda} \qquad \Lambda \to \Lambda' = R\,\Lambda\,R^T$$

whereas M_0 and Λ_{00} are *basis invariants*.

$$M^\mu \;\equiv\; \left(m_{11}^2 + m_{22}^2\,,\; 2\mathrm{Re}(m_{12}^2)\,,\; -2\mathrm{Im}(m_{12}^2)\,,\; m_{22}^2 - m_{11}^2\right) \;=\; (M_0\,,\; \vec{M})\,,$$

$$\Lambda^{\mu\nu} \;\equiv\; \begin{pmatrix} \frac{1}{2}(\lambda_1+\lambda_2)+\lambda_3 & -\mathrm{Re}\,(\lambda_6+\lambda_7) & \mathrm{Im}\,(\lambda_6+\lambda_7) & \frac{1}{2}(\lambda_2-\lambda_1) \\ -\mathrm{Re}\,(\lambda_6+\lambda_7) & \lambda_4+\mathrm{Re}\,(\lambda_5) & -\mathrm{Im}\,(\lambda_5) & \mathrm{Re}\,(\lambda_6-\lambda_7) \\ \mathrm{Im}\,(\lambda_6+\lambda_7) & -\mathrm{Im}\,(\lambda_5) & \lambda_4-\mathrm{Re}\,(\lambda_5) & -\mathrm{Im}\,(\lambda_6-\lambda_7) \\ \frac{1}{2}(\lambda_2-\lambda_1) & \mathrm{Re}\,(\lambda_6-\lambda_7) & -\mathrm{Im}\,(\lambda_6-\lambda_7) & \frac{1}{2}(\lambda_1+\lambda_2)-\lambda_3 \end{pmatrix}$$

$$\Lambda^{\mu\nu} = \begin{pmatrix} \Lambda_{00} & \vec{\Lambda} \\ \vec{\Lambda}^T & \Lambda \end{pmatrix}$$

Running of parameters of the 2HDM

Basis transformation invariants:

$$I_{1,1} = \Lambda_{00}\,, \qquad I_{1,2} = \mathrm{Tr}\Lambda$$
$$I_{2,1} = \vec{\Lambda}\cdot\vec{\Lambda}\,, \qquad I_{2,2} = \mathrm{Tr}\Lambda^2$$
$$I_{3,1} = \vec{\Lambda}\cdot\Lambda\vec{\Lambda}\,, \qquad I_{3,2} = \mathrm{Tr}\Lambda^3$$
$$I_{4,1} = \vec{\Lambda}\cdot\Lambda^2\vec{\Lambda}\,,$$

To all orders of perturbation theory,

$$\beta_{\vec{\Lambda}} = a_0\,\vec{\Lambda} + a_1\,\Lambda\,\vec{\Lambda} + a_2\,\Lambda^2\,\vec{\Lambda}$$

- $\vec{\Lambda} = \vec{0}$ *is a fixed point to all orders of perturbation theory.*

where the a_i are polynomial expressions involving invariants,

see A.V. Bednyakov, "On three-loop RGE for the Higgs sector of 2HDM", JHEP 11 (2018) 154, e-Print: 1809.04527

Running of parameters of the 2HDM

$$\beta_{\vec{M}} = c_0\,\vec{M} + c_1\,\Lambda\,\vec{M} + c_2\,\Lambda^2\,\vec{M} + c_3\,I_{M3}\,\vec{\Lambda} + c_4\,I_{M4}\,\Lambda\,\vec{\Lambda} + c_5\,I_{M5}\,\Lambda^2\,\vec{\Lambda}$$

- *If $\vec{\Lambda} = \vec{0}$, then $\vec{M} = \vec{0}$ is a fixed point to all orders*

$$\beta_{M_0} = b_0\,M_0 + b_1\,\vec{\Lambda}\cdot\vec{M} + b_2\,\vec{\Lambda}\cdot\left(\Lambda\vec{M}\right) + b_3\,\vec{\Lambda}\cdot\left(\Lambda^2\vec{M}\right)$$

- *If $\vec{\Lambda} = \vec{0}$, then $M_0 = 0$ is a fixed point to all orders.*

where the c_i are polynomial expressions involving invariants,

see A.V. Bednyakov

Running of parameters of the 2HDM

Two all-order fixed points of the 2HDM RG equations:

- $\{\vec{M} = \vec{0}\,,\, \vec{\Lambda} = \vec{0}\}$.

$$m_{11}^2 = m_{22}^2 \;\;,\;\; m_{12}^2 = 0 \;\;,\;\; \lambda_1 = \lambda_2 \;\;,\;\; \lambda_6 = -\lambda_7\,.$$

Who are they?

- $\{M_0 = 0\,,\, \vec{\Lambda} = \vec{0}\}$.

$$M_0 \equiv m_{11}^2 + m_{22}^2 = 0 \;\;,\;\; \lambda_1 = \lambda_2 \;\;,\;\; \lambda_6 = -\lambda_7\,.$$

These are the conditions mentioned before and are basis invariant, so they are *not* a basis change of the previous ones.

Running of parameters of the 2HDM

Two all-order fixed points of the 2HDM RG equations:

- $\{\vec{M} = \vec{0}\,,\ \vec{\Lambda} = \vec{0}\}$.

$$m_{11}^2 \ = \ m_{22}^2 \quad , \quad m_{12}^2 = 0 \quad , \quad \lambda_1 = \lambda_2 \quad , \quad \lambda_6 = -\lambda_7\ .$$

They are exactly the CP2 symmetry conditions.

- $\{M_0 = 0\,,\ \vec{\Lambda} = \vec{0}\}$.

$$M_0 \equiv m_{11}^2 + m_{22}^2 = 0 \quad , \quad \lambda_1 = \lambda_2 \quad , \quad \lambda_6 = -\lambda_7\ .$$

These are the conditions mentioned before and are basis invariant, so they are *not* a basis change of the previous ones.

The r_0 symmetry

$$V = M_0\, r_0 + \Lambda_{00}\, r_0^2 - \vec{M} \cdot \vec{r} - 2\left(\vec{\Lambda} \cdot \vec{r}\right) r_0 + \vec{r} \cdot (\Lambda\, \vec{r})$$

- $\{\vec{M} = \vec{0}\,,\ \vec{\Lambda} = \vec{0}\}$. These are exactly the CP2 ($\vec{r} \to -\vec{r}$).
- $\{M_0 = 0\,,\ \vec{\Lambda} = \vec{0}\}$ These are new, perhaps $r_0 \overset{?}{\to} -r_0$

The r_0 symmetry

$$\Phi_1 = \begin{pmatrix} \phi_1 + i\phi_2 \\ \phi_3 + i\phi_4 \end{pmatrix}, \quad \Phi_2 = \begin{pmatrix} \phi_5 + i\phi_6 \\ \phi_7 + i\phi_8 \end{pmatrix},$$

The transformation

$$\begin{pmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \\ \phi_4 \\ \phi_5 \\ \phi_6 \\ \phi_7 \\ \phi_8 \end{pmatrix} \to \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & i & 0 & 0 \\ 0 & 0 & 0 & 0 & i & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & i \\ 0 & 0 & 0 & 0 & 0 & 0 & i & 0 \\ 0 & -i & 0 & 0 & 0 & 0 & 0 & 0 \\ -i & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -i & 0 & 0 & 0 & 0 \\ 0 & 0 & -i & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \\ \phi_4 \\ \phi_5 \\ \phi_6 \\ \phi_7 \\ \phi_8 \end{pmatrix}$$

implies

$$r_0 \to -r_0 \qquad r_i \to +r_i$$

The r_0 symmetry

$$\Phi_1 \to -\Phi_2^* \quad \Phi_1^\dagger \to \Phi_2^T,$$
$$\Phi_2 \to \Phi_1^*, \quad \Phi_2^\dagger \to -\Phi_1^T.$$

- Higgs kinetic terms

$$\mathcal{L}_k = (D_\mu\Phi_1)^\dagger(D^\mu\Phi_1) + (D_\mu\Phi_2)^\dagger(D^\mu\Phi_2),$$

where

$$D^\mu = \partial^\mu + \frac{ig}{2}\sigma_i W_i^\mu + i\frac{g'}{2}B^\mu,$$

$\mathcal{L}_k$ remains invariant if the above transformation of $\Phi_{1,2}$ is supplemented by

$$\partial_\mu \to -i\partial_\mu, \qquad (x^\mu \to ix^\mu),$$
$$B_\mu \to iB_\mu,$$
$$W_{1\mu} \to iW_{1\mu}, \quad W_{2\mu} \to -iW_{2\mu}, \quad W_{3\mu} \to iW_{3\mu}.$$

- Gauge kinetic terms

$$\mathcal{L}^B = -\frac{1}{4}B_{\mu\nu}B^{\mu\nu} - \frac{1}{4}W_{i\mu\nu}W_i^{\mu\nu},$$

where $B^{\mu\nu} = \partial^\nu B^\mu - \partial^\mu B^\nu$ and $W_i^{\mu\nu} = \partial^\nu W_i^\mu - \partial^\mu W_i^\nu + g\epsilon_{ijk}W_j^\mu W_k^\nu$.
Under r_0 transformation

$$B^{\mu\nu} \to B^{\mu\nu},$$
$$W_1^{\mu\nu} \to W_1^{\mu\nu}, \quad W_2^{\mu\nu} \to -W_2^{\mu\nu}, \quad W_3^{\mu\nu} \to W_3^{\mu\nu}$$

Symmetries and 1-loop CW effective potential

$$\Phi_1 = \begin{pmatrix} \phi_1 + i\phi_2 \\ \phi_3 + i\phi_4 \end{pmatrix}, \quad \Phi_2 = \begin{pmatrix} \phi_5 + i\phi_6 \\ \phi_7 + i\phi_8 \end{pmatrix}$$

$$V_{\rm CW}^{(1-loop)}(\phi_a) = \frac{1}{2}\int \frac{d^4 p_E}{(2\pi)^4} {\rm Tr}\left[\ln(p_E^2 + M_S^2)\right] = -\frac{1}{2}\int \frac{d^4 p_E}{(2\pi)^4}\left[{\rm Tr}\sum_{n=1}^{\infty}\frac{(-1)^n}{n}\left(\frac{\mathrm{M}_S^2}{p_E^2}\right)^n\right]$$

$$\left(\mathrm{M}_S^2\right)_{ab} \equiv \frac{\partial^2 V}{\partial\phi_a \partial\phi_b}$$

$a, b = 1, \cdots, 8$

- Q.-H. Cao, K. Cheng, and C. Xu, "Global Symmetries and Effective Potential of 2HDM in Orbit Space", Phys.Rev.D 108 (2023) 055036, arXiv:2305.12764 [hep-ph].
- A. Pilaftsis, "Dirac Algebra Formalism for Two Higgs Doublet Models: the One-Loop Effective Potential", e-Print: 2408.04511 [hep-ph].

Symmetries and 1-loop CW effective potential

$$r_0 \to -r_0 \qquad\qquad r_i \to +r_i$$

Is $V_{CW}^{(1-loop)}(\phi_a)$ invariant under the r_0 transformation?

At the new fixed point $M_0 = 0$ and $\vec{\Lambda} = 0$ ($m_{11}^2 + m_{22}^2 = 0$, $\lambda_1 = \lambda_2$ and $\lambda_6 = -\lambda_7$):

$$\Lambda^{\mu\nu} \equiv \begin{pmatrix} \frac{1}{2}(\lambda_1 + \lambda_2) + \lambda_3 & -\mathrm{Re}\,(\lambda_6 + \lambda_7) & \mathrm{Im}\,(\lambda_6 + \lambda_7) & \frac{1}{2}(\lambda_2 - \lambda_1) \\ -\mathrm{Re}\,(\lambda_6 + \lambda_7) & \lambda_4 + \mathrm{Re}\,(\lambda_5) & -\mathrm{Im}\,(\lambda_5) & \mathrm{Re}\,(\lambda_6 - \lambda_7) \\ \mathrm{Im}\,(\lambda_6 + \lambda_7) & -\mathrm{Im}\,(\lambda_5) & \lambda_4 - \mathrm{Re}\,(\lambda_5) & -\mathrm{Im}\,(\lambda_6 - \lambda_7) \\ \frac{1}{2}(\lambda_2 - \lambda_1) & \mathrm{Re}\,(\lambda_6 - \lambda_7) & -\mathrm{Im}\,(\lambda_6 - \lambda_7) & \frac{1}{2}(\lambda_1 + \lambda_2) - \lambda_3 \end{pmatrix}$$

$$\Lambda^{\mu\nu} \equiv \begin{pmatrix} \Lambda_{00} & \vec{\Lambda} \\ \vec{\Lambda}^T & \Lambda \end{pmatrix}$$

Symmetries and 1-loop CW effective potential

$$r_0 \to -r_0 \qquad r_i \to +r_i$$

$$V_{\text{CW}}^{(1-loop)}(\phi_a) = -\frac{1}{2}\int \frac{d^4p_E}{(2\pi)^4}\left[\text{Tr}\sum_{n=1}^{\infty}\frac{(-1)^n}{n}\left(\frac{\text{M}_S^2}{p_E^2}\right)^n\right]$$

For $M_0 = 0$ and $\vec{\Lambda} = 0$ ($m_{11}^2 + m_{22}^2 = 0$, $\lambda_1 = \lambda_2$ and $\lambda_6 = -\lambda_7$):

$$\text{Tr}(M_S^2) = 4[5\Lambda_{00} + \text{tr}(\Lambda)]r_0 \xrightarrow{r_0} -\text{Tr}(M_S^2) = -4[5\Lambda_{00} + \text{tr}(\Lambda)]r_0$$

$$\begin{aligned} n = 1: &\qquad \text{Tr}\left[M_S^2\right] &&\qquad \text{odd} \\ n = 2: &\qquad \text{Tr}\left[(M_S^2)^2\right] &&\qquad \text{even} \\ &\vdots && \\ n = 2k: &\qquad \text{Tr}\left[(M_S^2)^{2k}\right] &&\qquad \text{even} \\ n = 2k+1: &\qquad \text{Tr}\left[(M_S^2)^{2k+1}\right] &&\qquad \text{odd} \end{aligned}$$

Symmetries and 1-loop CW effective potential

The cut-off regularization implies that

$$V_{\rm CW}^{(S)} = -\frac{1}{64\pi^2}\sum_{n=1}^{\infty}\left\{\frac{(-1)^n}{n}\int_0^{\Lambda_{UV}^2} d\rho\,\rho\,\frac{\mathrm{Tr}\left[M_S^2\right]^n}{\rho^n}\right\},$$

where $\rho \equiv |p_E|^2$.

Replacing Λ_{UV}^2 by $-\Lambda_{UV}^2$ this is equivalent of switching sign in front of ρ by $-\rho$ under the trace. Consequently, for odd n a minus sign appears. Since for odd n $\mathrm{Tr}\left[M_S^2(r_0)\right]^n$ is an odd function of r_0, therefore supplementing the r_0 transformation by $\Lambda_{UV}^2 \to -\Lambda_{UV}^2$ we arrive at an invariant 1-loop CW potential.

The toy model - 2RSM

$$\mathcal{L} = \frac{1}{2}(\partial_\mu\phi_1\partial^\mu\phi_1 + \partial_\mu\phi_2\partial^\mu\phi_2) - V(\phi_1,\phi_2)\,,$$

with

$$V(\phi_1\,\phi_2) = \frac{1}{2}m_1^2(\phi_1^2-\phi_2^2)+m_{12}^2\phi_1\phi_2+\frac{1}{2}\lambda_1(\phi_1^4+\phi_2^4)+\lambda_3(\phi_1\phi_2)^2+\lambda_6(\phi_1^2-\phi_2^2)\phi_1\phi_2\,.$$

The model is invariant under the following r_0-like transformation

$$\boldsymbol{x^\mu \to x'^{\,\mu} \equiv ix^\mu,\quad \phi_1(x) \to \phi_1'(x') \equiv i\phi_2(x),\quad \phi_2(x) \to \phi_2'(x') \equiv -i\phi_1(x)}$$

It is possible to choose (ϕ_1,ϕ_2)-basis such that $\lambda_6 = 0$.

The mass^2 matrix

$$\left(M_S^2\right)_{ij}(\phi_1,\phi_2) = \begin{pmatrix} m_1^2 + 6\lambda_1\phi_1^2 + 2\lambda_3\phi_2^2 & m_{12}^2 + 4\lambda_3\phi_1\phi_2 \\ m_{12}^2 + 4\lambda_3\phi_1\phi_2 & -m_1^2 + 6\lambda_1\phi_2^2 + 2\lambda_3\phi_1^2 \end{pmatrix}$$

The toy model - 2RSM

One can express the potential in terms of bilinear variables:

$$r_0 \equiv \frac{1}{2}(\phi_1^2 + \phi_2^2)$$

$$r_1 \equiv \phi_1 \phi_2$$

$$r_2 \equiv \frac{1}{2}(\phi_1^2 - \phi_2^2)\,.$$

Upon the r_0 transformation

$$(r_0, r_1, r_2) \xrightarrow{r_0} (-r_0, r_1, r_2)$$

The potential could be written as

$$V(r^\mu) = M_\mu r^\mu + \Lambda_{\mu\nu} r^\mu r^\nu$$

for $\mu, \nu = 0, 1, 2$ with $M_\mu = (0, m_{12}^2, m_1^2)$ and

$$\Lambda_{\mu\nu} = \begin{pmatrix} \Lambda_{00} & 0 & 0 \\ 0 & \Lambda_{11} & \Lambda_{12} \\ 0 & \Lambda_{21} & \Lambda_{22} \end{pmatrix} = \begin{pmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_3 & 0 \\ 0 & 0 & \lambda_1 \end{pmatrix}\,.$$

The toy model - 2RSM

$M_0 = 0$ and $\vec{\Lambda} = 0$ are implied by the r_0 symmetry
$(m_1^2 + m_2^2 = 0,\ \lambda_1 = \lambda_2$ and $\lambda_6 = -\lambda_7 = 0)$.

$$\mathrm{Tr}\left(M_S^2\right) = 4(3\lambda_1 + \lambda_3)r^0$$

Under the r_0 transformation the trace is odd:

$$\mathrm{Tr}\left(M_S^2\right) \xrightarrow{r_0} -\mathrm{Tr}\left(M_S^2\right)\ ,$$

Two local minima:

$$(v_1^2 - v_2^2) = \frac{-m_1^2}{\lambda_1}, \qquad v_1 v_2 = \frac{-m_{12}^2}{(\lambda_1 + \lambda_3)}$$

where $\langle\phi_{1,2}\rangle \equiv v_{1,2}/\sqrt{2}$.

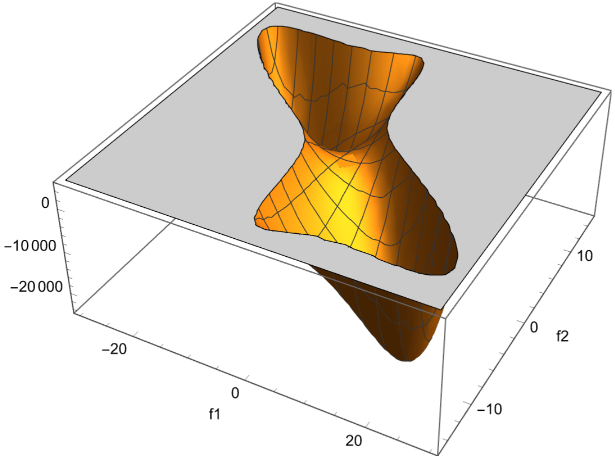

Figure 1: Scalar potential of the toy model, $m_1 = 10$, $m_{12} = 20$, $\lambda_1 = 1$, $\lambda_3 = 2$.

The toy model - 2RSM

The eigenvalues of M_S^2 could be expressed through bilinears

$$M_1^2(r_\mu) = 2(3\lambda_1 + \lambda_3)r_0 + \sqrt{\Delta}$$
$$M_2^2(r_\mu) = 2(3\lambda_1 + \lambda_3)r_0 - \sqrt{\Delta}\,,$$

where

$$\Delta = m_1^4 + m_{12}^4 + 4m_1^2(3\lambda_1 - \lambda_3)r_2 + 8m_{12}^2\lambda_3 r_1 + 16\lambda_3^2 r_0^2 + 12(3\lambda_1 + \lambda_3)(\lambda_1 - \lambda_3)r_2^2$$

$$M_1^2 \overset{r_0}{\longleftrightarrow} -M_2^2$$

$$\Box\phi_1 + \frac{\partial V(\phi_1,\phi_2)}{\partial\phi_1} = 0 \quad \overset{r_0}{\longleftrightarrow} \quad \Box\phi_2 + \frac{\partial V(\phi_1,\phi_2)}{\partial\phi_2} = 0$$

$$S = \int_{x_-}^{x_+} d^4x\,\mathcal{L}\left[\phi(x), \partial_\mu\phi(x), A_\mu(x), \partial_\nu A_\mu(x)\right]\ r_0 - \text{invariant}$$

The toy model - 2RSM

The 1-loop effective potential parametrized by the cut-off Λ_{cut}:

$$V_{CW}^{\text{1-loop}}(r_\mu) = \frac{\Lambda_{\text{cut}}^2}{32\pi^2} \sum_{i=1,2} M_i^2(r_\mu) + \frac{1}{64\pi^2} \sum_{i=1,2} M_i^4(r_\mu) \left\{ \log \left[\frac{M_i^2(r_\mu)}{\Lambda_{\text{cut}}^2} \right] - \frac{1}{2} \right\} .$$

$$\sum_{i=1,2} M_i^2(r_\mu) \xrightarrow{r_0} - \sum_{i=1,2} M_i^2(r_\mu)$$

$$\sum_{i=1,2} M_i^4(r_\mu) \xrightarrow{r_0} + \sum_{i=1,2} M_i^4(r_\mu)$$

$$\sum_{i=1,2} M_i^4(r_\mu) \log \frac{M_i^2(r_\mu)}{\Lambda_{\text{cut}}^2} \xrightarrow{r_0} \sum_{i=1,2} M_i^4(r_\mu) \log \left[\frac{-M_i^2(r_\mu)}{\Lambda_{\text{cut}}^2} \right]$$

The 1-loop effective potential is invariant under the r_0 transformation if ???

The toy model - 2RSM

$$M_1^2 \xrightarrow{r_0} -M_2^2$$
$$M_2^2 \xrightarrow{r_0} -M_1^2$$

The 1-loop effective potential parametrized by the cut-off Λ_{cut}:

$$V_{CW}^{\text{1-loop}}(r_\mu) = \frac{\Lambda_{\text{cut}}^2}{32\pi^2} \sum_{i=1,2} M_i^2(r_\mu) + \frac{1}{64\pi^2} \sum_{i=1,2} M_i^4(r_\mu) \left\{ \log\left[\frac{M_i^2(r_\mu)}{\Lambda_{\text{cut}}^2}\right] - \frac{1}{2} \right\} .$$

$$\sum_{i=1,2} M_i^2(r_\mu) \xrightarrow{r_0} - \sum_{i=1,2} M_i^2(r_\mu)$$
$$\sum_{i=1,2} M_i^4(r_\mu) \xrightarrow{r_0} + \sum_{i=1,2} M_i^4(r_\mu)$$
$$\sum_{i=1,2} M_i^4(r_\mu) \log \frac{M_i^2(r_\mu)}{\Lambda_{\text{cut}}^2} \xrightarrow{r_0} \sum_{i=1,2} M_i^4(r_\mu) \log\left[\frac{-M_i^2(r_\mu)}{\Lambda_{\text{cut}}^2}\right]$$

The 1-loop effective potential is invariant under the r_0 transformation if $\Lambda_{\text{cut}}^2 \xrightarrow{r_0} -\Lambda_{\text{cut}}^2$

The toy model - 2RSM

The model considered in this section indeed is stable under 1-loop RGE running.

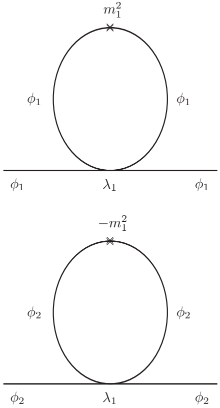

Figure 2: Diagrams which generate mass² beta functions: $\beta_{m_1^2}$ and $\beta_{m_2^2}$.

Summary and conclusions

- A set of constraints on 2HDM scalar parameters which is RG invariant to all orders with bosonic contributions to the β-functions – and which can be invariant to at least two loops if fermions are also included, have been found.
- The constraints are

$$m_{11}^2 + m_{22}^2 = 0 \quad , \quad \lambda_1 = \lambda_2 \quad , \quad \lambda_6 = -\lambda_7 \, ,$$

- The constraints are basis invariant.
- The constraints are fixed points of RGE equations for corresponding quantities, however they do not imply presence of any known symmetry.
- The constraints could be seen as emerging from the "r_0 symmetry": $r_0 \to -r_0$ defined in terms of the bilinears $r_0 \equiv \frac{1}{2}\left(\Phi_1^\dagger \Phi_1 + \Phi_2^\dagger \Phi_2\right)$.

Summary and conclusions

- The r_0 symmetry can not be obtained in terms of unitary transformation acting upon Higgs doubles, except for an unorthodox transformation (i.e. r_0 transformation) that involves $x_\mu \to i\, x_\mu$ and perhaps $p^\mu \to i\, p^\mu$.
- $$V_{\rm CW}^{(1-loop)}(\phi_a) = -\frac{1}{2}\int \frac{d^4 p_E}{(2\pi)^4}\left[{\rm Tr}\sum_{n=1}^{\infty}\frac{(-1)^n}{n}\left(\frac{M_S^2}{p_E^2}\right)^n\right]$$

  If the r_0 transformation is supplemented by cut-off sign change, $\Lambda^2_{\rm cut} \xrightarrow{r_0} -\Lambda^2_{\rm cut}$ then the 1-loop CW effective potential is invariant.
- Application: finding RGE stable relations between parameters in BSM models.

Backup slides

$$\Lambda^3 = (\mathrm{Tr}\Lambda)\Lambda^2 - \frac{1}{2}\left[(\mathrm{Tr}\Lambda)^2 - \mathrm{Tr}\Lambda^2\right]\Lambda + \frac{1}{6}\left[(\mathrm{Tr}\Lambda)^3 - 3\mathrm{Tr}\Lambda\,\mathrm{Tr}\Lambda^2 + 2\mathrm{Tr}\Lambda^3\right]$$

Running of parameters of the 2HDM

Symmetry	m_{11}^2	m_{22}^2	m_{12}^2	λ_1	λ_2	λ_3	λ_4	λ_5	λ_6	λ_7
r_0		$-m_{11}^2$			λ_1					$-\lambda_6$
0CP1		$-m_{11}^2$	real		λ_1			real	real	$-\lambda_6$
0Z_2		$-m_{11}^2$	0		λ_1				0	0
0U(1)		$-m_{11}^2$	0		λ_1			0	0	0
0CP2	0	0	0		λ_1					$-\lambda_6$
0CP3	0	0	0		λ_1			λ_{134}	0	0
0$SO(3)$	0	0	0		λ_1		$\lambda_1 - \lambda_3$	0	0	0

Table 2: Relations between 2HDM scalar potential parameters for each of the new seven symmetries discussed, $\lambda_{134} \equiv \lambda_1 - \lambda_3 - \lambda_4$.

Fermionic digression

Remarks:

- The two fixed points
  - $\{\vec{M} = \vec{0}\,,\ \vec{\Lambda} = \vec{0}\}$.
  - $\{M_0 = 0\,,\ \vec{\Lambda} = \vec{0}\}$.

  imply the same quartic scalar couplings, i.e. CP2 invariant.
- Yukawa couplings consistent with CP2 are known, see P. M. Ferreira and J. P. Silva, "A Two-Higgs Doublet Model With Remarkable CP Properties," Eur. Phys. J. C **69** (2010), 45-52, [arXiv:1001.0574 [hep-ph]].
- r_0 transformations of fermions are unknown,
- in the following we will adopt CP2 invariant Yukawas to calculate fermionic contributions to beta functions.

Fermionic digression

$$-\mathcal{L}_Y = \bar{q}_L(\Gamma_1\Phi_1+\Gamma_2\Phi_2)n_R + \bar{q}_L(\Delta_1\tilde{\Phi}_1+\Delta_2\tilde{\Phi}_2)p_R + \bar{l}_L(\Pi_1\Phi_1+\Pi_2\Phi_2)l_R + \mathrm{H.c.}$$

- For the CP2 symmetry:

$$\Gamma_1 = \begin{pmatrix} a_{11} & a_{12} & 0 \\ a_{12} & -a_{11} & 0 \\ 0 & 0 & 0 \end{pmatrix} , \quad \Gamma_2 = \begin{pmatrix} -a_{12}^* & a_{11}^* & 0 \\ a_{11}^* & a_{12}^* & 0 \\ 0 & 0 & 0 \end{pmatrix} .$$

  Similarly for Δ and Π matrices, with different coefficients b_{ij} and c_{ij} instead of a_{ij}.

Fermionic digression

For the most general 2HDM

$$\begin{aligned}
\beta^{F,1L}_{m^2_{11}} &= \left[3\,\mathrm{Tr}(\Delta_1\Delta_1^\dagger) + 3\,\mathrm{Tr}(\Gamma_1\Gamma_1^\dagger) + \mathrm{Tr}(\Pi_1\Pi_1^\dagger)\right]\, m^2_{11} \\
&\quad - \left\{\left[3\,\mathrm{Tr}(\Delta_1^\dagger\Delta_2) + 3\,\mathrm{Tr}(\Gamma_1^\dagger\Gamma_2) + \mathrm{Tr}(\Pi_1^\dagger\Pi_2)\right]\, m^2_{12} + \mathrm{h.c.}\right\}\,, \\
\beta^{F,1L}_{m^2_{22}} &= \left[3\mathrm{Tr}(\Delta_2\Delta_2^\dagger) + 3\mathrm{Tr}(\Gamma_2\Gamma_2^\dagger) + \mathrm{Tr}(\Pi_2\Pi_2^\dagger)\right]\, m^2_{22} \\
&\quad - \left\{\left[3\,\mathrm{Tr}(\Delta_1^\dagger\Delta_2) + 3\,\mathrm{Tr}(\Gamma_1^\dagger\Gamma_2) + \mathrm{Tr}(\Pi_1^\dagger\Pi_2)\right]\, m^2_{12} + \mathrm{h.c.}\right\}\,,
\end{aligned}$$

It turns out that

$$\mathrm{Tr}(\Delta_1\Delta_1^\dagger) = \mathrm{Tr}(\Delta_2\Delta_2^\dagger)\ ,\quad \mathrm{Tr}(\Gamma_1\Gamma_1^\dagger) = \mathrm{Tr}(\Gamma_2\Gamma_2^\dagger)\ ,\quad \mathrm{Tr}(\Pi_1\Pi_1^\dagger) = \mathrm{Tr}(\Pi_2\Pi_2^\dagger)\,,$$

as well as

$$\mathrm{Tr}(\Delta_1\Delta_2^\dagger) = \mathrm{Tr}(\Gamma_1\Gamma_2^\dagger) = \mathrm{Tr}(\Pi_1\Pi_2^\dagger) = 0.$$

Hence,

$$\beta^{F,1L}_{m^2_{11}+m^2_{22}} = \left[3\,\mathrm{Tr}(\Delta_1\Delta_1^\dagger) + 3\,\mathrm{Tr}(\Gamma_1\Gamma_1^\dagger) + \mathrm{Tr}(\Pi_1\Pi_1^\dagger)\right]\,\left(m^2_{11} + m^2_{22}\right)$$

Fermionic digression

It could be shown that

$$\beta^{F,1-loop}_{m^2_{11}+m^2_{22}} \propto \left(m^2_{11} + m^2_{22}\right)$$

and

$$\beta^{F,2-loop}_{m^2_{11}+m^2_{22}} \propto \left(m^2_{11} + m^2_{22}\right)$$

So $m^2_{11} + m^2_{22} = 0$ is preserved by fermionic contributions up to 2 loops.

Phenomenology of the r_0 symmetry (semisymmetry)

The set of 11 independent physical parameters of 2HDM:

$$\mathcal{P} \equiv \{M_{H^\pm}^2, M_1^2, M_2^2, M_3^2, e_1, e_2, e_3, q_1, q_2, q_3, q\}$$

The kinetic Lagrangian:

$$\mathcal{L}_k = (D_\mu \Phi_1)^\dagger (D^\mu \Phi_1) + (D_\mu \Phi_2)^\dagger (D^\mu \Phi_2)$$

$$\text{Coefficient}\left(\mathcal{L}_k, Z^\mu \left[H_j \overleftrightarrow{\partial_\mu} H_i\right]\right) = \frac{g}{2v\cos\theta_W} \epsilon_{ijk} e_k$$

$$\text{Coefficient}(\mathcal{L}_k, H_i Z^\mu Z^\nu) = \frac{g^2}{4\cos^2\theta_W} e_i\, g_{\mu\nu}$$

$$\text{Coefficient}\left(\mathcal{L}_k, H_i W^{+\mu} W^{-\nu}\right) = \frac{g^2}{2} e_i\, g_{\mu\nu}$$

$$q_i \equiv \text{Coefficient}(V, H_i H^+ H^-)$$

$$q \equiv \text{Coefficient}(V, H^+ H^+ H^- H^-)$$

Phenomenology of the r_0 symmetry (semisymmetry)

CP-sensitive invariants in the bilinear notation

$$I_1 = \left(\vec{M} \times \vec{\Lambda}\right) \cdot \left(\Lambda \vec{M}\right)$$
$$I_2 = \left(\vec{M} \times \vec{\Lambda}\right) \cdot \left(\Lambda \vec{\Lambda}\right)$$
$$I_3 = \left[\vec{M} \times \left(\Lambda \vec{M}\right)\right] \cdot \left(\Lambda^2 \vec{M}\right)$$
$$I_4 = \left[\vec{\Lambda} \times \left(\Lambda \vec{\Lambda}\right)\right] \cdot \left(\Lambda^2 \vec{\Lambda}\right)$$

Since the r_0 symmetry implies $\vec{\Lambda} = \vec{0}$ the invariants $I_{1,2,4}$ are automatically zero. However

$$I_3 = -16\lambda_5\, m_{11}^2\, \mathrm{Im}(m_{12}^2)\, \mathrm{Re}(m_{12}^2)\, \left[(\lambda_1 - \lambda_3 - \lambda_4)^2 - \lambda_5^2\right] \neq 0$$

explicit violation of CP

Phenomenology of the r_0 symmetry (semisymmetry)

Stationary-point equations:

$$
\begin{aligned}
m_{11}^2 &= \frac{1}{2}\lambda_1\left(v_2^2 - v_1^2\right), \\
\mathrm{Re}\, m_{12}^2 &= \frac{1}{2} v_1 v_2 \cos\xi \,(\lambda_1 + \lambda_3 + \lambda_4 + \lambda_5), \\
\mathrm{Im}\, m_{12}^2 &= -\frac{1}{2} v_1 v_2 \sin\xi \,(\lambda_1 + \lambda_3 + \lambda_4 - \lambda_5).
\end{aligned}
$$

The neutral sector rotation matrix is then given by

$$
R = \begin{pmatrix} \frac{v_2 \cos\xi}{v} & \frac{v_1 \cos\xi}{v} & -\sin\xi \\ -\frac{v_1}{v} & \frac{v_2}{v} & 0 \\ \frac{v_2 \sin\xi}{v} & \frac{v_1 \sin\xi}{v} & \cos\xi \end{pmatrix},
$$

yielding masses

$$
M_1^2 = \frac{1}{2} v^2 (\lambda_1 + \lambda_3 + \lambda_4 + \lambda_5), \quad M_2^2 = \lambda_1 v^2,
$$

$$
M_3^2 = \frac{1}{2} v^2 (\lambda_1 + \lambda_3 + \lambda_4 - \lambda_5), \quad M_{H\pm}^2 = \frac{1}{2}(\lambda_1 + \lambda_3)\, v^2
$$

No decoupling limit!

Phenomenology of the r_0 symmetry (semisymmetry)

Assuming that M_2 is the SM-like Higgs boson, we obtain from unitarity and boundedness-from-below constraints:

$$M_{H^\pm} \leq 711 \text{ GeV}\,,$$
$$M_3 \leq 712 \text{ GeV}\,,$$
$$M_1 \leq 711 \text{ GeV}$$

Input parameters:

$$\mathcal{P} \equiv \{M_{H^\pm}^2, M_1^2, M_2^2, M_3^2, e_1, e_2, e_3, q_1, q_2, q_3, q\}$$

Constraints implied by the r_0 symmetry:

$$v^2(e_1 q_2 - e_2 q_1) + e_1 e_2(M_2^2 - M_1^2) = 0, \quad v^2(e_1 q_3 - e_3 q_1) + e_1 e_3(M_3^2 - M_1^2) = 0,$$

$$v^2(e_2 q_3 - e_3 q_2) + e_2 e_3(M_3^2 - M_2^2) = 0, \quad q = \frac{1}{2v^4}(e_1^2 M_1^2 + e_2^2 M_2^2 + e_3^2 M_3^2),$$

$$M_{H^\pm}^2 = \frac{1}{2}(e_1 q_1 + e_2 q_2 + e_3 q_3) + \frac{1}{2v^2}(e_1^2 M_1^2 + e_2^2 M_2^2 + e_3^2 M_3^2),$$