

Semisymmetries in scalar field theories

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based on

- P.M. Ferreira , B.G., O.M. Ogreid, P. Osland, "New Symmetries of the Two-Higgs-Doublet Model", *Eur.Phys.J.C* 84 (2024) 3, 234, e-Print: 2306.02410
- work in progress

The Two-Higgs Doublet Model (2HDM) in the bilinear notation

$$V = m_{11}^2 \Phi_1^\dagger \Phi_1 + m_{22}^2 \Phi_2^\dagger \Phi_2 - [m_{12}^2 \Phi_1^\dagger \Phi_2 + \text{h.c.}] + \frac{1}{2} \lambda_1 (\Phi_1^\dagger \Phi_1)^2 + \frac{1}{2} \lambda_2 (\Phi_2^\dagger \Phi_2)^2 + \lambda_3 (\Phi_1^\dagger \Phi_1) (\Phi_2^\dagger \Phi_2) + \lambda_4 (\Phi_1^\dagger \Phi_2) (\Phi_2^\dagger \Phi_1) + \left\{ \frac{1}{2} \lambda_5 (\Phi_1^\dagger \Phi_2)^2 + [\lambda_6 (\Phi_1^\dagger \Phi_1) + \lambda_7 (\Phi_2^\dagger \Phi_2)] \Phi_1^\dagger \Phi_2 + \text{h.c.} \right\},$$

where m_{12}^2 and $\lambda_{5,6,7}$ might be complex.

An alternative notation uses four gauge-invariant bilinears constructed from the doublets (Velhinho 1994, Nagel 2004, Ivanov 2005, Maniatis 2006, Nishi 2006):

$$\begin{aligned} r_0 &\equiv \frac{1}{2} \left(\Phi_1^\dagger \Phi_1 + \Phi_2^\dagger \Phi_2 \right), \\ r_1 &\equiv \frac{1}{2} \left(\Phi_1^\dagger \Phi_2 + \Phi_2^\dagger \Phi_1 \right) = \text{Re} \left(\Phi_1^\dagger \Phi_2 \right), \\ r_2 &\equiv -\frac{i}{2} \left(\Phi_1^\dagger \Phi_2 - \Phi_2^\dagger \Phi_1 \right) = \text{Im} \left(\Phi_1^\dagger \Phi_2 \right), \\ r_3 &\equiv \frac{1}{2} \left(\Phi_1^\dagger \Phi_1 - \Phi_2^\dagger \Phi_2 \right). \end{aligned}$$

The Two-Higgs Doublet Model (2HDM) in the bilinear notation

The potential of may be written as

$$V = M_\mu r^\mu + \Lambda_{\mu\nu} r^\mu r^\nu,$$

where

$$r^\mu \equiv (r_0, r_1, r_2, r_3) = (r_0, \vec{r}),$$

$$M^\mu \equiv (m_{11}^2 + m_{22}^2, 2\text{Re}(m_{12}^2), -2\text{Im}(m_{12}^2), m_{22}^2 - m_{11}^2) = (M_0, \vec{M}),$$

$$\Lambda^{\mu\nu} \equiv \begin{pmatrix} \frac{1}{2}(\lambda_1 + \lambda_2) + \lambda_3 & -\text{Re}(\lambda_6 + \lambda_7) & \text{Im}(\lambda_6 + \lambda_7) & \frac{1}{2}(\lambda_2 - \lambda_1) \\ -\text{Re}(\lambda_6 + \lambda_7) & \lambda_4 + \text{Re}(\lambda_5) & -\text{Im}(\lambda_5) & \text{Re}(\lambda_6 - \lambda_7) \\ \text{Im}(\lambda_6 + \lambda_7) & -\text{Im}(\lambda_5) & \lambda_4 - \text{Re}(\lambda_5) & -\text{Im}(\lambda_6 - \lambda_7) \\ \frac{1}{2}(\lambda_2 - \lambda_1) & \text{Re}(\lambda_6 - \lambda_7) & -\text{Im}(\lambda_6 - \lambda_7) & \frac{1}{2}(\lambda_1 + \lambda_2) - \lambda_3 \end{pmatrix}$$

$$\Lambda^{\mu\nu} \equiv \begin{pmatrix} \Lambda_{00} & \vec{\Lambda} \\ \vec{\Lambda}^T & \Lambda \end{pmatrix}$$

Basis transformations

Weak-basis transformation, $U(2)$:

$$\begin{pmatrix} \Phi'_1 \\ \Phi'_2 \end{pmatrix} = e^{i\psi} \underbrace{\begin{pmatrix} \cos \theta & e^{-i\tilde{\xi}} \sin \theta \\ -e^{i\chi} \sin \theta & e^{i(\chi-\tilde{\xi})} \cos \theta \end{pmatrix}}_{U(2)} \begin{pmatrix} \Phi_1 \\ \Phi_2 \end{pmatrix}$$

$$\mathcal{L}_k = (D_\mu \Phi_1)^\dagger (D^\mu \Phi_1) + (D_\mu \Phi_2)^\dagger (D^\mu \Phi_2),$$

- The Higgs kinetic terms remain invariant.
- Physical observables are basis-independent.

Basis transformations

$$V = M_{\mu} r^{\mu} + \Lambda_{\mu\nu} r^{\mu} r^{\nu}$$

The basis rotation matrix

$$R_{ij}(U) \equiv \frac{1}{2} \text{Tr} (U^{\dagger} \sigma_i U \sigma_j),$$

where σ_i ($i = 1, 2, 3$) are the Pauli matrices.

The basis transformations:

$$\begin{aligned}\vec{r} \rightarrow \vec{r}' &= R \vec{r} \\ \vec{M} \rightarrow \vec{M}' &= R \vec{M} \\ \vec{\Lambda} \rightarrow \vec{\Lambda}' &= R \vec{\Lambda} \\ \Lambda \rightarrow \Lambda' &= R \Lambda R^T\end{aligned}$$

whereas r_0 , M_0 and Λ_{00} do not change under basis transformations – they are *basis invariants*.

Global symmetries of the 2HDM

- *Higgs-family symmetries*, unitary transformations mix both doublets,

$$\Phi_i \rightarrow \Phi'_i = \sum_{j=1}^2 U_{ij} \Phi_j, \quad U \in U(2)$$

e.g. Z_2 :

$$\Phi_1 \rightarrow \Phi_1, \quad \Phi_2 \rightarrow -\Phi_2,$$

prevents the occurrence of tree-level flavour-changing neutral currents (FCNC).

- *generalized CP (GCP)*, transformations:

$$\Phi_i \rightarrow \Phi'_i = \sum_{j=1}^2 X_{ij} \Phi_j^*, \quad X \in U(2)$$

e.g. "standard" CP transformation (CP1):

$$\Phi_i \rightarrow \Phi_i^*$$

Global symmetries of the 2HDM

In the bilinear formalism, symmetries are represented by rotations in the 3-dimensional space defined by the vector $\vec{r}$:

$$\vec{r} \rightarrow \vec{r}' = S \vec{r},$$

where $S \in O(3)$.

$$S_{Z_2} = \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad S_{CP1} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\text{CP2: } \Phi_1 \rightarrow \Phi_2^*, \quad \Phi_2 \rightarrow -\Phi_1^*$$

$$S_{CP2} = \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix},$$

A parity transformation about the three axes.

Global symmetries of the 2HDM

S	m_{11}^2	m_{22}^2	m_{12}^2	λ_1	λ_2	λ_3	λ_4	λ_5	λ_6	λ_7
CP1			real					real	real	real
Z_2			0						0	0
U(1)			0					0	0	0
CP2		m_{11}^2	0		λ_1					$-\lambda_6$
CP3		m_{11}^2	0		λ_1			λ_{134}	0	0
SO(3)		m_{11}^2	0		λ_1		$\lambda_1 - \lambda_3$	0	0	0

Table 1: Relations between 2HDM scalar potential parameters for each of the six symmetries discussed, $\lambda_{134} \equiv \lambda_1 - \lambda_3 - \lambda_4$.

$$\begin{aligned}
 V = & m_{11}^2 \Phi_1^\dagger \Phi_1 + m_{22}^2 \Phi_2^\dagger \Phi_2 - [m_{12}^2 \Phi_1^\dagger \Phi_2 + \text{h.c.}] + \\
 & \frac{1}{2} \lambda_1 (\Phi_1^\dagger \Phi_1)^2 + \frac{1}{2} \lambda_2 (\Phi_2^\dagger \Phi_2)^2 + \lambda_3 (\Phi_1^\dagger \Phi_1)(\Phi_2^\dagger \Phi_2) + \lambda_4 (\Phi_1^\dagger \Phi_2)(\Phi_2^\dagger \Phi_1) + \\
 & \left\{ \frac{1}{2} \lambda_5 (\Phi_1^\dagger \Phi_2)^2 + [\lambda_6 (\Phi_1^\dagger \Phi_1) + \lambda_7 (\Phi_2^\dagger \Phi_2)] \Phi_1^\dagger \Phi_2 + \text{h.c.} \right\},
 \end{aligned}$$

Running of parameters of the 2HDM

The 1-loop β -functions for the quadratic couplings

$$\beta_{m_{11}^2} = 3\lambda_1 m_{11}^2 + (2\lambda_3 + \lambda_4) m_{22}^2 - 3 (\lambda_6^* m_{12}^2 + \text{h.c.}) - \frac{1}{4} (9g^2 + 3g'^2) m_{11}^2 + \beta_{m_{11}^2}^F,$$

$$\beta_{m_{22}^2} = (2\lambda_3 + \lambda_4) m_{11}^2 + 3\lambda_2 m_{22}^2 - 3 (\lambda_7^* m_{12}^2 + \text{h.c.}) - \frac{1}{4} (9g^2 + 3g'^2) m_{22}^2 + \beta_{m_{22}^2}^F,$$

$$\beta_{m_{12}^2} = -3 (\lambda_6 m_{11}^2 + \lambda_7 m_{22}^2) + (\lambda_3 + 2\lambda_4) m_{12}^2 + 3\lambda_5 m_{12}^{2*} - \frac{1}{4} (9g^2 + 3g'^2) m_{12}^2 + \beta_{m_{12}^2}^F,$$

Running of parameters of the 2HDM

and 1-loop β functions for the quartic ones,

$$\begin{aligned}
 \beta_{\lambda_1} &= 6\lambda_1^2 + 2\lambda_3^2 + 2\lambda_3\lambda_4 + \lambda_4^2 + |\lambda_5|^2 + 12|\lambda_6|^2 \\
 &\quad + \frac{3}{8}(3g^4 + g'^4 + 2g^2g'^2) - \frac{3}{2}\lambda_1(3g^2 + g'^2) + \beta_{\lambda_1}^F, \\
 \beta_{\lambda_2} &= 6\lambda_2^2 + 2\lambda_3^2 + 2\lambda_3\lambda_4 + \lambda_4^2 + |\lambda_5|^2 + 12|\lambda_7|^2 \\
 &\quad + \frac{3}{8}(3g^4 + g'^4 + 2g^2g'^2) - \frac{3}{2}\lambda_2(3g^2 + g'^2) + \beta_{\lambda_2}^F, \\
 \beta_{\lambda_3} &= (\lambda_1 + \lambda_2)(3\lambda_3 + \lambda_4) + 2\lambda_3^2 + \lambda_4^2 + |\lambda_5|^2 + 2(|\lambda_6|^2 + |\lambda_7|^2) + 8\text{Re}(\lambda_6\lambda_7^*) \\
 &\quad + \frac{3}{8}(3g^4 + g'^4 - 2g^2g'^2) - \frac{3}{2}\lambda_3(3g^2 + g'^2) + \beta_{\lambda_3}^F, \\
 \beta_{\lambda_4} &= (\lambda_1 + \lambda_2)\lambda_4 + 4\lambda_3\lambda_4 + 2\lambda_4^2 + 4|\lambda_5|^2 + 5(|\lambda_6|^2 + |\lambda_7|^2) + 2\text{Re}(\lambda_6\lambda_7^*) \\
 &\quad + \frac{3}{2}g^2g'^2 - \frac{3}{2}\lambda_4(3g^2 + g'^2) + \beta_{\lambda_4}^F, \\
 \beta_{\lambda_5} &= (\lambda_1 + \lambda_2 + 4\lambda_3 + 6\lambda_4)\lambda_5 + 5(\lambda_6^2 + \lambda_7^2) + 2\lambda_6\lambda_7 \\
 &\quad - \frac{3}{2}\lambda_5(3g^2 + g'^2) + \beta_{\lambda_5}^F, \\
 \beta_{\lambda_6} &= (6\lambda_1 + 3\lambda_3 + 4\lambda_4)\lambda_6 + (3\lambda_3 + 2\lambda_4)\lambda_7 + 5\lambda_5\lambda_6^* + \lambda_5\lambda_7^* \\
 &\quad - \frac{3}{2}\lambda_6(3g^2 + g'^2) + \beta_{\lambda_6}^F, \\
 \beta_{\lambda_7} &= (6\lambda_2 + 3\lambda_3 + 4\lambda_4)\lambda_7 + (3\lambda_3 + 2\lambda_4)\lambda_6 + 5\lambda_5\lambda_7^* + \lambda_5\lambda_6^* \\
 &\quad - \frac{3}{2}\lambda_7(3g^2 + g'^2) + \beta_{\lambda_7}^F,
 \end{aligned}$$

where the β_x^F terms contain all contributions coming from fermions.

Running of parameters of the 2HDM

$$Z_2 \text{ symmetry} \Rightarrow \lambda_6 = \lambda_7 = 0 \Rightarrow \beta_{\lambda_6} = \beta_{\lambda_7} = 0$$

a symmetry-based conditions on λ 's are preserved by RGE running at the one-loop order.

For the Z_2 symmetry

$$\beta_{\lambda_5} = \left[\lambda_1 + \lambda_2 + 4\lambda_3 + 6\lambda_4 - \frac{3}{2} (3g^2 + g'^2) \right] \lambda_5$$

$\lambda_5 = 0$ is a *fixed point* of this RG equation: if at any scale $\lambda_5 = 0$, that coupling will remain zero for all renormalization scales. Such fixed points of RG equations are usually fingerprints of symmetries, and indeed that is the case here: if $\lambda_6 = \lambda_7 = 0$, the extra constraint $\lambda_5 = 0$ takes us from Z_2 -symmetric model to $U(1)$ -symmetric.

Running of parameters of the 2HDM

We have noticed that

$$\{m_{11}^2 + m_{22}^2 = 0, \lambda_1 - \lambda_2 = 0, \lambda_6 + \lambda_7 = 0\}$$

- constitutes a fixed point of the 1-loop RG equations,
- are basis transformation invariants.

$$\begin{aligned}\beta_{m_{11}^2 + m_{22}^2} &= 3(\lambda_1 m_{11}^2 + \lambda_2 m_{22}^2) + (2\lambda_3 + \lambda_4)(m_{11}^2 + m_{22}^2) \\ &\quad - 3 [(\lambda_6^* + \lambda_7^*)m_{12}^2 + \text{h.c.}] - \frac{1}{4}(9g^2 + 3g'^2)(m_{11}^2 + m_{22}^2) \\ \beta_{\lambda_1 - \lambda_2} &= 6(\lambda_1^2 - \lambda_2^2) + 12(|\lambda_6|^2 - |\lambda_7|^2) - \frac{3}{2}(\lambda_1 - \lambda_2)(3g^2 + g'^2) \\ \beta_{\lambda_6 + \lambda_7} &= 6(\lambda_1\lambda_6 + \lambda_2\lambda_7) + (3\lambda_3 + 2\lambda_4)(\lambda_6 + \lambda_7) + 6\lambda_5(\lambda_6^* + \lambda_7^*) \\ &\quad - \frac{3}{2}(\lambda_6 + \lambda_7)(3g^2 + g'^2)\end{aligned}$$

It turns out that

$$\{m_{11}^2 + m_{22}^2 = 0, \lambda_1 - \lambda_2 = 0, \lambda_6 + \lambda_7 = 0\}$$

is also the 2-loop fixed point.



Conclusion:

Perhaps there is a symmetry behind the fixed point:

$$\{m_{11}^2 + m_{22}^2 = 0, \lambda_1 - \lambda_2 = 0, \lambda_6 + \lambda_7 = 0\}$$

Running of parameters of the 2HDM

$$V = M_\mu r^\mu + \Lambda_{\mu\nu} r^\mu r^\nu$$

The rotation matrix $R_{ij}(U) = \text{Tr}(U^\dagger \sigma_i U \sigma_j) / 2$, and the basis transformations:

$$\vec{M} \rightarrow \vec{M}' = R \vec{M} \quad \vec{\Lambda} \rightarrow \vec{\Lambda}' = R \vec{\Lambda} \quad \Lambda \rightarrow \Lambda' = R \Lambda R^T$$

whereas M_0 and Λ_{00} are basis invariants.

$$M^\mu \equiv (m_{11}^2 + m_{22}^2, 2\text{Re}(m_{12}^2), -2\text{Im}(m_{12}^2), m_{22}^2 - m_{11}^2) = (M_0, \vec{M}),$$

$$\Lambda^{\mu\nu} \equiv \begin{pmatrix} \frac{1}{2}(\lambda_1 + \lambda_2) + \lambda_3 & -\text{Re}(\lambda_6 + \lambda_7) & \text{Im}(\lambda_6 + \lambda_7) & \frac{1}{2}(\lambda_2 - \lambda_1) \\ -\text{Re}(\lambda_6 + \lambda_7) & \lambda_4 + \text{Re}(\lambda_5) & -\text{Im}(\lambda_5) & \text{Re}(\lambda_6 - \lambda_7) \\ \text{Im}(\lambda_6 + \lambda_7) & -\text{Im}(\lambda_5) & \lambda_4 - \text{Re}(\lambda_5) & -\text{Im}(\lambda_6 - \lambda_7) \\ \frac{1}{2}(\lambda_2 - \lambda_1) & \text{Re}(\lambda_6 - \lambda_7) & -\text{Im}(\lambda_6 - \lambda_7) & \frac{1}{2}(\lambda_1 + \lambda_2) - \lambda_3 \end{pmatrix}$$

$$\Lambda^{\mu\nu} = \begin{pmatrix} \Lambda_{00} & \vec{\Lambda} \\ \vec{\Lambda}^T & \Lambda \end{pmatrix}$$

Running of parameters of the 2HDM

Basis transformation invariants:

$$I_{1,1} = \Lambda_{00},$$

$$I_{1,2} = \text{Tr}\Lambda$$

$$I_{2,1} = \vec{\Lambda} \cdot \vec{\Lambda},$$

$$I_{2,2} = \text{Tr}\Lambda^2$$

$$I_{3,1} = \vec{\Lambda} \cdot \Lambda \vec{\Lambda},$$

$$I_{3,2} = \text{Tr}\Lambda^3$$

$$I_{4,1} = \vec{\Lambda} \cdot \Lambda^2 \vec{\Lambda},$$

To all orders in perturbation theory,

$$\beta_{\vec{\Lambda}} = a_0 \vec{\Lambda} + a_1 \Lambda \vec{\Lambda} + a_2 \Lambda^2 \vec{\Lambda}$$

$\vec{\Lambda} = \vec{0}$ is a fixed point to all orders in perturbation theory.

where the a_i are polynomial expressions involving invariants,

see A.V. Bednyakov, "On three-loop RGE for the Higgs sector of 2HDM", JHEP 11 (2018) 154, e-Print: 1809.04527

Running of parameters of the 2HDM

$$\beta_{\vec{M}} = c_0 \vec{M} + c_1 \Lambda \vec{M} + c_2 \Lambda^2 \vec{M} + c_3 I_{M3} \vec{\Lambda} + c_4 I_{M4} \Lambda \vec{\Lambda} + c_5 I_{M5} \Lambda^2 \vec{\Lambda}$$

• If $\vec{\Lambda} = \vec{0}$, then $\vec{M} = \vec{0}$ is a fixed point to all orders

$$\beta_{M_0} = b_0 M_0 + b_1 \vec{\Lambda} \cdot \vec{M} + b_2 \vec{\Lambda} \cdot (\Lambda \vec{M}) + b_3 \vec{\Lambda} \cdot (\Lambda^2 \vec{M})$$

• If $\vec{\Lambda} = \vec{0}$, then $M_0 = 0$ is a fixed point to all orders.

where the b_i and c_i are polynomial expressions involving invariants,
see A.V. Bednyakov

Running of parameters of the 2HDM

Two all-order fixed points of the 2HDM RG equations:

- $\{\vec{M} = \vec{0}, \vec{\Lambda} = \vec{0}\}.$

$$m_{11}^2 = m_{22}^2, \quad m_{12}^2 = 0, \quad \lambda_1 = \lambda_2, \quad \lambda_6 = -\lambda_7.$$

- $\{M_0 = 0, \vec{\Lambda} = \vec{0}\}.$

$$M_0 \equiv m_{11}^2 + m_{22}^2 = 0, \quad \lambda_1 = \lambda_2, \quad \lambda_6 = -\lambda_7.$$

Who are they?

These are the conditions mentioned before and are basis invariant, so they are *not* a basis change of the previous ones.

Running of parameters of the 2HDM

Two all-order fixed points of the 2HDM RG equations:

- $\{\vec{M} = \vec{0}, \vec{\Lambda} = \vec{0}\}.$

$$m_{11}^2 = m_{22}^2, \quad m_{12}^2 = 0, \quad \lambda_1 = \lambda_2, \quad \lambda_6 = -\lambda_7.$$

They are exactly the CP2 symmetry conditions.

- $\{M_0 = 0, \vec{\Lambda} = \vec{0}\}.$

$$M_0 \equiv m_{11}^2 + m_{22}^2 = 0, \quad \lambda_1 = \lambda_2, \quad \lambda_6 = -\lambda_7.$$

These are the conditions mentioned before and are basis invariant, so they are *not* a basis change of the previous ones.

The r_0 symmetry

$$V = M_0 r_0 + \Lambda_{00} r_0^2 - \vec{M} \cdot \vec{r} - 2 \left(\vec{\Lambda} \cdot \vec{r} \right) r_0 + \vec{r} \cdot (\Lambda \vec{r})$$

- $\{\vec{M} = \vec{0}, \vec{\Lambda} = \vec{0}\}$. These are exactly the CP2 ($\vec{r} \rightarrow -\vec{r}$).
- $\{M_0 = 0, \vec{\Lambda} = \vec{0}\}$ These are new, perhaps $r_0 \xrightarrow{?} -r_0$

The r_0 symmetry

$$\Phi_1 = \begin{pmatrix} \phi_1 + i\phi_2 \\ \phi_3 + i\phi_4 \end{pmatrix}, \quad \Phi_2 = \begin{pmatrix} \phi_5 + i\phi_6 \\ \phi_7 + i\phi_8 \end{pmatrix},$$

The transformation

$$\begin{pmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \\ \phi_4 \\ \phi_5 \\ \phi_6 \\ \phi_7 \\ \phi_8 \end{pmatrix} \rightarrow \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & i & 0 & 0 \\ 0 & 0 & 0 & 0 & i & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & i \\ 0 & 0 & 0 & 0 & 0 & 0 & i & 0 \\ 0 & -i & 0 & 0 & 0 & 0 & 0 & 0 \\ -i & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -i & 0 & 0 & 0 & 0 \\ 0 & 0 & -i & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \\ \phi_4 \\ \phi_5 \\ \phi_6 \\ \phi_7 \\ \phi_8 \end{pmatrix}$$

implies

$$r_0 \rightarrow -r_0 \quad r_i \rightarrow +r_i$$

The r_0 symmetry

$$\begin{aligned}\Phi_1 &\rightarrow -\Phi_2^* & \Phi_1^\dagger &\rightarrow \Phi_2^T, \\ \Phi_2 &\rightarrow \Phi_1^*, & \Phi_2^\dagger &\rightarrow -\Phi_1^T.\end{aligned}$$

- Higgs kinetic terms

$$\mathcal{L}_k = (D_\mu \Phi_1)^\dagger (D^\mu \Phi_1) + (D_\mu \Phi_2)^\dagger (D^\mu \Phi_2),$$

where

$$D^\mu = \partial^\mu + \frac{ig}{2} \sigma_i W_i^\mu + i \frac{g'}{2} B^\mu,$$

$\mathcal{L}_k$ remains invariant if the above transformation of $\Phi_{1,2}$ is supplemented by

$$\partial_\mu \rightarrow -i\partial_\mu, \quad (x^\mu \rightarrow ix^\mu),$$

$$B_\mu \rightarrow iB_\mu,$$

$$W_{1\mu} \rightarrow iW_{1\mu}, \quad W_{2\mu} \rightarrow -iW_{2\mu}, \quad W_{3\mu} \rightarrow iW_{3\mu}.$$

The r_0 symmetry

- Gauge kinetic terms

$$\mathcal{L}^B = -\frac{1}{4}B_{\mu\nu}B^{\mu\nu} - \frac{1}{4}W_{i\mu\nu}W_i^{\mu\nu},$$

where $B^{\mu\nu} = \partial^\nu B^\mu - \partial^\mu B^\nu$ and $W_i^{\mu\nu} = \partial^\nu W_i^\mu - \partial^\mu W_i^\nu + g\epsilon_{ijk}W_j^\mu W_k^\nu$.

Under r_0 transformation

$$B^{\mu\nu} \rightarrow B^{\mu\nu}, \\ W_1^{\mu\nu} \rightarrow W_1^{\mu\nu}, \quad W_2^{\mu\nu} \rightarrow -W_2^{\mu\nu}, \quad W_3^{\mu\nu} \rightarrow W_3^{\mu\nu}$$

Symmetries and 1-loop CW effective potential

$$V_{\text{eff}}^{(1)}(\phi_c) \equiv - \sum_{n=2}^{\infty} \frac{\phi_c^n}{n!} \Gamma^{(n)}(p_{\text{ext}} = 0) = -\frac{1}{2} \int \frac{d^4 p_E}{(2\pi)^4} \left[\text{Tr} \sum_{n=1}^{\infty} \frac{(-1)^n}{n} \left(\frac{M_S^2}{p_E^2} \right)^n \right]$$

for

$$(M_S^2)_{ab} \equiv \frac{\partial^2 V}{\partial \phi_a \partial \phi_b}$$

2HDM:

$$\Phi_1 = \begin{pmatrix} \phi_1 + i\phi_2 \\ \phi_3 + i\phi_4 \end{pmatrix}, \quad \Phi_2 = \begin{pmatrix} \phi_5 + i\phi_6 \\ \phi_7 + i\phi_8 \end{pmatrix}$$

- Q.-H. Cao, K. Cheng, and C. Xu, "Global Symmetries and Effective Potential of 2HDM in Orbit Space", Phys.Rev.D 108 (2023) 055036, arXiv:2305.12764 [hep-ph].
- A. Pilaftsis, "Dirac Algebra Formalism for Two Higgs Doublet Models: the One-Loop Effective Potential", Phys.Lett.B 860 (2025) 139147, arXiv: 2408.04511 [hep-ph].

Symmetries and 1-loop CW effective potential

$$r_0 \rightarrow -r_0 \quad r_i \rightarrow +r_i$$

Is $V_{\text{eff}}^{(1)}(r_\mu)$ invariant under the r_0 transformation?

At the new fixed point $M_0 = 0$ and $\vec{\Lambda} = 0$ ($m_{11}^2 + m_{22}^2 = 0$, $\lambda_1 = \lambda_2$ and $\lambda_6 = -\lambda_7$):

$$\text{Tr}(M_S^2) = 4[5\Lambda_{00} + \text{tr}(\Lambda)]r_0 \xrightarrow{r_0} -\text{Tr}(M_S^2) = -4[5\Lambda_{00} + \text{tr}(\Lambda)]r_0$$

$$n = 1 : \quad \text{Tr} [M_S^2] \quad \text{odd}$$

$$n = 2 : \quad \text{Tr} [(M_S^2)^2] \quad \text{even}$$

$$\vdots$$

$$n = 2k : \quad \text{Tr} [(M_S^2)^{2k}] \quad \text{even}$$

$$n = 2k + 1 : \quad \text{Tr} [(M_S^2)^{2k+1}] \quad \text{odd}$$

$$\vdots$$

Symmetries and 1-loop CW effective potential

The cut-off regularization implies that

$$V_{\text{eff}}^{(1)} = -\frac{1}{64\pi^2} \sum_{n=1}^{\infty} \left\{ \frac{(-1)^n}{n} \int_0^{\Lambda_{UV}^2} d\rho \, \rho \frac{\text{Tr} [M_S^2]^n}{\rho^n} \right\},$$

where $\rho \equiv |p_E|^2$.

Replacing Λ_{UV}^2 by $-\Lambda_{UV}^2$ is equivalent of switching sign in front of ρ by $-\rho$. Consequently, for odd n a minus sign appears. Since for odd n $\text{Tr} [M_S^2(r_0)]^n$ is an odd function of r_0 , therefore supplementing the r_0 transformation by $\Lambda_{UV}^2 \rightarrow -\Lambda_{UV}^2$ we arrive at an invariant 1-loop effective potential.

The toy model - 2RSM

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi_1 \partial^\mu \phi_1 + \partial_\mu \phi_2 \partial^\mu \phi_2) - V(\phi_1, \phi_2),$$

with

$$V(\phi_1, \phi_2) = \frac{1}{2} m_1^2 (\phi_1^2 - \phi_2^2) + m_{12}^2 \phi_1 \phi_2 + \frac{1}{2} \lambda_1 (\phi_1^4 + \phi_2^4) + \lambda_3 (\phi_1 \phi_2)^2 + \lambda_6 (\phi_1^2 - \phi_2^2) \phi_1 \phi_2$$

The model is invariant under the following r_0 -like transformation

$$x^\mu \rightarrow x'^\mu \equiv ix^\mu, \quad \phi_1(x) \rightarrow \phi'_1(x') \equiv i\phi_2(x), \quad \phi_2(x) \rightarrow \phi'_2(x') \equiv -i\phi_1(x)$$

It is possible to choose (ϕ_1, ϕ_2) -basis such that $\lambda_6 = 0$.

The mass² matrix

$$(M_S^2)_{ij}(\phi_1, \phi_2) = \begin{pmatrix} m_1^2 + 6\lambda_1\phi_1^2 + 2\lambda_3\phi_2^2 & m_{12}^2 + 4\lambda_3\phi_1\phi_2 \\ m_{12}^2 + 4\lambda_3\phi_1\phi_2 & -m_1^2 + 6\lambda_1\phi_2^2 + 2\lambda_3\phi_1^2 \end{pmatrix}$$

The toy model - 2RSM

One can express the potential in terms of bilinear variables:

$$r_0 \equiv \frac{1}{2}(\phi_1^2 + \phi_2^2)$$

$$r_1 \equiv \phi_1 \phi_2$$

$$r_2 \equiv \frac{1}{2}(\phi_1^2 - \phi_2^2).$$

Upon the r_0 transformation

$$(r_0, r_1, r_2) \xrightarrow{r_0} (-r_0, r_1, r_2)$$

The potential could be written as

$$V(r^\mu) = M_\mu r^\mu + \Lambda_{\mu\nu} r^\mu r^\nu$$

for $\mu, \nu = 0, 1, 2$ with $M_\mu = (0, m_{12}^2, m_1^2)$ and

$$\Lambda_{\mu\nu} = \begin{pmatrix} \Lambda_{00} & 0 & 0 \\ 0 & \Lambda_{11} & \Lambda_{12} \\ 0 & \Lambda_{21} & \Lambda_{22} \end{pmatrix} = \begin{pmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_3 & 0 \\ 0 & 0 & \lambda_1 \end{pmatrix}.$$

The toy model - 2RSM

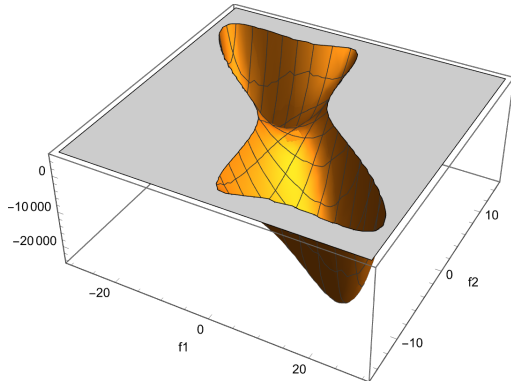


Figure 1: Scalar potential of the toy model, $m_1 = 10$, $m_{12} = 20$, $\lambda_1 = 1$, $\lambda_3 = 2$.

Two local minima:

$$(v_1^2 - v_2^2) = \frac{-m_1^2}{\lambda_1}, \quad v_1 v_2 = \frac{-m_{12}^2}{(\lambda_1 + \lambda_3)}$$

where $\langle \phi_{1,2} \rangle \equiv v_{1,2}/\sqrt{2}$.

The toy model - 2RSM

The eigenvalues of M_S^2 could be expressed through bilinears

$$M_1^2(r_\mu) = 2(3\lambda_1 + \lambda_3)r_0 + \sqrt{\Delta}$$

$$M_2^2(r_\mu) = 2(3\lambda_1 + \lambda_3)r_0 - \sqrt{\Delta},$$

where

$$\Delta = m_1^4 + m_{12}^4 + 4m_1^2(3\lambda_1 - \lambda_3)r_2 + 8m_{12}^2\lambda_3r_1 + 16\lambda_3^2r_0^2 + 12(3\lambda_1 + \lambda_3)(\lambda_1 - \lambda_3)r_2^2$$

$$M_1^2 \xleftrightarrow{r_0} -M_2^2$$

$$\square\phi_1 + \frac{\partial V(\phi_1, \phi_2)}{\partial\phi_1} = 0 \quad \xleftrightarrow{r_0} \quad \square\phi_2 + \frac{\partial V(\phi_1, \phi_2)}{\partial\phi_2} = 0$$

$$S = \int_{x_-}^{x_+} d^4x \mathcal{L} [\phi(x), \partial_\mu\phi(x), A_\mu(x), \partial_\nu A_\mu(x)] \quad r_0 - \text{invariant}$$

Stability under the 1-loop RGE running?

$$\beta_{m_1^2+m_2^2} = 12(\lambda_1 m_1^2 + \lambda_2 m_2^2) + 2\lambda_3(m_1^2 + m_2^2) + 6(\lambda_6 + \lambda_7)m_{12}^2$$

$$\beta_{m_{12}^2} = 3(\lambda_6 m_1^2 + \lambda_7 m_2^2) + 4\lambda_3 m_{12}^2$$

When the r_0 symmetry is imposed $\beta_{m_1^2+m_2^2}$ vanishes at the 1-loop level.

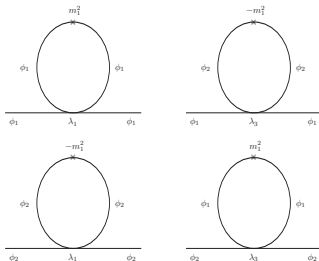


Figure 2: Diagrams which generate mass² beta functions: $\beta_{m_1^2}$ and $\beta_{m_2^2}$.

The toy model - 2RSM

The 1-loop effective potential parametrized by the cut-off Λ_{cut} :

$$V_{\text{CW}}^{(1)}(r_\mu) = \frac{\Lambda_{\text{cut}}^2}{32\pi^2} \sum_{i=1,2} M_i^2(r_\mu) + \frac{1}{64\pi^2} \sum_{i=1,2} M_i^4(r_\mu) \left\{ \log \left[\frac{M_i^2(r_\mu)}{\Lambda_{\text{cut}}^2} \right] - \frac{1}{2} \right\}.$$

$$\sum_{i=1,2} M_i^2(r_\mu) \xrightarrow{r_0} - \sum_{i=1,2} M_i^2(r_\mu)$$

$$\sum_{i=1,2} M_i^4(r_\mu) \xrightarrow{r_0} + \sum_{i=1,2} M_i^4(r_\mu)$$

$$\sum_{i=1,2} M_i^4(r_\mu) \log \frac{M_i^2(r_\mu)}{\Lambda_{\text{cut}}^2} \xrightarrow{r_0} \sum_{i=1,2} M_i^4(r_\mu) \log \left[\frac{-M_i^2(r_\mu)}{\Lambda_{\text{cut}}^2} \right]$$

The 1-loop effective potential is invariant under the r_0 transformation if ???

The toy model - 2RSM

$$M_1^2 \xrightarrow{r_0} -M_2^2$$

$$M_2^2 \xrightarrow{r_0} -M_1^2$$

The 1-loop effective potential parametrized by the cut-off Λ_{cut} :

$$V_{\text{CW}}^{(1)}(r_\mu) = \frac{\Lambda_{\text{cut}}^2}{32\pi^2} \sum_{i=1,2} M_i^2(r_\mu) + \frac{1}{64\pi^2} \sum_{i=1,2} M_i^4(r_\mu) \left\{ \log \left[\frac{M_i^2(r_\mu)}{\Lambda_{\text{cut}}^2} \right] - \frac{1}{2} \right\}.$$

$$\sum_{i=1,2} M_i^2(r_\mu) \xrightarrow{r_0} - \sum_{i=1,2} M_i^2(r_\mu)$$

$$\sum_{i=1,2} M_i^4(r_\mu) \xrightarrow{r_0} + \sum_{i=1,2} M_i^4(r_\mu)$$

$$\sum_{i=1,2} M_i^4(r_\mu) \log \frac{M_i^2(r_\mu)}{\Lambda_{\text{cut}}^2} \xrightarrow{r_0} \sum_{i=1,2} M_i^4(r_\mu) \log \left[\frac{-M_i^2(r_\mu)}{\Lambda_{\text{cut}}^2} \right]$$

The 1-loop effective potential is invariant under the r_0 transformation if $\Lambda_{\text{cut}}^2 \xrightarrow{r_0} -\Lambda_{\text{cut}}^2$

Summary and conclusions

- A set of constraints on 2HDM scalar parameters which is RG invariant to all orders with bosonic contributions to the β -functions – and which can be invariant to at least two loops if fermions are also included, have been found.
- The constraints are $m_{11}^2 + m_{22}^2 = 0$, $\lambda_1 = \lambda_2$, $\lambda_6 = -\lambda_7$.
- The constraints are basis invariant.
- The constraints are fixed points of RG equations for corresponding quantities, however they do not imply presence of any hitherto known symmetry.
- The constraints could be seen as emerging from the " r_0 symmetry": $r_0 \rightarrow -r_0$ for $r_0 \equiv \frac{1}{2} \left(\Phi_1^\dagger \Phi_1 + \Phi_2^\dagger \Phi_2 \right)$.
- The r_0 symmetry seems to be anomalous, as the 1-loop effective potential breaks the symmetry by quadratic and quartic divergence.
- The minimal r_0 -symmetric model has been found (the toy model).
- Application: finding RGE stable relations between parameters.

$$\Lambda^3 = (\text{Tr}\Lambda)\Lambda^2 - \frac{1}{2} [(\text{Tr}\Lambda)^2 - \text{Tr}\Lambda^2] \Lambda + \frac{1}{6} [(\text{Tr}\Lambda)^3 - 3\text{Tr}\Lambda \text{Tr}\Lambda^2 + 2\text{Tr}\Lambda^3]$$

Running of parameters of the 2HDM

Symmetry	m_{11}^2	m_{22}^2	m_{12}^2	λ_1	λ_2	λ_3	λ_4	λ_5	λ_6	λ_7
r_0		$-m_{11}^2$			λ_1					$-\lambda_6$
oCP1		$-m_{11}^2$	real		λ_1			real	real	$-\lambda_6$
oZ ₂		$-m_{11}^2$	0		λ_1				0	0
oU(1)		$-m_{11}^2$	0		λ_1			0	0	0
oCP2	0	0	0		λ_1					$-\lambda_6$
oCP3	0	0	0		λ_1			λ_{134}	0	0
oSO(3)	0	0	0		λ_1		$\lambda_1 - \lambda_3$	0	0	0

Table 2: Relations between 2HDM scalar potential parameters for each of the new seven symmetries discussed, $\lambda_{134} \equiv \lambda_1 - \lambda_3 - \lambda_4$.

Remarks:

- The two fixed points
 - $\{\vec{M} = \vec{0}, \vec{\Lambda} = \vec{0}\}.$
 - $\{M_0 = 0, \vec{\Lambda} = \vec{0}\}.$

imply the same quartic scalar couplings, i.e. CP2 invariant.

- Yukawa couplings consistent with CP2 are known, see P. M. Ferreira and J. P. Silva, “A Two-Higgs Doublet Model With Remarkable CP Properties,” Eur. Phys. J. C **69** (2010), 45-52, [arXiv:1001.0574 [hep-ph]].
- r_0 transformations of fermions are unknown,
- in the following we will adopt CP2 invariant Yukawas to calculate fermionic contributions to beta functions.

$$-\mathcal{L}_Y = \bar{q}_L(\Gamma_1\Phi_1+\Gamma_2\Phi_2)n_R + \bar{q}_L(\Delta_1\tilde{\Phi}_1+\Delta_2\tilde{\Phi}_2)p_R + \bar{l}_L(\Pi_1\Phi_1+\Pi_2\Phi_2)l_R + \text{H.c.}$$

- For the CP2 symmetry:

$$\Gamma_1 = \begin{pmatrix} a_{11} & a_{12} & 0 \\ a_{12} & -a_{11} & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \Gamma_2 = \begin{pmatrix} -a_{12}^* & a_{11}^* & 0 \\ a_{11}^* & a_{12}^* & 0 \\ 0 & 0 & 0 \end{pmatrix}.$$

Similarly for Δ and Π matrices, with different coefficients b_{ij} and c_{ij} instead of a_{ij} .

Fermionic digression

For the most general 2HDM

$$\begin{aligned}\beta_{m_{11}^2}^{F,1L} &= \left[3 \operatorname{Tr}(\Delta_1 \Delta_1^\dagger) + 3 \operatorname{Tr}(\Gamma_1 \Gamma_1^\dagger) + \operatorname{Tr}(\Pi_1 \Pi_1^\dagger) \right] m_{11}^2 \\ &\quad - \left\{ \left[3 \operatorname{Tr}(\Delta_1^\dagger \Delta_2) + 3 \operatorname{Tr}(\Gamma_1^\dagger \Gamma_2) + \operatorname{Tr}(\Pi_1^\dagger \Pi_2) \right] m_{12}^2 + \text{h.c.} \right\} , \\ \beta_{m_{22}^2}^{F,1L} &= \left[3 \operatorname{Tr}(\Delta_2 \Delta_2^\dagger) + 3 \operatorname{Tr}(\Gamma_2 \Gamma_2^\dagger) + \operatorname{Tr}(\Pi_2 \Pi_2^\dagger) \right] m_{22}^2 \\ &\quad - \left\{ \left[3 \operatorname{Tr}(\Delta_1^\dagger \Delta_2) + 3 \operatorname{Tr}(\Gamma_1^\dagger \Gamma_2) + \operatorname{Tr}(\Pi_1^\dagger \Pi_2) \right] m_{12}^2 + \text{h.c.} \right\} ,\end{aligned}$$

It turns out that

$$\operatorname{Tr}(\Delta_1 \Delta_1^\dagger) = \operatorname{Tr}(\Delta_2 \Delta_2^\dagger) , \quad \operatorname{Tr}(\Gamma_1 \Gamma_1^\dagger) = \operatorname{Tr}(\Gamma_2 \Gamma_2^\dagger) , \quad \operatorname{Tr}(\Pi_1 \Pi_1^\dagger) = \operatorname{Tr}(\Pi_2 \Pi_2^\dagger) ,$$

as well as

$$\operatorname{Tr}(\Delta_1 \Delta_2^\dagger) = \operatorname{Tr}(\Gamma_1 \Gamma_2^\dagger) = \operatorname{Tr}(\Pi_1 \Pi_2^\dagger) = 0 .$$

Hence,

$$\beta_{m_{11}^2 + m_{22}^2}^{F,1L} = \left[3 \operatorname{Tr}(\Delta_1 \Delta_1^\dagger) + 3 \operatorname{Tr}(\Gamma_1 \Gamma_1^\dagger) + \operatorname{Tr}(\Pi_1 \Pi_1^\dagger) \right] (m_{11}^2 + m_{22}^2)$$

It could be shown that

$$\beta_{m_{11}^2+m_{22}^2}^{F,1-loop} \propto (m_{11}^2 + m_{22}^2)$$

and

$$\beta_{m_{11}^2+m_{22}^2}^{F,2-loop} \propto (m_{11}^2 + m_{22}^2)$$

So $m_{11}^2 + m_{22}^2 = 0$ is preserved by fermionic contributions up to 2 loops.

Phenomenology of the r_0 symmetry (semisymmetry)

The set of 11 independent physical parameters of 2HDM:

$$\mathcal{P} \equiv \{M_{H^\pm}^2, M_1^2, M_2^2, M_3^2, e_1, e_2, e_3, q_1, q_2, q_3, q\}$$

The kinetic Lagrangian:

$$\mathcal{L}_k = (D_\mu \Phi_1)^\dagger (D^\mu \Phi_1) + (D_\mu \Phi_2)^\dagger (D^\mu \Phi_2)$$

$$\text{Coefficient} \left(\mathcal{L}_k, Z^\mu \left[H_j \overleftrightarrow{\partial}_\mu H_i \right] \right) = \frac{g}{2v \cos \theta_W} \epsilon_{ijk} e_k$$

$$\text{Coefficient}(\mathcal{L}_k, H_i Z^\mu Z^\nu) = \frac{g^2}{4 \cos^2 \theta_W} e_i g_{\mu\nu}$$

$$\text{Coefficient}(\mathcal{L}_k, H_i W^{+\mu} W^{-\nu}) = \frac{g^2}{2} e_i g_{\mu\nu}$$

$$q_i \equiv \text{Coefficient}(V, H_i H^+ H^-)$$

$$q \equiv \text{Coefficient}(V, H^+ H^+ H^- H^-)$$

Phenomenology of the r_0 symmetry (semisymmetry)

CP-sensitive invariants in the bilinear notation

$$I_1 = (\vec{M} \times \vec{\Lambda}) \cdot (\Lambda \vec{M})$$

$$I_2 = (\vec{M} \times \vec{\Lambda}) \cdot (\Lambda \vec{\Lambda})$$

$$I_3 = [\vec{M} \times (\Lambda \vec{M})] \cdot (\Lambda^2 \vec{M})$$

$$I_4 = [\vec{\Lambda} \times (\Lambda \vec{\Lambda})] \cdot (\Lambda^2 \vec{\Lambda})$$

Since the r_0 symmetry implies $\vec{\Lambda} = \vec{0}$ the invariants $I_{1,2,4}$ are automatically zero. However

$$I_3 = -16\lambda_5 m_{11}^2 \operatorname{Im}(m_{12}^2) \operatorname{Re}(m_{12}^2) [(\lambda_1 - \lambda_3 - \lambda_4)^2 - \lambda_5^2] \neq 0$$

explicit violation of CP

Phenomenology of the r_0 symmetry (semisymmetry)

Stationary-point equations:

$$\begin{aligned}m_{11}^2 &= \frac{1}{2}\lambda_1 (v_2^2 - v_1^2), \\ \text{Re } m_{12}^2 &= \frac{1}{2}v_1 v_2 \cos \xi (\lambda_1 + \lambda_3 + \lambda_4 + \lambda_5), \\ \text{Im } m_{12}^2 &= -\frac{1}{2}v_1 v_2 \sin \xi (\lambda_1 + \lambda_3 + \lambda_4 - \lambda_5).\end{aligned}$$

The neutral sector rotation matrix is then given by

$$R = \begin{pmatrix} \frac{v_2 \cos \xi}{v} & \frac{v_1 \cos \xi}{v} & -\sin \xi \\ -\frac{v_1}{v} & \frac{v_2}{v} & 0 \\ \frac{v_2 \sin \xi}{v} & \frac{v_1 \sin \xi}{v} & \cos \xi \end{pmatrix},$$

yielding masses

$$\begin{aligned}M_1^2 &= \frac{1}{2}v^2 (\lambda_1 + \lambda_3 + \lambda_4 + \lambda_5), & M_2^2 &= \lambda_1 v^2, \\ M_3^2 &= \frac{1}{2}v^2 (\lambda_1 + \lambda_3 + \lambda_4 - \lambda_5), & M_{H^\pm}^2 &= \frac{1}{2}(\lambda_1 + \lambda_3) v^2\end{aligned}$$

No decoupling limit!

Phenomenology of the r_0 symmetry (semisymmetry)

Assuming that M_2 is the SM-like Higgs boson, we obtain from unitarity and boundedness-from-below constraints:

$$M_{H^\pm} \leq 711 \text{ GeV},$$

$$M_3 \leq 712 \text{ GeV},$$

$$M_1 \leq 711 \text{ GeV}$$

Input parameters:

$$\mathcal{P} \equiv \{M_{H^\pm}^2, M_1^2, M_2^2, M_3^2, e_1, e_2, e_3, q_1, q_2, q_3, q\}$$

Constraints implied by the r_0 symmetry:

$$v^2(e_1 q_2 - e_2 q_1) + e_1 e_2 (M_2^2 - M_1^2) = 0, \quad v^2(e_1 q_3 - e_3 q_1) + e_1 e_3 (M_3^2 - M_1^2) = 0,$$

$$v^2(e_2 q_3 - e_3 q_2) + e_2 e_3 (M_3^2 - M_2^2) = 0, \quad q = \frac{1}{2v^4} (e_1^2 M_1^2 + e_2^2 M_2^2 + e_3^2 M_3^2),$$

$$M_{H^\pm}^2 = \frac{1}{2} (e_1 q_1 + e_2 q_2 + e_3 q_3) + \frac{1}{2v^2} (e_1^2 M_1^2 + e_2^2 M_2^2 + e_3^2 M_3^2),$$